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# Hedging Decisions of Importing Firms for U.S. Commodity with Multiple Risks: The case of Soybeans

Qiang Zhang, Michael Reed, and Leigh Maynard<sup>1</sup>

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<sup>1</sup> Authors are, respectively, Research Assistant, Professor, and Associate Professor, Department of Agricultural Economics, University of Kentucky. Corresponding author: qzhan6@uky.edu

## **Introduction**

Risk management has become increasingly important to importing firms engaged in international commodity trade. Hedging the exchange rate, the commodity price, and the ocean freight risks with futures contracts are widely used to manage risks for many importing firms. A hedge can be employed through taking opposite positions in the spot and futures market simultaneously.

Through a hedge, when any unfavorable price changes happen in one market and firms have to suffer the loss, they can be offset by any favorable price changes in the other market. This study focuses on the joint hedging decisions of the commodity importing firms in the U.S. based commodity price, freight rate, and exchange rate futures. The main U.S. based futures markets for commodities and currencies include the Chicago Board of Trade (CBOT), the Chicago Mercantile Exchange (CME), the New York Board of Trade (NYBOT), and the New York Mercantile Exchange (NYMEX). The freight futures market - The Baltic International Freight Futures Exchange - which was based on the Baltic Freight Index, was established in 1986. Since then, it had been modestly successful in some years, but it was closed in 2001 due to lack of liquidity. However, as the largest component of variable shipping cost, the price of fuel oil is volatile and it impacts the ocean shipping costs significantly. The fuel oil futures (NYMEX) can be considered as a substitute for hedging the ocean freight risk in practice.

The general objective of this study is to investigate the hedging effectiveness of using foreign exchange futures, commodity futures, and fuel oil futures simultaneously. Several combinations of hedging instruments can be considered to manage risks by using futures markets. The two main methods of measuring the optimal hedge ratio, the traditional approach and time series techniques, are used in this study. The traditional hedge ratios are estimated based on the time invariant variances and covariance of the related variables. Time series techniques, such as

the VAR model, the VECM, and Multivariate GARCH model, can be used to calculate the hedge ratios. Furthermore, this study involves empirical comparisons of hedging effectiveness and optimal hedge ratios in these futures markets for different hedging scenarios and different estimation methods. Based on the importance of Asian markets for U.S. commodities, a Japanese soybean importing firm which encounters all three risks is selected in this study. The results of this analysis have implications for any importing firms which face similar quantifiable market risks (for example, a Chinese company which encounters commodity price and ocean freight risks and a Mexican company that faces foreign exchange and commodity price risks) in choosing an effective hedge strategy.

## **Literature Review**

### *1. Study in General Economics*

There are two main hedging hypotheses in economics that concern the motives of hedging. One is from the ideas of Keynes (1930) and Hicks (1946) (Keynes-Hicks hypothesis). They think the prime motive of hedging is risk reduction and that risk can be transferred from risk-averse agents to risk-seeking agents (speculators). Unfortunately, hedgers cannot eliminate all risk due to the existence of basic risk. Working (1953) asserts the main motive of hedging is not to reduce risk but rather to profit from changes in basis. Johnson (1960) is the first to suggest that hedging and speculative activities can be combined in financial markets. He gives a theoretical framework to hedgers who want to reduce price risk and collect a premium in financial markets.

Modern hedging theory has been developed based on a combination of the above two hedging hypotheses. According to much of the literature, hedging can be considered as a motivation to reduce risks, as suggested by Keynes-Hicks theory, but the levels of hedging are determined by expected profits, as Working mentioned.

## *1.1 Hedge Strategies*

In modern hedging literature, some basic concepts have been discussed for research. In a volatile price world, hedging is a motive to buy (long) or sell (short) futures contracts to reduce unfavorable price changes. To achieve a hedge, the most important point in a firm's hedging decision is to calculate the appropriate hedge ratio which is defined as the ratio of the futures position to the spot position. Previous research has distinguished three basic hedge strategies: the naïve hedge, the minimum variance hedge, and the mean-variance hedge. If the spot and futures prices are perfectly correlated, the naïve (full-hedge) hedging strategy, which takes a futures position exactly equal to the spot position in magnitude with an opposite sign, is enough to eliminate the price risk. However, no spot and futures positions are perfectly correlated in the real world, so a naïve hedging strategy is not optimal for reducing risk. Many studies focus on this imperfect correlation and develop the minimum variance hedge and the mean-variance hedge.

### *1.1.1. Minimum Variance Hedge*

Ederington (1979) develops the analyses of the portfolio theory from Johnson (1960) and Stein (1961) and adopts a risk (variance) minimizing objective to establish a model which can examine how firms manage their price risk with futures markets. Ederington's empirical study also finds it is not necessary for a firm which wants to minimize risk to use a naïve hedging strategy. Since risk in the spot market can be defined as the uncertainty or variability of returns, as measured by the variance of expected returns, it is reasonable to calculate an optimal hedge ratio based on solving the problem of minimizing the variance of the expected return. If an investor goes short in the commodity futures market to hedge a long position in the cash market, his hedged portfolio return  $p_t$  in the time  $t$  can be considered as :  $p_t = (c_t - c_{t-1}) - h_t(f_t - f_{t-1})$ , where

$(c_t - c_{t-1})$  and  $(f_t - f_{t-1})$  are the return from the cash market and the futures market, respectively, and  $h_t$  is the hedge ratio. The variance of the returns on his hedged portfolio is

$Var(p_t) = Var(c_t) + h_t^2 Var(f_t) - 2h_t Cov(c_t, f_t)$ ; the first order condition of this variance with

respect to  $h_t$  is  $h_t^* = \frac{Cov(c_t, f_t)}{Var(f_t)}$ . The minimum variance hedge ratio is defined as the ratio of the

covariance between cash and futures prices to the variance of the futures prices.

### 1.1.2. Mean-variance Hedge

Another method to calculate optimal hedge ratio is based on the basic microeconomic theory that an investor wants to maximize his expected utility from his portfolio. His expected utility is a linear function that is increasing in expected return and decreasing in return variance, that is,

$MaxEU(p_t) = E(p_t) - \gamma V(p_t) = Max\{E(p_t) - \gamma[Var(c_t + h_t^2 Var(f_t) - 2h_t Cov(c_t, f_t))]\}$ , where  $\gamma$

is the relative measure of risk preference. The first order condition of this variance with respect

to  $h_t$  is  $h_t^* = \frac{Cov(c_t, f_t)}{2Var(f_t)} - \frac{E(f_t - f_{t-1})}{\gamma Var(f_t)}$ .

The second term in the above equation can be considered as speculative demand for futures. If the mean-variance hedge ratio is consistent with the minimum variance hedge ratio, either expected returns on the futures market needs to be zero or investors are infinitely risk averse. Benninga, Eldor and Zilcha (1984) find that the minimum variance hedge ratio from a risk minimization model is an optimal hedge ratio and is consistent with expected utility maximization if the futures market is unbiased. Theoretically, the unbiasedness hypothesis is a property of many futures markets. Furthermore, based on a random walk model or its alternative, the martingale model, the expected return of the futures market is equal to zero, that is, the second term of the above equation is zero. The unbiasedness hypothesis of futures markets has

been verified by many empirical studies (Baillie and Myers, 1991; McKenzie and Holt, 2002). If this unbiasedness hypothesis holds, the optimal hedge ratio is identical for the minimum variance hedge method and the mean-variance method.

### *1.2 Econometric Methods to Calculate Optimal Hedge Ratio*

Many economists adopt the minimum variance method instead of the expected utility maximization of the profit based on a mean-variance objective function. Through the minimum risk (variance) approach, it is convenient to apply several kinds of econometric methods based on basic economic theories to calculate the optimal hedge ratio (Lence, 1995). The basic calculation methods can be divided into two groups: static methods and dynamic methods.

#### *1.2.1. Static Methods*

As shown above, the naïve hedge which fails to consider the correlation between spot and futures markets is a traditionally static approach.

There has been much empirical research on the calculation of the optimal hedge ratio using the traditional ordinary least squares (OLS) method. The minimum variance hedge ratio is the slope coefficient of futures price when the return on the spot market is regressed on the return on the futures market by the OLS method, that is,  $\Delta c_t = a + h^* \Delta f_t + \varepsilon_t$ . The slope coefficient is the OLS minimum variance hedge ratio. Empirically, the seemingly unrelated regression (SUR) method is employed as an alternative of the OLS method. The OLS method has been criticized based on two main reasons: (1) they are based on unconditional variance and covariance and the conditional information is omitted (Myers and Thompson, 1989); and (2) the OLS method ignores the time-varying characteristics in time series (e.g., Baillie and Myers, 1991).

#### *1.2.2. Dynamic Methods*

##### *1.2.2.1. Generalized Hedge Approach*

Myers and Thompson (1989) develop a generalized approach to measure the optimal hedge ratio. They allow the conditional variance and covariance matrix to change overtime and show a generalized approach is not difficult to apply to this issue and that this approach is more reliable.

The optimal hedge ratio is  $(h_t^*|X_{t-1}) = \frac{Cov(c_t, f_t|X_{t-1})}{Var(f_t|X_{t-1})}$ , where  $X_{t-1}$  is a vector of variables

known at  $t-1$  that the variance and covariance rely on it. Ferson and Schadt (1996) employ a conditional approach which uses the predetermined variables to represent information and time-variation in asset pricing model. Miffre (2004) modifies the traditional OLS approach by using a conditional OLS model to estimate optimal hedge ratio in the exchange rate futures market. The conditional optimal hedge ratio is obtained from the regression function:

$\Delta c_t = a + a_1 X_{t-1} + h^* \Delta f_t + b \Delta f_t X_{t-1} + \varepsilon_t$ , where  $X_{t-1}$  is a vector of mean zero predetermined instruments available at time  $t-1$ . The conditional OLS hedge ratio is simple to obtain. In the econometrics literature, some macroeconomic factors are used as appropriate predetermined instruments variables. However, it is not easy to find such instrument variables to represent the actual information used by hedgers for all the futures markets. Some researchers find that the most important factors that impact variances and covariances of prices in the financial time series seem to be the lagged endogenous variables themselves.

#### *1.2.2.2. Time Series Techniques*

Vector autoregressive moving average model (VARMA) is one of the most successful models to analyze the multivariate time series. It has been proven to be especially useful for analyzing the dynamic behavior of time series and is the basis of building the other multivariate models. When the presence of a cointegration relationship among the variables is tested, the vector error correction model (VECM) is an appropriate model to deal with such time series. The VECM can



be considered as an augmented VARMA model with the addition of linear combinations of these time series which are stationary as additional regressors.

Engle (1982) develops the autoregressive conditional heteroskedasticity (ARCH) model which is used in financial time series to model asset price volatility over time. The ARCH model estimates future volatility as a function of prior volatility. Suppose a standard autoregressive integrated moving average (ARIMA) model:  $\phi(B)(1-B)^d Z_t = \theta_0 + \theta(B)a_t$ , where  $B$  is backshift operator,  $Z_t$  is the time series,  $a_t$ 's are independent random variables with zero mean and  $\sigma_a^2$  variance but conditionally,  $Var(a_t|\Omega_{t-1}) = h_t$  which depends on  $t$ . To model  $h_t$  on the history of the  $Z_t$  process, Engle (1982) postulates that

$$a_t = b_t \sqrt{h_t}, \quad h_t = \omega + \sum_{i=1}^p a_i a_{t-i}^2, \quad \omega > 0, a_i > 0 (i = 1, 2, \dots, p), \sum_{i=1}^p a_i < 1, \text{ where } b_t \text{'s are } i.i.d.(0,1),$$

and are independent of  $a_{t-1}, a_{t-2}, \dots$ . This is known as the ARCH (p) model. After the introduction of the ARCH model there are enormous theoretic and empirical developments in modeling financial time series. Bollerslev (1986) extends Engle's ARCH model to the generalized ARCH (GARCH) model for  $a_t$  as  $a_t = b_t \sqrt{h_t}$ ,

$$h_t = \omega + \sum_{i=1}^p a_i a_{t-i}^2 + \sum_{i=1}^r \gamma_i h_{t-i}, \quad \omega > 0, a_i > 0, \gamma_i > 0 (i = 1, 2, \dots, p), \sum_{i=1}^p a_i + \sum_{i=1}^r \gamma_i < 1, \text{ where } b_t \text{'s are } i.i.d.(0,1), \text{ and are independent of } a_{t-1}, a_{t-2}, \dots. \text{ This is known as the GARCH (r,p).}$$

Since this GARCH model was established, the econometric techniques in estimating the optimal hedge ratio have become increasingly complicated, and more advanced time series techniques have been used. Cechetti, Cumby, and Figlewski (1988) estimate the optimal hedge ratio based on maximizing the firm's objective function within a mean-variance framework by using (ARCH) methods. More importantly, the univariate GARCH methods and extensions of

them, such as the Bivariate GARCH method (Baillie and Myers, 1991), and the Multivariate GARCH method (Engle and Kroner, 1995), are also employed for this issue. One of the most important applications of the Multivariate GARCH models is the study of the time-varying hedge ratio (Bauwens, Laurent, and Rombouts, 2006). Other time series techniques, such as the VECM (Lien, 1993 and 1996; Yang and Allen, 2004), and the VAR model (Yang and Allen, 2004) are also used to estimate optimal hedge ratio. Even though the Multivariate GARCH method is considered to have better performance compared with other methods (Brooks et al, 2002), some studies find these methods based on the GARCH methods are too complicated, cost more, and require greater skills. On the other hand, the OLS method can achieve similar performance in many cases (Myers, 1991; Miffre, 2001).

## *2. Study in Agricultural Economics*

Empirically, numerous previous studies have employed hedging strategies to manage price risk, production risk, or other manageable risks for individual commodities in agricultural production and trade. Empirical studies have covered almost every individual commodity with a commodity futures market either from the producers' perspective or the international traders' perspective. Early studies focus their attention on the optimal way for hedgers who encounter one risk, such as price risk or yield risk, with financial instruments. Some more complex hedging situations have also been studied, for example, crossing hedging for some commodities which have no actual futures markets (Anderson and Danthine, 1981) and multiproduct hedging for multiple commodities (Fackler and Mcnew, 1993).

Thompson and Bond (1987) first extend the standard commodity hedging framework to exchange rate uncertainty for offshore commodity trade. Thuong and Visscher (1991) estimate the hedge effectiveness of dry-bulk freight rate futures. When more risks encountered by

producers or traders can be hedged through futures markets, researchers have paid more attention to hedging effectiveness by combining several risks and future contracts.

Kawai and Zilcha's (1986) and Fung and Lai (1991) first develop the theoretical models which deal with multiple risks for a firm. Zilcha and Broll (1993) further expanded models for multiple risks. Vukina, Li, and Holthausen (1996) examine the effects of hedging yield and price risks from the producers' perspective in futures markets simultaneously, and they find such joint strategies are more effective when compared to using price futures alone. Through extending Vukina, Li, and Holthausen and Heifner and Coble models, Nayak and Turvey (2000) investigate the hedging effectiveness of joint hedging in price, yield, and foreign exchange futures for a Canadian firm in the U.S. futures market. They also find simultaneously hedging price and yield can reduce more revenue risk than only price hedging. Manfredo, Garcia, and Leuthold (2000) estimate time-varying multiproduct hedge ratios in the soybean complex. They develop a model which is used to hedge multiproduct (soybeans, soybean oil, and soybean meal) risks simultaneously for soybean processors.

From an international trader perspective, Haigh and Holt (2000) estimate the optimal hedge ratio of joint hedging price and freight rate futures for a grain exporter (wheat and soybean) who is shipping grain from the U.S. Gulf to Rotterdam. They employ the OLS, the SUR, and the Multivariate GARCH methods for optimal hedging ratio estimation and they find that the Multivariate GARCH method has superior performance. Haigh and Holt (2002) extend this model to investigate the hedge effectiveness of a European grain importer who imports grain (wheat, soybean, and corn) from the U.S. Gulf to Rotterdam and hedges price, freight, and foreign exchange risks simultaneously in COBT, BIFFEX, and foreign exchange futures. They

find that the optimal hedging ratio from the Multivariate GARCH model outperforms the OLS and SUR in this study.

### *3. Needed Extensions on Previous Work*

Foreign exchange rate, commodity price, and ocean freight costs can be considered three main risks that an importing firm encounters. Some previous researchers have investigated how to hedge these three risks simultaneously for a U.S. exporter or a European importer through different econometric methods (Haich and Holt, 2000; Haigh and Holt, 1999). But some problems appear when using BIFFEX as a futures market to hedge ocean shipping costs. First, BIFFEX closed in 2002 due to a lack of liquidity. Secondly, the freight futures price was reported in index form by BIFFEX and had to be converted into U.S. dollars for research, and for a specific shipping route, the index may not reflect real market conditions.

The volatility in fuel oil prices has become increasingly common. Oil price volatility, which directly impacts traders' transaction cost volatility, harms the international commodity trade. In the empirical model, fuel oil futures can be considered as an alternative market for an importing company hedging freight rate risk. This idea that the fuel oil futures market can be considered as a suitable market to hedge ocean shipping costs has been mentioned by Thuong and Visscher (1990). The effectiveness of involving heating oil<sup>2</sup> hedge in the traders' decision needs to be empirically tested.

The differences between hedging three risks simultaneously and hedging any two of three and only hedging price risk are not mentioned in the international trader model.

Specifically, this study focuses on U.S. soybean exports to Asian markets because Asian markets are the most important markets for U.S. soybean exports. Finding an effective hedge

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<sup>2</sup> Heating oil futures are used to hedge the diesel fuel in the futures market.

strategy to reduce market risks for Asian importers will contribute to importing more commodities from the U.S.

For data frequency, most of the previous studies use daily or weekly data. Even though some researchers apply the monthly data to estimate the optimal hedge ratio for financial markets, little research employing the monthly data focuses on the market risks for the commodity importers. Data frequency is an important issue that impacts the empirical results. It is important to select the appropriate data frequency that reflects the characteristics in the cash market and the futures market in the empirical studies. Monthly data is more appropriate to reflect the characteristics of U.S. soybean trade in nature.

## **Model Description**

### *1. The Timing of Hedging Decisions*

Suppose the importing firm produces final goods for a domestic demand by using an imported commodity as an input. The timing of hedging decisions, production, and selling final goods is illustrated in figure 1. This firm receives orders for its domestic output at  $t - m - n$ , and output is priced and sold at  $t + o + p$ . At  $t - m - n$  this firm wants to have a shipment of soybeans for delivery at  $t + o$ . It begins to calculate the basis by subtracting the futures price from importing price and review the historic basis records. With current prices on the low side at  $t - m$ , it decides it is a good time to lock in a price for soybeans and it purchases a certain amount of soybean futures contracts through calculating the optimal hedge ratio in the futures market. Then this firm continues to survey its supplies to fulfill its cash market requirement. At time  $t$  the basis changes in its favor and he decides it is time to accept the exporter's offer for delivery at  $t + o$  and sell futures contracts simultaneously. The firm can produce its domestic output and fulfill the domestic demand at  $t + o + p$ . The duration of time from  $m + n$  to  $o + p$  depends on

industry practices and transaction conditions. Hedge ratios can be calculated at different values at time  $t$ , as well as for different periods forward or backward from  $m + n$  to  $o + p$ . However, it is difficult to hedge exchange rate and freight rate risks according to the commodity payment date because the magnitude or timing of exchange rate and heating oil transactions are not consistent with soybean futures market.

Even though the calculated hedging parameters would vary depending on the duration of these periods, the analytical solution is unchanged. Specifically,  $m$  is equal to 30 in this study.

## 2. Minimum Variance Hedging Decisions

Following basic optimal hedging strategies, a mean-variance framework as mentioned in Myers and Thompson (1989) can be described as:

$$\underset{\alpha, \beta, \delta}{\text{Max}} E(\pi_t | X_{t-1}) = E(\pi_t | X_{t-1}) - \gamma V(\pi_t | X_{t-1})$$

Where,  $X_{t-1}$  is the set of information available in the first period, and  $\gamma$  is the relative measure of risk preference. The importing firm needs to select the optimal hedging ratio in order to maximize second period conditional utility. It is necessary to clarify that the major aim of the importing country is to reduce risks instead of speculating in the futures market. It is reasonable to suppose that the expected return from the futures market is zero, and the minimum variance hedge is identical with the equation above, maximizing expected second period utility under the mean-variance framework. Theoretically, the importing firm goes long in commodity futures markets and fuel oil markets to offset the cash price increases in these spot markets and goes

short in the foreign exchange futures market to purchase the U.S. dollar to pay for commodity and ocean shipping costs<sup>3</sup>.

According to the minimum variance hedging model, the first step is to calculate the variance of revenue. If importers use commodity futures, oil futures, and currency futures simultaneously (3-way hedge), the hedged price revenue at the end of second period invoiced in domestic currency can be estimated as (excluding transaction cost):

$$(1) HR = -(p^d + p^o)R + \beta(F' - f')R + \delta(\tilde{F} - \tilde{f})R + \alpha(R' - r')(p^d + P^o)$$

$p^d$  is the price of imports in U.S. dollars.

$p^o$  is the price of heating oil (a proxy for the freight rate)

$R$  is the spot exchange rate on the date of payment

$\beta$  is the hedging ratio in the soybean futures market

$F'$  is the soybean futures market price at the first period.

$f'$  is the soybean futures market price at the second period.

$\tilde{F}$  is the heating oil futures market price at the first period.

$\tilde{f}$  is the heating oil futures market price at the second period.

$\delta$  is the hedge ratio in the heating oil futures market

$\alpha$  is the hedging ratio for foreign exchange futures market

$R'$  is the futures exchange rate at the first period

$r'$  is the futures exchange rate at the second period

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<sup>3</sup> The optimal hedge positions in the soybean, heating oil, and exchange rate futures market can either be short or long depending on the covariance between the spot market and futures market. If hedging the multiple risks simultaneously, the sign of the optimal hedge position in each futures market is more complex to decide.

For the last term of equation (1), the hedging return from a foreign exchange futures market is multiplied by commodity price and freight (oil) price in order to put them in equivalent units to the other two hedged returns (Haigh and Holt, 2002).

Defining  $M = -JR$  as the unhedged importer's price revenue,  $N = CR$  as the return from the soybean futures market,  $S = OR$  as the return from the heating oil futures market, and  $T = ZJ$  as the return from the currency futures market, they are measured in the importing country's currency, where  $J = (P^d + P^o)$ ,  $F' - f' = C$ ,  $\tilde{F} - \tilde{f} = O$ ,  $R' - r' = Z$ .

$$(2) HR = -JR + \beta CR + \delta OR + \alpha ZJ$$

The variance of the hedged revenue can be described as:

$$(3) \sigma_{HR}^2 = \sigma_{JR}^2 + \beta^2 \sigma_{CR}^2 + \delta^2 \sigma_{OR}^2 + \alpha^2 \sigma_{ZJ}^2 - 2\beta \sigma_{JR.CR} - 2\delta \sigma_{JR.OR} - 2\alpha \sigma_{JR.ZJ} + 2\beta \delta \sigma_{CR.OR} + 2\delta \alpha \sigma_{OR.ZJ} + 2\beta \alpha \sigma_{CR.ZJ}$$

Where  $\sigma_{JR}^2 = \text{Var}(JR)$ ,  $\sigma_{CR}^2 = \text{Var}(CR)$ ,  $\sigma_{OR}^2 = \text{Var}(OR)$ ,  $\sigma_{ZJ}^2 = \text{Var}(ZJ)$ ,  $\sigma_{JR.CR} = \text{Cov}(JR.CR)$ ,  $\sigma_{JR.OR} = \text{Cov}(JR.OR)$ ,  $\sigma_{JR.ZJ} = \text{Cov}(JR.ZJ)$ ,  $\sigma_{CR.OR} = \text{Cov}(CR.OR)$ ,  $\sigma_{OR.ZJ} = \text{Cov}(OR.ZJ)$ ,  $\sigma_{CR.ZJ} = \text{Cov}(CR.ZJ)$ .

The optimal hedge ratio can be obtained by minimizing variance of hedged revenue in the equation (3) with respect to  $\alpha, \beta, \delta$ , respectively. The first order conditions are

$$(4) \frac{\partial \sigma_{HR}^2}{\partial \alpha} = 2\alpha \sigma_{ZJ}^2 - 2\sigma_{JR.ZJ} + 2\delta \sigma_{OR.ZJ} + 2\beta \sigma_{CR.ZJ} = 0 \text{ or}$$

$$(5) \alpha \sigma_{ZJ}^2 - \sigma_{JR.ZJ} + \delta \sigma_{OR.ZJ} + \beta \sigma_{CR.ZJ} = 0$$

$$(6) \frac{\partial \sigma_{HR}^2}{\partial \beta} = 2\beta \sigma_{CR}^2 - 2\sigma_{JR.CR} + 2\delta \sigma_{CR.OR} + 2\alpha \sigma_{CR.ZJ} \text{ or}$$

$$(7) \beta \sigma_{CR}^2 - \sigma_{JR.CR} + \delta \sigma_{CR.OR} + \alpha \sigma_{CR.ZJ} = 0$$



$$(8) \frac{\partial \sigma_{HR}^2}{\partial \delta} = 2\delta\sigma_{OR}^2 - 2\sigma_{JR,OR} + 2\beta\sigma_{CR,OR} + 2\alpha\sigma_{OR,ZJ} \text{ or}$$

$$(9) \delta\sigma_{OR}^2 - \sigma_{JR,OR} + \beta\sigma_{CR,OR} + \alpha\sigma_{OR,ZJ} = 0$$

The importing firm's hedging decision can be obtained from  $\alpha, \beta, \delta$  through solving equations (5), (7), and (9). The risk-minimizing hedging positions in soybean price futures, heating oil futures, and currency futures are

(10)

$$\delta = \frac{\sigma_{JR,CR}(\sigma_{ZJ}^2\sigma_{CR,OR} - \sigma_{CR,ZJ}\sigma_{OR,ZJ}) - \sigma_{JR,ZJ}(\sigma_{CR,ZJ}\sigma_{CR,OR} - \sigma_{CR}^2\sigma_{OR,ZJ}) - \sigma_{JR,OR}(\sigma_{ZJ}^2\sigma_{CR}^2 - \sigma_{CR,ZJ}^2)}{\omega}$$

(11)

$$\beta = \frac{\sigma_{JR,OR}(\sigma_{ZJ}^2\sigma_{CR,OR} - \sigma_{OR,ZJ}\sigma_{CR,ZJ}) - \sigma_{JR,ZJ}(\sigma_{OR,ZJ}\sigma_{CR,OR} - \sigma_{OR}^2\sigma_{CR,ZJ}) - \sigma_{JR,CR}(\sigma_{ZJ}^2\sigma_{OR}^2 - \sigma_{OR,ZJ}^2)}{\omega}$$

(12)

$$\alpha = \frac{\sigma_{JR,CR}(\sigma_{OR}^2\sigma_{CR,ZJ} - \sigma_{OR,ZJ}\sigma_{CR,OR}) - \sigma_{JR,OR}(\sigma_{CR,ZJ}\sigma_{CR,OR} - \sigma_{CR}^2\sigma_{OR,ZJ}) - \sigma_{JR,ZJ}(\sigma_{CR}^2\sigma_{OR}^2 - \sigma_{CR,OR}^2)}{\omega}$$

Where  $\omega = \sigma_{ZJ}^2\sigma_{CR,OR}^2 + \sigma_{CR}^2\sigma_{OR,ZJ}^2 - \sigma_{OR}^2\sigma_{CR,ZJ}^2 - \sigma_{ZJ}^2\sigma_{CR}^2\sigma_{OR}^2 - 2\sigma_{OR,ZJ}\sigma_{CR,ZJ}\sigma_{CR,OR}$

Other combinations of hedging instruments can be considered to manage risks for commodity importers. The corresponding adjustments to equations (11), (12), and (13) are explained as:

2-way hedge

1) Combining soybean futures and exchange rate futures but no heating oil futures

( $\sigma_{CR,OR} = 0, \sigma_{OR,ZJ} = 0$  in equations (11) and (12)):

$$(13) \alpha = \frac{\sigma_{CR}^2\sigma_{JR,ZJ} - \sigma_{JR,CR}\sigma_{CR,ZJ}}{\sigma_{ZJ}^2\sigma_{CR}^2 - \sigma_{CR,ZJ}^2}$$

$$(14) \beta = \frac{\sigma_{ZJ}^2\sigma_{JR,CR} - \sigma_{JR,ZJ}\sigma_{CR,ZJ}}{\sigma_{ZJ}^2\sigma_{CR}^2 - \sigma_{CR,ZJ}^2}$$

2) Combining soybean futures and heating oil futures but no exchange rate futures

( $\sigma_{CR,ZI} = 0, \sigma_{OR,ZI} = 0$  in equations (10) and (11)):

$$(15) \delta = \frac{\sigma_{CR}^2 \sigma_{JR,OR} - \sigma_{JR,CR} \sigma_{CR,OR}}{\sigma_{CR}^2 \sigma_{OR}^2 - \sigma_{CR,OR}^2}$$

$$(16) \beta = \frac{\sigma_{OR}^2 \sigma_{JR,CR} - \sigma_{JR,OR} \sigma_{CR,OR}}{\sigma_{CR}^2 \sigma_{OR}^2 - \sigma_{CR,OR}^2}$$

1-way hedge

3) Only soybean futures ( $\sigma_{CR,OR} = 0, \sigma_{OR,ZI} = 0, \sigma_{CR,ZI} = 0$  in the equation (11)):

$$(17) \beta = \frac{\sigma_{JR,CR}}{\sigma_{CR}^2}$$

In order to investigate the impact of exchange rate on the hedge decisions, this part also examines the hedge decisions based on the hedge price revenue that is denominated in U.S. dollars. From equations (15) and (16), the hedge ratios for 2-way (soybeans and heating oil) are:

$$(18) \delta_{us} = \frac{\sigma_C^2 \sigma_{J,O} - \sigma_{J,C} \sigma_{C,O}}{\sigma_C^2 \sigma_O^2 - \sigma_{C,O}^2}$$

$$(19) \beta_{us} = \frac{\sigma_O^2 \sigma_{J,C} - \sigma_{J,O} \sigma_{C,O}}{\sigma_C^2 \sigma_O^2 - \sigma_{C,O}^2}$$

From equation (17), the hedge ratio for 1-way (soybeans) is:

$$(20) \beta_{us} = \frac{\sigma_{J,C}}{\sigma_C^2}$$

### 3. Estimation Procedures

The estimation procedures can be divided into two parts. In the first part, the estimations for hedge ratios for different combinations of futures markets are based on the time invariant

variances and covariances. Then the best hedging strategy can be identified by comparing the hedge effectiveness among these combinations of derivative securities.

The risk-minimizing hedge decision of an importer is determined by the variances and covariances of the unhedged importer's price revenue and the returns from the soybeans, heating oil, and exchange rate futures market. According to the traditional approach, the time invariant variances and covariances in the above equations can be calculated directly. Furthermore, based on these calculated variances and covariances, the optimal hedging ratio can be obtained for equations (10) to (17). The benefit of this method is its simplicity. Furthermore, it is necessary to compare the hedge effectiveness among the 3-way hedge, 2-way hedge, and 1-way hedge by using this approach and finding the best hedge method for the importer<sup>4</sup>. As mentioned above, this procedure might be statistically inefficient because it ignores time-varying characteristics which imply that estimates could not explain information flow in variables for time series.

In the second part, this analysis will choose an appropriate combination of futures markets based on the comparison in the first part to estimate the variances and covariances of the variables by using of time series techniques, including the VARMA model, conintegration analysis and the VECM, and the multivariate GARCH model.

First, the VARMA model is a very popular tool for analyzing the dynamic relationships for multivariate time series. The  $d$  - time series  $(Z_{1t}, Z_{2t}, \dots, Z_{dt})$  can be jointly modeled as

$$\phi(B)Z_t = \theta_0 + \theta(B)a_t,$$

Where  $Z_t = (Z_{1t}, Z_{2t}, \dots, Z_{dt})^T$ ,  $a_t = (a_{1t}, a_{2t}, \dots, a_{dt})^T$ ,  $\phi(B) = I - \phi_1 B - \dots - \phi_p B^p$ ,

$\theta(B) = I - \theta_1 B - \dots - \theta_q B^q$ ,  $\phi_i = (\phi_{i,jk})$  is an  $m \times m$  matrix, and  $\theta_i = (\theta_{i,jk})$  is an  $m \times m$  matrix.

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<sup>4</sup> The comparisons for 3-way, 2-way, and 1-way hedge methods are not reliable based on time series techniques because the selected multivariate models might be different based on the model identification, building, and diagnostic checking when different variables are involved.

$a_t$  is a vector white noise process with  $a_t = (a_{1t}, a_{2t}, \dots, a_{dt})^T$  such that  $E(a_t) = 0, E(a_t a_t^T) = \Sigma$ , and  $E(a_t a_s^T) = 0$  for  $t \neq s$ . The VAR (p) model can be considered as a seemingly unrelated regression (SUR) model with lagged variables and deterministic terms as common regressors.

Secondly, the cointegration analysis should be performed because if there are cointegrating relationships between the series, the VECM is more appropriate to analyze time series. The Johansen-Juselius method is used for the cointegration rank test. The VECM may be used according to the results of this test.

Finally, due to the existence of multi-markets in this study, the multivariate GARCH model, which is derived from the univariate GARCH model, might be appropriate. It is straightforward to extend the univariate GARCH models to the multivariate case. For the error term  $a_t$  of a  $d$ -dimensional time series model, its conditional mean is zero and the conditional covariance matrix is given by the positive definite  $d \times d$  matrix  $H_t$ , i.e.,  $a_t = \sqrt{H_t} b_t$ , where  $b_t$  is a *i.i.d.* vector, and its mean is zero and the covariance matrix is equal to the identity matrix  $I_d$ . As mentioned before, in the univariate case,  $H_t$  depends on lagged error terms  $a_{t-i}$ ,  $i = 1, \dots, p$ , and on lagged conditional covariance matrices  $H_{t-i}$ ,  $i = 1, \dots, r$ . Bollerslev, Engle, and Wooldridge (1988) develop a VEC-GARCH model. Suppose  $\text{vech}(\cdot)$  denote the operator that stacks the lower triangular part of a symmetric  $d \times d$  matrix as a  $d(d+1)/2 \times 1$  vector and uses the notation  $h_t = \text{vech}(H_t)$  and  $\eta_t = \text{vech}(a_t a_t^T)$ . The VEC specification of a multivariate GARCH (p, r) model is given by  $h_t = \omega + \sum_{i=1}^p A_i \eta_{t-i} + \sum_{i=1}^r B_i h_{t-i}$ , where  $A_i$  and  $B_i$  are parameter matrices with each one containing  $[d(d+1)/2]^2$  parameters. The vector  $\omega$  represents constant components of the covariances and contains  $d(d+1)/2$  parameters. However, the VEC-GARCH model cannot

ensure the conditional covariance matrix of the returns from spot and futures markets to be positive semi-definite (Lien and Tse, 2002). In order to solve this problem, Engle and Kroner (1995) develop the BEKK (named after Bata, Engle, Kraft, and Kroner) specification of a multivariate GARCH model as a special case of the VEC model (Bauwens, Laurent, and

Rombouts, 2006), 
$$h_t = C^T C + \sum_{k=1}^k \sum_{i=1}^p A_{ki}^T a_{t-i} a_{t-i}^T A_{ki} + \sum_{k=1}^k \sum_{j=1}^r B_{kj}^T H_{t-j} B_{kj}$$
, where  $C$  is an upper

triangular matrix and  $A_{ki}$  and  $B_{kj}$  are  $d \times d$  parameter matrices. A specific BEKK multivariate GARCH model will be applied in this study.

The SAS system 9.1 for Windows is used for the time series analysis.

## Data Sources

Monthly data for spot and futures prices from January 1989 to March 2007 are used in this analysis to estimate optimal hedge ratios and evaluate the hedge effectiveness. A Japanese soybean importing firm was chosen in this empirical study<sup>5</sup>. Monthly data were used in this analysis for several reasons. Firstly, monthly data are more appropriate to reflect the characteristics of U.S. soybean trade in nature because soybean trading contracts are typically based on monthly frequency and the contract months in the soybean futures market are based on monthly frequency as well. Secondly, it is reasonable to assume that financial portfolio adjustment occurs on a monthly basis because it takes approximately 30 days when shipping grain from the U.S. to its Asian markets. Finally, monthly data are used because the export prices to the U.S.'s destination markets are only available at this frequency. It is common in the previous studies to use weekly or monthly price data to study futures market prices. For the

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<sup>5</sup> Comparing with the importing firm in China and Mexico, the Japanese soybean importing firm which encounters all three risks is more typical in the empirical studies. Furthermore, U.S. soybean exports are consistent every month to Japan over 20 years and it guarantees there are enough data for in-sample hedge ratios estimations and out-of-sample hedge effectiveness comparisons.

weekly or monthly data, it is not the weekly or monthly average prices which are used in the studies. For example, a random trading day of the week or month may be chosen for the collected data set. The reason for such method been selected can be explained by Working (1960). He points out averaged daily prices over a week or month would not follow a random walk even though the daily price series can be considered as a random walk. All the price information except export prices<sup>6</sup> are taken as close as possible to the same end-of-month point, and the nearby contract prices are used in this analysis in order to meet the random walk assumption in the futures market.

For soybean spot market price, the data used are based on U.S monthly value (1000 U.S. dollar) and quantity (1000 MT) of soybean exports to Japan. Export prices of soybeans are obtained by dividing the export value by the quantity exported.<sup>7</sup> The data source is the Foreign Agricultural Service (FAS) of the USDA. The spot market exchange rates are obtained from the Economic Research- Federal Reserve Bank of St. Louis. The spot market heating oil prices are the prices of New York harbor No.2 heating oil which are obtained from the Energy Information Administration (EIA) (<http://tonto.eia.doe.gov/dnav/pet/hist/rhonyhd.htm>).

Monthly futures market prices<sup>8</sup> for soybeans, heating oil, and the Japanese yen per U.S. dollar are obtained from the published CD-ROM of the Commodity Research Bureau (CRB)<sup>9</sup>.

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<sup>6</sup> It should be note that U.S. soybean exports to its destination market do not take place everyday. Furthermore, the export prices may not be available for the U.S. Department of Agriculture (USDA) to compare with the actual daily prices in the futures market. Instead, only monthly export prices are available from USDA-FAS.

<sup>7</sup> According to "Guide to Foreign Trade Statistics", the export price is F.A.S. (free alongside ship). The value excludes the cost of loading the merchandise aboard the exporting carrier and freight, insurance, and any charges or transportation costs beyond the port of exportation.

<sup>8</sup> Soybean futures prices are monthly prices in the Chicago Board of Trade (CBOT) futures market; heating oil futures prices are monthly prices of New York harbor No.2 heating oil in the New York Mercantile Exchange (NYMEX) futures market; futures market prices for exchange rate are monthly prices in the Chicago Mercantile Exchange (CME) futures market.

<sup>9</sup> The data from January 1989 to March 2006 are obtained from this CD-ROM. Since March 2006, soybean futures market prices and futures market values for Japanese yen are retrieved from <http://futures.tradingcharts.com/menu.html>, and heating oil futures prices are obtained from EIA.

Data on every spot and futures market are compiled from several different sources. It is necessary to convert spot market price and futures market price to the same units. For soybean prices, both the spot and futures market prices are quoted in U.S. dollars per bushel based on the formula that one bushel is equal to 0.027216 ton. It is difficult to measure the exact volume of oil that is used to transport one bushel soybean from the U.S. to Japan. Alternatively, the prices in the heating oil spot and futures markets are quoted in U.S. dollars per gallon. All price series used in this study are plotted in figure 2. Figure 2.1 plots the time series for the soybean export prices from the U.S. to Japan and the CBOT soybean futures price. In general, these prices tend to move together (coefficient of correlation,  $\rho = 0.84$ ). Figure 2.2 plots the spot and the NYMEX futures markets prices for New York harbor No.2 heating oil. They are very highly correlated ( $\rho \approx 1$ ). Finally, figure 2.3 plots the spot and the CME futures markets values for Japanese yen. These two series are very highly correlated ( $\rho \approx 1$ ).

Only the first 195 observations<sup>10</sup> are used to estimate the optimal hedge ratio, leaving the last 24 observations, starting from April 2005, for the hedge ratio performance comparison in the various hedging strategies. The descriptive statistics of these price variables are shown in table 1. The coefficient of variation (CV) represents the ratio of the standard deviation to the mean, and it is a useful statistic for comparing the degree of variation from one data series to another, even if they have significantly different mean values from each other. The low value of CV means the low volatility for trade quantities in this case.

### **Empirical Results and Hedge Effectiveness Evaluation**

As shown before, the market risks for the soybean importers are hedged by using four possible combinations of derivative securities: soybean price futures, soybean price and heating oil

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<sup>10</sup> 194 observations are used finally after calculating  $M, N, S, T$ .

futures, soybean price and currency futures, and soybean price, heating oil, and currency futures. The importers' optimal hedge ratios in the futures markets are determined by the unhedged importer's price revenue ( $M$ ), and the returns ( $N, S, T$ ) from the futures market for soybeans, heating oil, and exchange rate. These price revenue and returns can be calculated by the prices on every spot and futures market. These variables are plotted in Figure 3.1 – 3.3. The descriptive statistics and variance-covariance matrices of these four variables are shown in table 2. Table 3 illustrates the optimal hedge ratios for equations (10) – (20).

There are two methods to evaluate the hedge effectiveness: the minimum variance comparison and utility-based comparison.

#### *1. Minimum variance comparison*

The variances of the unhedged and hedged revenue for an importer are calculated based on equation (3):

3-way hedge:

$$Var(UR) = \sigma_{UR}^2 = \sigma_{JR}^2 \text{ and}$$

$$Var(HR) = \sigma_{HR}^2 = \sigma_{JR}^2 + \beta^2 \sigma_{CR}^2 + \delta^2 \sigma_{OR}^2 + \alpha^2 \sigma_{ZJ}^2 - 2\beta\sigma_{JR.CR} - 2\delta\sigma_{JR.OR} - 2\alpha\sigma_{JR.ZJ} + 2\beta\delta\sigma_{CR.OR} + 2\delta\alpha\sigma_{OR.ZJ} + 2\beta\alpha\sigma_{CR.ZJ}$$

The hedged revenue can be adjusted based on the different combinations of hedging instruments.

2-way hedge

1) Soybean price and exchange rate hedge:

$$Var(HR) = \sigma_{HR}^2 = \sigma_{JR}^2 + \beta^2 \sigma_{CR}^2 + \alpha^2 \sigma_{ZJ}^2 - 2\beta\sigma_{JR.CR} - 2\alpha\sigma_{JR.ZJ} + 2\beta\alpha\sigma_{CR.ZJ}$$

2) Soybean price and heating oil price hedge:

$$Var(HR) = \sigma_{HR}^2 = \sigma_{JR}^2 + \beta^2 \sigma_{CR}^2 + \delta^2 \sigma_{OR}^2 - 2\beta\sigma_{JR.CR} - 2\delta\sigma_{JR.OR} + 2\beta\delta\sigma_{CR.OR}$$

Only soybean price hedge:



$$Var(HR) = \sigma_{HR}^2 = \sigma_{JR}^2 + \beta^2 \sigma_{CR}^2 - 2\beta \sigma_{JR.CR}$$

Ederington (1979) first proposes the method for the measure of the hedge effectiveness. The hedge effectiveness can be measured by the percentage reduction in the variance of the hedged

returns to the unhedged returns:  $1 - \frac{\sigma_{HR}^2}{\sigma_{UR}^2}$ . This hedge effectiveness measure assumes time-

invariant mean and variance. In order to compare the hedge effectiveness among the different combinations of hedging instruments, the variances and covariances of in-sample and out-of-sample hedge periods of 8, 16, 24 months and the entire in-sample period are used, respectively. The results are presented in table 4. The left part of this table illustrates the hedge comparisons, which are denominated in the Japanese yen. The hedge ratio estimates reduce the variance of the unhedged portfolio across most of the hedge periods. However, the most complex hedge strategy which all three futures markets uses, does not perform the best for both in-sample and out-of-sample evaluations. 2-way hedge ratios are superior to other hedge ratio estimates in terms of portfolio variance reduction for both in-sample and out-of-sample comparisons. Furthermore, the soybean price and currency hedge estimates are most preferred for in-sample comparisons, and the soybean price and heating oil hedge estimates provide the most variance reductions for out-of-the sample comparisons.

The right part of table 4 displays the hedge comparisons that are denominated in U.S. dollars. Comparison of soybean price and heating oil hedge and soybean price hedge that are calculated in U.S. dollar with those calculated in the Japanese yen shows the exchange rate affects the hedging effectiveness of a Japanese importing firm significantly. The exchange rate has a significant effect on the hedging effectiveness for a Japanese importing firm. For example, in a 24-month period hedge, the percentage variance reduction is 38 percent by using soybean

and heating oil hedge when variances and covariances are measured in the Japanese yen, while the percentage variance reduction is 52 percent using the same combination of the futures markets when the exchange rate is not considered.

## 2. Utility-based comparison

Minimum variance comparison fails to consider the impact of the importers' risk aversion on the measure of hedge effectiveness. Some researchers use a utility-based comparison approach which incorporates the hedger's degree of risk aversion in the measure of the hedge effectiveness (Cecchetti et al., 1988; Gagnon et al., 1998; Yang and Allen, 2004). As shown above, maximizing expected second period utility under the mean-variance framework<sup>11</sup> for an importer is the following equation:  $Max_{\alpha, \beta, \delta} E(HR_t | X_{t-1}) = E(HR_t | X_{t-1}) - \gamma V(HR_t | X_{t-1})$ . Table 5 illustrates

importers' utility comparisons associated with the different combinations of the futures markets over both an in-sample and out-of-sample 24 months forecasting period. For all the degrees of the risk aversion, all combinations of hedge ratio estimates increase the importer's utility across most of the hedge periods. They further indicate that the 2-way hedge estimates provide the greatest level of utility. The soybean and currency hedge outperforms the others for the in-sample period while the soybean and heating oil hedge is superior to the other hedge methods for the out-of-sample period. The results are consistent with the findings from the minimum variance comparison.

## Time Series Techniques

Based on the findings from the hedge effectiveness comparisons, the 2-way hedge (the soybean price and exchange rate hedge) will be applied in the time series data analysis.

### 1. Primary Time Series Analysis

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<sup>11</sup>  $HR_t$  is the importer's net purchase price at time  $t$ , and the utility is negative in this case.

It is necessary to investigate the time series properties of the variables. First, there are three methods to check for stationarity of the variables: analyzing time plot, examining autocorrelations, and performing unit root test by using the Dickey-Fuller test. 1) From the most reliable and direct method - time plot, no visible trend exists in means of the series. There are a few spikes in the variations around the means, especially for the returns from the futures markets, but there is no obvious general trend in mean related to time for any of the series. The variances of these variables might be nonstationary. 2) If the autocorrelation dies out very slowly, then the series is non-stationary in the mean. By comparing autocorrelation,  $r_k$ , with  $2SE(r_k) = 2/\sqrt{194} = 0.14$ , the autocorrelations for each series but  $M$  die out very quickly. Furthermore, the partial autocorrelation coefficient cuts off at lag one for  $M$ . All the series but  $M$  are stationary in mean and  $M$  follows a random walk. 3). Unit root. The estimated coefficients and standard errors of  $\beta_i$  along with the calculated test statistics, are shown in Table 6. The test statistic is calculated using t-value:  $(b_i - 1)/SE(b_i)$  and the following hypothesis which needs to be tested:  $H_0: \beta_1=1$ , and  $H_a: \beta_1<1$ . The Dickey-Fuller value for  $n=194$  and probability = 0.05 is equal to -2.90. Since the calculated value for every series is less than the critical value, the null hypothesis cannot be rejected, indicating the presence of a unit root and that the series might be stationary in mean. However, the test value for  $M$  is marginal. Based on the autocorrelation test, it might be non-stationary in mean for variable  $M$ .

## *2. Model Identification and Building for the VARMA model*

### *2.1. Model Identification*

To determine the orders of  $p$  and  $q$  for the stationary VARMA ( $p, q$ ) model, it is necessary to analyze the lag auto- and cross-correlation matrices,  $R(k)$  and the partial autoregression matrices,  $P(k)$ , at lag one through ten.

1) Lag Auto- and Cross-Correlations. The joint significance of these elements in each matrix can be tested by using the  $Q$ -test. The test hypotheses formula is given below:

$$H_0 : R(k) = 0, H_a : R(k) \neq 0.$$

$$Q = (n - k) \sum_{ij} [r_{ij}(k)]^2$$

where  $r_{ij}(k)$  are the elements in the lag  $k$  matrix in the  $i^{th}$  row and  $j^{th}$  column.  $Q$  is the Chi-square distribution with 9 degree of freedom in this case. At the 5% level of significance, the critical value of Chi-square with degree of freedom 9 is 16.9. The null hypotheses may be rejected if the test statistic is greater than 16.9.

2) Lag Partial Autocorrelation Coefficients.

The likelihood Ratio test is approximated by the  $M$ -test using the follow the hypotheses and formula:

$$H_0 : R(k) = 0, H_a : R(k) \neq 0.$$

$$2(L_u - L_c) \cong M = -(n - \frac{1}{2} - km) \ln \left\{ \frac{|S(k)|}{|S(k-1)|} \right\}$$

where  $n$  is the number of the observations,  $k$  is the order, and  $m$  is the number of variables.

$S(k)$  and  $S(k-1)$  are determinants of variance-covariance matrix of the residuals.  $M$  is distributed Chi-sq with degree of freedom equal to  $m^2$ . The critical value at 5% significance is 16.9. Table 7 shows the calculated values for  $Q$  and  $M$  tests. Intuitively, it might to say that autocorrelations tail off and partial autocorrelations cut off at lag 5 by comparing the values of  $Q$ -test and  $M$ -test. Thus, the VAR (4) is considered as the final model. The parameter estimation and diagnostic checking on the VAR (4) model can be performed.

### 2.1. Model Building

There are three methods to check the validity of the VARMA model: (1).Significance of the parameter estimates, (2) Multicollinearity of the parameters, and (3) White noise of the residuals. First, it is possible to simplify the identified model by comparing the  $t$ -value against the cut off rule of 1.00 and eliminating some of the insignificant parameters. The second test is for multicollinearity. The correlation matrix of the parameters needed to be checked, and the parameters which have high correlation with other estimates need to be dropped. No obvious multicollinearity problem appears by checking the cross-correlation coefficient. The last diagnostic test of the model is to check if the residuals obtained from the model are white noise. If the model is acceptable, its residuals should be white noise. This white noise test needs to rely on the joint test-  $Q$ -test at each lag. Table 8 reports the  $Q$  values for residuals. Since the cross-correlation matrices are significant at lags 5 and 7, the residuals are not white noise. Since SAS cannot provide the appropriate MA model, the corresponding AR model can be used to approximate it. The VAR (7) with  $\phi_6 = 0$  is performed. Further diagnostic tests verify this model is appropriate in this analysis.

### *3. Investigation of Cointegration and the VECM*

First, the VAR (2) model is chosen through VARMA model identification for the original time series. Second, the Johansen-Juselius Method is used as the cointegration rank test. The trace test statistics in the fourth column are computed by  $-T \sum_{i=r+1}^k \log(1 - \lambda_i)$  where  $T$  is the available number of observations and  $\lambda_i$  is the eigenvalue in the third column. From the results in table 9, the time series are cointegrated with rank=2. Finally, the VECM (2) with rank=2 is used in this analysis to estimate the variances and covariances of the time series.

### *4. The MGARCH Model*

Based on the VAR model, The VAR (7)-ARCH (1) with  $\phi_6 = 0$  might be an appropriate model in this analysis. Unfortunately, SAS cannot complete this process through the Quasi-Newton optimization<sup>12</sup>. The BEKK model has been defined in the above section and adopted by some researchers to estimate hedge ratios. Even though the BEKK model ensures the conditional variance - covariance matrix of the spot and futures returns to be positive semi-definite and reduces the numbers of estimation parameters in the conditional variance – covariance structure, this specification has been shown to produce the least hedging effectiveness compared to the hedge ratios from the OLS and random coefficient (RC) models (Lien and Tse, 2002). From a practical standpoint, the VAR (1) – GARCH (1, 1) for the BEKK representation is adopted as a specification of the Multivariate GARCH models in this analysis<sup>13</sup>.

#### *5. Hedge Effectiveness comparisons for Two-way Hedge*

Three different models based on the time series analysis techniques have been proposed to estimate the optimal hedge ratios for the 2-way hedge. Table 10 shows the optimal hedge ratios calculated from these three models. The major difference between the conventional method and the time series method is the sign of the soybean price hedge ratio. The importer should go short hedge based on the conventional method while going long hedge according to the time series techniques. For optimal hedge position in exchange rate, the importer should go long hedge based on both methods, but the magnitude of the hedge position is different.

The performances of the different hedging models previously discussed are evaluated and compared using the minimum variance comparison and utility-based comparison. Table 11 illustrates in-sample and out-of-sample minimum variance comparisons of the various models.

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<sup>12</sup> The SAS developers state that the PROC VARMAX procedure is the “experimental or production” in the latest version 9.1. QUANEW Optimization cannot be completed.

<sup>13</sup> The constant-correlation GARCH (CC-CARCH) model is an alternative to the BEKK model for ensuring the positive semi-definite conditional variance-covariance matrix. However, this specification cannot be estimated by SAS 9.1.

The results demonstrate that all hedge strategies permit achieving risk reductions compared to the unhedged position in most of the periods except a 16-month hedge from out-of-sample periods. The models based on the time series analysis techniques seem not to provide hedge performances superior to the conventional method. The 8-month hedge from the in-sample case and the 24-month hedge from the out-of-sample case were the only ones where the time series analysis models offer performances slightly superior to the conventional methods. The hedge performances for the VAR model and VECM do not differ very much and outperform the MGARCH model. Furthermore, table 12 presents the utility-based hedging performance comparisons for both an in-sample and out-of-sample 24-month forecasting period. In accordance with the findings from minimum variance comparisons, the hedge ratio calculated from the conventional methods is most preferred for in-sample analysis, and the hedge ratios based on the time series techniques perform better than the conventional hedge ratio for out-of-sample analysis. Among the time series models, the VAR model and the VECM outperform the MGARCH model for providing the greatest level of utility.

### **Summary and Conclusion**

Commodity price, foreign exchange rate, and fuel oil price, which directly impacts ocean freight cost significantly, are generally more volatile in this era, and the volatility for these prices fluctuates over time. The importing firms encounter these price risks when they import commodities from the U.S. Obtaining the optimal hedge ratios for multiple volatilities through an effective method will reduce the adverse impacts of multiple risks on import demand for U.S. commodities. It is more comprehensive to hedge commodity price, exchange rate, and freight rate in futures markets simultaneously. This study is concerned with estimating futures hedge ratios for an importing firm which imports from the U.S. Specifically, this study develops the

optimal risk-minimizing hedge ratios for the joint hedging decision for a Japanese soybean importing firm based on the monthly data. A theoretical analysis of the hedged price revenue has been constructed according to the minimum variance hedging model. The hedge ratios of a variety of hedging scenarios, including 3-way hedge, 2-way hedge, and 1-way hedge, are derived. They are determined by the variances and covariances of the unhedged importer's price revenue and the returns from the soybean, heating oil, and exchange rate futures markets.

Empirical results are achieved by using the conventional method and the time series techniques. The hedging effectiveness is compared by using in-sample and out-of-sample hedge periods based on two approaches: the minimum-variance reduction method and the utility-maximization method. These two comparison approaches yield consistent results for both in-sample and out-of-sample periods. The empirical results presented make a contribution to developing an effective hedge strategy for the importing firm, which imports commodities from the U.S. First, this study compares the hedge effectiveness for a variety of hedging scenarios using the conventional method and finds that 2-way hedge scenarios are more effective than 3-way and 1-way hedges. Moreover, the result shows the exchange rate affects the hedging effectiveness of a Japanese importing firm significantly. Second, this analysis estimates the optimal hedge ratio for 2-way (soybean price and exchange rate) hedge from a VAR model, a VECM, and a MGARCH model through SAS 9.1 and then compares the hedge effectiveness of these hedge ratios with the hedge ratio calculated from the conventional method. The results show that the hedge ratios estimated from the time series techniques do not outperform the hedge ratio from the conventional method. Among the time series models, the VAR hedge ratio and the VECM hedge ratio have the similar performances. SAS fails to establish a more appropriate MGARCH model in this analysis, and it might be the reason that the VAR model and VECM



model are better than the MGARCH model in estimating the hedge ratios. Finally, the key results of this study are that an importing firm jointly hedge soybean price and exchange rate or jointly hedge soybean price and heating oil price can reduce more revenue risk than a 3-way hedge and a 1-way hedge, and the conventional method is more effective than the time series techniques.

Similar methods can be used to analyze soybean or other individual commodities exporting to Japan or other importing countries based on data availability. Further research is definitely needed in this area based on the findings of this paper. The effectiveness of heating oil price as a proxy of freight rate needs to be tested in further study because it is difficult to measure the exact volume of oil that is used to transport one bushel of soybean. Furthermore, the timing of hedging decisions impacts the hedge effectiveness significantly. How to decide the appropriate timing of hedging decisions for hedging multiple risks simultaneously is an important issue for further research.

## Reference

- Anderson, R. W. and J. P. Danthine “Cross hedging.” *Journal of Political Economy* 89(1981): 1182–1196.
- Baillie, R. T. and R. J. Mayers. “Bivariate GARCH Estimation of the Optimal Commodity Futures Hedges.” *Journal of Applied Econometrics* 6(1991): 109-24.
- Bauwens, L., S. Laurent, and J. V. K. Rombouts. “Multivariate GARCH Models: A Survey” *Journal of Applied Econometrics* 21(2006): 79-109.
- Benniga, S., R. Eldor, and I. Zilcha. “Optimal Hedging in the Futures Market under Price Uncertainty.” *Economic Letter* 13(1983): 141-45.
- Bollerslev, T. “Generalized Autoregressive Conditional Heteroskedasticity.” *Journal of Econometrics* 31(1986): 307-27.
- Brooks, C., Henry, O. T. and Persaud, G. “The Effects of Asymmetries on Optimal Hedge Ratios.” *Journal of Business* 75(2002): 333-52.
- Cecchetti, S. G., R. E. Cumby, and S. Figlewski. “Estimation of the Optimal Futures Hedge”. *Economics and Statistics* 70(1988): 623-30.
- Ederington, L.. “The Hedging Performance of the New Futures Markets”. *Journal of Finance* 34(1979): 157-70.
- Engle, R. F., and K. F. Kroner. “Multivariate Simultaneous Generalized ARCH.” *Econometric Theory* 11(1995): 122-50.
- Fackler, P. L., and K. P. McNew. “Multiproduct hedging: Theory, Estimation, and An Application.” *Review of Agricultural Economics* 15(1993): 521-35.
- Ferson W, and R. Schadt. “Measuring Fund Strategy and Performance in Changing Economic Conditions.” *Journal of Finance* 51(1996): 425-61.
- Haigh, M.S., and H. Bryant. “The Effect of Barge and Ocean Freight Price Volatility in International Grain Markets.” *Agricultural Economics* 21(2001): 41–58.
- Haigh, M. M., and T. Holt. “Hedging Multiple Price Uncertainty in International Grain Trade”. *American Journal of Agricultural Economics* 82(2000): 881-96.
- Haigh, M. S., and M. T. Holt. “Hedging Foreign Currency, Freight and Commodity Futures Portfolios: A Note”. *The Journal of Futures Markets* 22(2002):1205-1221.

- Heifer, R., and K. Coble. "The Effectiveness of Yield and Price Hedge in Reducing Corn Revenue Risks". Paper presented at the Chicago Board of Trade Spring Research Seminar, 1996.
- Hicks, J. R. 1946. *Value and Capital*. 2<sup>nd</sup> ed. London, UK: Oxford University Press.
- Johnson, L. L. The Theory of Hedging and Speculation in Commodity Futures. *Review of Economic Studies* 27 (1960): 139-51.
- Kawai, M., and I. Zilcha. "International Trade with Forward-Futures Markets under Exchange Rate and Price Uncertainty." *Journal of International Economics* 20(1986): 83-98.
- Keynes, J. M. 1930. *A treatise on Money*. New York, NY: Harcourt Brace Jovanovich.
- Lence, S. H. "The Economic Value of Minimum-variance Hedges." *American Journal of Agricultural Economics*. 77 (1995): 353-64.
- Lien, D. "The Effect of the Cointegration Relationship on Futures Hedging." *Journal of Futures Markets* 16(1996): 773-80.
- Lien, D., and Y. K. Tse. "Some Recent Developments in Futures Hedging." *Journal of Economic Surveys* 16(2002): 357-96.
- Manfredo, M. R., P. Garcia, and R. M. Leuthold. "Time-varying Multiproduct Hedge Ratio Estimation in the Soybean Complex: A simplified Approach." Presented paper at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, Chicago, Illinois, April 17-18, 2000.
- McKenzie, A. M., and M.T. Holt. "Market Efficiency in Agricultural Futures markets." *Applied Economics* 34 (2002):1519-32.
- Miffre, J. "Efficiency in the Pricing of the FTSE100 Futures Contract." *European Financial Management* 7(2001): 9-22.
- \_\_\_\_\_. "Conditional OLS minimum variance hedge ratios." *Journal of Futures Markets* 24(2004): 945-64.
- Myers, R. T., and S. R., Thompson. "Generalized Optimal Hedging Estimation. *American Journal of Agricultural Economics* 71(1989): 858-67.
- Myers, R. J. "Estimating Timing-varying Hedge Ratio on Futures Markets." *Journal of Futures Markets* 11(1991): 39-53.
- Nayak, G., and C. G. Turvey. "Simultaneous Hedging of Price, Crop Yield and Currency Risk". *Canadian Journal of Agricultural Economics* 48(2000): 123-40

- Stein, L. J. "The Simultaneous Determination of Spot and Futures Prices." *American Economic Review* 51(1961): 139-51.
- Thompson, S. R., and G. E. Bond. "Offshore Commodity Hedging under Floating Exchange Rates." *American Journal of Agricultural Economics* 69(1987): 58-65.
- Thoung, L. T., and S. V. Visscher. "The Hedging Effectiveness of Dry-Bulk Freight Rate Futures." *Transportation Journal* 29(1990): 58-65.
- Vukina T., D. Li, and D. M., Holthausen. "Hedging with Crop Yield Futures: A Mean-variance Analysis." *American Journal of Agricultural Economics* 78(1996): 1015-25.
- Working, H. "Hedging Reconsidered." *Journal of Farm Economics* 35(1953): 544-61.
- \_\_\_\_\_. "Note on the Correlation of the First Differences of Averages in the Random Chain." *Econometrica* 28(4): 916-8.
- Yang, W., and D.E. Allen. "Multivariate GARCH Hedge Ratios and Hedging Effectiveness in Australian Futures Markets." *Accounting and Finance* 45(2004): 301-21.
- Zilcha, I., and U. Broll. "Optimal Hedging by Firms with Multiple Sources of Risk Revenues." *Economics Letter* 39(1992): 473-77.

Figure 1. Timeline of hedging and producing periods

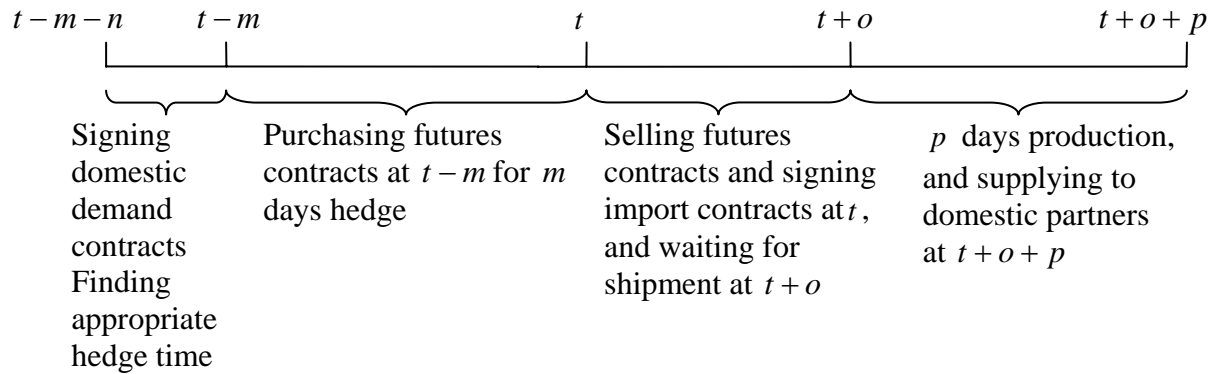


Figure 2. The Spot and Futures Market Prices

Figure 2.1 Soybean export prices (U.S. to Japan) and CBOT soybean prices

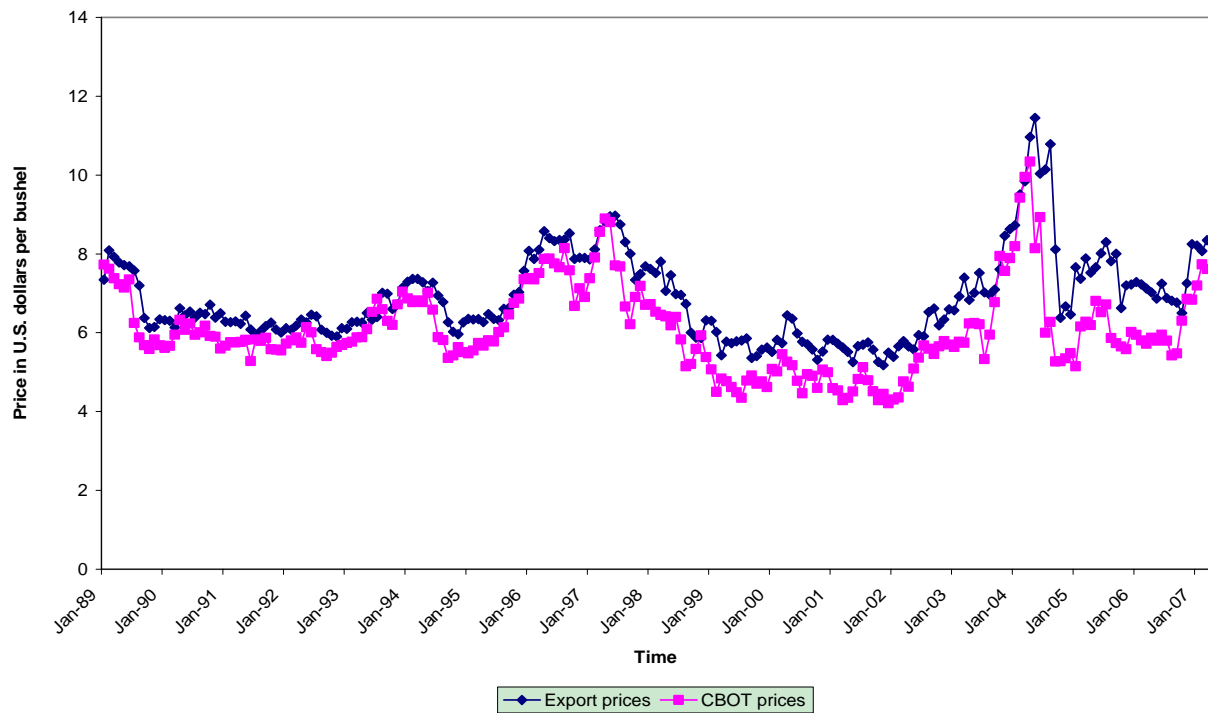


Figure 2.2 Heating oil spot prices and NYMEX prices



Figure 2.3 U.S. Dollar/Japanese Yen spot and futures exchange rate



Figure 3. Unhedged Price Revenue and Returns from Futures Markets

Figure 3.1 Unhedged importer's price revenue

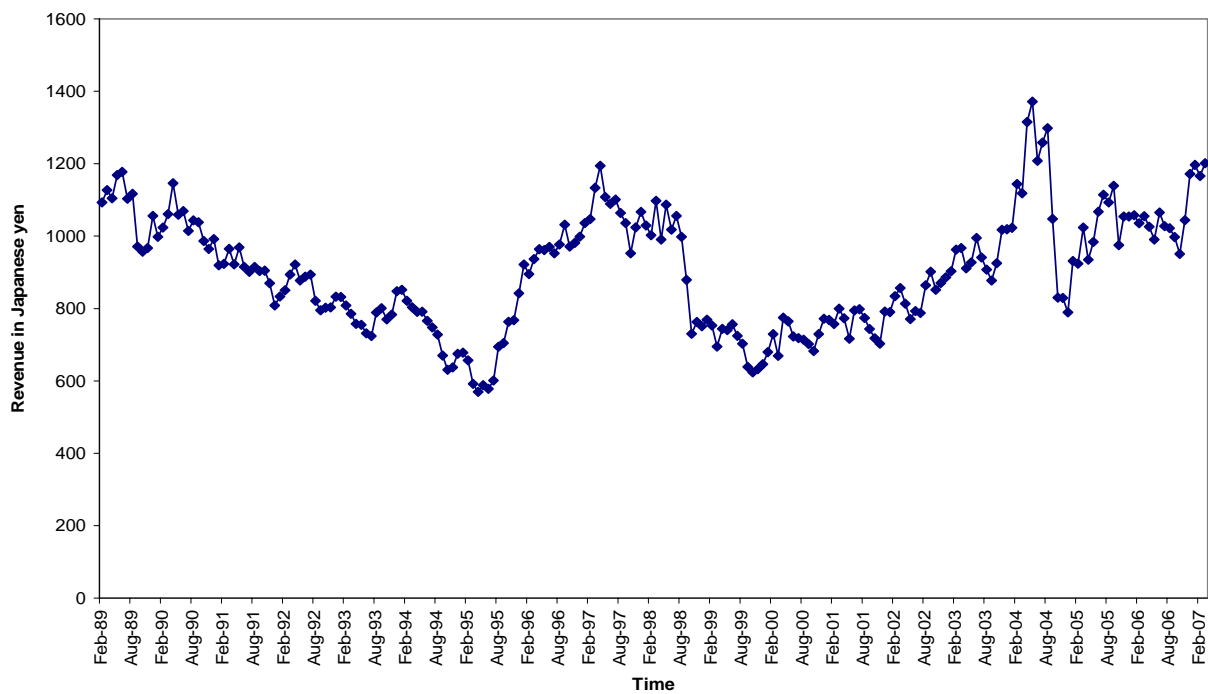


Figure 3.2 Revenue from CBOT soybean market

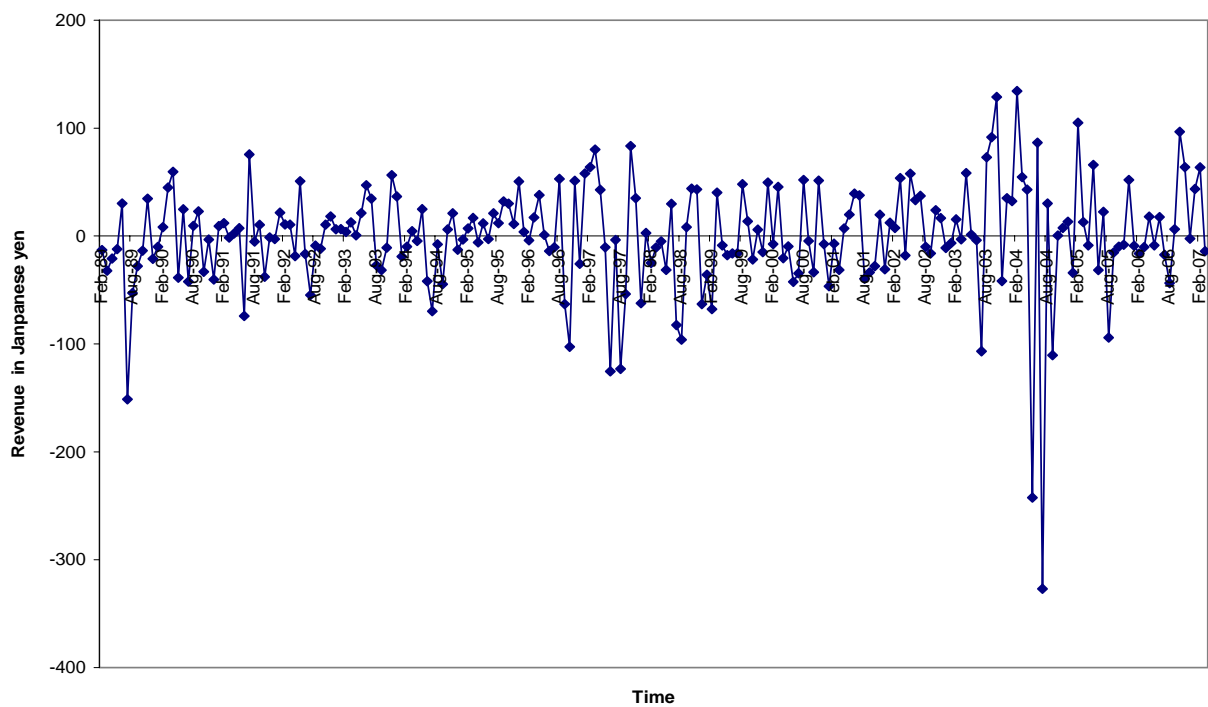


Figure 3.3 Revenue from the NYMEX heating oil market

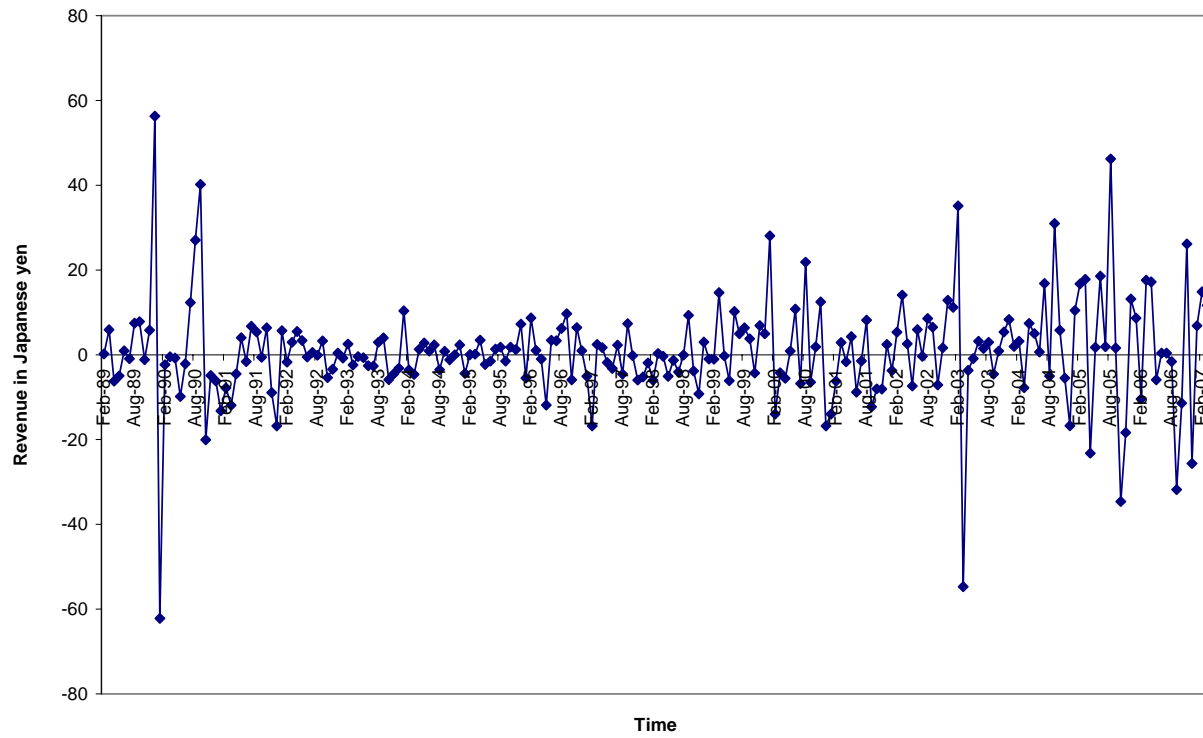
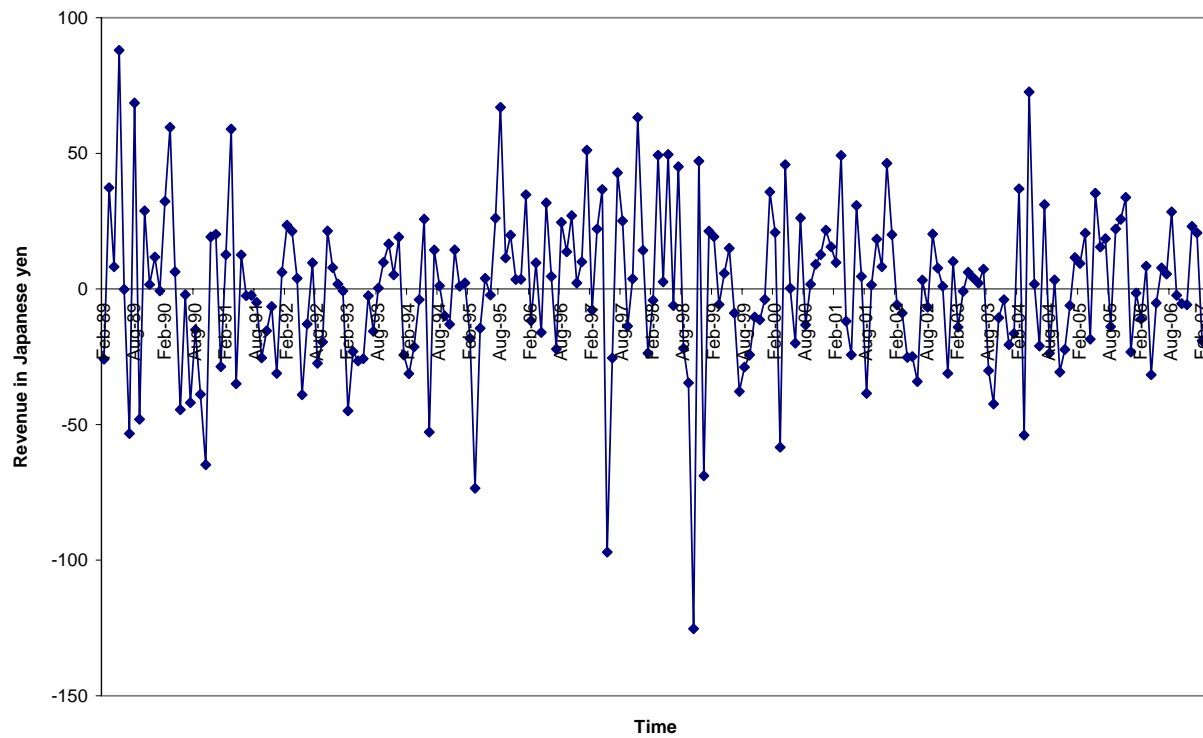


Figure 3.4 Revenue from the CME Japanese yen futures market





**Table 1. Descriptive Statistics of the Data Used in the Empirical Analysis**

	Soybean export price (\$/Bu)	CBOT soybean price (\$/Bu)	New York harbor No.2 heating oil (\$/Ga)	NYMEX heating oil price (\$/Ga)	Exchange rate (Yen/US\$)	CME exchange rate (Yen/US\$)
Mean	6.80	6.04	0.66	0.66	118.71	118.35
Standard deviation	1.14	1.12	0.23	0.23	0.23	14.77
Coefficient of Variation (CV)	0.17	0.19	0.34	0.35	0.12	0.12

**Table 2. Variance- covariance Matrix Used in Calculating Hedge Levels and Risk Reduction**

	Unhedged importer's price revenue (M)	Returns from soybean futures market(N)	Returns from heating oil futures market (S)	Returns from exchange rate futures market (T)
M	24841.79			
N	-1440.64	2686.59		
S	108.02	-16.42	123.63	
T	832.70	-55.40	-13.85	892.02

**Table 3. Optimal Hedge Ratio**

	In the Japanese yen				In U.S. dollar	
	3-way soybean, heating oil, and currency	2-way soybean and currency	2-way soybean and heating oil	1-way soybean only	2-way soybean and heating oil	1-way soybean only
Soybean price	-0.51	-0.52	-0.54	-0.54	-0.38	-0.39
Heating Oil	0.82		0.70		1.39	
Exchange rate	0.90	0.89				

**Table 4. Risk Return Hedging Performances Comparison for Different Scenarios**

		In the Japanese yen				In U.S. dollar	
		3-way	2-way		1-way	2-way	1-way
		soybean, heating oil, and currency	soybean and currency	soybean and heating oil	soybean only	soybean and heating oil	soybean only
<b>In-sample</b>							
8-month	VAR(UR)	3487.24	3487.24	3487.24	3487.24	0.24	0.24
	VAR(HR)	3068.70	2867.47	5069.06	4871.85	0.21	0.28
	Percentage variance reduction	0.12	0.18	-0.45	-0.40	0.13	-0.15
16-month	VAR(UR)	4764.74	4764.74	4764.74	4764.74	0.47	0.47
	VAR(HR)	4698.63	3262.12	3986.92	3975.81	0.22	0.20
	Percentage variance reduction	0.01	0.32	0.16	0.17	0.53	0.58
24-month	VAR(UR)	4862.89	4862.89	4862.89	4862.89	0.34	0.34
	VAR(HR)	4269.12	3010.98	3755.11	3893.92	0.23	0.21
	Percentage variance reduction	0.12	0.38	0.23	0.20	0.32	0.39
194-month	VAR(UR)	25454.02	25454.02	25454.02	25454.02	1.45	1.45
	VAR(HR)	23997.15	2186.95	2883.80	2995.88	0.16	0.21
	Percentage variance reduction	0.06	0.91	0.89	0.88	0.89	0.86
<b>Out-of-sample</b>							
8-month	VAR(UR)	4664.31	4664.31	4664.31	4664.31	0.33	0.33
	VAR(HR)	2716.08	2494.78	2329.11	2992.64	0.08	0.21
	Percentage variance reduction	0.42	0.47	0.50	0.36	0.77	0.35
16-month	VAR(UR)	2602.39	2602.39	2602.39	2602.39	0.20	0.20
	VAR(HR)	1803.25	3321.16	2985.23	3395.40	0.16	0.23
	Percentage variance reduction	0.31	-0.28	-0.15	-0.30	0.18	-0.13
24-month	VAR(UR)	5273.22	5273.22	5273.22	5273.22	0.31	0.31
	VAR(HR)	4960.17	3864.63	3263.57	3731.70	0.15	0.23
	Percentage variance reduction	0.06	0.27	0.38	0.29	0.52	0.27

**Table 5. Utility-maximization Hedge Performance Comparison for Different Hedge Scenarios**

		In the Japanese yen				
		3-way	2-way		1-way	
		soybean, heating oil, and currency	soybean and currency	soybean and heating oil	soybean only	
Risk performance	Unhedged	Hedged	Hedged	Hedged	Hedged	
In-sample						
	0.1	-1534.49	-1467.69	-1342.78	-1416.86	-1431.53
	0.5	-3479.65	-3175.33	-2547.17	-2918.90	-2989.10
	1	-5911.09	-5309.89	-4052.66	-4796.46	-4936.06
	2	-10773.98	-9579.01	-7063.64	-8551.57	-8829.98
	3	-15636.87	-13848.13	-10074.61	-12306.67	-12723.90
	4	-20499.77	-18117.24	-13085.59	-16061.78	-16617.82
Out-of-sample						
	0.1	-1586.63	-1554.45	-1445.81	-1388.52	-1436.07
	0.5	-3695.91	-3538.52	-2991.67	-2693.95	-2928.75
	1	-6332.52	-6018.60	-4923.98	-4325.74	-4794.61
	2	-11605.74	-10978.77	-8788.61	-7589.31	-8526.31
	3	-16878.96	-15938.94	-12653.24	-10852.89	-12258.02
	4	-22152.18	-20899.11	-16517.88	-14116.46	-15989.72

**Table 6. Unit Root Test Results**

Variables	bi	Std Err	t-value
M	0.920	0.027	-2.981
N	0.008	0.141	-7.045
S	-0.447	0.176	-8.199
T	0.021	0.141	-6.947

**Table 7. Q and M test for VARMA model Identification**

Lag	1	2	3	4	5	6	7	8	9	10
Q-test.	25.27	26.51	12.47	6.40	32.47	7.29	25.45	9.03	13.57	4.94
M-Test	27.97	44.1	20.15	36.97	4.1	13.38	6.48	4.38	11.98	8.15

**Table 8. Q test for Checking if the Residual is White Noise**

Lag	1	2	3	4	5	6	7	8	9	10
Q-test.	4.19	6.41	7.92	1.56	41.6	5.06	22.65	11.01	15.99	4.19

**Table 9. Cointegration Rank Test Using Trace**

Eigenvalue	Trace	5% Critical Value	Drift in ECM	Drift in Process
0.51	203.56	24.08	NOINT	Constant
0.30	67.42	12.21		
0.0013	0.24	4.14		

**Table 10. Optimal Hedge Ratios for Different Estimation Methods**

	Conventional Method	VAR Model	VECM	MGARCH Model
Soybean Price	-0.52	0.08	0.07	0.15
Exchange rate	0.89	1.08	1.09	1.11

**Table 11. Risk Return Hedging Performances Comparison for Different Estimation Methods**

		In the Japanese yen			
		Conventional Method	VAR Model	VECM	MGARCH Model
<b>In sample</b>					
8-month	VAR(UR)	3487.24	3487.24	3487.24	3487.24
	VAR(HR)	2867.47	2573.59	2582.33	2689.09
	Percentage variance reduction	0.18	0.26	0.26	0.23
16-month	VAR(UR)	4764.74	4764.74	4764.74	4764.74
	VAR(HR)	3262.12	3893.60	3883.24	4089.81
	Percentage variance reduction	0.32	0.18	0.19	0.14
24-month	VAR(UR)	4862.89	4862.89	4862.89	4862.89
	VAR(HR)	3010.98	3343.74	3338.80	3477.88
	Percentage variance reduction	0.38	0.31	0.31	0.28
194-month	VAR(UR)	25454.02	25454.02	25454.02	25454.02
	VAR(HR)	2186.95	2826.18	2780.37	3002.78
	Percentage variance reduction	0.91	0.89	0.89	0.88
<b>Out-of-sample</b>					
8-month	VAR(UR)	4664.31	4664.31	4664.31	4664.31
	VAR(HR)	2494.78	3785.83	3732.76	4012.72
	Percentage variance reduction	0.47	0.19	0.20	0.14
16-month	VAR(UR)	2602.39	2602.39	2602.39	2602.39
	VAR(HR)	3321.16	3688.47	3673.50	3784.09
	Percentage variance reduction	-0.28	-0.42	-0.41	-0.45
24-month	VAR(UR)	5273.22	5273.22	5273.22	5273.22
	VAR(HR)	3864.63	3712.98	3710.33	3769.73
	Percentage variance reduction	0.27	0.30	0.30	0.29

**Table 12. Utility-maximization Hedging Performances Comparison for Different Estimation Models**

		Conventional			MGARCH	
		Method	VAR Model	VECM	Model	
Risk performance	Unhedged	Hedged	Hedged	Hedged	Hedged	
<b>In-sample</b>						
	0.1	-1534.49	-1342.78	-1382.70	-1381.99	-1396.79
	0.5	-3479.65	-2547.17	-2720.19	-2717.51	-2787.94
	1	-5911.09	-4052.66	-4392.06	-4386.91	-4526.89
	2	-10773.98	-7063.64	-7735.79	-7725.70	-8004.77
	3	-15636.87	-10074.61	-11079.53	-11064.50	-11482.65
	4	-20499.77	-13085.59	-14423.27	-14403.30	-14960.54
<b>Out-of-sample</b>						
	0.1	-1586.63	-1445.81	-1425.34	-1425.08	-1431.02
	0.5	-3695.91	-2991.67	-2910.53	-2909.21	-2938.91
	1	-6332.52	-4923.98	-4767.02	-4764.37	-4823.77
	2	-11605.74	-8788.61	-8480.00	-8474.70	-8593.50
	3	-16878.96	-12653.24	-12192.98	-12185.03	-12363.22
	4	-22152.18	-16517.88	-15905.96	-15895.35	-16132.95