Groundwater Pumping by Heterogeneous Users

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Abstract

Farm size is a significant determinant of both groundwater irrigated farm acreage and groundwater irrigation application rates per acre. This paper analyzes the patterns of groundwater exploitation when resource users in the area overlying a common aquifer are heterogeneous. In the presence of user heterogeneity, the common resource problem consists of inefficient dynamic and spatial allocation of groundwater because it impacts income distribution not only across periods but also across farmers. Under competitive allocation, smaller farmers pump groundwater faster if farmers have a constant marginal periodic utility of income. However, it is possible that larger farmers pump faster if the Arrow-Pratt coefficient of relative risk-aversion is sufficiently decreasing in wealth. A greater farm-size inequality may either moderate or amplify income inequality among farmers. Its effect on welfare depends on the curvature properties of the agricultural output function and the farmer utility of income. Also, it is shown that a flat-rate quota policy that limits the quantity of groundwater extraction per unit land may have unintended consequences for the income distribution among farmers.

Keywords: common property resource, groundwater, majorization

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1. Introduction

Theoretical models of groundwater extraction typically assume that the resource is non-exclusive or that the resource users are identical. This, along with the assumption of instantaneous interseasonal transmissivity, simplifies the analysis because there exists a representative user. However, this approach does not take into account the spatial distribution of users, and the dependence of individual groundwater stocks on the history of past extractions (Brozovic et al 2003, Koundouri 2004). As a result, the existing models have relatively little to say about the patterns of groundwater exploitation when resource users are heterogeneous.

This is an important issue because irrigated agriculture, one of the major consumers of groundwater, and by far the largest consumer of fresh surface water, is comprised of farms of different sizes (Schaible 2004). Farm size is a significant determinant of both groundwater irrigated farm acreage and groundwater irrigation application rates per acre (see Table 1).[1] On average, larger farms tend to be the more intensive and extensive-margin irrigation operations. But in some states the relationships are either non-monotone or inversely monotone. For example, in Nevada and Oregon, smaller farms using groundwater have, on average, higher application rates per acre.

It is well known that, to the extent that groundwater is a common property resource, private decisions lead to inefficient allocation.[2] However, it is not clear whether user heterogeneity alleviates or exacerbates the so-called curse of the commons. Furthermore, the effects of water management policies on irrigation efficiency and farm incomes likely depend on farmland ownership structure. Specifically, we ask the following questions. What are the determinants of the relationship between farm size and groundwater use intensity? How does the distribution of farm sizes in the area influence the efficiency of groundwater allocation? What are the distributional impacts of farmland ownership structure and water management policies?

[1] A farm is categorized as small, medium, large, and very large, based on total annual farm sales, respectively, less than $100,000, between $100,000 and $250,000, between $250,000 and $500,000, and in excess of $500,000. For example, in 1998 the distribution of irrigated farms in the western states consisted of 65 percent small, 15.6 percent medium, 9.7 percent large, and 9.5 percent very large farms (USDA).

[2] This result holds unless the aquifer is relatively large in comparison to total groundwater use, users can cooperate, or hydraulic conductivities are so small that the resource is effectively private (Feinerman and Knapp 1983).

Table 1. Average groundwater applied per acre (acre feet per acre) and groundwater irrigated acres, by farm size and state, for farms using groundwater in selected western states surveyed in 1998

	Farm size class based on farm sales							
State	Small		Medium		Large		Very Large	
	Per acre	Acres	Per acre	Acres	Per acre	Acres	Per acre	Acres
Arizona	**2.96 ***	53 ***	**3.93 ***	255 *	**3.79 ***	378 **	**3.71 ***	879 *
California	**1.63 ***	38 ***	**1.73 ***	125 **	**2.08 ***	343 *	**2.51 ***	628 *
Colorado	**1.68 ***	98 **	**1.35 ***	335 *	**1.70 ***	604 *	**1.58 ***	1,184 *
Idaho	**1.33 ***	113 **	**1.96 ***	347 **	**1.36 ***	258 **	**1.64 ***	1,548 *
Kansas	**0.88 ***	342 **	**1.49 ***	290 *	**1.40 ***	531 *	**1.35 ***	1,258 *
Montana	**1.61 ***	10 **	**3.23 ****	269 *	**1.09 ***	432 **	**0.91 ****	781 **
Nebraska	**0.64 ***	183 *	**0.76 ***	325 *	**0.87 ***	686 *	**0.93 ***	895 *
Nevada	**3.51 ***	129 **	**3.17 ***	360 *	**2.89 ***	582 *	**2.79 ***	1,612 *
New Mexico	**1.79 ***	53 **	**1.86 ***	371 *	**2.32 ***	471 *	**2.44 ***	972 *
Oklahoma	**1.39 ***	113 *	**1.35 ***	339 **	**1.47 ***	702 *	**1.48 ***	1,247 *
Oregon	**1.84 ***	30 **	**1.76 ***	169 *	**1.75 ***	257 *	**1.54 ***	405 *
Texas	**1.30 ***	154 **	**1.31 ***	456 *	**1.30 ***	715 *	**1.43 ***	1,550 *
Farm average:	**1.30 ***	85 *	**1.26 ***	304 *	**1.33 ***	552 *	**1.71 ***	932 *

Note: * , **, and *** indicate the coefficient of variation (CV), respectively, between 0 to 25, 25 to 50, and 50 to 100. CV values were computed as the [Standard Error of Estimate/Estimate] x 100.
Source: Farm and Ranch Irrigation Survey (1998), National Agricultural Statistics Service, U.S. Department of Agriculture (1999). Data were summarized by the Economic Research Service, USDA.

In a two-period framework, we show that the relationship between farm size and pumping rates depends on the farmers' preferences over income. If the periodic marginal utility of income is constant, smaller farmers pump groundwater faster. However, the converse may hold, if the periodic marginal utility is concave and the Arrow-Pratt measure of relative risk aversion is decreasing in wealth.[3] This reflects the trade-off between two effects. On the one hand, a farmer operating on a smaller acreage effectively owns a smaller share of the aquifer, and perceives groundwater as a "more" common resource compared with a farmer with a larger acreage. Therefore, smaller farmers tend to pump faster. On the other hand, they may face a greater differential

[3] Even though there is no uncertainty in the environment studied in the paper, it is convenient to formulate our results using measures of risk aversion and the intensity of the precautionary saving motive such as, respectively, the Arrow-Pratt coefficient of risk aversion and prudence (Gollier 2004). In a non-stochastic framework, these coefficients measure the strengths of the inter-temporal income smoothing motive (i.e., the diminishing marginal utility of income). Adding uncertainty will not change the qualitative nature of our results. There is an empirical literature on the relationship between farmers' risk preferences and their dynamic use of groundwater (e.g., Antle (1983), Antle (1987), and Koundouri et al (forthcoming)) as well as on the effects of risk preferences on farmer's reaction to water quota policies (e.g., Groom et al (forthcoming)).

between marginal utilities of present and future income, and therefore, have a greater incentive to save groundwater for future use.

Turning to the effects of greater inequality in farm sizes on welfare, we find that there are *dynamic* and *spatial* components. The dynamic component refers to the effect of farm-size inequality on the average pumping rate, or the speed with which the aquifer is depleted. The spatial component refers to the effect of farm-size inequality on the distribution of pumping rates and income across farmers in each irrigation season. The conditions under which these effects can be signed require structure on farmers' marginal utility of income, and the degree of concavity of output and marginal output functions.

The speed with which the aquifer is depleted can either increase or decrease with farm-size inequality. The average pumping rate increases if the acreage is shifted towards farmers who pump faster than the average farmer. The converse holds, if the acreage is shifted towards more efficient users. Conditions under which the effect of *any* increase in acreage inequality on the average pumping rate is known, assure that farm size bears a monotone relationship with the pumping rate and order the sensitivities of the pumping rate to farm size for slow and fast users. Suppose that farmers are risk-neutral and the small farms under the new land ownership distribution are not "too small". Then the conditions can be conveniently stated in terms of the curvature properties of the first derivative of the function that relates income to water use for a fixed stock of groundwater (i.e., the inverse of the agricultural output function with respect to demand for groundwater). We find that the average pumping rate decreases (increases) with farm-size inequality depending on whether the marginal inverse output is log-concave (log-convex).

Even if the average pumping rate does not change, social welfare can also either increase or decrease with farm-size inequality. On the one hand, keeping the groundwater allocation fixed, a greater farm-size inequality implies a *greater* income inequality among farmers, and hence, a decline in social welfare. On the other hand, a greater farm-size inequality may imply a *lesser* income inequality. This is because smaller farmers gain a greater *strategic* advantage as they are able to poach more groundwater per unit land than their larger neighbors.

For methodological reasons, we differentiate between the cases when farmers' marginal periodic utility of income is constant, and when it is decreasing. In the former case, under constant returns to scale farming technology, farm-size heterogeneity does not contribute to welfare loss beyond that caused by the curse of the commons. In the latter case, the analysis needs to disentangle the pure income redistribution effect of the land ownership structure, keeping the allocation of groundwater fixed, from its effects on the equilibrium average pumping rate and the spatial distribution of groundwater withdrawals across farmers.

Notably, this insight may contribute to the continuing debate on the magnitude of the welfare difference between optimal control rules and competitive outcomes (Gisser 1983, Gisser and Sanchez 1980, Koundouri 2004). Provencher and Burt (1993) identify three sources of inefficiency associated with groundwater use in agriculture: stock, pumping cost, and risk externalities. In the presence of user heterogeneity, *income distribution* externality is added to this list. The income distribution externality arises when the rates of groundwater extraction differ across farms overlying a common aquifer. This externality can be both positive and negative, depending on whether smaller farms appropriate, on a per acre basis, a greater share of the common resource. Small and large farmers can be thought of as, respectively, low and high income groups. And so, a common resource such as groundwater may become a natural vehicle for wealth transfer, and can either *neutralize* or *amplify* income inequality caused by the inequality in farmland holdings.

Taking a policy perspective, we find that policies that affect the farmland ownership and management structures may have unexpected repercussions on the efficiency of groundwater exploitation. It turns out that, even within the simple framework analyzed in the paper, one needs a rather detailed knowledge of the farmers' utility function and farming and irrigation technologies to make useful policy recommendations. Using an example of a flat-rate quota policy, we show that policy-induced gains and losses are unequally distributed across farmers. This heterogeneity, and the arising political economy considerations, may adversely influence the adoption of policies that lead to overall efficiency gains and raise the average farm income.

Literature Review

Knapp and Vaux (1982) and Feinerman (1988) are among the few studies that consider equity and distributional effects of groundwater management schemes. Knapp and Vaux (1982) consider groups of farmers differentiated by their derived demand for water, and present an empirical example that demonstrates that some users may suffer substantial losses from quota allocation policies even though the group as a whole benefits. Feinerman (1988) extends their analysis and considers a variety of management tools including pump taxes, quotas, subsidies, and markets for water rights. Using simulations calibrated to Kern County, California, the author concludes that while the welfare distributional effects on user groups may be substantial, the negotiations between the policy-makers and the users are likely to be difficult because the attractiveness of policies varies across users and is sensitive to the parameters. However, following Gisser and Sanchez (1980), these studies ignore the stock externality, and assume that under competition users behave myopically and base their decisions solely on the consideration of their immediate (periodic) profits. Also, there is no investigation of the effect of the extent of user heterogeneity on the properties of competitive allocation.

There is a rather thin literature in development economics that is concerned with the effect of inequality in land holdings on groundwater exploitation. Motivated by the role of groundwater in sustaining the Green revolution and developing agrarian economies, Foster and Rosenzweig (2005) consider the patterns of groundwater extraction in rural India. They develop a dynamic model of groundwater extraction that captures the relationships between growth in agricultural productivity, the distribution of land ownership, water table depth, and tubewell failure. Using data on household irrigation assets including tubewell depth as a proxy for irrigation intensity, they find that large landowners are more likely to construct tubewells, but their tubewells tend to be less deep than those dug by smaller landowners. The authors conclude that this is indicative of a free-riding effect in the sense that large farmers are less able to effectively poach the water from neighboring farmers by lowering the water-table under their own lands. They also find evidence of land consolidation as a way to improve efficiency of groundwater exploitation.

In the context of irrigated agriculture in developed economies, where farmers have better access to capital markets and modern irrigation technologies, we focus on the simpler irrigation application rate decision. A two-period framework with a "quasi-bathtub" aquifer is particularly well suited to fully work out the equilibrium effects of farm-size inequality on the welfare difference between the competitive and efficient allocations. Given the seasonality of production in irrigated agriculture, we assume that a groundwater resource is quasi-bathtub as it acquires features of a common property resource over time.[4] This assumption is justified if all wells are spaced so that the localized cones of depression caused by pumping from neighboring wells do not overlap within each irrigation season. However, the main insights and policy implications obtained in this framework carry on to more realistic settings.

The rest of the paper is organized as follows. In Section 2, we present a simple two-period model of groundwater extraction in the presence of farm-size heterogeneity. In Section 3, we consider the social planner's solution. In Section 4, we analyze equilibrium allocation and the effect of farm-size inequality on the pumping rates and farm income when farmers' marginal periodic utility of income is constant. In Section 5, we consider equilibrium allocation when farmers' marginal periodic utility of income is decreasing. In Section 6, we consider a flat-rate quota policy that illustrates political economy issues that arise in the presence of user heterogeneity.

2. Model

For simplicity, we focus on the stock, cost, and income distribution externalities. We consider intensive-margin decisions taking the distribution of irrigated acres across farmers as exogenous. With slight modifications, the model can be extended to include extensive-margin (the share of acreage allocated to irrigated crops) decisions. Farmers

[4] This happens when the time period during which groundwater is extracted is relatively short, and does not allow for seepage from one point in the aquifer (such as a well or a pool) to another. However, the water level tends to be more uniform throughout the aquifer in the long run. We refer to the latter as a "quasi-bathtub" property: the resource at each extraction point is private within each period, but the aquifer becomes a "bathtub" across periods. This seems plausible whenever the irrigation season is considerably shorter than the time that elapses between the two seasons. However, there is a large variation in local hydrologic properties such as the aquifer's storativity and transmissivity values as well as well-spacing requirements that vary from 4 miles in parts of Kansas to less than 300 feet in Texas (e.g., Brozovic et al 2003, Kaiser and Skiller 2001).

are identical except for the distribution of land ownership, and irrigation technology is constant returns to scale. All profits are derived from agricultural outputs using groundwater for irrigation on a fixed land area, and farmers hold exclusive pumping rights on their land. The individual groundwater stocks are private during each irrigation season because there is no *intra-seasonal* well interference. However, the groundwater is an *inter-seasonal* common property resource based on the groundwater hydrology over a longer time interval. The following assumptions are standard (e.g., Negri 1989):

1. (**Fixed land ownership**) The distribution of farmland ownership does not change over time.
2. (**Constant returns to scale and homogenous land quality**) The agricultural production function is constant returns to scale. Land quality is identical across all farms. Inputs other than groundwater including the choice of irrigation technology, fertilize, crops, etc., are optimized conditional on the rate of water extraction. Output and input prices, including energy costs, are exogenous.
3. (**Pumping cost**) The total cost of groundwater extraction per acre increases with the pumping rate and decreases with the level of the water table (or the stock of groundwater).
4. (**User location is irrelevant**) The aquifer is confined, non-rechargeable, homogenous, and isotropic. The groundwater basin has parallel sides with a flat bottom.
5. (**Quasi-bathtub**) There are no intra-seasonal lateral flows of groundwater across farms. However, inter-seasonal changes in groundwater level are transmitted instantaneously to all users (i.e., the groundwater has an infinite rate of transmissivity during the time elapsed from one irrigation season until next).[5]
6. (**Two periods**) There are only two periods (irrigation seasons), and farmer preferences over income are additively separable across periods.

Provencher and Burt (1994) also consider and provide justifications for a two-period framework. The assumption that the aquifer is non-renewable is for expositional convenience, and a positive rate of recharge can be easily incorporated. The groundwater

[5] Brozovic et al (2003) provide a detailed discussion of the consequences of this assumption.

extractions are the gross quantity of water withdrawn if some fraction of the water percolates back to the stock. Next we introduce the notation.

Aquifer

The total stock of groundwater stored in the aquifer in the beginning of period 1 is $x_1 = Ah_1$, where h_1 is the height of the water table in period 1, and A is the size of the area measured in acres. Let $L = \{1,..., A\}$ denote the set of acres. The hydraulic heads of the water table under each acre are the same in the beginning of each period, $h_{i,t} = h_{j,t}$ $= h_t$ $\forall i, j \in L$ and $t = 1,2$. Let $u_{i,t}$ denote the quantity of groundwater applied in period t on acre i. By the quasi-bathtub assumption, the per acre quantity of groundwater withdrawn in each period cannot exceed the per acre stock or h_t acre-feet:

$$u_{i,t} \le h_t \text{ for all } i \in L \text{ and } t = 1,2. \tag{1}$$

Let $u_1 = A^{-1}\sum_{i=1}^{A} u_{i,1}$ denote the average pumping in period 1. Since there is no recharge, the stock of groundwater in the aquifer in period 2 is $x_2 = x_1 - Au_1$, and the level of the water table is

$$h_2 = h_1 - u_1. \tag{2}$$

Land ownership

There are n farmers (users of groundwater) who are located in the area overlying the aquifer and grow irrigated crops. Farmer k farms acres $L_k \subseteq L$, and let $A_k = | L_k |$ denote the number of irrigable acres owned by farmer k, where $\sum_{k=1}^{n} A_k = A$. We will refer to the set of acres L_k as "farm k" or "farmer k". For concreteness, we hold that farm indices are ordered by farm size, $A_1 \le A_2 \le ... \le A_n$. In what follows, in the doubly subscripted variables, the first symbol identifies the acre, and the second identifies the period, $t = 1,2$. Variables with one subscript typically refer to the aggregate values in the specified period, unless they are farm-specific and invariant across periods. We will use letters i, j to index acres, and letters k, l to index farmers.

Production technology

The periodic per acre benefit of water consumption net of all costs including groundwater pumping cost is

$$g(u_{i,t}, h_t), \tag{3}$$

where g is strictly increasing and concave. While irrigation increases yield, a higher groundwater stock decreases the cost of pumping due to a decrease in pumping lift, and increases the efficiency of irrigation by permitting a more flexible application schedule. Agricultural production technology is constant returns to scale (farm size), and land quality is homogenous: $g(u,h,A_k,i) = g(u,h,A_l,j)$. For simplicity, the rainfall and surface water supply are the same on all farms in both periods. For example, (3) can take the following form:

$$g(u,h) = \max_z \; py(u,h,z) - c(u,h) - qz,$$

where p is the per unit price of the crop, y is yield, and c is the cost of pumping groundwater, z is the vector of other inputs, and q is the price vector of other inputs. For notational convenience, let

$$f(h) = g_u(h,h) + g_h(h,h) \tag{4}$$

denote the marginal per acre benefit of water consumption evaluated at the point of depletion of an individual groundwater stock.[6] By concavity of g, $f'(h) < 0$ $\forall h \in (0, h_1)$.[7]

Let v denote the periodic utility of farm income, $v' > 0, v'' \leq 0$. Each farmer maximizes the sum of utilities of the whole-farm revenue in each period:

$$\pi_k = \max_{\{u_{i,t}\}_{i\in L_k}} \sum_{t=1,2} v\Big(\sum_{i\in L_k} g(u_{i,t}, h_t)\Big) \text{ subject to (1) and (2).} \tag{5}$$

For simplicity, there is no discounting of future income.

3. Social planner

Before we turn to the analysis of the competitive allocation by non-cooperating users, we

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[6] The subscripts on functions denote differentiation with respect to the lettered arguments.

[7] All of our results continue to hold under weaker technical conditions: $g_{uu} < 0$, $g_{hh} < 0$, and $f'(h) = g_{uu}(h,h) + g_{hh}(h,h) - 2g_{uh}(h,h) < 0$, which are implied by concavity of g.

characterize the efficient allocation. The social planner chooses $\{u_{i,t}^s\}$ to maximize producer welfare conditional on the land ownership distribution:

$$W^s = \max_{\{u_{i,t}^s\}} \sum\nolimits_{t=1,2} \sum\nolimits_{k=1}^{n} v(\sum\nolimits_{i \in L_k} g(u_{i,t}^s, h_t)) \text{ subject to (1) and (2).} \tag{6}$$

The following result shows that the efficient allocation of groundwater compensates for income inequality caused by the inequality in farm sizes. The common resource may serve as a vehicle to decrease income inequality by redistributing income from larger farmers to smaller farmers. This effect is absent if either farm sizes are identical, or farmers are risk-neutral (periodic utility is linear in income). Note that optimal groundwater consumption in the final period exhausts the remaining stock on each farm, and hence, must be identical on all acres, $u_{i,2}^s = u_{j,2}^s = h_2 \;\; \forall i, j \in L$, because the income utility and water benefit functions are strictly increasing. And so, the focus is solely on period 1 pumping. All proofs that are not in the text are in the Appendix.

Proposition 1. (Efficient pumping) *Efficient allocation of groundwater is*

a) invariant across acres, $u_{i,1}^s = u_{j,1}^s \;\; \forall i, j \in L$, *and is determined by*

$$g_u(u_{i,1}^s, h_1) - f(h_1 - u_{i,1}^s) = 0, \tag{7}$$

if either farmers are risk-neutral, $v'' = 0$, *or acreage is uniformly distributed across farmers,* $A_k = A/n$ for $k = 1,...,n$;

(b) characterized by smaller farmers pumping groundwater faster, $u_{j,1}^s \geq u_{i,1}^s$, for $j \in L_k$, $i \in L_l$, $k < l$, *if farmers are risk-averse (decreasing marginal utility of income),* $v'' \leq 0$.

(7) is easiest to interpret for the special case when the water benefit depends only on water use, *u*. In this case, it is efficient to equalize the marginal benefits of water use in the two periods: $g_u(u_{i,1}^s) = g_u(h_1 - u_{i,1}^s)$, which implies that $u_{i,1}^s = h_1 / 2 \;\; \forall i \in L$. This is equivalent to the assertion that, in the absence of a pumping cost externality and inequality of income across farmers, the efficient solution distributes the available water equally across the two periods on each farm.

It is convenient to differentiate between the case when farmers' per period marginal utility of income is (1) constant (i.e., farmers are risk-neutral), and (2) decreasing (i.e., farmers are risk-averse). In the former case, from the social planner's point of view, there is no inherent inefficiency of income distribution due to a non-uniform distribution of acreage across farmers. However, such inefficiency may still arise in competitive equilibrium. In the latter case, as is demonstrated in Part (b) of Proposition 1, the social planner faces a trade-off between dynamic and distributional sources of inefficiencies.

From a policy perspective, an important insight of the analysis to follow is that, in the presence of farmer heterogeneity, competitive allocations go beyond the *curse of the commons*, and affect *income inequality* as well. The welfare difference between the optimal and competitive allocations may be particularly large, when, from the societal point of view, the income distribution matters. This happens when the equilibrium distribution of pumping rates across heterogeneous farmers *amplifies* the income inequality caused by size inequality. However, the competitive allocation may also *moderate* the inherent inequality in income distribution caused by the inequality in land ownership, or even change its sign, whereas total incomes over two periods earned by smaller farmers exceed that of larger ones (see footnote 10).

4. Risk-neutral producers

In this section, we consider the case of risk-neutral farmers, $v'' = 0$. In Section 4.1, we characterize competitive equilibrium. In Section 4.2, we analyze the effect of inequality in farm sizes on the groundwater stock and the distribution of income.

4.1. Equilibrium

Farmers are non-cooperative, and each farmer takes the quantity of water pumped by others in each period as given. In period 2, all farmers exhaust the available stocks of groundwater on each acre, so that $u^*_{i,2} = h_2$ for $\forall i \in L$. By (5), in period 1 farmer k's payoff is

$$\pi_k = \max_{\{u_{i,1}\}_{i \in L_k}} \sum_{i \in L_k} g(u_{i,1}, h_1) + g(h_2, h_2) \text{ subject to (1) and (2).} \quad (8)$$

Next we characterize competitive allocation. Differentiating (8), the best response by farmer k on acre $i \in L_k$, $u_{i,1}^*$, satisfies

$$g_u(u_{i,1}^*, x) - a_k f(h_2) = 0, \text{ if } u_{i,1}^* \le h_1, \text{ and } u_{i,1}^* = h_1, \text{ if otherwise} \qquad (9)$$

where $a_k = A_k / A$ is the share of the aquifer that can be captured by farmer k. (9) can be written in a more compact form

$$u_{i,1}^* = \min[h_1, g_u^{-1}(a_k f(h_2); h_1)], \quad \forall i \in L_k \qquad (10)$$

where $g_u^{-1}(.; h)$ is the inverse of $g_u(u, h)$ obtained by treating h as a parameter. Note that per acre pumping rates on each farm are identical $u_{i,1}^* = u_{j,1}^*$ $\forall i, j \in L_k$. Summing pumping rates (10) over all $k = 1,...,n$ and $i \in L_k$, and substituting (2), yields

$$u_1^* = \sum_{k=1}^{n} a_k \min[h_1, g_u^{-1}(a_k f(h_1 - u_1^*); h_1)], \qquad (11)$$

where $u_1^* = (1/A)\sum_{i=1}^{A} u_{i,1}^*$ is the equilibrium average pumping in period 1. By concavity of g, (11) uniquely determines the aggregate pumping in period 1, u_1^*. Together (10) and (11) prove the existence and uniqueness of equilibrium.

Proposition 2. (Competitive allocation) *Suppose that farmers are risk-neutral. Competitive equilibrium exists, it is unique, and is given by (10) and (11). The average pumping rate is higher than the socially efficient average rate,* $u_1^* \ge u_1^s$. *Also, smaller farmers pump faster than larger farmers,* $u_{i,1}^* \ge u_{j,1}^*$, for any $i \in L_k$, $j \in L_l$, $k < l$.

Comparing the first-order conditions that characterize the efficient and competitive allocations, (7) and (9), respectively, shows that the discrepancy between them arises along both *spatial* and *temporal* dimensions. That is the competitive allocation leads to an inefficiently high *aggregate* pumping in period 1, which entails an inefficient allocation of groundwater across periods. Nonetheless, it is possible that *individual* farmers extract groundwater at a *slower* rate than the socially efficient average rate, i.e. $u_{i,1}^* \le u_1^s$ for some i (see Section 4.2.4 and Figure 1b). Also, unless all farmers are identical, the competitive allocation results in inefficient pumping rates *across* farmers in period 1. Recall that, by Proposition 1(a), efficiency requires that the per acre irrigation application rates be identical when farmers are risk-neutral.

Under risk-neutrality, smaller farmers always deviate more from the socially efficient allocation. However, it is not clear whether the non-uniformity of the distribution of land ownership, in and of itself, leads to a loss or gain of total farm income. As we show next, the effects of the inequality in farm sizes on the groundwater stock and farm income depend on rather subtle properties of the agricultural production function.

4.2. Inequality in farm sizes

Next we introduce the measure of inequality that is used to model an increase in the concentration of land ownership (a smaller share of farmers owns a larger share of land). In Section 4.2.2, we analyze the effect of inequality in farm sizes on the remaining groundwater stock. In Section 4.2.3, we analyze the effect of inequality in farm sizes on total income. In Section 4.2.4, we present an example that illustrates our findings.

4.2.1. Measuring inequality

To model the effect of inequality in farm size on groundwater exploitation we use the majorization order. A comprehensive treatment of majorization can be found in Marshall and Olkin (1979). Let $B_1 \le B_2 \le ... \le B_n$ represent an alternative distribution of land ownership among farmers, where $\sum_{k=1}^{n} B_k = A$.

Definition. Real vector $\vec{A}$ is majorized by $\vec{B}$, denoted $\vec{A} \le^m \vec{B}$, if $\sum_{k=1}^{l} A_k \ge \sum_{k=1}^{l} B_k$ for $l = 1,...,n$, and $\sum_{k=1}^{n} A_k = \sum_{k=1}^{n} B_k$.

Majorization is a tool to compare the dissimilarity within the components of vectors, and is ideally suited to compare the inequality of income distributions. We will also need a related notion of Schur-convex and Schur-concave functions. A real-valued function $y(\vec{A})$ is called Schur-convex if $\vec{A} \le^m \vec{B}$ implies $y(\vec{A}) \le y(\vec{B})$, and $y(\vec{A})$ is Schur-concave, if $-y(\vec{A})$ is Schur-convex. In our analysis, we will appeal to the following important property of Schur-convex functions. Suppose that $y(\vec{A}) = \sum_{k=1}^{n} z(A_k)$. Then $y(\vec{A})$ is Schur-concave if and only if z is concave.

4.2.2. Inequality of farm sizes and groundwater stock

Let $R = -g_{uu}(u,h_1)/g_u(u,h_1)$ denote the index of concavity of agricultural output function, and $P = -g_{uuu}(u,h_1)/g_{uu}(u,h_1)$ denote the index of concavity of the marginal output function of a farmer with technology $g(u,h_1)$ in period 1. Note that R is the Arrow-Pratt coefficient of absolute risk aversion, and P is the coefficient of absolute prudence, if $g(u,h_1)$ is interpreted as the utility of wealth measured in acre-feet of water.[8]

Proposition 3. *Suppose that farmers are risk-neutral. Then under more unequal distribution of farm sizes, $\vec{A} \le^m \vec{B}$, the groundwater stock in period 2*

(a) increases, $h_2^*(\vec{A}) \le h_2^*(\vec{B})$, if $2R \ge P$;

(b) decreases, $h_2^*(\vec{A}) \ge h_2^*(\vec{B})$, if (i) $b_1 \ge g_u(h_1,h_1)/f(h_2^*(\vec{A}))$, *i.e. the smallest farm under the new land ownership distribution is not "too small" and* (ii) $2R \le P$.

The inequality in land ownership creates a trade-off in terms of its effect on the pumping decisions in period 1. A heavier left tail of the acreage distribution implies that there are more farmers, who own a smaller share of the aquifer and tend to pump faster than the average farmer. However, a heavier right tail implies the opposite. Therefore, ascertaining the effect of *any* increase in acreage inequality on the competitive allocation requires structure on the *farm-size sensitivity* of the difference in pumping rates between small and large farmers, $u_{i,1}^* - u_{j,1}^*$, where $i \in L_k$, $j \in L_l$, $A_k < A_l$.[9]

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[8] Comparative statics results for the portfolio selection problem, along with interesting intuition, that are stated in terms of the Arrow-Pratt coefficients of risk-aversion and prudence can be found in Gollier (2004).

[9] The *farm-size sensitivity* of the difference in pumping rates across farms is $a_k u''(a_k)/u'(a_k)$, where $u(a_k) = g_u^{-1}(a_k f(h_2); h_1) < h_1$. If the pumping rate differential, u', is increasing (decreasing), the sensitivity is negative (positive).

Condition (a) states that, when the aquifer is full, the agricultural output, $g(.,h_1)$, is in a sense more concave than the marginal output, $g_u(.,h_1)$. Then the perceived benefit from a more stable inter-seasonal groundwater use pattern increases with size at an accelerating rate, and a greater inequality stimulates, on average, a slower pumping rate. Note that condition $2R \leq (\geq) P$ is equivalent to log-concavity (log-convexity) of the first derivative of the demand for water with respect to output when the aquifer is full, $g_y^{-1}(y;h_1)$, where $g^{-1}(y;h_1) = \{u : y = g(u;h_1)\}$ is the inverse of agricultural output function obtained by treating the stock of groundwater, h_1, as a parameter.

To guarantee that the average pumping rate increases, we need an additional condition (i) in Part (b) because the aquifer is a quasi-bathtub (see constraint (1)). This condition puts a limit on the increase in the size of large farms. It implies that, under the new distribution of land ownership, the number of farmers, who grow irrigated crops is the same, $B_1 > 0$, and that, under the initial distribution of land ownership, no farmer depleted her stock of groundwater in period 1, $u_{i,1}^*(\vec{A}) < h_1$ for all $i \in L_1$.

4.2.3. Farm-size inequality and farm income

Here we consider the effect of the inequality in farm acreage on total farm income attained by the competitive allocation. In the case of risk-neutral farmers, (6) becomes

$$W^c(\vec{A}) = \sum_{k=1}^{n} \pi_k = \sum_{k=1}^{n} A_k \{g(\min[h_1, g_u^{-1}(a_k f(h_2^*); h_1)], h_1) + g(h_2^*, h_2^*)\}, \quad (12)$$

where $h_2^* = h_1 - u_1^*$ is given by (11), and $W^c(\vec{A})$ symbolizes the dependence of total farm income (agricultural output) on the distribution of land ownership among farmers.

The farm-size inequality affects both the groundwater stock in period 2 (*dynamic allocation*) and the distribution of groundwater application rates across farms in period 1 (*spatial allocation*). Keeping everything else equal, a more stable inter-seasonal pattern of groundwater use increases total farm income. The distributional effect of farm-size inequality on farm income is more difficult because a higher variability in farm sizes may or may not lead to a higher variability in the per acre pumping rates (see Proposition 3).

Proposition 4. *Suppose that farmers are risk-neutral. Then under more unequal distribution of farm sizes,* $\vec{A} \leq^m \vec{B}$*, total farm income*

(a) decreases, $W^c(\vec{A}) \geq W^c(\vec{B})$*, if* (i) $3R \geq P$ and (ii) $h_2^*(\vec{A}) \geq h_2^*(\vec{B})$*;*

(b) increases, $W^c(\vec{A}) \leq W^c(\vec{B})$*, if* (i) *the smallest farm under the new land ownership distribution is not "too small",* $b_1 \geq g_u(h_1, h_1) / f(h_2^*(\vec{A}))$, (ii) $3R \leq P$, and (iii) $h_2^*(\vec{A}) \leq h_2^*(\vec{B})$.

Conditions in (a) guarantee that the unequal distribution of farm acreage aggravates both the distributional (a(i)) and dynamic (a(ii)) inefficiencies, that are associated with the competitive allocation. Condition a(i) requires that the net benefit of irrigation when the aquifer is full, $g(u, h_1)$, is in a sense more concave than the marginal benefit, $g_u(u, h_1)$. Then a greater inequality in farm sizes stimulates a greater variability in (acreage-weighted) pumping rates and lowers total output. Observe that a(i) is less stringent than (a) in Proposition 3. This is because the net benefit of irrigation, $g(u, h_1)$, is concave in u, which adds additional curvature, and thus, on average, a smaller (or positive) *farm-size sensitivity* of the spatial pumping rate differential suffices to cause a total output loss.

Part (b) has a similar interpretation. Condition b(i) is the same as in Proposition 3. But now sufficient condition b(ii) is more stringent compared with b(ii) in Proposition 3. This is because a negative and "sufficiently" large (in absolute value) *farm-size sensitivity* of the spatial pumping rate differential is required in order to assuredly raise total output. Note that condition $3R \leq (\geq) P$ is equivalent to concavity (convexity) of the first derivative of the inverse output function (i.e., demand for water as a function of output) when the aquifer is full, $g_y^{-1}(y; h_1)$.

Combining Propositions 3(b) and 4(a) yields

Proposition 5. *Suppose that farmers are risk-neutral. Then under more unequal distribution of farm sizes,* $\vec{A} \leq^m \vec{B}$*, total farm income decreases,* $W^c(\vec{A}) \geq W^c(\vec{B})$*, if* $2R \leq P \leq 3R$.

Sufficient conditions under which more unequal distribution of farm sizes has an unambiguously positive effect on total farm income cannot be obtained in this way. To guarantee a lesser inequality in pumping rates, the pumping rate spatial differential, $u'(a_k)$, must be "sufficiently" decreasing (in absolute value) with farm size. In contrast, to guarantee a more stable average pumping rate, the pumping rate spatial differential must be increasing or "slightly" decreasing (in absolute value) with farm size.

Furthermore, as clear from the proof of Proposition 4 (see (A1) in Appendix), the sign of $\partial\pi_k / \partial A_k$ is ambiguous. Therefore, it is possible that smaller farmers earn more total income than larger farmers, $\pi_k \geq \pi_l$ for $k < l$.[10] This may happen because smaller farmers are in a better *strategic* position to take advantage of the common property resource as they are able to steal more groundwater *per unit* of land than their larger neighbors. The following example illustrates.

4.2.4. Small and large farms: an example

Let $g(u,h) = (u+z)^\gamma$, $\gamma \in (0,1)$, $z \geq -0.5h_1$, and $v'' = 0$. By Proposition 1, the efficient allocation of groundwater across acres and seasons is invariant to the distribution of land ownership, and is given by $u_{i,1}^s = 0.5h_1$ for $i \in L$. The maximal regional farm income is $W^s = 2A(0.5h_1 + z)^\gamma$.

For simplicity, all farms fall in one of the two categories: small and large. The size of small farms is s acres, $A_k = s$ for $k = 1,...,m$, and the size of large farms is l acres, $A_k = l$ for $k = m+1,...,n$, where $s \leq l$. The number of small farms is m, and the number of large farms is $n-m$, where $ms + (n-m)l = A$. By (10) and (11) equilibrium pumping in period 1 is

$$u_{i,1}^* = \min[h_1, (\frac{s}{A})^{1/(\gamma-1)}(h_1 - \frac{E + z(1-E)}{1+E}) + z(1-(\frac{s}{A})^{1/(\gamma-1)})] \text{ for } i \in L_k,\ k = 1,...,m,$$

[10] Of course, larger farmers always have higher total revenues in period 2. But smaller farmers have more intensive-margin operations and higher per acre revenues in period 1. The differential in total revenues between small and large farmers in period 1 can be positive, and even, exceed the magnitude of the negative differential in total revenues in period 2.

$$u_{i,1}^{*} = \frac{h_1 - smu_{m,1}^{*}/A + z((l/A)^{1/(1-\gamma)}) - 1)}{(l/A)^{1/(1-\gamma)} + l(n-m)/A} \text{ for } i \in L_k \text{ and } k = m+1,\ldots,n$$

where $E = m(s/A)^{\gamma/(\gamma-1)} + (n-m)(l/A)^{\gamma/(\gamma-1)})]$.

For concreteness, we consider a special case of an increase in farm size inequality whereas small farms get uniformly smaller and large farms get uniformly larger. Note that $\vec{A}(s';m,l(s')) \leq^{m} \vec{A}(s'';m,l(s''))$ for $s' > s''$, where $l(s) = (A - ms)/(n-s)$. Clearly, a uniform shift of acreage from small farms to large farms, keeping the number of farms in each size category fixed, constitutes an increase in farm size inequality. We consider the effect of increasing the gap between the acreage on small and large farms, $\Delta = l - s \geq 0$, keeping the number of each type of farms, m, fixed.

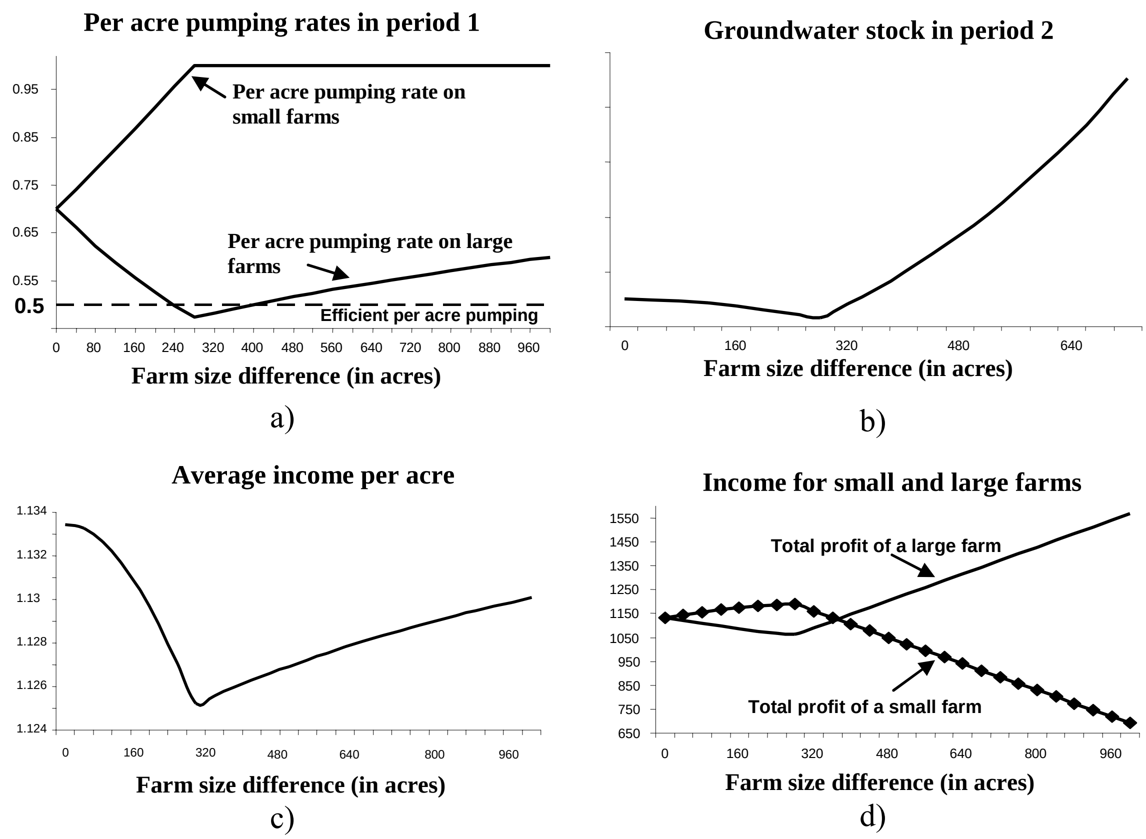

Figure 1. Inequality in farm sizes, pumping rates, and income

In Figure 1, parameters are: $\gamma = 0.8$, $z = -0.3$, $n = 100$, $m = 50$, $h_1 = 1$, and $A = 100{,}000$. Then the maximal farm income per acre is $W^{s}/A = 10 \times 0.2^{1.8}$. At $\Delta = 0$

(i.e., $s = l = 1000$), small and large farms are the same, and the distribution of land ownership is uniform across farmers. Next we analyze the effect of an increase in farm size inequality on the equilibrium groundwater stocks, pumping rates, and incomes.

As shown in Figure 1(a), when the difference in farm sizes is relatively small, $\Delta \leq 280$, the difference in the *pumping rates* increases until the small farmers deplete their wells in period 1, $u^*_{i,1} = h_1 = 1$ for $i \in L_k$ and $k = 1,...,50$. This limits the ability of small farmers to "steal" groundwater from their neighbors, and therefore, establishes an upper bound on the difference in the pumping rates. Curiously, the large farmers pump ***less*** than the efficient quantity, $u^*_{i,1} \leq 0.5h_1 = 0.5$ for $i \in L_k$ and $k = 51,...,100$, when $\Delta \in [220, 400]$. In this range, the gain in the dynamic efficiency for the large farmers outweighs the loss associated with letting the small farmers steal their groundwater. However, as the size of each large farm, and hence the total share of the aquifer farmed by large farms, increases, large farmers are able to more effectively "push" the aggregate groundwater use towards the efficient allocation. Even though the incentive to pump groundwater efficiently for each individual large farmer declines, the aggregate groundwater usage in period 1 decreases. This is because the distribution of total acreage is skewed more (less) heavily towards large (small) farmers, who pump slowly (who deplete their wells in period 1).

Figure 1(b) illustrates the non-monotone relationship between the *stock* of groundwater in period 2 and farm-size inequality. As explained earlier, when the gap between small and large farms is small, $\Delta \in [0, 280]$, the large farmers are relatively ineffective in raising the dynamic efficiency. This is because, even though they decrease their pumping rates in order to compensate for the higher pumping rates by small farmers, their weight in aggregate pumping is relatively light. And so, the negative effect of the aggressive pumping by small farms dominates, and the groundwater stock in period 2 falls. As the share of total acreage owned by small farmers declines, but their pumping rates remain constant ($u^*_{i,1} = h_1 = 1$ for $i \in L_k$ and $k = 1,...,50$), the large farmers need to give up less of period 1 pumping to push the region towards more dynamically efficient allocation. From the perspective of a large farmer, the groundwater resource is more private, which reinforces the diminished influence of aggressive pumping by small farmers. As a result, the average stock in period 2 increases, and the region moves towards a more dynamically (and spatially) efficient allocation.

Figure 1(c) shows the non-monotone effect of the inequality in farm sizes on *total income*. Proposition 4 shows that, in general, an increase in size inequality affects the total farm income in two distinct ways. First, it affects the groundwater stock in period 2. Second, it affects the variability of the pumping rates among farmers in period 1. When the gap is small, $\Delta \in [0, 280]$, both the "stock" and "pumping rate variability" effects work in the same direction. When the gap is "sufficiently" large, any further increase in farm-size inequality raises the total farm income. Note that the dip in the total income in Figure 1(c) has a rather pointed peak. This is because for $\Delta \geq 280$ there is an additional income gain associated with the gain in the *spatial efficiency* due to the *decline* in the heterogeneity of pumping rates. The period 1 pumping on large farms increases, while pumping on small farms remains constant (as they deplete their wells in period 1).

As shown in Figure 1(d), total *per farm* incomes are also non-monotone in the extent of farm-size inequality. Surprisingly, the total small farm income *increases* when the acreage on small farms *decreases* in the range $\Delta \in [0, 280]$. The converse holds for large farms. This is because small farms are in a better position to steal groundwater from their neighbors operating on large farms. However, the cap on the pumping in period 1, $u_{i,1}^* \leq 1$, eventually annuls this effect. Consequently, a further increase in farm-size inequality affects farm incomes in the expected direction because, keeping everything else equal, a smaller (larger) acreage entails a smaller (larger) whole-farm income.

5. Risk-averse farmers

So far, we considered the effect of farm-size heterogeneity on welfare in the case of risk-neutral farmers (constant marginal utility of income). As we show next, relaxing this assumption may lead to rather different conclusions. Even the property that, under the competitive allocation, smaller farmers pump faster may no longer hold.

We consider the case of risk-averse farmers with (strictly) concave per period utility of whole-farm revenues, $v'' < 0$. To highlight the role of risk-aversion, we assume that per acre profit (e.g., yield) is a linear function of the amount of water applied per

acre, and that pumping costs do not depend on the hydraulic head, $g(u,h)=u$.
Following the same steps as before, we can show that the equilibrium best response of farmer k on acre $i \in L_k$, $u^*_{i,1}$, is

$$u^*_{i,1} = \min[h_1, (1/A_k)v_1^{-1}(a_k v'(A_k(h_1 - u^*_1)))], \quad \forall i \in L_k \tag{13}$$

where $v_1^{-1}(.)$ is the inverse of v', and the average pumping in period 1, u^*_1, solves

$$u^*_1 = (1/A)\sum_{k=1}^{n} \min[A_k h_1, v_1^{-1}(a_k v'(A_k(h_1 - u^*_1)))]. \tag{14}$$

Let $r(u) = -uv''(u)/v'(u)$ denote the Arrow-Pratt coefficient of relative risk-aversion of a farmer with the periodic utility of income v.

Proposition 6. *Suppose that farmers are risk-averse. Then the average pumping rate is higher than the socially efficient average rate,* $u^*_1 \geq u^s_1$*, and for all* $i \in L_k$, $j \in L_l$, $k < l$

a) smaller farms pump faster, $u^*_{i,1} \geq u^*_{j,1}$*, if* $r' \geq 0$.

b) smaller farms pump slower, $u^*_{i,1} \leq u^*_{j,1}$*, if* $1 + r(v_1^{-1}(av'(ahA))) \leq r(ahA)$ $\forall a \in [a_k, a_l]$ *and* $h \in (0, 0.5h_1)$.

Farm size has two effects on the farmer's pumping decision. On the one hand, larger farmers view their stock of groundwater as a relatively more private resource. This provides them with a greater incentive to push the regional use towards a dynamically more efficient allocation. On the other hand, larger farmers may have a smaller (negative) difference in marginal utilities of income in periods 1 and 2. This diminishes their incentive to push the region towards a dynamically more efficient allocation compared with smaller farmers. The "private resource" effect dominates if the coefficient of relative risk-aversion is increasing in wealth. The "income scale" effect dominates if the coefficient of relative risk-aversion is "sufficiently" large and decreasing in wealth (in the sense of condition in Part (b)).

While not reported here due to space constraints, the counterparts of Proposition 3-5 carry over to the case of risk-averse farmers as well. Competitive allocations may either exacerbate or alleviate income inequality associated with the distribution of land holdings among farmers. If the coefficient of relative risk-aversion is increasing in wealth, small farmers pump more groundwater per acre than large farmers. This lessens the income inequality caused by an unequal distribution of acreage. The converse is true if larger farmers pump more aggressively (on a per acre basis), which is possible if the coefficient of relative risk-aversion is "sufficiently" large and decreasing.

Note that, in the absence of the effect of farm-size inequality on the disaggregated pumping rates, from the societal point of view, the heterogeneity in land holdings is immaterial if farmers are *risk-neutral* (i.e., they value marginal income in both periods independently of the number of acres they farm). When farmers are *risk-averse*, the heterogeneity in the pumping rates can be welfare-increasing, given that the per acre irrigation rates increase on smaller farms and decrease on larger ones, so that in period 1 income is redistributed from rich to poor farmers (see Proposition 1). However, because of the decreasing marginal per acre benefits of water, total income always decreases under a greater variability of the pumping rates. This may create a tension between the effects of farm-size inequality on *income distribution* and *total income (output)*. Next we take a policy perspective and investigate the workings of a very simple groundwater use policy in the presence of farmer heterogeneity.

6. Policy implications: an example of flat-rate quota policy

In this section, we consider some political economy aspects of implementing a simple policy that allocates per period per farm pumping quotas. Suppose that the policy takes the form

$$\sum_{i\in L_k} u_{i,1}^* \le A_k q \text{ and } \sum_{i\in L_k} u_{i,2}^* \le A_k q + \max[A_k q - \sum_{i\in L_k} u_{i,1}^*, 0] \text{ for } k = 1,...,n, \qquad (15)$$

where $q \in (0, h_1]$ is the per acre quota (measured in acre-feet), and the quota allocated to each farm is proportional to its size. The quota limits the quantity of groundwater extracted in each period, but allows farmers to carry over unused portions of their quota into the next period. There is no market for water rights, and the unused quotas cannot be bought or sold.

For concreteness, we consider the case of risk-neutral farmers and a strictly concave agricultural output function analyzed in Section 4. The following result establishes that, while this policy always slows the rate of the aquifer depletion, the effect on farmer incomes is likely heterogeneous. We consider equilibrium where the pumping rates decrease with time $u_{i,1}^* \geq u_{i,2}^* \;\; \forall i \in L$, so that $u_1^* \geq 0.5h_1 \geq u_2^*$. For example, this is always true if all farmers are sufficiently small relative to the aquifer, $a_n \leq$ $\inf_{u \in (0,h_1)} \{g_u(h_1 - u, h_1) / f(h_1 - u)\}$. Then, under quota policy (15), farmers do not transfer the unused portion of their quotas from period 1 to period 2: $q \geq u_{i,1}^* \geq u_{i,2}^*$, if $q \geq h_1 / 2$, and $u_{i,1}^* = u_{i,2}^* = q \;\; \forall i \in L$ if $q < h_1 / 2$. Hence, for $q \geq h_1 / 2$ equilibrium is given by

$$u_{i,1}^*(q) = \min[q,\, g_u^{-1}(a_k f(h_1 - u_1^*(q)); h_1)], \;\; \forall i \in L_k,\; k = 1,...,n \tag{16}$$

$$u_1^*(q) = \sum_{k=1}^{n} a_k \min[q,\, g_u^{-1}(a_k f(h_1 - u_1^*(q)); h_1)] \;. \tag{17}$$

The income of farmer k under the quota policy is

$$\pi_k(q) = A_k \{g(\, q,\, h_1) + g(q, h_1 - q)\}, \text{ if } q < h_1 / 2, \text{ and} \tag{18a}$$

$$\pi_k(q) = A_k \{g(\, \min[q,\, g_u^{-1}(a_k f(h_1 - u_1^*(q)); h_1)], h_1) + g(h_1 - u_1^*(q), h_1 - u_1^*(q))\}, \text{ if } q \geq h_1 / 2 . \tag{18b}$$

From (18a) it follows that all farmers lose (gain) from a more restrictive quota, if the initial quota is sufficiently small and the marginal benefit of a higher stock is "small" ("large") relative to the marginal benefit of water consumption: $\partial \pi_k(q) / \partial q = A_k$ $\{g_u(q, h_1) + g_u(q, h_1 - q) - g_h(q, h_1 - q)\} \geq (\leq) 0$ for all $k = 1,...,n$. On the other hand, from (18b) it follows that the income of large farmers, who are not bound by the quota, increases because the quota policy slows down the average pumping rate in period 1.

Let $m(q) = \sup\{k : a_k \leq g_u(q, h_1) / f(h_2^*(q)),\; 1 \leq k \leq n\}$. Note that $m(q)$ is a non-increasing function. Then farmers $k = 1,..., m(q)$ are bound by the quota in period 1. Also, farmers $k = 1,...,\, m(q = h_1)$ deplete their wells in period 1, where $q = h_1$ symbolizes the absence of the quota policy.

Proposition 7. *Suppose that the quota is applicable,* $u_{1,1}^*(q = h_1) > q'$. *Then under the groundwater quota policy* $q = q' < h_1$

a) the groundwater stock in period 2 increases, $h_2(q = h_1) < h_2(q = q') \;\; \forall q' < h_1$. *Suppose that the period 2 quota is not binding,* $q' \geq h_1 / 2$. *Then* *b) large farmers gain,* $\pi_k(q=h_1) \le \pi_k(q=q')$ *for* $k = m(q')+1,...,n$ *;*

c) small farmers lose, $\pi_k(q=h_1) \ge \pi_k(q=q')$ *for* $k=1,...,m(h_1)$*, if (i)* $g_{uuu} \ge 0$,

$g_{uuh} \ge 0$, $2g_{uh}(h,h) + g_{hh}(h,h) \le 0$*, and (ii)* $a_z \ge \sum_{k=1}^{z-1} a_k / \sum_{k=z+1}^{n} a_k^2$ *for all*

$z = m(h_1),...,m(q')$.

Farmers in the medium size range, $m(h_1) \le k \le m(q')$, may lose or gain from a quota. The intuition for this result is very clear: Small farmers, who pump faster than the average farmer, stand to lose the most from a quota policy. Large farmers, who are not restricted by the policy, strictly gain from the quota because of the more stable inter-seasonal allocation of groundwater induced by this policy.

This illustrates that policies that do not account for user heterogeneity, are likely to affect not only the inter-seasonal but also the spatial distribution of incomes among farmers. The ensuing political economy issues and the relative weight of small and large farmers in the policy-making process pose additional constraints on the design of efficient groundwater management policies.

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Appendix

Proof of Proposition 1: First, note that in period 2, the planner optimally exhausts the remaining stock on each farm because g and v are strictly increasing. This implies that constraint (1) binds for $t = 2$ (i.e., $u_{i,2}^s = h_2 \ \forall i \in L$), so that (6) can be written

$$W^s = \max_{\{u_{i,1}^s\}} \sum\nolimits_{k=1}^n (v(\sum\nolimits_{i\in L_k} g(u_{i,t}^s, h_t)) + v(A_k g(h_2, h_2))) .$$

Because $\sum_{i\in L_k} g(u_{i,t}^s, h_t)$ is symmetric and concave in $u_{i,1}^s$, and W^s is symmetric in $v(.)$, optimality requires that $u_{i,1}^s = u_{j,1}^s$ for any $i \in L_k$ and $j \in L_l$ if $A_k = A_l$. Additionally, corner solutions are ruled out because v and g are increasing and concave in each argument. Substituting the law of motion (2), $h_2 = 1 - u_1^s$, into the objective function and differentiating, the first-order conditions for a maximum are

(A) $$v'(A_k g(u_{i,1}^s, h_1)) g_u(u_{i,1}^s, 1) - \frac{f(h_1 - u_1^s)}{A} \sum\nolimits_{l=1}^n A_l v'(A_l g(h_1 - u_1^s, h_1 - u_1^s)) = 0,$$

if $u_{i,1}^s \le h_1$, and $u_{i,1}^s = h_1$, otherwise, for all $i \in L_k$ and $k = 1,...,n$. Part (a) follows by observing that (A) reduces to (7) when $v'' = 0$ because $\sum_{l=1}^n A_l = A$. Part (b) follows by observing that only the first term in (A) depends on farm size A_k, and, by concavity of utility function, v, it decreases with A_k. Then by concavity of yield function, g, this implies that $u_{i,1}^s$ is a non-increasing function of farm acreage. ■

Proof of Proposition 2: Suppose that $u_1^s > u_1^*$. Then, by (11)

$$u_1^* = \sum\nolimits_{k=1}^n a_k \min[h_1, g_u^{-1}(a_k f(h_1 - u_1^*); h_1)] \ge \sum\nolimits_{k=1}^n a_k \min[h_1, g_u^{-1}(a_k f(h_1 - u_1^s); h_1)]$$

$$\ge \sum\nolimits_{k=1}^n a_k \min[h_1, g_u^{-1}(f(h_1 - u_1^s); h_1)] = g_u^{-1}(f(h_1 - u_1^s); h_1) = u_1^s .$$

The inequalities follow by concavity of g. The equality follows by (7). And so, we obtained a contradiction. Also, $u_{i,1}^* = \min[h_1, g_u^{-1}(a_k f(h_2); h_1)] \ge \min[h_1,$ $g_u^{-1}(a_l f(h_2); h_1)] = u_{j,1}^*$ for any $i \in L_k$, $j \in L_l$, $k < l$. ■

Proof of Proposition 3:

Part (a). Suppose that $h_2^*(\vec{A}) > h_2^*(\vec{B})$. Then, by (11),

$$u_1^*(\vec{A}) = \sum_{k=1}^{n} a_k \min[h_1,\ g_u^{-1}(a_k f(h_1 - u_1^*(\vec{A})); h_1)]$$

$$\geq \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{A})); h_1)]$$

$$\geq \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{B})); h_1)] = u_1^*(\vec{B}).$$

The first inequality follows because the sum of compositions of two concave functions: $\min[a_k h_1, a_k g_u^{-1}(a_k f(.); h_1)]$, is Schur-concave in $a_1,...,a_n$. To show this, we need to check that $a g_u^{-1}(af)$ is concave in a. Differentiating twice yields

$$\frac{\partial^2 [a g_u^{-1}(af)]}{\partial a^2} = \frac{f}{R(u) g_{uu}(u,h_1)}(2R(u) - P(u)) \leq 0,$$

where the inequality follows by condition in (a). The second inequality follows by concavity of g. And so, we obtained a contradiction.

Part (b). Suppose that $h_2^*(\vec{A}) < h_2^*(\vec{B})$. Then, by (11),

$$u_1^*(\vec{A}) = \sum_{k=1}^{n} a_k\ g_u^{-1}(a_k f(h_1 - u_1^*(\vec{A})); h_1) \leq \sum_{k=1}^{n} b_k\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{A})); h_1)$$

$$= \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{A})); h_1)]$$

$$\leq \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{B})); h_1)] = u_1^*(\vec{B}).$$

The equalities follow because, by b(i) and concavity of g, $g_u^{-1}(a_k f(h_2^*(\vec{A}); h_1) \leq h_1$ and $g_u^{-1}(b_k f(h_2^*(\vec{A}); h_1) \leq h_1$ for all $k = 1,...,n$, since $\vec{A} \leq^m \vec{B}$ implies $a_1 \geq b_1$. The first inequality follows because, by b(ii), $\sum_{k=1}^{n} a_k\ g_u^{-1}(a_k f(h_1 - u_1^*(\vec{A})); h_1)$ is Schur-convex (see Part (a)). The second equality follows by assumption. And so, we obtained a contradiction. ■

Proof of Proposition 4:

To show parts (a) and (b), we need two facts.

Fact 1. (i) $\pi_k(a_k) = A a_k g(\min[h_1, u(a_k)])$ is concave in a_k when $3R \leq P$.

(ii) $\pi_k(a_k)\big|_{u(a_k)<h_1}$ is convex in a_k when $3R \geq P$, where $u(a_k) = g_u^{-1}(a_k f(h_2^*(\vec{A}); h_1)$.

Proof: To verify, differentiate twice with respect to $a_k = a$:

(A1) $$\left.\frac{\partial \pi_k(a)}{\partial a}\right|_{u(a)<h_1} = A\frac{\partial[ag(u(a),h_1)]}{\partial a} = A(g(u,h_1) + \frac{(af)^2}{g_{uu}(u,h_1)})$$, and

(A2) $$\left.\frac{\partial^2 \pi_k(a)}{\partial a^2}\right|_{u(a)<h_1} = A\frac{\partial^2[ag(u(a),h_1)]}{\partial a^2} = A\frac{af^2}{R(u)g_{uu}(u,h_1)}(3R(u) - P(u)) \le (\ge) 0 .$$

depending on whether $3R \le (\ge) P$. This proves Fact 1(ii). To show Fact 1(i), note that $a_k g(\min[h_1, u(a_k)]) = \min[a_k g(h_1,h_1), a_k g(u(a_k),h_1)]$ by monotonicity of g. Hence, $a_k g(\min[h_1, u(a_k)])$ is concave in a_i when $3R \le P$ as a composition of concave functions.

Fact 2. $\partial W^c / \partial h_2^* > 0$.

Proof: $\partial W^c / \partial h_2^*$ inherits the sign of $\partial\{g(g_u^{-1}(af(h_2);h_1)],h_1) + g(h_2,h_2)\}/\partial h_2$ $= af'(h_2)/g_{uu}(u,h_1) + f(h_2) > 0$, where the inequality follows by concavity of g. Keeping everything else equal, as the extent of dynamic inefficiency of the competitive allocation increases, welfare falls.

Part (a). By (12),

$$W^c(\vec{A}) = \sum_{k=1}^n A_k \{g(\min[h_1,\ g_u^{-1}(a_k f(h_2^*(\vec{A});h_1)],h_1) + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\ge \sum_{k=1}^n B_k \{g(\min[h_1,\ g_u^{-1}(b_k f(h_2^*(\vec{A});h_1),h_1)] + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\ge \sum_{k=1}^n B_k \{g(\min[h_1, g_u^{-1}(b_k f(h_2^*(\vec{B});h_1)],h_1) + g(h_2^*(\vec{B}),h_2^*(\vec{B}))\} = W^c(\vec{B}).$$

The first inequality follows because function $W(\vec{A})$ is Schur-concave as the sum of concave functions by a(i) and Fact 1(i). The second inequality follows by a(ii) and Fact 2.

Part (b). By b(i), $u_{i,1}^*(\vec{A}) < h_1$ for all $i \in L$ because $\vec{A} \le^m \vec{B}$ implies that $a_1 \ge b_1$ so that $g_u^{-1}(a_k f(h_2^*(\vec{A});h_1) \le h_1$ and $g_u^{-1}(b_k f(h_2^*(\vec{A});h_1) \le h_1$ for all $k = 1,...,n$. Then, by (12),

$$W^c(\vec{A}) = \sum_{k=1}^n A_k \{g(\ g_u^{-1}(a_k f(h_2^*(\vec{A});h_1)],h_1) + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\le \sum_{k=1}^n B_k \{g(\ g_u^{-1}(b_k f(h_2^*(\vec{A});h_1),h_1) + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$= \sum_{k=1}^n B_k \{g(\min[h_1,\ g_u^{-1}(b_k f(h_2^*(\vec{A});h_1),h_1)] + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\leq \sum_{k=1}^{n} B_k \{g(\ \min[h_1, g_u^{-1}(b_k f(h_2^*(\vec{B}); h_1)], h_1) + g(h_2^*(\vec{B}), h_2^*(\vec{B}))\} = W^c(\vec{B}).$$

The first inequality follows because, function $W(\vec{A})$ is Schur-convex by Fact 1(ii). The equality follows by b(ii). The second inequality follows by b(iii) and Fact 2. ■

Proof of Proposition 6: Suppose that $u_1^s \geq u_1^*$. Then, by (A) and (14) in the text,

$$u_1^s = (1/A)\sum_{k=1}^{n} \min[A_k h_1, v_1^{-1}(\sum_{l=1}^{n} a_l v'(A_l(h_1 - u_1^s)))]$$

$$< (1/A)\sum_{k=1}^{n} \min[A_k h_1, v_1^{-1}(\sum_{l=1}^{n} a_l v'(A_l(h_1 - u_1^*)))]$$

$$\leq (1/A)\sum_{k=1}^{n} \min[A_k h_1, v_1^{-1}(a_k v'(A_k(h_1 - u_1^*)))] = u_1^*.$$

The inequalities follow by concavity of v. And so, we obtained a contradiction.

Part (a). Let $i \in L_k$. First, consider $u_{i,1}^*(A_k) < h_1$. By (13), differentiation yields

$$\partial u_{i,1}^* / \partial A_k = v'(A_k h_2)/(AA_k v''(A_k u_{i,1}^*))[1 + R(A_k u_{i,1}^*) - R(A_k h_2)] \leq 0$$

The inequality follows because, by (13), $u_{i,1}^* \geq h_1 - u_1^*$, and so $1 + R(A_k u_{i,1}^*)$ $- R(A_k(h_1 - u_1^*)) \geq 1 > 0$. If $u_{i,1}^* = h_1$ then $u_{j,1}^* \leq h_1$ for $j \in L_l$, $k < l$.

Part (b). Proof is analogous. ■

Proof of Proposition 7:

Part (a). Note that this trivially true when the quota is binding in period 2, $q' < h_1/2$, because then $u_{i,1}^* = q$, and $u_{i,2}^* = h_2 = h_1 - q \ \ \forall i \in L$. So consider the case when $q' \geq h_1/2$ and suppose that $u_1^*(q = h_1) < u_1^*(q = q')$. Then, by (17),

$$u_1^*(q = h_1) = \sum_{k=1}^{n} a_k \min[h_1, g_u^{-1}(a_k f(h_1 - u_1^*(q = h_1)); h_1)]$$

$$\geq \sum_{k=1}^{n} a_k \min[q', g_u^{-1}(a_k f(h_1 - u_1^*(q = h_1)); h_1)]$$

$$\geq \sum_{k=1}^{n} a_k \min[q', g_u^{-1}(a_k f(h_1 - u_1^*(q = q')); h_1)] = u_1^*(q = q'),$$

where the last inequality follows by concavity of g. And so, we obtained a contradiction.

Part (b). By (18b), farmer k's income for $k = m(q') + 1, ..., n$ is

$$\pi_k(q=q') = A_k\{g(\ g_u^{-1}(a_k f(h_1 - u_1^*(q')); h_1), h_1) + g(h_1 - u_1^*(q'), h_1 - u_1^*(q'))\}$$

$$\geq A_k\{g(g_u^{-1}(a_k f(h_1 - u_1^*(h_1)); h_1), h_1) + g(h_1 - u_1^*(h_1), h_1 - u_1^*(h_1))\} = \pi_k(q = h_1),$$

where the inequality follows by Part (a), and monotonicity and concavity of g.

Part (c). By (18b), farmer k's income is $\pi_k(q') = A_k\{g(\ q'\ , h_1) + g(h_1 - u_1^*, h_1 - u_1^*)\}$ for $k = 1,...,m(h_1)$. Differentiation yields

(A3) $$\frac{\partial \pi_k(q')}{\partial q} = A_k\{g_u(q,h_1) - f(h_1 - u_1^*)\frac{\partial u_1^*}{\partial q}\} \geq A_k f(h_1 - u_1^*)\{a_{m(q')} - \frac{\partial u_1^*}{\partial q}\}$$

$$\geq A_k f(h_1 - u_1^*)\{a_{m(q')} - \frac{\sum_{l=1}^{m(q')} a_l}{1 + \sum_{l=m(q')+1}^{n} a_l^2}\} \geq 0.$$

The first inequality follows because $m(h_1) \leq m(q')$, which follows by concavity of g.

The second inequality follows because, by (17), $u_1^*(q) = q\sum_{l=1}^{m(q')} a_l + \sum_{l=m(q')+1}^{n} a_l$ $g_u^{-1}(a_l f(h_1 - u_1^*(q)); h_1)$, and implicit differentiation yields

$$\frac{\partial u_1^*(q)}{\partial q} = \frac{\sum_{l=1}^{m(q')} a_l}{1 + \sum_{l=m(q')+1}^{n} a_l^2 f'(h_1 - u_1^*(q)) / g_{uu}(u_{i,1}^*(A_l); h_1)} \leq \frac{\sum_{l=1}^{m(q')} a_l}{1 + \sum_{l=m(q')+1}^{n} a_l^2},$$

since, by c(i),

$$f'(h_1 - u_1^*) = g_{uu}(h_1 - u_1^*, h_1 - u_1^*) + 2g_{uh}(h_1 - u_1^*, h_1 - u_1^*) + g_{hh}(h_1 - u_1^*, h_1 - u_1^*)$$

$$\leq g_{uu}(u_{i,1}^*; h_1).$$

The third inequality in (A3) follows by c(ii). Hence, $\pi_k(q = q') \leq \pi_k(q = h_1)$ for $k = 1,...,m(h_1)$ because $\partial \pi_k(q)/\partial q \geq 0$ for all $q \in [q', h_1]$. ■