The Law of the Minimum and Sources of Nonzero Skewness for Crop Yield Distributions

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Abstract: Crop yields are not commonly found to be normally distributed, but the cause of the non-normal distribution is unclear. The non-normality might be due to weather variables and/or an underlying von Liebig law of the minimum (LoM) production function. Our objective is to determine the degree to which an underlying linear response stochastic plateau production function can explain the skewness of Oklahoma wheat yields at varied nitrogen rates. We use farm-level wheat data from a long-term experiment in Oklahoma, which is a unique data set to the literature. The Tembo et al. (2008) production function provides negative skewness at all levels of nitrogen with skewness near zero for both very high and very low levels of nitrogen. Observed skewness for wheat yields, however, is positive. The variation in the plateau by year shows positive skewness. Skewness in yield potential related to weather should be considered as a possible explanation of skewness.

Keywords: linear plateau model, non-normal distributions, skewness, wheat, yield distribution

Introduction

Research has recognized that crop yields are not normally distributed and several different models have been developed to estimate crop yield distributions (Norwood, Roberts and Lusk 2004). Day (1965) analyzed corn, oat, and cotton yields from experiments conducted in the Mississippi delta. He found evidence of positive skewness for the logarithm of cotton and corn yields and evidence of negative skewness for the logarithm of oat yields. Day (1965) also states that skewness was found to increase as the quantity of nitrogen applied to the crops increased.

The majority of the more recent work has reported yield distributions to be negatively skewed for various crops across the U.S. (Nelson and Preckel 1989; Gallagher 1987; Moss and Shonkwiler 1993; Ramirez, Misra, and Field 2003; Harri et al. 2008; Hennessey 2009) ,but Harri et al. (2008) do find positive skewness in some western U.S. dryland production areas.

There is no consensus regarding the underlying cause of the observed non-normal crop distributions (Harri et al. 2008; Hennessey 2009b). Researchers have sought to provide rationale for the observed nonzero skewness in crop yield distributions and have developed two primary hypotheses. The first hypothesis is that the von Liebig law of the minimum (LoM) production function might cause crop yields to be non-normal (Hennessy 2009a). Hennessy (2009a) conjectures that the non-normal yield distribution is associated with minimum quantities of a limiting resource such as fertilizers, and that the degree of skewness decreases as the level of the limiting resource availability increases. Hennessy (2009a) related yield skewness to the LoM production function as used by Berck and Helfand (1990), and Paris (1992), and concluded that asymmetries in resource availability can determine skewness of crop yield distributions. According to Hennessy, when resource availability is stochastic, the linear response plateau (LRP) production function can support positive or negative skewness. Positive skewness is supported when there is uniform resource availability to the crop and negative skewness occurs whenever production is tightly controlled so that the left tails of some resource availability distributions are thin.

Berck and Helfand (1990) and Paris (1992) find the LRP production technology is appropriate to indentify the deterministic component of the production function. Tembo et al. (2008) extend the conventional LRP developed by Berck and Helfand (1990) and Paris (1992) by including a plateau year random effect. Tembo et al. (2008) emphasize the effect of stochastic weather on crop yield response to some factor inputs by including a plateau year random effect and year random effect. Tembo et al. (2008) used wheat grain yield data, and showed that the linear response model with a stochastic plateau fit data better than the conventional LRP or the Berck and Helfand version of a stochastic plateau.

The second hypothesis is that non-normality in yields is due to non-normality in weather (Gallagher 1987; Kaylen and Koroma 1991; Goodwin and Ker 1998; Ker and McGowan 2000). Weather conditions can be critical factors determining yield distribution outcomes, but few studies have attempted to determine how weather variables affect crop yield distributions. Studies that have taken into account weather effects in characterizing yield distributions considered long-term climate cycles such as El Niño and La Niña (Ker and McGowan 2000; Nadolnyak, Vedenov, and Novak 2008). Kaylen and Koroma (1991) explicitly determined the impact of weather variables such as monthly rainfall and temperature on yield distributions and find weather variables help describe U.S. crop yield distributions. However, their study uses aggregate yield data instead of farm-level data, which is a limitation found in several studies.

A problem commonly encountered by researchers is the lack of reliable farm-level yield data (Taylor 1990; Just and Weninger 1999; Atwood, Shaik, and Watts 2002). Researchers often resort to using aggregate yield data at county or regional levels to describe farm-level yield distributions. Aggregate data can underestimate farm-level yield variability, which may lead to mis-specified distributions as the moments of aggregate data may not accurately reflect farm-level yield distribution (Just and Weninger 1999; Atwood, Shaik, and Watts 2002; Ker and McGowan 2000). Methods to control for potential bias arising from aggregation of data have emerged (Wang and Zhang 2002; Rudstrom et al. 2002; Popp, Rudstrom and Manning 2005), but complete control of spatial heterogeneities is practically impossible. Thus, using yield distributions based on aggregate data may lead to inaccurate characterization of farm-level yield distributions.

The objective of this study is to determine whether the LRP model of Tembo et al. (2008) can explain the observed skewness. We use a unique data set of wheat grain yields from a farm-level experiment spanning from 1971 to 2009. The experiment was conducted near Lahoma, Oklahoma, and was intended to investigate the effects of nitrogen rates on yields. This relatively long-term data can potentially allow assessing the impact of weather effects on yield distributions at the farm-level, while avoiding the data aggregation problem. The 39 years is larger than the longest dataset considered by Du, Hennessy, and Yu (2010), which included 24 years, and numerous years in a dataset is critical in testing hypotheses about skewness. The results from our study should provide a better understanding of the source(s) of non-normality, which is important for estimating accurate yield distributions as well as assist in setting accurate crop insurance rates and making other farm planning decisions involving uncertainty and risk.

Experimental Data

The data are from a long-term experiment (experiment 502) conducted at the North Central Oklahoma agricultural research station near Lahoma under conventional tillage. The soil is Grant silt loam (fine-silty, mixed, thermic Udic Argiustoll). The study was established in 1971 to determine hard red winter wheat grain yield response to fertilizer application, using a randomized complete block design (Raun et al. 2002). For the past 39 years, treatment levels of nitrogen include 0, 20, 40, 60, 80, and 100 lb N ac^{-1} yr^{-1} with each treatment being replicated four times. The experiment included additional plots with 60 lb N ac^{-1} yr^{-1} and varying levels of P and K. Only the plots with 40 lb P_2O_5 ac^{-1} yr^{-1} are included. More detail on this experiment is available from Department of Plant and Soil Sciences (2009). Data from the same experiments was analyzed by Tembo et al. (2008) and Brorsen and Richter (2011). The present study uses additional years of data that includes 2003 and 2008 when yields were extremely high. While Lahoma is in Garfield County, the experiment station itself is in Major County. County level yields from Major County are used as a comparison.

Production Function Hypothesis

As Hennessy (2009a, 2009b) and Du, Hennessy, and Yu (2010) have shown, the effects of increasing levels of nitrogen on skewness depend on the assumed production function. Day (1965) proposed that skewness would vary with the level of nitrogen. We offer a theory of skewness decreasing and then increasing with increasing levels of nitrogen. Hennessy (2009a) argued uniform resource availability will lead to positive skewness of yields; while negative skewness is observed whenever production is tightly controlled so that the left tails of resource availability distributions are thin. Du, Hennessy, and Yu offered an explanation of skewness decreasing with increases in nitrogen. We test these hypotheses as Day (1965) and Du, Hennessy, and Yu have done by calculating skewness for different levels of nitrogen.

Past studies have shown that the distribution of output is a unique function of its distribution moments (Day 1965; Antle 1983; Rosegrant and Roumasset 1985). Hence, a production technology can be uniquely represented as a stochastic process where the distribution of crop yields is conditional on input levels. For estimation of input effects on output distribution, one widely applied model is by Just and Pope (1978, 1979). Just and Pope developed a stochastic production function that allows estimation of input effects such as capital, fertilizers and labor on the mean and variance of output distribution. However, the method by

Just and Pope has been criticized since it imposes restrictions on higher moments of the output distribution (Antle 1983).

To determine the effect of inputs on output distribution, we consider a simpler moment based approach, similar to that proposed by Antle (1983). The approach involves estimating not only the mean output as a function of inputs, but also specifying and estimating the variance, third moment, and fourth moment. This method imposes no restrictions and is general such that the distribution of output can be measured at any point of input level. Higher order moment functions are estimated by hypothesizing that the moments are functions of inputs (or input levels). The wheat yield data include nitrogen application of 0, 20, 40, 60, 80, and 100 lbs ac^{-1}. We calculate the skewness and relative kurtosis at each level of nitrogen.

Methods

To accomplish our objectives, we perform several statistical tests and empirical estimations. First, we test the data for a deterministic time trend, which would have to be removed before testing the data for normality. Then to test the crop yield distributions for normality, the K^2 and Jarque-Bera (JB) tests are used. Next, we calculate skewness by nitrogen rate as in Day (1965) to see if results are as predicted by the stochastic plateau model.

A univariate linear response with a stochastic plateau is

$$y_{ti} = \min(\alpha + \alpha_1 N_{ti}, P + v_t) + u_t + \varepsilon_{ti} \tag{1}$$

where y_{it} is yield of the i^{th} observation in year t, N_{it} is the amount of nitrogen applied , α_0 and α_1 are coefficients, P is the plateau, $v_t \sim N(0,\sigma_v^2)$, $u_t \sim N(0,\sigma_u^2)$, $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$, and the three error terms are independent. Figure 1 shows how skewness changes with nitrogen using the estimates of Tembo et al. (2008). The Paris (1992) stochastic plateau can generate positive or negative skewness, but skewness with the Tembo et al. (2008) model is always negative. The negative skewness is due to the plateau model cutting off the upper tail of the distribution of the plateau error. Skewness is near zero for either very high or very low levels of nitrogen resulting in a quadratic-like function in figure 1. Skewness approaches zero as high levels of nitrogen are applied, but skewness will not necessarily go to zero when zero nitrogen is applied (figure 1). Also, note that producers will   apply levels of nitrogen that would lead to negative skewness.

Time Trend

The crop yield data need to be tested for a deterministic (or stochastic[1]) time trend and if one is found it needs to be removed. Yields can be upward trending, often due to technology change over time. A visual inspection of our data (figure 2) reveals the possibility of a time trend (especially the farm level yield series). Oklahoma wheat yields, however, have not shown a consistent upward trend (Epplin 1997) like corn yields. Determining the presence of a deterministic trend often involves regressing yield against time variable(s). The choice of appropriate time trend structures is a subject of debate. Just and Weninger (1999) suggest starting with a higher order polynomial structure and move to a linear trend until the appropriate structure is found. In most studies however, a deterministic trend is often approximated by a low-order trend function usually of the quadratic order. We also assume a quadratic time function and fit the equation

$$y_t = S_t + e_t \tag{2}$$

[1] Issues related to a stochastic trend (often introduced into data by permanent shocks) are not explored in this study given that most past literature does not support a stochastic trend for grain yields.

where y_t is average wheat yield in bu/acre obtained in year t (t = 1,2,…,T), $S_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2$, and e_t the idiosyncratic disturbances. We estimate equation (2) using the PROC ROBUSTREG procedure in SAS (SAS Institute Inc. 2003).

Normality test

Various tests of normality have been discussed and used in previous studies on yield distributions. The most commonly used are the Jarque-Bera (JB) test (Jarque and Bera 1980), and the K^2 test (D'Agostino and Stephens 1986). These tests combine both skewness and kurtosis, and are more powerful than individual tests of skewness or kurtosis (David and Mckinnon 1993). Just and Weninger (1999) recommend using the K^2. Deb and Sefton (1996) investigate the power of the JB test in both large and small samples using Monte Carlo simulations, and concluded the JB test has good small sample properties. We present normality tests with both the JB and K^2 tests.

The K^2 statistic is defined as $K^2 = [Z(b1)^{0.5}]^2 + [Z(b2)]^2$ where $Z(b1)^{0.5}$ is the skewness statistic and $Z(b2)$ is the kurtosis statistic. Under the null hypothesis of normality, $Z(b1)^{0.5}$ and $Z(b2)$ are distributed approximately normal. The JB test is defined as

$JB = N[(\sqrt{b1})^2 / 6 + (b2-3)^2 / 24]$ where $\sqrt{b1} = m_3 / m_2^{1.5}$, $b2 = m_4 / m_2$, and

$m_i = \sum_{t=1}^{N} (\hat{e}_t)^i / N \; i = 1,2,3,4$. Both the K^2 and JB statistic are distributed asymptotically chi-squared with two degrees of freedom so the .05 critical value is $\chi^2_{2,0.05} = 5.99$. The null hypothesis of normality is tested with both the county and farm-level yield data.

Weather and Plateau Skewness

We are also interested in testing the effects of weather on the expected yield distribution in a given year. To estimate weather effects, a stochastic LRP function is estimated following Tembo et al. (2008). We also estimate a model that does not impose normality by estimating the fixed effects for each of the 39 years instead of using a year random effect. The LRP function with fixed effects for year is specified as

$$y_{it} = \min(a + bN_i, P + \sum_{k=1}^{38} \beta_k T_{kt}) + e_{it} \tag{3}$$

where y_{it} is the yield in year t due to the i^{th} level of nitrogen N, P the expected plateau yield, β_k is the fixed effect for year, T_{kt} is an indicator variable for year, and $e_{it} \sim N(0, \sigma_e^2)$ is the random error term. The parameters a, b and P are estimated from equation (3) using NLMIXED procedure in SAS (SAS Institute Inc. 2003).

Generally, winter wheat in the southern Great Plains is planted beginning in early September through the middle of November. According to crop weather summary in Oklahoma, wheat begins to double ridge and joint in February. In April, anthesis is begun, and finally wheat harvest begins approximately in the second half of May and continues until about the middle of July. Using the described general relationship of weather and wheat growth stages, the study selected the months of February through April as the most critical stages for weather variables to affect wheat yield. Weather variables included in the analysis are monthly average rainfall, monthly average maximum and minimum temperatures. Data were obtained from the Mesonet station located in Enid, OK, which is relatively close to the experimental site (Lahoma, OK). This data will be considered in future research.

Results

Table 1 presents the results from the time trend model. The coefficients for the farm-level data are not significant at the 0.05 level. The same result is found for the county level data; however, the coefficients are significant at the 0.10 level, suggesting modest evidence of a quadratic time trend for the county level data. This displays the difference discussed above in county and farm-level and how data aggregation can impact the crop yield distribution. This result of no deterministic trend[2] is not surprising, as literature suggests the pace of technological improvement has been slower for wheat than for other field crops such as corn and soybeans (Epplin 1997; USDA Wheat Baseline 2010).

We cannot reject the null hypothesis that Oklahoma wheat yield is normally distributed for the county-level data (Table 2). Both the K^2 and the JB test statistic were smaller than the χ^2_2 critical value. However, the null hypothesis is rejected for the farm-level data (Table 2). The experiment station wheat yield is positively skewed (Table 2). The positive skewness in crop yield is not commonly found in the literature but is possible (Day 1965; Hennessy 2009a). Again, this result demonstrates the potential differences in using aggregate yield data and farm-level data.

Table 3 shows the skewness and kurtosis of crop yields at the different levels of nitrogen applied. Note that the average yields across all observations in a year are used in calculating Table 3. Thus we only consider time-series variability and do not mix cross-sectional variability and time-series variability as is done in most past research. We find skewness much different than that predicted by the Tembo et al. (2008) model. Skewness is positive and increases with the

[2] If ordinary least squares is used to estimate the trend model, the trend is significant. The residuals from this regression still exhibit the same pattern of positive skewness, but the skewness is less pronounced.

level of nitrogen. We estimated a stochastic LRP model following Tembo et al. (2008), which assumes normality, for the data and found estimates similar to that of Tembo et al. (Table 4). It appears that this production function cannot explain the observed skewness.

A model was estimated that used fixed effects rather than normally distributed random effects to capture year to year variability in the plateau. The results from the K^2 and the JB tests indicated the null hypothesis that fixed effects are not normally distributed and exhibit strong positive skewness (Table 5). Future research will test whether this positive skewness can be related to weather variables. A linear response stochastic plateau is a viable production function, but not one that assumes normality for the distribution of plateau random effects.

Conclusions

It is widely known that crop yield distributions are not normal; however, the conclusions on what causes the non-normality are not robust. We tested the hypothesis that a non-normal yield distribution can be explained by the linear response stochastic plateau of Tembo et al. (2008). We use farm-level data from a long-term experiment on Oklahoma winter wheat. These data are unique to the literature, which commonly uses aggregate data to estimate crop yield distributions.

The stochastic LRP developed by Tembo et al. (2008) has negatively skewed yields when normality is assumed. The actual skewness is positive and increases with the level of nitrogen applied. Thus, the Tembo et al. production function cannot be the source of the positive skewness. Future research will consider the degree to which weather data can explain the positive skewness in the plateau yield.

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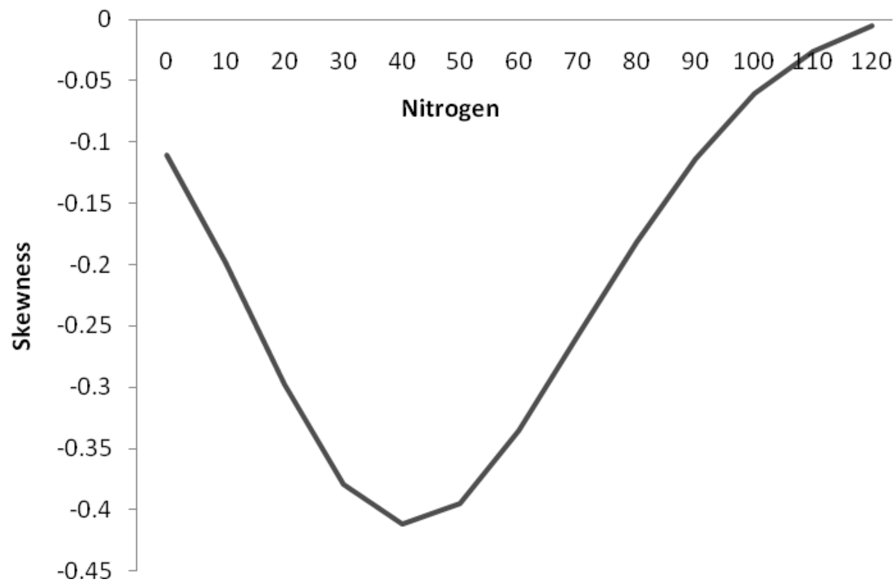

Figure 1. Simulated skewness of yield from Tembo et al. (2008) estimated model

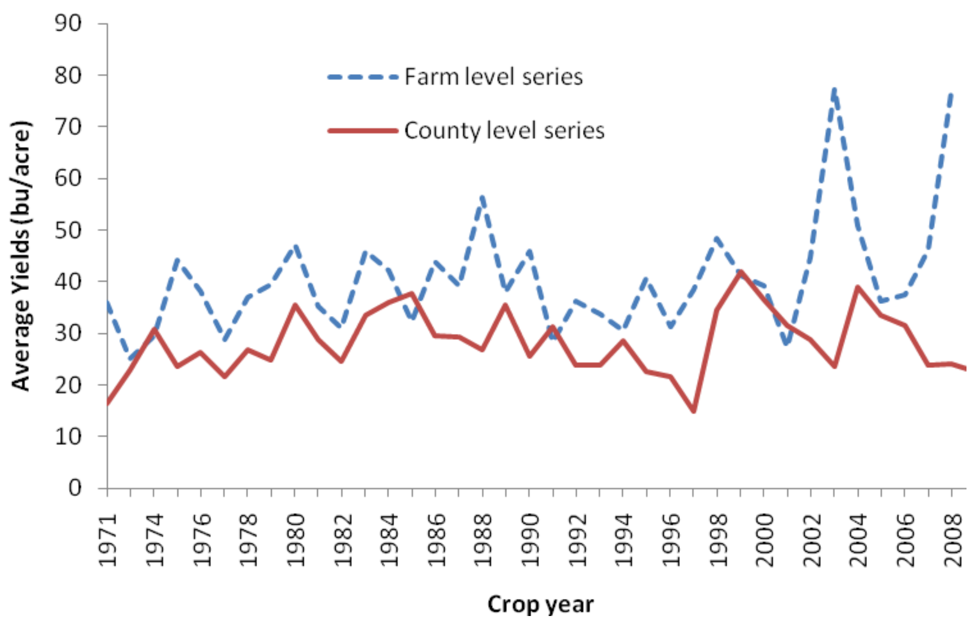

Figure 2. Average wheat yield over time

Table 1. Estimates of the Regression of Wheat Yield (bu/ac) Against a Quadratic Time Trend

	Farm Level			County Level		
Parameter	Estimate	SE	Pr > χ_1^2	Estimate	SE	Pr > χ_1^2
Intercept	34.38	4.28	<0.0001	22.31	3.19	<.0001
t	0.11	0.48	0.8116	0.73	0.37	0.0515
t^2	0.001	0.01	0.8691	-0.02	0.01	0.0751

Table 2. Test of Normality of Wheat Yields

Spatial level	Skewness	Kurtosis	K^2	JB
Farm	1.65	4.05	21.83	47.35
County	0.16	-0.37	0.34	0.38

Note: The critical value of the test statistic is $\chi^2_{2,0.05} = 5.99$.

Table 3. Skewness and Kurtosis of Annual Wheat Yield by Nitrogen Rate

N rate	Skewness	Kurtosis
0	0.38	-0.89
20	0.68	-0.06
40	1.49	2.78
60	1.73	4.11
80	1.53	3.49
100	2.18	1.26

Table 4. Estimated Wheat Yield Response to Nitrogen

Parameter	Estimate	SE	Pr > t
Intercept	25.71	0.87	<.0001
N rate slope	0.45	0.02	<.0001
Plateau yield	45.94	0.60	<.0001
Plateau random effect	148.36	18.82	<.0001
Year random effect	74.29	5.43	<.0001
Random error term	31.52	1.79	<.0001
Plateau N rate	44.96		
-2 log likelihood	12394		

Table 5. Test of Normality of Plateau Yearly Fixed Effects

Spatial level	Skewness	Kurtosis	K^2	JB
Farm	1.98	3.30	28.34	60.79

Note: The critical value of the test statistic is $\chi^2_{2,0.05} = 5.99$.