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**Estimation and Analysis of Rational Expectations Model of International Cotton Market**

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# Estimation and Analysis of Rational Expectations Model of International Cotton Market

Oleksiy Tokovenko Lewell F. Gunter Dmitry V. Vedenov†

**Abstract**

The paper outlines an approach to estimation and analysis of rational expectations international cotton market. A multiple model bootstrap filter is used to compute unobserved market expectations and their distributions. Estimation results are used to analyze the welfare effects of exogenous trade shocks and government programs, with application to the national market security.

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## Introduction

The contemporary theory of commodity markets attempts to model the behavior of commodity prices in order to explain the factors that generate price fluctuations and thus to make predictions of future prices. Assumptions about rationality of price expectations have been widely used in empirical studies. Although the rational expectations of market prices are often approximated through the observed futures prices on the relevant commodities, this approach is more appropriate to studying of contemporaneous or past market response to policy changes. On the other hand, endogenous modeling of market expectations allows one to estimate the effects of

†Authors thank Nando de Freitas for graciously providing his computer codes.

structural changes in the model and thus analyze market performance under alternative policies (e.g. Miranda and Helmberger (1988)). In addition, in international trade studies, it is often impossible to use futures prices due to aggregation issues or simple data unavailability. Applications of endogenous rational expectations models to the analysis of agricultural commodity markets in a fully stochastic-dynamic setting can be found, for example, in Miranda and Glauber (1993) and Peterson and Tomek (2005). The main issue with this class of models is using parameterized expectations as a function of the current value of state variables, such as carryover of commodity. Most of the existing models typically analyze closed single region markets as computational complexity increases dramatically with the dimensionality of the model due to parametrization. In international trade analysis, this often happens as the number of regions increases. Makki, Tweeten and Miranda (1996, 1998) present studies that explain the behavior of a real market in a three region model. However, only two of regions are active market participants while the third serves as the balancing price taker. In order to make the rational expectations framework more applicable for applied analysis of international trade, we propose to treat the values of future prices as unobserved, thus following the idea behind the state-space approach to time-series analysis. In such a framework expected values of prices at a future period can be learned through the information available in the current period. We impose the weaker condition for rationality of the model behavior (such as consistency of price expectations) that will allow for stochastic differences in values of expected and realized prices at any given point in time as long as the both processes converge.

## Objectives

The objectives of this paper are to develop an alternative estimation algorithm for the international commodity market model with nonlinear rational expectations, and to use the underlying structural model to assess the supply response and the welfare effects of government programs that force supply changes, both through direct price controls and by affecting the intertemporal conditions for storage. In the empirical application, a model of the international cotton market is studied as represented by U.S., China and other countries aggregated conventionally to the Rest of The World. The U.S. market is modeled as an active consumer, producer and storer of cotton, while China and the ROW countries enter the model as the active importers. The analysis is performed mainly in terms of the U.S. market with special focus on unexpected international policy disturbances, such as significant increases in foreign levels of carryover above the projected levels due to foreign government import decisions, which are exogenous to the model. We will also discuss the stability of different intervention programs to such disturbances and implications of these results to national market security.

## Particle Filter

The general parametrized state-space model can be described as

$$\boldsymbol{x}_{t+1} = g_I(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{1t}) \tag{1}$$

$$\boldsymbol{y}_t = f_I(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{2t}) \tag{2}$$

where $g_I$ and $f_I$ are the parametrized state transition and measurement equations, $\boldsymbol{y}_t$, $\boldsymbol{x}_t$, $\boldsymbol{u}_t$ are the state, control and measurement vectors, and $\boldsymbol{\epsilon}_{1t}$ and $\boldsymbol{\epsilon}_{2t}$ are the process and measurement noise vectors, all at period $t$. The model index parameter $I$ can take on any of $N$ discrete values $I = \overline{1, N}$. The parametrized model in (1) and (2) can have exogenous or endogenous switching mechanism. The former implies that the regime switching process is Markov chain $[\Pr(I_{t-1} = 1), \ldots, \Pr(I_{t-1} = N)]\mathbf{\Lambda} = [\Pr(I_t = 1), \ldots, \Pr(I_t = N)]$ with transition matrix $\mathbf{\Lambda}$ while the latter assumes that the regime switching mechanism involves dependent variables of regression equations so that the transition probabilities are no longer time invariant. In this case, the model index $I$ as the conditioning parameter can branch at the next time step according to the endogenous transition probability matrix $\mathbf{\Lambda}_t(\boldsymbol{x}_t, \boldsymbol{u}_{t+1})$. Due to this specific nature of the endogenous switching models, they proved to be a useful tool in analyzing the dynamic disequilibrium models described e.g. in Quandt (1988). Since the state-space systems in (1) and (2) are often non-linear and have non-Gaussian disturbances the tool known as the multiple model bootstrap filter (MMBF) can be used for estimation puropses[1]. Here, in the discussion of MMBF we follow McGinnity and Irwin (2001), while looking at the special case of univariate state transition process. The extension of the bootstrap filter suggested in Gordon, Salmond and Ewing (1995) to multiple models is developed by considering a single sample, augmented by an index vector representing the model parameter, $I_t$. Let us denote the information set at time $t$ as $\mathfrak{F}_t = \{\boldsymbol{y}_t, \boldsymbol{u}_t\}$. Then at any period $t$ the unconditioned posterior density for unobserved state variable $p(x_t|I_t, \mathfrak{F}_t)$ is given by the $I$-augmented sample from the state space[2] which is represented by the set of connected pairs of random realizations $x_{t|t}$ of the state variable and the index $I_{t|t}$ of the model which generated this realization of

[1]Hendry and Richard (1992) introduced alternative approach by simulation methods for the class of dynamic latent-dependent variables models based on factorization of the sequential joint density of the observable and latent-dependent variables. This approach was further discovered from the simulated maximum likelihood (Lee(1997)) and nonlinear particle filtering (Liu and Chen (1998)) estimation perspectives.

[2]The time invariant parameter vector $\boldsymbol{\theta}$ is suppressed for the simplicity of description.

$x$. As suggested in McGinnity and Irwin (2001), the approximate posterior $j$'s model probabilities for the period $t$ can be obtained by computing the proportion of the trajectories associated with the given regime $j$, that is $p(I_{t|t} = j|\mathfrak{F}_t) \approx n(I_{t|t} = j)/K$ for $j = \overline{1, N}$. It is necessary to have $K$ large enough to obtain an accurate approximation of model probabilities. Since the evolution of regimes is actually a branching process that generally would require an exponential growth in number of trajectories, the following two stage procedure can be used to keep the number of estimators constant. First, to receive the branched prior distribution $p(x_t|I_t, I_{t+1}, \mathfrak{F}_t)$ McGinnity and Irwin (2001) suggest augmenting the posterior vector of $\boldsymbol{x}_{t|t}$ with an additional index vector $\boldsymbol{I}_{t+1,t}$ of possible effective future regimes. The values of elements of the index vector are generated using endogenous transition probabilities implied by the state-space system of interest. Second, the merged prior density $p(x_t|I_{t+1}, \mathfrak{F}_{t+1})$ is obtained by discarding the index vector $\boldsymbol{I}_{t,t}$ which is irrelevant now due to the Markovian nature of the process. The prediction density $p(x_t|I_{t+1} = j, \mathfrak{F}_{t+1})$ can then be approximated by transforming each element of $\boldsymbol{x}_{t|t}$ using the transition equation conditionally on the associated model $I_{t+1}$. At the next step, the posterior density of the state variable is formed using the resampling algorithm utilized in the single model bootstrap filter with the importance weights based on the likelihood of the prediction set. Using relevant posterior densities and recurrent relations, the simulated likelihood function of interest can be written as

$$L(\boldsymbol{y}|\boldsymbol{u}) = \sum_{I_1=1}^{N} \cdots \sum_{I_T=1}^{N} \int_{x_1} \cdots \int_{x_T} \prod_{t=1}^{T} p(y_t|x_t, u_t, I_t) p(x_t|u_t, I_t) p(I_t|u_t) dx_t \tag{3}$$

Note, that ranges of integration in the likelihood function are usually restricted for endogenous dynamic switching models, as it is pointed out by Lee (1997). While forming likelihood function in (3) we follow the idea of the expectation-maximuzation

(EM) algorithm introduced by Dempster, Laird and Rubin (1977)[3] with the unknown parameters being replaced by their expectations. However, since in many cases the estimation equations are nonlinear and distributions of unobserved variables change over time, more complicated approaches, involving the simulated likelihood function estimation should be used.

## Model

In any period $t$ the supply $q_t$ initially available in a given region is composed of a carryover from the preceding period $s_{t-1}$ and new production, which is determined by an exogenous random per-acre yield $y_t$ on the acreage $a_{t-1}$, planted in the preceding period such that $q_t = a_{t-1}y_t + s_{t-1}$. The region must allocate total supply available $q_t$ among consumption $c_t$, future storage $s_t$ and amount traded $z_t$. Therefore the decisions of market agents must obey the following aggregate balance equation $q_t = c_t + s_t + z_t$. In general, if $q_t > c_t + s_t$, the excess supply is observed in a given region which is then a net exporter at that period and $z_t > 0$. Otherwise, in the case of $q_t < c_t + s_t$, the region is a net importer, with $z_t$ being negative. We assume that each group of market participants makes decisions independently. Define the market clearing price $p_t \in R_{++}$ as the strictly decreasing function of a current consumption $c_t \in R_{++}$:

$$p_t = \alpha_0 c_t^{\alpha_1} \exp(\varepsilon_{1t}) \tag{4}$$

where consumers are assumed to have quasi-linear preferences. The acreage $a_t \in R_{++}$ planted by rational producers is a strictly increasing function of the effective future

[3]See McLachlan and Krishnan (1997) and Small and Wang (2003) for detailed discussion including the stochastic version of EM algorithm.

price, expected at the harvest time $x^*_{t+1}$

$$a_t = \gamma_0 {x^*_{t+1}}^{\gamma_1} \exp(\varepsilon_{2t}) \tag{5}$$

The effective future price is defined as the maximum of the expected future price and the target price announced by the government for the period $t$. This condition introduces the disequilibrium dynamics in the price expectation process, such that two possible models can be used to define the acreage decisions

$$a_t = \gamma_0 x^{\gamma_1}_{t+1} \exp(\varepsilon_{2t}) \quad \text{if} \quad x_{t+1} > \tilde{x}_{t+1} \quad I_{t+1} = 1 \tag{6}$$

$$a_t = \gamma_0 \tilde{x}^{\gamma_1}_{t+1} \exp(\varepsilon_{2t}) \quad \text{if} \quad x_{t+1} \leq \tilde{x}_{t+1} \quad I_{t+1} = 2 \tag{7}$$

Given the announced target price $\tilde{x}_{t+1}$, the prior model probabilities can be defined as $p(I_{t+1|t} = 1) = 1 - \Phi(x_{t+1|t} = \tilde{x}_{t+1})$ and $p(I_{t+1|t} = 2) = \Phi(x_{t+1|t} = \tilde{x}_{t+1})$, where $\Phi(x_{t+1|t})$ is the unconditioned prior distribution function of unobserved state variable. Storage decisions are made by competitive storers. Assuming that marginal cost $k_t$ of storage increases, private demand for stocks is defined as

$$s_t = \beta_0 w^{\beta_1}_{t+1} \exp(\varepsilon_{3t}) \tag{8}$$

where $w_{t+1} = \delta_t f_{t+1}/p_t$ is the expected rate of appreciation of stocks and $\delta_t = 1/1+r_t$ is the annual discount factor, and $r_t$ is the annual interest rate. The market model is closed by assuming that market agents make their decisions consistently using information available at the present time.

The motion of price signals is limited by the following linear transition equation based on market efficiency condition

$$x_{t+1} = b_0 + b_1 x_t + \eta_{t+1} \tag{9}$$

Equations (4), (5) (in the form of switching models (6) and (7)) and (8) form the system of measurement equations (2) of the general model, while equation (9) corresponds to the state transition rule (1). In this case, the application of the MMBF to the model recursion can be described with the following pseudo code

Estimation algorithm

**Step 0a: Initialization** Define $\tau$, $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_0$ and set $j = 1$.

**Step 0b: Initialization** Set $t = 0$ and define the initial conditioned distribution of $x$ as vectors $\boldsymbol{x}_0$ and $\boldsymbol{I}_0$.

**Step 1: Prediction** Generate the unconditioned distribution of $\boldsymbol{x}_{t+1}$ from $\boldsymbol{x}_t$ using state transition equation. Given target value $\tilde{x}_{t+1}$ compute the prior regime probabilities $p(I_{t+1} = j)$ for $j = 1, 2$ using unconditioned empirical distribution function of $\boldsymbol{x}_{t+1}$. Generate new index vector $\boldsymbol{I}_{t+1}$ using prior regime probabilities. Augment $\boldsymbol{x}_t$ index vector $\boldsymbol{I}_{t+1}$ and discard old index vector $\boldsymbol{I}_t$. Generate prediction density for state variable $x$ conditionally on prior model distribution.

**Step 2: Update** Form posterior conditioned distribution of $\boldsymbol{x}_{t|t}$ using the sampling importance resampling scheme.

**Step 3: Counter check** If $t < T$ set $t = t + 1$ and go to **Step 1**.

**Step 4: Maximization** Maximize full information simulated likelihood for $\boldsymbol{\theta}^*$ given the distributions computed in steps 1 - 3.

**Step 5: Convergence check** If $\| \boldsymbol{\theta}^* - \boldsymbol{\theta}_j \| > \tau$ set $j = j + 1$ and $\boldsymbol{\theta}_j = \boldsymbol{\theta}^*$ and go to **Step 0b**. Otherwise stop.

The forecast based on the model can be obtained with the use of the following algorithm, that combines bayesian inference and classical optimization routines

Forecast algorithm

**Step 0a: Initialization** Set $\boldsymbol{\theta} = \boldsymbol{\theta}^*$.

**Step 0b: Initialization** Set $t = T + 1$ define the posterior vectors $\boldsymbol{x}_{T|T}$ and $\boldsymbol{I}_{T|T}$ as the initial conditioned distribution of $x$.

**Step 1: Prediction** Construct prediction density for state variable $x$ conditionally on prior model distribution for period $T + 1$ using schema from **Step 1** of estimation algorithm.

**Step 2: Prior forecast** Compute bayesian optimal prior point estimate of $\hat{x}_{T+1|T}$

**Step 3: Optimization** Solve for equilibrium values of $\boldsymbol{y}_{T+1}$ given $\hat{x}_{T+1}$, $\tilde{x}_{T+1}$, $\boldsymbol{u}_{T+1}$, $\boldsymbol{\theta}$ subject to balance conditions.

**Step 4: Update** Form posterior conditioned distribution of $\boldsymbol{x}_{t|t}$ using the sampling importance resampling scheme.

**Step 5: Posterior forecast** Compute bayesian optimal posterior point estimate of $\hat{x}_{T+1|T+1}$

## Computation

Bayesian recursive estimation is implemented using the sequential Sampling Importance Resampling (SIR) algorithm based on the original program code provided by Nando de Freitas as the part of SIR Demo package. The algorithm is modified to account for model specific likelihood and multiple regime resampling. The full information simulated likelihood is then maximized by genetic algorithm (GA) from The Genetic Algorithm Optimization Toolbox for MatLab developed by C. Houck, J. Joines and M. Kay (1995). All computations are done on Pentium 4 2.8 GHz IBM PC computer using Mathworks MatLab R2006b programming environment.

## Data

The data used for the study are annual time-series from 1972 to 2005 for cotton. The relevant data have been collected from the Cotton and Wool Yearbook and Cotton and Wool Outlook published by the USDA Economic Research Service. Additional information on missing quantities and prices were collected from 1996 to 2005 Cotton: World Markets and Trade reports provided by USDA Foreign Agricultural Service. The market prices are measured as season average prices adjusted for inflation using Producer Price Index (PPI) for cotton products.

## Expected Results

At the time of this writing we have run the simulations while correcting and improving the SIR algorithm in terms of efficiency. Obtained estimates should be consistent with modern commodity markets theory, which require the price expectations to be negatively related to the value of commodity storage and have highly nonlinear form due to the disequilibrium effects of the government intervention programs. Introduction of switching mechanisms and multiple models will yield more complex distributions of variables that should better describe the real market processes. Analytical results will include welfare estimates for the alternative levels of policy along with stability considerations based on safety first criteria. Also, we expect to compute a set of critical values and probabilities associated with different market scenarios. Additional results will include tests for rationality of market behavior such as the nonparametric tests on the difference in distributions and series of expected and realized prices in order to justify the modeling results in statistical and economic sense. Obtained results should be more accurate than the approximate computations based on probability inequalities, since the optimal distributions of variables will be directly available.

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