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Single payment scheme and dual values of land in PMP models

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Abstract
Land dual values are one of the important aspects of the results of mathematical programming models used to evaluate the impact of agricultural policy measures at regional and farm level. When the decoupling of direct payments and the payment entitlements per hectare are included in PMP models in the context of the Single Payment Scheme (SPS), the analysis of the land dual values is more complex than in models which do not take these aspects into account. In this paper, we present a theoretical analysis of the land dual values when the SPS is included in PMP farm models. This theoretical analysis is carried out for the base year (linear model) and for a simulated year (quadratic model).The results of this analysis are illustrated by comparing numerically the land opportunity costs obtained in the case of partial decoupling and in the case of full decoupling of direct payments.

Keywords: *Positive mathematical programming, Single Payment Scheme or Single Farm Payment, Land dual values.*

JEL classification: C61, Q18

1. INTRODUCTION

To obtain the values of the parameters of the non linear objective function of a positive mathematical programming (PMP) farm model it is necessary to previously estimate the opportunity costs of the limited resources.

The standard PMP (Howitt, 1995) uses a linear programming model in the so called first stage of the PMP to estimate these opportunity costs. This model, which maximizes the gross margin of the farm, includes the calibration constraints, that is, the constraints limiting the area of each crop to the area existing in the baseline situation of the farm plus a small positive number.

When the only limited resource of the farm is the land, and the specificity of the Single Payment Scheme (SPS) (or Single Farm Payment (SFP)) introduced in the Mid-Term Review (MTR) of the Common Agricultural Policy (CAP) is not taken into account, the land dual value is equal to the gross margin of the marginal crop, that is, the crop for which the calibration constraint is not binding.

The land dual value is not so simple when the SPF is explicitly included in the model. In this case the land dual value is different if the eligible area for SPF is or is not less than a reference area.

The objective of this paper is to present a theoretical analysis of the land dual values when the SFP is taken into account in PMP models

The first part of this paper, Section 2, concerns the analysis of the land dual values in the linear model of the first stage of the PMP. In Section 3, the expressions to calculate the land dual values are obtained for the PMP non linear model. Finally, in Section 4, an example illustrates the theoretical results obtained in the previous sections. In this example the results in the case in which direct payments are partially decoupled are compared to the case in which these payments are fully decoupled.

2. LAND DUAL VALUES IN THE LINEAR MODEL OF THE FIRST STAGE OF PMP.

Before introducing the model to be used for the analysis of the land dual values in different hypotheses, it is necessary to take a brief look at the main characteristics of the single farm payment (SFP) defined in the Mid-Term Review (MTR) of the CAP of 2003 to be included in the model.

With the CAP measures of the Agenda 2000 which were previous to the MTR, the direct payments received for each farm were coupled to production. With the MTR in place these payments are totally or partially decoupled from production. The decoupled payments are received by each farm as a single farm payment (SFP) on the basis of an entitlement payment per hectare, which is calculated by dividing the amount (or to a proportion of the amount) of direct payments received in a reference period (years 2000, 2001 and 2002 in the MTR) by the area cultivated with eligible crops for the SFP during the said period. The result obtained by dividing this area by the number of years of the reference period will be called: reference area.

Not all crops are eligible for the SFP: most fruits, vegetables and potatoes are not eligible.

In one year the SFP can be:

- Equal to the amount of the entitlement payment per hectare multiplied by the reference area if the area of the farm, cultivated with eligible crops for SFP, is greater than or equal to the reference area. Henceforth, we will say that the SFP is generated by the reference area in this case

- Equal to the amount of the entitlement payment per hectare multiplied by the area of the farm farming with eligible crops for SFP, if this area is smaller than the reference area. In this case we will say henceforth that the SFP is generated by the eligible area of the farm (the area cultivated with eligible crops in the solution of the PMP model).

Our analysis also take into account the modulation of direct payments, included in the MTR, as it may affect the direct payments received and therefore the opportunity cost of the land. The modulation leads to the reduction of a percentage of the total (coupled and decoupled) direct payments exceeding €5000.

We will obtain the expressions of the land dual values in different cases taking into account the SFP and the modulation using the model that follows, based on the formulation of Henry de Frahan et al. (2007).

Let I be a set of crops i, eligible and not eligible for the SFP grown on the farm, $I_1 \subset I$ the sub-set of crops which are eligible for the SFP and $I_2 \subset I$ the sub-set of crops which are not eligible for the SFP.

Defining X_i as the area, in ha, of crop *i*; XES as the area, in ha, growing eligible crops; XE as the area in ha generating the SFP; XP1 as the amount in € in the first payment interval exempt from modulation (regarded to be less than or equal to €5000); and XP2 as the amount, in €, in the second payment interval (more than €5000) subject to a modulation discount, assumed to be *mod**100% , the farm model for the baseline situation (base year) is:

$$\max \sum_{i\in I}(r_i - c_i)X_i + XP1 + (1 - mod) * XP2 \qquad (1)$$

$$\sum_{i\in I} X_i \le A \qquad (2) \qquad \lambda_2$$

$$\sum_{i\in I_2} X_i + XES \le A \qquad (3) \qquad \lambda_3$$

$$XE \le REF \qquad (4) \qquad \lambda_4$$

$$XE - XES \le 0 \qquad (5) \qquad \lambda_5$$

$$-\sum_{i\in I_1} a_i * X_i - d * XE + XP1 + XP2 \le 0 \qquad (6) \qquad \lambda_6$$

$$XP1 \le 5000 \qquad (7) \qquad \lambda_7$$

$$X_i \le X_i^0 + \varepsilon_i \quad \forall i \quad (8) \qquad \pi_i$$

where: r_i : revenue per ha, net of direct payments of crop *i*, in €; c_i : variable cost per ha of crop *i* in €; a_i: coupled payment per ha of crop *i*, in €; *d*: payment entitlement per ha in €; *A*: farm area, in ha.; REF: reference area; X_i^0: area in ha of crop i in the base year; ε_i: small positive numbers.

The objective function (1) maximizes the farm gross margin. Equation (2) limits the cultivated area on the farm. Equation (3) defines the area cultivated with eligible crops, *XES*. Equations (4) and (5) define the area, *XE*, which generates the SFP. This area is the minimum area between the reference area (*REF*) and the area growing eligible crops (*XES*). Equation (6) defines the total amount of (coupled and decoupled) payments, *XP1+XP2.* Equation (7) limits the amount of direct payments exempt from reduction for modulation. Finally, equations (8) are the calibration constraints for each crop.

The dual variables associated to each constraint are represented on the right of the constraint. The opportunity cost of the land is the sum of λ_2 and λ_3.

From the relationships between the primal and the dual problems we have obtained the expressions of λ_2 and λ_3, shown in Table 1, combining the following cases:

- Category of the marginal crops ($\pi_i = 0$). Two categories are possible: $i \in I_1$, that is, the marginal crop is eligible for the SFP and $i \in I_2$, that is, the marginal crop is not eligible for SFP.
- Area generating the SFP: XE. This area can be the reference area (REF) in which case: $\lambda_3 = \lambda_5 = 0$ or the eligible area in the solution (XES) in which case $\lambda_4 = 0$.

- Total amount of direct payments (XP1+XP2), that can be greater or less than €5000. In the last case $\lambda_7 = 0$.

Table 1 presents these expressions.

Table 1: Expresions of λ_2 and λ_3. For the linear model of the first stage of PMP in different cases

		Direct Payments $> €5000$	Direct Payments $< €5000$
$XE = REF < XES$	Marginal crop eligible ($\pi_i = 0 \; i \in I_1$)	$\lambda_i = r_i - c_i + (1 - mod)\, a_i$ $\lambda_3 = 0$	$\lambda_i = r_i - c_i + a_i$ $\lambda_3 = 0$
	Marginal crop no eligible ($\pi_i = 0 \; i \in I_2$)	$\lambda_i = r_i - c_i$ $\lambda_3 = 0$	$\lambda_i = r_i - c_i$ $\lambda_3 = 0$
$XE = XES < REF$	Marginal crop eligible ($\pi_i = 0 \; i \in I_1$)	$\lambda_i = r_i - c_i - (1 - mod)\, a_i$ $\lambda_3 = (1 - mod)\, d$	$\lambda_i = r_i - c_i + a_i$ $\lambda_3 = d$
	Marginal crop no eligible ($\pi_i = 0 \; i \in I_2$)	$\lambda_i = r_i - c_i - (1 - mod)\, d$ $\lambda_3 = (1 - mod)\, d$	$\lambda_i = r_i - c_i - d$ $\lambda_3 = d$

This Table shows:

- The only difference in the expressions of λ_2 and λ_3 when the total amount of direct payments is greater or less than €5000 is that in the first case the coefficient of a_i and *d* is *1* and in the second it is *1-mod*.

- λ_2 and/or λ_3 are a function of the payment entitlement per hectare (*d*) only if the area generating the SFP is less than the reference area. The sum $\lambda_2 + \lambda_3$ is a function of *d* when the marginal crop is eligible for the SFP.

3. LAND DUAL VALUES IN THE NON LINEAR MODEL

After estimating the dual values of the limiting resources of the farm (only land in our case) by the first stage of PMP or by another procedure, it is possible to define the non linear function of a PMP model in such a way that the model is able to reproduce, without the calibration constraints, the crop distribution existing in the baseline situation of the farm. In a general formulation this model can be formulated as follows:

$$max: Q(X) + XP1 + (1 - mod) * XP2 \qquad (9)$$

subject to constraints: (2)-(7)

where $\boldsymbol{X}$ is the vector of components X_i and $Q(\boldsymbol{X})$ a non linear function (generally quadratic) whose parameters are a function of the dual values of the resources.

The structure of the model allows the land dual values λ_2 and λ_3 to be obtained considering, for each possible case, only the subset of constraints (2)-(7) that are binding (we assume that constraint (2) is always binding) ,and ignoring the rest of the constraints whose dual values are null. So, in each case λ_2 and λ_3 can be obtained from the necessary optimal conditions derived from the Lagrangian formed by the objective function and the binding constraints. The results are shown in Table 2. In this Table the vector $\bar{\boldsymbol{X}}$ represents the optimal solution of the vector $\boldsymbol{X}$.

Table 2. Expressions of λ_2 and λ_3 for the non linear model in different cases.

	Direct Payments $> €5000$	Direct Payments $< €5000$
$XE = REF < XES$	$\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} + (1-mod)a_i \;\; \forall i \in I_1$ and also: $\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} \;\; \forall i \in I_2$ $\lambda_3 = 0$	$\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} + a_i \;\; \forall i \in I_1$ and also: $\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} \;\; \forall i \in I_2$ $\lambda_3 = 0$
$XE = XES < REF$	$\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} + (1-mod)a_i \;\; \forall i \in I_1$ and also: $\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} - (1-mod)d \;\; \forall i \in I_2$ $\lambda_3 = (1-mod)\,d$	$\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} + a_i \;\; \forall i \in I_1$ and also: $\lambda_2 = \left(\frac{\partial Q(X)}{\partial X_i}\right)_{X=\bar{X}} - d \;\; \forall i \in I_2$ $\lambda_3 = d$

The ignored constraints in each case are the following:
Case of direct payments $> €5000$ *and* $XE = REF < XES$: constraints (3) and (5).
Case of direct payments $< €5000$ *and* $XE = REF < XES$: constraints (3), (5) and (7).
Case of direct payments $> €5000$ *and* $XE = XES < REF$: constraint (4).
Case of direct payments $< €5000$ *and* $XE = XES < REF$: constraints (4) and (7).

The results of Table 3 are consistent with those obtained in Table 2 for the marginal crops using the lineal model of the first stage of PMP.

4. AN ILLUSTRATIVE NUMERICAL EXAMPLE.

The expressions of λ_2 and λ_3 obtained in Sections 2 and 3 in the case of direct payments of more than €5000 are very similar to the case of the direct payments of less than €5000, the only difference being the coefficients of d and a_i in these expressions. We will consider in this numerical example only the case of direct payments of more than €5000.

4.1. The data

To obtain illustrative numerical results for the theoretical expressions of λ_2 and λ_3 shown in Sections 2 and 3 we consider a hypothetical farm of 40 hectares growing two eligible crops for SFP: barley (*i=1*) and corn (*i=2*) and one non eligible crop: potatoes (*i=3*). It is assumed that in the baseline situation the percentage of direct payments coupled is 25%.

The area of these crops in the baseline situation and their characteristics *in the case in which the marginal crop is an eligible crop* for the SFP (barley), are presented in Table 3.

Table 3. Area in the baseline situation (base year) and characteristics of crops in the case in which barley is the marginal crop.

	Area, in ha, in the base year: X_i^0	Yield, in tn/ha, y_i	Price, in €/ton, p_i	Coupled payments, in €, a_i	Variable costs, in €, c_i
Barley (*i=1*)	20	4.70	125	55.13	325
Corn (*i=2*)	5	11.58	144	122.85	1000
Potato (*i=3*)	15	22.74	150	-	1960

$r_i=p_i*y_i$

To simulate *the case in which the marginal crop is not eligible* for the SFP (potatoes), the characteristics of the crops are those of Table 3 except for the price of the potatoes which is €100/ton instead of €150/ton.

To simulate *the case in which the area generating the SFP is the reference area (REF)* we consider *REF=23* hectares and *d* (the entitlement payments per hectare corresponding to 75% of the total direct payments received by the farm in the reference period) equal to €200.71.

To simulate *the case in which the area generating the SFP is the eligible area in the solution (XES),* it is considered: *REF=27* hectares and *d* equal to €210.53.

The reduction for modulation of the total direct payments exceeding €5000 is 5%. So, *mod=0.05*.

4.2. Specification of the quadratic function

For this numerical illustration the objective function (9) will be:

$$max\colon \sum_{i\in I}\left(r_i-\frac{1}{2}q_i*X_i\right)*X_i+XP1+(1-mod)*XP2$$

We have chosen this function because it is very simple and all crops have a positive quadratic term.

The Kuhn-Tucker necessary conditions for the optimal solution to be $X_i = X_i^0 \ \forall i$ give:

$$q_i=\frac{r_i-\sum_{j=2}^{7}b_{ij}*\lambda_j}{X_i^0} \qquad (10)$$

where b_{ij} is the coefficient of X_i in in the constrain (*j*).

If the first step of the PMP is used (model (1)-(8)), as in this numerical illustration, to obtain the dual values λ_j, the following relationship holds:

$$\sum_{j=2}^{7} b_{ij} * \lambda_j + \pi_i = r_i - c_i \qquad (11)$$

So, replacing in (10) $\sum_{j=2}^{7} b_{ij} * \lambda_j$ by its expression in (11) q_i has the following expression proposed by Paris (1988) in the earliest stages of the development of the PMP: $q_i = \frac{\pi_i + c_i}{x_i^p}$.

4.3 Results

Numerical results are presented in Table 4, firstly for the baseline situation in which

Table 4: Land dual values in the baseline situation (base year) and in the simulated year

		Base year (partial decoupling)	Simulated year (full decoupling)	
			Value	Variation (%)
	Barley (ha)	20	19,97	-0,15
	Corn (ha)	5	4,78	-4,40
$XE = REF$	Potato (ha)	15	15,25	1,67
	λ_2 (€)	314,87	263	-16,47
$\pi_i = 0\ i \in I_1$	λ_3(€)	0	0	-
	$\lambda_2 + \lambda_3$ (€)	314,87	263	-16,47
	Barley (ha)	20	19,85	-0,75
	Corn (ha)	5	4,77	-4,60
$XE = REF$	Potato (ha)	15	15,38	2,53
	λ_2 (€)	314	264,14	-15,88
$\pi_i = 0\ i \in I_2$	λ_3(€)	0	0	-
	$\lambda_2 + \lambda_3$ (€)	314	264,14	-15,88
	Barley (ha)	20	20,26	1,30
	Corn (ha)	5	4,80	-4,00
$XE = XES$	Potato (ha)	15	14,95	-0,33
	λ_2 (€)	314,87	258,32	-17,96
$\pi_i = 0\ i \in I_1$	λ_3(€)	200	266,67	33,34
	$\lambda_2 + \lambda_3$ (€)	514,87	514,99	0,02
	Barley (ha)	20	20,24	1,20
	Corn (ha)	5	4,83	-3,40
$XE = XES$	Potato (ha)	15	14,94	-0,40
	λ_2 (€)	114	55,42	-51,39
$\pi_i = 0\ i \in I_2$	λ_3(€)	200	266,67	33,34
	$\lambda_2 + \lambda_3$ (€)	314	322,09	2,58

the farm receives direct payments partially decoupled. These results are compared with the results simulated for the case in which the partially decoupled direct payments become fully decoupled.

To carry out the full decoupling simulation, the coupled payments received by the farm for barley and corn in the baseline situation become null and the amount of the entitlement payment per hectare (d) is €267.61 instead of €200.71 when the area that generates the SFP is the reference area (*REF*), and is €280.70 instead of €210.53 when the SFP is generated by XES.

The results were obtained with GAMS/CONOPT for the baseline situation and for the simulated scenario. Besides the values of λ_2 and λ_3, and λ_3, the Table also gives the values of X_i for the baseline situation and for the optimal solution of the full decoupling simulation.

As this Table shows when the area generating the SFP is the reference area (*XE=REF*) the opportunity cost of land $(\lambda_2 + \lambda_3)$ decreases when partial decoupling changes to full decoupling, that is, when $a_i = 0 \ \forall i$.This change barely affects the land opportunity cost if the area generating the SFP is the area cultivated with eligible crops (*XE=XES*).

These results can be explained by the expressions in Table 2 of λ_3 and λ_2 for the case of the eligible crops $(i \in I_1)$. These expressions show that in the case in which *XE=REF*: $\lambda_3 = 0$ and the value of λ_2 decrease if $a_i = 0$. When *XE=XES*, the decrease in λ_2 is offset by the increase in λ_3 due to an increment in the amount of the payment entitlement per hectare (d) that occurs when full decoupling replaces the partial decoupling.

5. CONCLUSIONS.

In this paper we have obtained the theoretical expressions of land dual values in the various cases that may arise in a PMP farm model explicitly including the single farm payment and the modulation of the direct payments. These expressions concern the linear model used in the first stage of the PMP and the PMP non linear model.

Our theoretical approach can help to understand the meaning of the dual value of land when the first stage of the PMP is used and to give consistent dual values to the constraints associated with the land when these values are provided exogenously to the model.

In this paper we have also studied the changes in the opportunity cost of the land when the degree of decoupling of the direct payments increases. The study shows that the variations are a function of the rate of modulation applied to the direct payments and of the area for which the amount of the payment entitlement (the reference area or the area cultivated with eligible crops) has to be multiplied to obtain the single farm payment.

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