PHASE SPACE RECONSTRUCTION AND NONLINEAR EQUILIBRIUM DYNAMICS IN THE UNITED STATES BEEF MARKET

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Abstract

This paper investigates dynamic interactions in the US beef market using phase space reconstruction, which has been developed to analyze nonlinear dynamical systems. This approach provides important and unique empirical insights into consumers' behavior in the beef market. Our results from a phase space reconstruction analysis demonstrate distinct differences between intertemporal short run impacts from food safety outbreaks (e.g., E. Coli) and longer run health effects (e.g., cholesterol). Adjustments due to factors such as cholesterol are permanent changes and do not affect the manner by which people consume, while consumers react to food safety scares by adjusting consumption for a short period of time and then returning to their normal steady state cycle of consumption.

Key Words: nonlinear time series, phase space reconstruction, food safety, health effects

Introduction

There have been many attempts to extract measures of consumer behavior from time series data. As a result, substantial literature has been developed theorizing structural elements of consumer behavior. Based upon this literature many empirical applications have been completed like extensions of Becker's theory of rational addiction or Grossman's model of the demand for health (Becker et al., 1994, Grossman, 1972). These empirical applications require certain assumptions like functional forms of utility. However, when dealing with consumer behavior assumptions that have been proven to hold for certain goods will not necessarily hold for others. In response to such pitfalls, techniques that require very little restrictions on the front end of model building have become very popular. One such technique has been developed to analyze nonlinear dynamical systems. Phase space reconstruction allows for the extraction of the underlying structural system generating the data when one observes only data. It is an innovative empirical tool to provide unique insights into behavioral characteristics.[1]

Understanding dynamic behavior is particularly important in consumer reactions to food scares and health effects in the beef industry. Zhen (2005) theorized a model based upon Becker's theory of rational addiction, providing evidence that beef consumption in the United States is persistent. He argued that the more myopic the consumer the less responsive they are to food scares and health effects. However, his a priori assumption that health effects are continuous decreases in consumption may be constraining the model to underestimate later food scare reactions. Mazzocchi created a dynamic almost ideal demand system to determine the time-varying reactions consumers

[1] Phase space reconstruction does not necessarily lead to a final definitive model in-of-itself, but rather can provide information to specify a more complete structural model.

have to outbreaks. He showed that the inclusion of autoregressive parameters as consumer reactions provided decent short term forecast ability (Mazzocchi, 2006). Piggott and Marsh examined the impacts of public food safety information on US meat demand. They found small and short lived, but statistically significant impacts of food safety information on meat demand. The translation of the models to the beef industry provides useful information but requires a substantial amount of constraining assumptions.[2] Phase space reconstruction allows for an accurate illustration of consumer reactions to food safety and health effects with limited prior assumptions. Through this reconstruction the differences between food safety and health effects becomes apparent bringing to light previous misspecification. With more insight into consumer behavioral reactions a more accurate model may be built.

Previous empirical evidence has suggested that short run impacts from food safety outbreaks (e.g., E. Coli) are structurally different from longer run health effects (Kinnucan et al., 1997; Piggott and Marsh, 2004). Moreover, jointly estimating shorter-run and longer-run impacts is difficult in a static demand system. Indeed, our results from a phase space reconstruction analysis demonstrate distinct differences among the intertemporal responses. Through phase space reconstruction we are able to delineate the effects of different types of contaminate outbreaks on beef consumption. This adds valuable insight into the behavior of consumers when faced with differing health aspects. We show that the nonlinear dynamics present in American beef consumption have remained somewhat similar over the last twenty years with the exception of the health effect of cholesterol.

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[2] General AIDS models such as these are very useful in determining short term adjustments in behavior, as both Mazzocchi and Piggott and Marsh did, but the models are fundamentally restricted to this type of analysis. They have difficulty delineating long run dynamics from short run dynamics.

Since its development phase space reconstruction has become an essential part of nonlinear dynamics (Packard et al., 1980, Takens, 1981). It has been incorporated into various areas of research from Schaffer and Kot's SIER model of epidemics to Zaldívar's forecasting of Venice water levels, phase space reconstruction is the qualitative benchmark for nonlinear analysis (Schaffer and Kot, 1985, Zaldívar et al., 2000). It allows for the basic properties of the system to be determined and subsequent qualitative analysis to be performed without any prior knowledge of a system. This is analogous to nonparametric regression, which allows for the relationship of variables to be determined without imposing restrictions or prior functional form. Indeed, phase space reconstruction could be thought of as a nonparametric approach to nonlinear time series modeling.

By incorporating phase space reconstruction into the analysis of beef consumption we have provided the economic discipline with not only a better understanding of consumer behavior in general but with a new tool to be used in all types of future research. The techniques involved with phase space reconstruction, discussed below in the Theoretical Framework section, have been improved upon making them asymptotically efficient and more easily understood for those wishing to use the technique.

The study proceeds in the following manner; an overview of nonlinear time series methods is presented followed by the theory and method for reconstructing phase space in the Theoretical Framework. The applied phase space reconstruction of the United States beef consumption follows in Consumer Beef Demand with results and discussion.

Consumer reactions to health effects and food safety are then delineated and compared to existing research with concluding remarks for future modeling purposes.

Theoretical Framework

The nonlinear time series methods discussed in this paper are motivated and based on the theory of dynamical systems (Takens, 1981). The general idea is that a single time series has sufficient information with which to reconstruct the entire dynamical system, much like a single stain of DNA contains sufficient information to reproduce an entire organism. As such, time evolution is defined in a phase space.[3] Dynamical systems are usually defined by a set of first-order ordinary differential equations. The mathematical theory of ordinary differential equations ensures the existence and uniqueness of the trajectories, if certain conditions are met (Packard et al., 1980).

Data are often observed as a temporal sequence of scalar values. For any event, n outcomes are observed as a subset of the total population and are denoted by the time series vector $X_t = [x(t), x(t-1), \ldots, x(t-n)]'$. For future reference the τ^{th} lag of this vector will be referred to as $X_{t-\tau} = [x(t-\tau), x(t-1-\tau), \ldots, x(t-n-\tau)]'$. The challenge is to convert the observations into state vectors and reproduce dynamics in phase space. This is the problem of phase space reconstruction, which we solve using the method of delays. Phase space reconstruction is a diffeomorphism that reproduces a time series on a plane

[3] Typically phase space is defined as the space in which some geometric structure lives. In very general terms every trajectory of the structure of question may be represented as a coordinate in its particular phase space. For qualitative analysis we will always be referring to phase spaces of two and/or three dimensions.

that mirrors the phase portrait of the underlying system.[4] This reproduction makes it possible to delineate short or long run behavioral processes that evolve over time and generally better understand the nature of the dynamical system.

Embedding

The idea of embedding attractors onto different spaces and in different dimensions is an important concept in the theory of dynamical systems.[5] It was not until Packard first proposed that this be done from measured time series that the idea of phase space reconstruction was formed (Packard et al., 1980). Packard proved that the embedding of the geometry of a strange attractor may be correctly represented by a series of differential equations.[6] Takens extended this proof to encompass what is now known as the Method of Delays (Takens, 1981). The Method of Delays was proved to be a diffeomorphism of the attractor with dimension m onto a phase space of dimension n where $n \geq 2m+1$. The Method of Delays requires an optimal time lag τ, be chosen followed by a minimum embedding dimension λ. Combining the two parameter estimates, the time series X_t will generate a reconstructed phase space matrix $Y_\lambda = \lfloor X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(\lambda-1)\tau} \rfloor$ with dimension $[(n-\lambda\tau)\times\lambda]$.[7]

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[4] A diffeomorphism is a one-to-one mapping of a geometrical figure from one space to another. This mapping preserves all geometric properties of the original figure. Much like a topographical map preserves all geometric properties of the earth.

[5] An embedding is the mapping process used to reproduce geometric figures onto different spaces. Again it is analogous to creating a two-dimensional map of the three-dimensional world. Not all embeddings are diffeomorphisms, just like not all maps contain all the properties of the area they cover. Nonetheless, even though road maps don't usually contain elevation gain they provide a great deal of information.

[6] An attractor is a subset of a space onto which a system evolves to over time. The strange attractor allows for a greater degree of flexibility in that the subset of the space may be fractal, i.e., the dimension of the space does not have to be a real integer.

[7] As discussed before, the process of phase space reconstruction is much like map making. The reconstruction is a map containing all geometric properties of the original system that drives the dynamics of the observed time series. Through Takens' embedding theorem it is possible to extract this map of the underlying dynamics from a single time series.

The time lag τ is paramount to empirical applications of Takens' theorem. While his condition $n \geq 2m+1$ is sufficient, it is not necessary. By choosing a time lag that yields the highest independence between the column vectors in matrix Y_λ the geometry of the original manifold will be preserved even when the time series is contaminated with noise.

This time lag needs to be chosen optimally. If it is too small the approximation will be smooth but there will exist a high degree of correlation between components. This has the potential to force the trajectories of the attractor to lie on the diagonal in the embedding space (Broomhead and King, 1986). If, on the other hand, the time lag is chosen to be too large the dynamics of the system may unfold between components and therefore be unobserved. The optimal time lag is that which preserves the largest amount of information between components while achieving the largest degree of independence.

Time Lag for Embedding - Mutual Information

The mutual information coefficient was developed as a global measure of dependencies between two random variables (Fraser and Swinney, 1986). The dependencies measured are both linear and nonlinear making this the ideal candidate for estimating the optimal time lag for embedding. The mutual information function is defined as the combination of joint and marginal probabilities of the outcomes of an event in a sequence while increasing the time lag τ between components:

$$I(X_t, X_{t-\tau}) = \sum_{n-\tau} \sum_{n-\tau} P(X_t, X_{t-\tau}) \log\left[P(X_t, X_{t-\tau}) / P(X_t) P(X_{t-\tau})\right].$$

Choosing the time lag that yields the first local minimum of the mutual information function ensures independence of components with a maximum amount of new information (Fraser and Swinney, 1986).

Estimating the mutual information coefficient hinges on estimating the probability density function of a time series and its lagged values. This has traditionally been done using histogram estimators but may be improved upon drastically (Mittelhammer, et al., 2000). The histogram method of estimating density functions uniformly weights points within a predetermined window. If the time series contains a large portion of observations located close together and some that are spread out, the histogram method will inconsistently estimate the probability density function. Algorithms have been developed that vary the window size based upon how close observations are located to each other but they are computationally intense and not easily programmed. By using kernel density approximations we have developed a method for estimating the mutual information coefficient that takes substantially less computational time. In addition to being faster, based on asymptotic mean square error, our method of estimating the mutual information function is also asymptotically efficient. By using kernel weights the possible inefficiencies encountered with the histogram method of estimating the mutual information function are minimized.

Embedding Dimension - False Nearest Neighbors

With the chosen optimal time lag the minimum embedding dimension λ can be estimated. Kennel and Brown developed the False Nearest Neighbors technique (discussed below) for choosing a minimum embedding dimension (Kennel, et al., 1992). With a graphical development by Aittokallio (1999) the embedding dimension must be chosen properly or the reconstruction my not reflect the original manifold. If λ is too small the reconstruction cannot unfold the geometry of the possible strange attractor.[8] The coordinate vectors of Y_λ may be forced close by projection rather than by the underlying dynamics. They may belong to very different trajectories while appearing close in smaller dimensions. If λ is too large procedures used to determine basic properties of the system and qualitative analysis may become unreliable (Aittokallio, et al., 1999, Kennel, et al., 1992).

The False Nearest Neighbors technique uses Euclidean distances to determine if the vectors of Y_λ are still "close" as the dimension of the phase space is increased. By calculating the Euclidean distance between Y_λ vectors before and after an increase in dimension, it is possible to determine if the vectors are actual nearest neighbors or "false" nearest neighbors. The test statistic developed by Kennel and Brown defining neighbors to be false is: $\frac{\left|x(t+d)-x(n(t)+d)\right|}{\left\|y_t - y_{n(t)}\right\|} > R_{tol}$ where $x(t+d)$ denotes the last coordinate in the t^{th} row of the phase space reconstruction matrix $Y_{\lambda+1}$, $n(t)$ denotes the nearest neighbor in Euclidean distance of t for each row vector y_t in matrix Y_λ, and R_{tol} is the desired tolerance level usually set between 10 and 20. When the percentage of false nearest neighbors is minimized or drops below a preset threshold for the entire system, the minimum embedding dimension for phase space reconstruction is found (Kennel, et al., 1992).

The graphical false nearest neighbor test works in similar fashion. The test plots the density of false nearest neighbors in time delay space. Using the same notation as

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[8] An example of an embedding dimension being too small would be a 2-dimensional representation of a cube. In 2-dimensional space the cube appears to be a square. In 3-dimensional space the true geometry of the cube is clearly not a square but a much more complex figure.

above; $\left|x(t+d)-x(n(t)+d)\right|$ is plotted on the y-axis against $\left\|y_t - y_{n(t)}\right\| / \sqrt{d}$ on the x-axis. The first embedding dimension which yields the entire density of points contained below a 90° line is the minimum embedding dimension (Aittokallio, et al., 1999).

Consumer Beef Demand

Beef is a well-known staple in the American diet, and has been extensively studied (Chavas, 2000, Kinnucan, et al., 1997, Mazzocchi, 2006, Patil, et al., 2005, Piggott and Marsh, 2004; Zhen, 2006; Zhen, 2005). United States consumers eat roughly as much beef as they did forty years ago; see Figure 1 for time series graph and Table 1 for descriptive statistics, seasonal trends, health effects, and food scares. Figure 1 is the time series graph of quarterly beef consumption from 1960-2005. It illustrates the seasonal trend that occurs throughout the history of beef consumption. The peaks of the cycle occur in summer months and the troughs in the winter. The average difference between the first and third quarter is .71, see Table 1. This difference increases slightly for the period after 1980 and then stays relatively consistent.

In addition to the clear seasonal behavior, there appears to be a consistent level of consumption since 1990. Many of the previously mentioned studies have focused on determining why beef consumption might be so consistent and why there might exist deviations from this steady state. By and large the conclusion that is drawn theorizes that Americans consume a predetermined level of beef each year on which external factors create deviations.

There are four deviations from normal behavior that stand out in the consumption time series. First is the reaction consumers had to the information regarding the negative health effects of cholesterol. The cholesterol health effect is represented by the downward trend that consumption takes in the mid-eighties and from first glance appears quite significant. The next two deviations can be attributed to food safety scares regarding E. coli outbreaks and the last deviation is due to the BSE outbreak in 20003. These deviations are also colored in on Figure 1.

The application of phase space reconstruction makes it possible to more accurately qualitatively analyze the deviations from normal consumption behavior. The first minimum of the mutual information function is used to determine the optimal time lag for the phase space reconstruction. The optimal time lag for beef consumption is estimated to be $\tau = 6$ and is shown in Figure 2. The graphical false nearest neighbors test is implemented, shown in Figure 3, to determine the minimum embedding dimension for the phase space reconstruction. Clearly the entire density of points is contained below a line of degree less than 90 for the two-dimension case making the minimum embedding dimension $\lambda = 2$. Using the two parameter estimates we can create a graphical representation of the underlying dynamics that drive the system, see Figure 4.

Interpreting the phase space reconstruction may seem a bit confusing at first but by properly incorporating knowledge of the original time series it becomes clearer. The horizontal axis is observed time series and the vertical axis is time series lagged six periods. If any interesting dynamics exist in the reconstruction, we can separate them by pinpointing the starting date as the horizontal coordinate of the first observation of the outlying structure. The ending date of the dynamic occurrence will be the horizontal coordinate of the last observation of the outlying structure. Figure 4 makes the distinction between the two dynamic phases from 1960-1980 and 1980-Present.

Seasonality of beef consumption is exhibited in the original time series; people consistently eat more beef in the summer than in the winter. The reason, traditional American winter dishes consist mostly of beef's biggest substitutes poultry and pork, turkey for Thanksgiving; duck and ham for the holidays, while the traditional American summer dish is primarily barbequed beef. What is apparent from the reconstruction is that the difference between winter and summer consumption has been relatively consistent over time.

The phase space reconstruction in Figure 4 shows the period from 1960-1980 experiencing a large transition. The consumer started to incorporate beef as a large part of the daily diet. Fast food restaurants such as McDonalds founded in the early fifties were beginning to takeoff. Then in the late seventies the price of beef began to increase dramatically. The consumer decreased the amount of beef she ate until a steady equilibrium path emerged. The equilibrium path developed in the early eighties has remained the typical American beef consumption behavior. The difference between the transition period from 1960 and the equilibrium cycle from 1980 can be seen in the phase space reconstruction of Figure 4.

The two dynamic periods, Pre-1980 and Post-1980, create difficulties in modeling. As illustrated, the two periods are fundamentally different. Therefore, if a single model is created for the entire time series it will most certainly incorrectly specify one of the dynamic periods. Indeed, if consumer behavior is different between periods one modeling technique will not suffice to explain them both.

An important purpose of this application is to explain consumer reactions to the health concern cholesterol and food safety concerns Escherichia coli (E. coli) and bovine spongiform encephalopathy (BSE). Given that both health and food safety effects have had major impacts on beef consumption in the last dynamic period from 1980, analysis is performed on the post transition period beginning in the late 1970's. Removing the transition period from the phase space reconstruction makes the effects of cholesterol quite apparent.

Consumer Reactions to Health Effects

During the late eighties information was published on the negative effects cholesterol in beef has on health. These health effects resulted in a decrease in U.S. beef consumption. The behavioral response consumers had can be seen in Figure 5 as the adjustment period. The reconstruction shows a period of consistent consumption, the cluster in the northwest sector of the graph, a transition period (indicated as the consumer reaction to cholesterol), and a lower level of consumption in the southeast corner. Previously empiricists have treated health effects similarly to food safety effects. The phase space reconstruction illustrates that the effects of negative information concerning health and food safety are fundamentally different. Consumers reacted to the information by adjusting their levels of consumption permanently. This aligns with the recent empirical findings of Piggott and Marsh that food safety effects are short run and therefore fundamentally different from permanent health effects (Piggott and Marsh, 2004). The adjustment period illustrated in Figure 5 is a long run behavioral response while responses to food safety tend to be much shorter lived.

Modeling consumer reactions to health effects requires proper behavioral responses to be delineated. Zhen's (2006) analysis of consumer response to contaminate outbreaks in beef, modeled the health effects of cholesterol as a downward trend occurring since the negative information was released. A trend implies that the reaction consumers have had to cholesterol is constantly decreasing. The phase space reconstruction shows that consumption is not continually decreasing. Rather, an adjustment period occurred after which consumption returned to its regular equilibrium cycle at a permanently lower level. The two clusters in Figure 5 are almost identical in shape indicating that the cycle, post health effects, is almost identical to the cycle before the health effect information was released. The phase space reconstruction suggests a long run shift in consumption levels opposed to a change in behavior, as a trend adjustment would imply. An incorrect specification of health effects in the 1980's will change the estimated consumption impact of later food scares. Given that a permanent shift in consumption is the proper consumer response; modeling health effects with a downward trend in consumption will cause the underestimation of subsequent food safety effects.

Consumer Reactions to Food Safety

With the effects of cholesterol controlled for, people's reactions to food safety concerns are more easily viewed. As can be seen in the phase space reconstruction of the period from 1980 – Present, the reactions from E. coli and BSE outbreaks are significantly different as Figures 6 and 7 show.

During an outbreak of E. coli or BSE, beef consumption is shifted down for a period of time until consumers deem it acceptable to return to normal behavior. This is evident in the phase space reconstruction. The reconstruction shows a steady cycle of consumption until the outbreak period. During the outbreak consumption juts outward and then tends back to steady state. Visualizing the phase space reconstruction makes it possible to differentiate the perturbation magnitude to different attributes of each particular contaminate.

Figure 7 shows the phase space reconstruction of beef consumption when effects of E. coli and BSE are isolated. In the reconstruction the first outbreak of E. coli is noticeably larger than the second. This can be attributed to the fact that E. coli can be cooked out of beef, which will decrease the impact of the outbreak if the consumer takes the proper measures. The difference between the two reactions is evidence of consumer's ability to learn and adjust behavior.

The 2003 BSE outbreak results in a greater decrease in consumption than the shifts due to E. coli. Unlike E. coli, BSE cannot simple be cooked out of beef. There are no preventions for being contaminated by BSE other than abstaining from eating beef. In addition to the lack of preventative measures, evidence has been found for a causal link between BSE and Creutzfeldt-Jakob disease. However, the risk of becoming contaminated by BSE is much lower than E. coli. BSE is shown to be present in the central nervous system and bone marrow of cattle, portions not normally consumed, while E. coli may be found in meat. This gives evidence that people's behavior is affected more by the hazard once contaminated rather than the probability of being contaminated.

Concluding Remarks

The implementation of phase space reconstruction has opened a door for future research in econometrics. Just as it is in nonparametric regression, it is no longer necessary to make as many restrictive assumptions a priory. Nonlinear time series analysis may be based upon observations or predictions generated through phase space reconstruction. This process is applicable to all types of analysis. It has been in use for over two decades in the physics and biometric literature. Whenever there might exist a strange attractor driving the dynamics of a time series it may be reproduced with a great deal of accuracy and very little effort through phase space reconstruction.

In addition to highlighting the usefulness of phase space reconstruction, we have contributed to its empirical foundation. The method developed to estimate the minimum embedding dimension has been improved making it both asymptotically efficient and computationally simple. The mutual information coefficient is not only useful in phase space reconstruction. It is being used in place of the correlation coefficient in a variety of analysis. Kraskov et al., (2005) use it as a coefficient to base a hierarchical clustering algorithm with a high degree of accuracy. Being that it is both a linear and nonlinear global determinant of dependency between two random variables, its application is sure to be wide spread.

The recent empirical evidence suggesting the different consumer responses to health effects and food safety is evident in the phase space reconstruction. The long run health effect of cholesterol has caused consumers to shift their consumption behavior to a lower level. This lower level contains the same behavioral dynamics present before the health information was released. Consumers did not fundamentally adjust behavior as has been suggested.

The effects of food safety information in the phase space reconstruction are shown as temporary adjustments from the consumer equilibrium cycle. These adjustments are solely dependent on the particular contaminate. Consumers are apparently learning to better prepare food so that the impact of E. coli outbreaks have lessened over time. The evidence regarding the dramatic reaction to the BSE outbreak shows that consumers place higher risk aversion on the severity of illness once contaminated over the probability of contamination. For a predetermined consumption good such beef, outbreaks of contaminates have short run negative impacts. Consumers return to a steady equilibrium cycle when they believe the risk of contamination has decreased to a significant level.

The phase space reconstruction has allowed for a more accurate representation of consumer reactions to long run health effects and short run food safety effects. It has confirmed the previous empirical evidence that consumer reactions to health effects are fundamentally different than reactions to food safety. The reconstruction has also suggested some possible modeling techniques to better describe the consumer reactions. The degree to which a good is a staple or predetermined good affects the consumer response. As well, consumer response appears to be much larger when the severity of illness is larger, regardless of probability of infection.

In addition to the insights on beef consumption this analysis provides a framework for better understanding consumer behavior in general. As shown, if the consumption good, contaminate, or factor that affects perceived quality is relatively new to a society they will react in a much more severe manner. Decreasing consumption in accordance with knowledge. This suggests the need for further research into the effects that increased consumer knowledge pre-outbreak has on decreasing the impact of contaminates during an outbreak.

Tables

Descriptive Statistics		*Length of Health Effects and Food Scares in Quarters*	
Mean	18.62		
Standard Deviation	2.14	Cholesterol, 1987	14
Minimum	14.95	E. Coli 1, 1992	5
Maximum	24.38	E. Coli 2, 1996	5
Mean post Health Effects	16.61	BSE, 2003	9
Average Seasonal Adjustment	0.71		
Average Seasonal Adjustment 1980 - 1990	1.01		
Average Seasonal Adjustment 1990 - Present	0.96		

Table 1: Descriptive statistics, seasonal trends, health effects, and food scares.

Figures

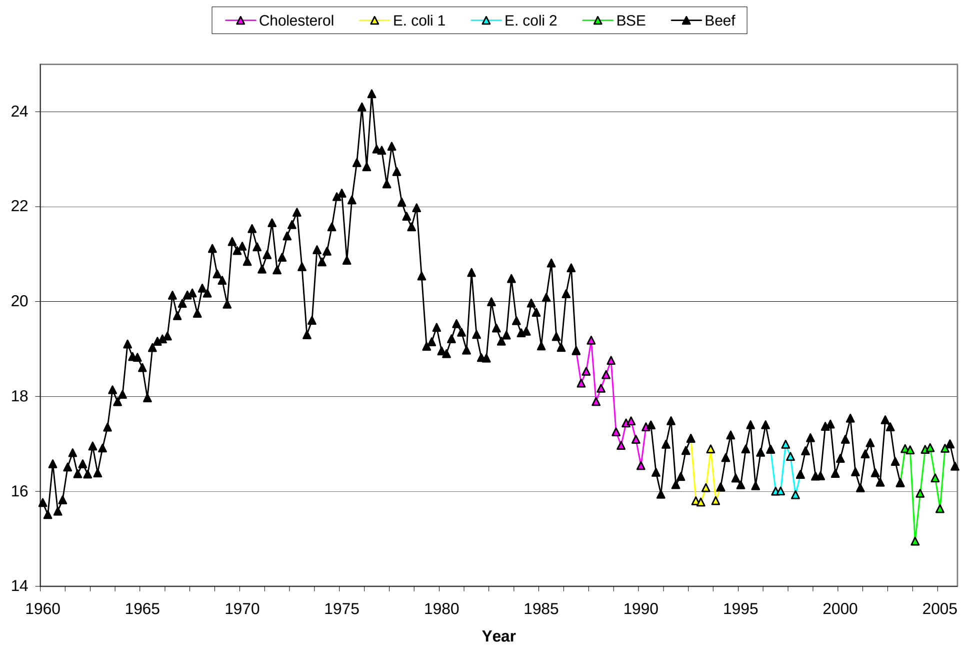

Figure 1: United States consumption of beef with Health Effects and Food Scares indicated by different colored line segments.

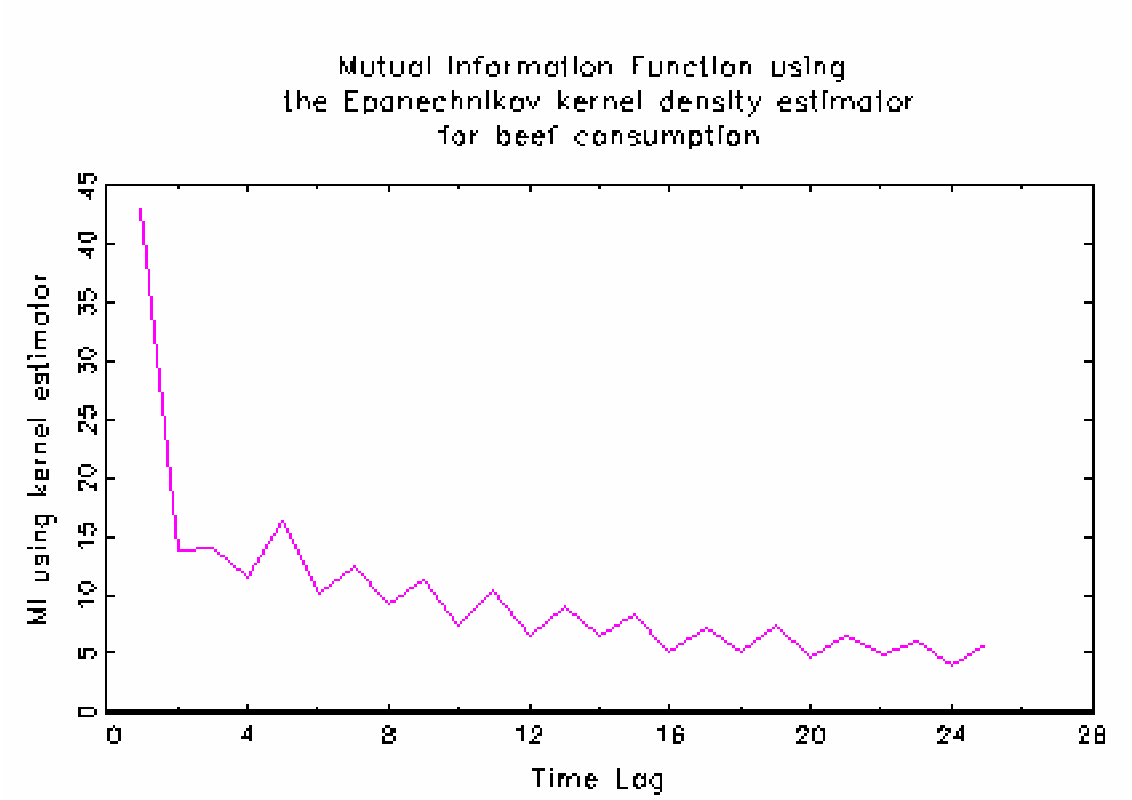

Figure 2: The Mutual Information Function for US beef consumption. The optimal time lag is chosen $\tau = 6$.

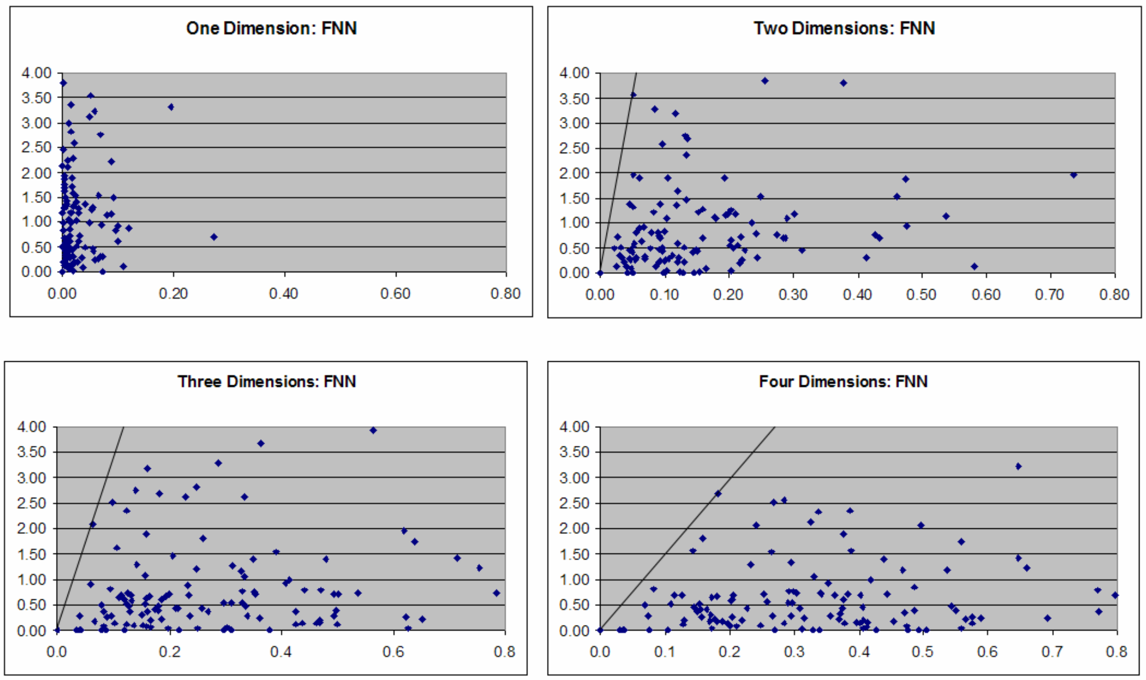

Figure 3: Graphical false nearest neighbors test for minimum embedding dimension indication a dimension of $\lambda = 2$.

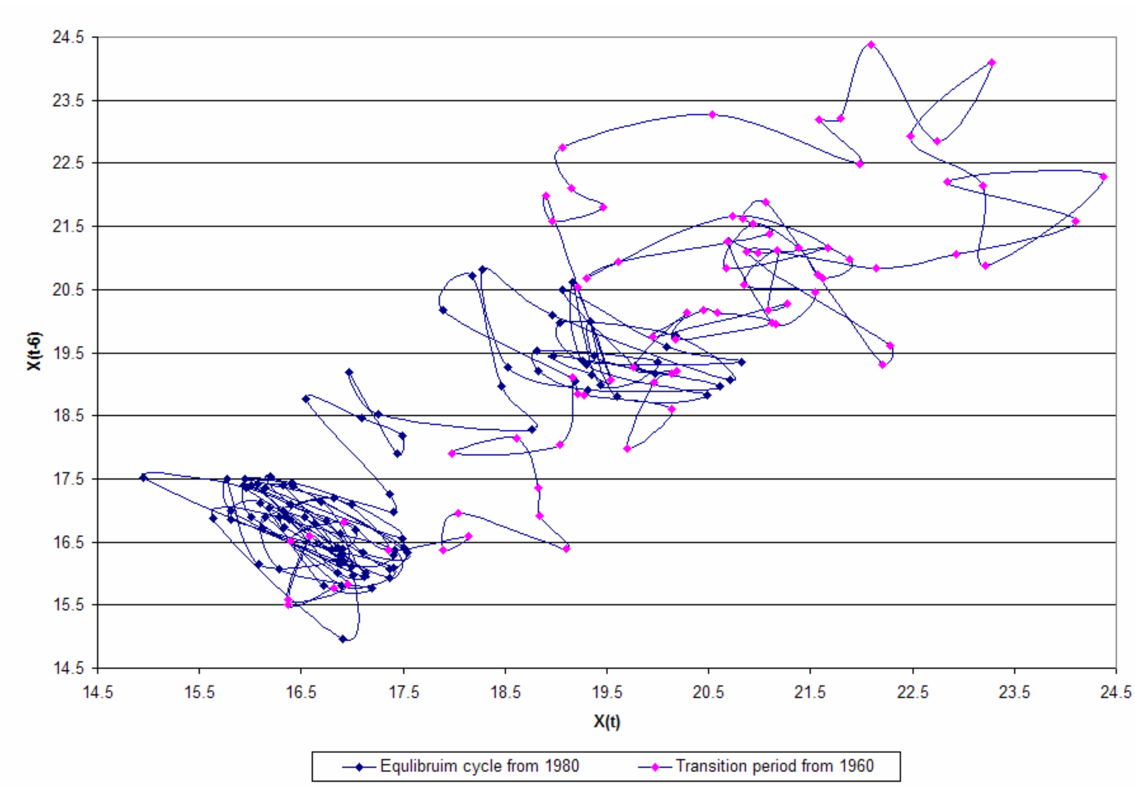

Figure 4: Reconstructed phase space for U.S. beef consumption.

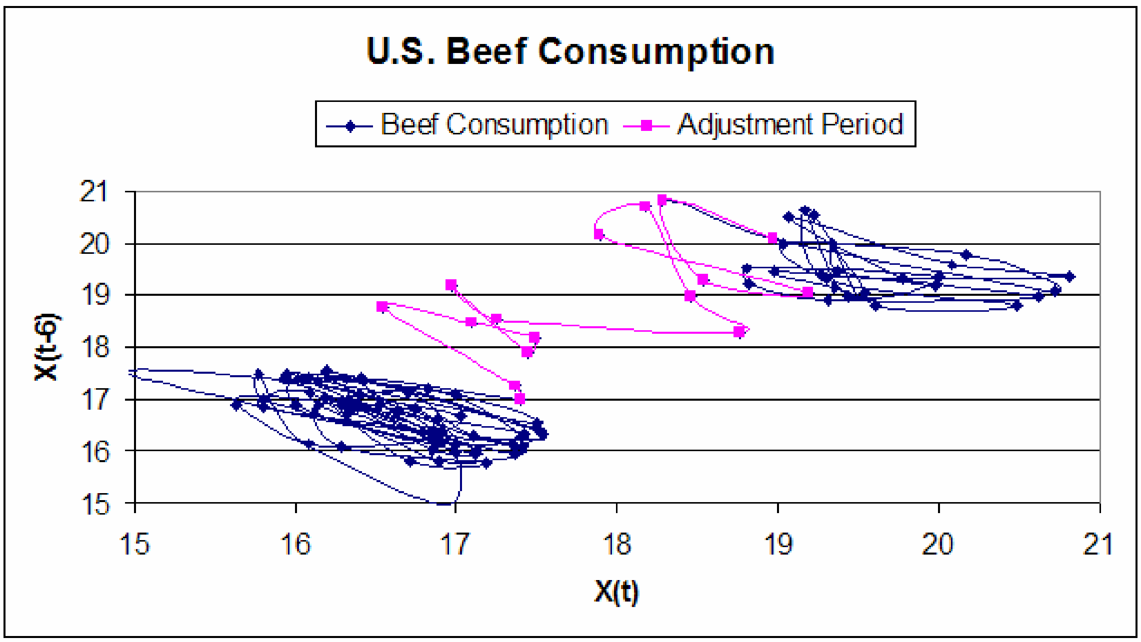

Figure 5: The effects of cholesterol on U.S. Beef consumption

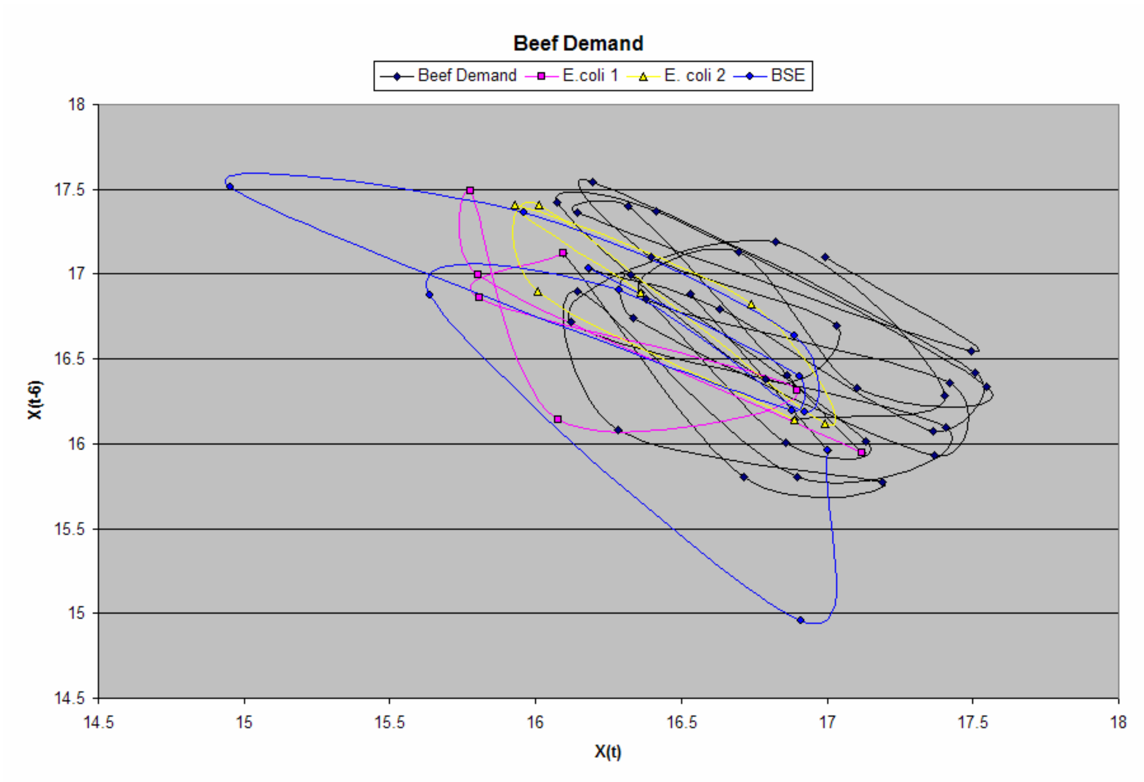

Figure 6: Reconstructed phase space with cholesterol effects controlled for.

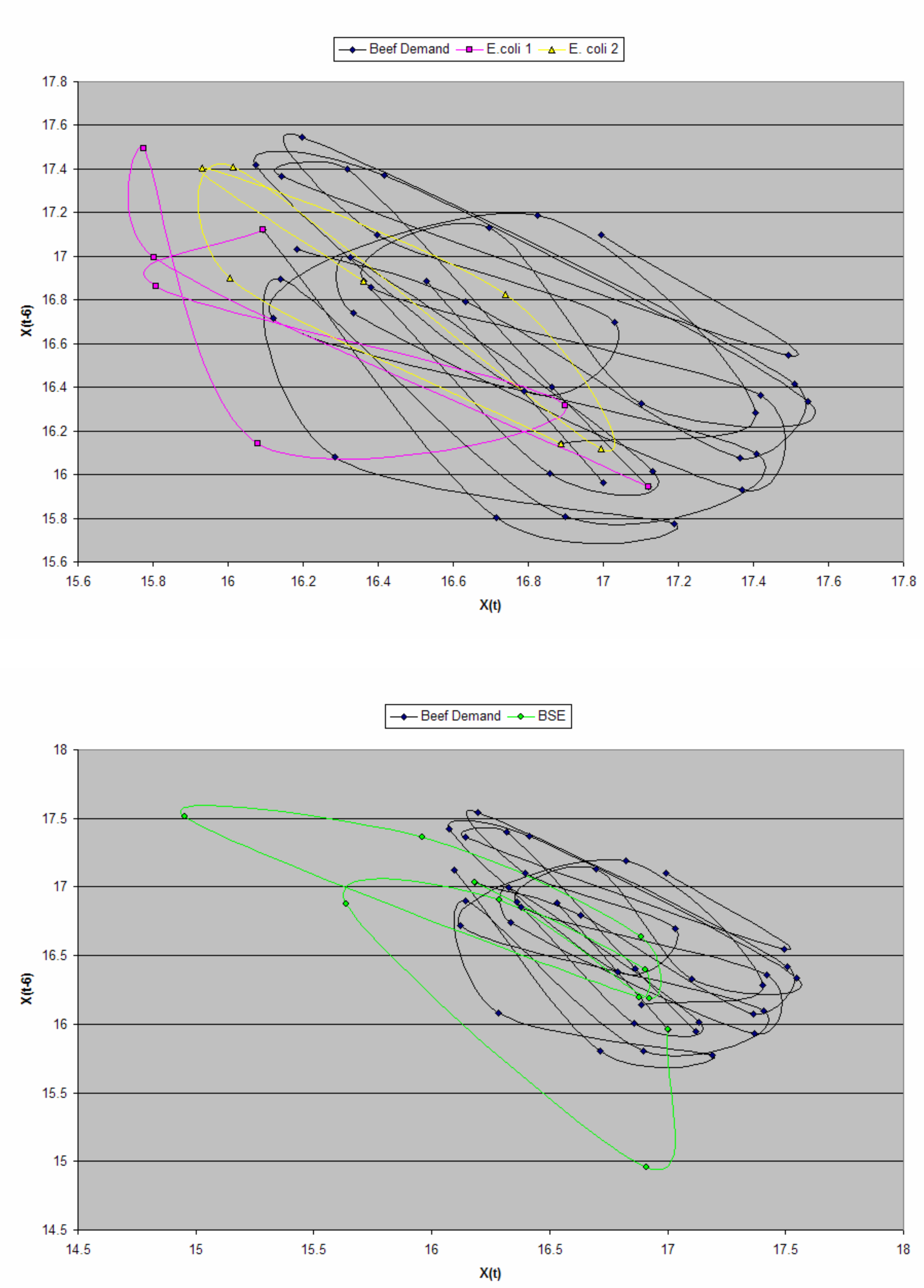

Figure 7: Reconstructed phase space isolating the effects of E. coli and BSE.

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