Comparative Statics Effects for Supermarket Oligopoly with Applications to Sales Taxes and Slotting Allowances

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May 31, 2007

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1 Introduction

There are significant economies of scope in the U.S. retailing industry. Over the period 1990 to 2004, the number of stock keeping units (SKUs) in U.S. supermarkets increased 52.4% (from 16,500 to 25,153) (Progressive Grocer) and the number of SKUs at Home Depot and Lowe's in the Do-It-Yourself segment of the building product retail industry increased 50% from 30,000 to over 45,000 (Home Channel News). Supercenters, the class of retail stores that encompasses both grocery and general merchandise categories, represented the fastest-growing segment of the U.S. retail industry in 2006, offering a range of products between 100,000 and 225,000 SKUs (Supercenter Industry). The goal of this paper is to frame a simple model of oligopoly supermarket competition that is capable of generating comparative statics predictions on the effect of business practices (e.g., slotting allowances) and public policies (e.g., excise taxes) on equilibrium prices, the equilibrium number of products sold at supermarkets, and the equilibrium number of retailers in the supermarket industry.

Does a large product assortment stimulate retail sales? Considerable controversy exists on the effect of the number of products sold at supermarkets on retail sales. On the one hand, much of the empirical literature on efficient assortment suggests that retailers can dramatically reduce product variety without compromising sales (Dreze, Hoch and Purk, 1994; Broniarczyk, Hoyer and McAlister, 1998; Boatwright, and Nunes, 2001). On the other hand, it is often argued that a larger product assortment supports higher retail prices by facilitating consumer loyalty, a claim that is supported by the recent finding of Borle, Boatwright, Kadane, Nunes and Shmueli (2005) that customers shop less frequently and purchase fewer items per visit in response to smaller retail product lines. A difficulty in sorting out the elements of this controversy is that both retail prices and retail product variety are endogenous variables. Our model illuminates the theoretical underpinnings of this issue by assessing the correspondence between these variables in response to changes

in various exogenous variables: following a change in most industry parameters, for instance wholesale prices, per-product fixed marketing costs, or the rate of sales taxation, we find changes in retail sales to be positively correlated with changes in the length of product lines; however, following retailer exit, for instance through merger, retail variety may rise in conjunction with decreased retail sales. Moreover, emerging retail industry practices such as slotting allowances, which serve to shift retailer per-product fixed costs backwards onto manufacturers, increase the length of retailer product lines but, at the same time, decrease retail sales when the transfer of fixed costs is accompanied by cost-neutral increases in wholesale prices (as would occur in a competitive manufacturing environment).

What is the incidence of public policies in multi-product retail environments? Surprisingly, almost all we know about the effect of important public policies in oligopoly markets, for instance farm programs and excise taxes, stems from markets with single-product firms. Do the standard policy implications derived in single-product oligopoly markets carry over to multi-product markets? Consider the case of excise taxes. Following Suits and Musgrave (1953), it is well known that ad valorem (sales) taxes welfare-dominate specific (unit) taxes that are equivalent in terms of total tax yield. The reason is that the tax burden under ad valorem taxes is smaller at lower market prices, which makes residual (after-tax) demand facing firms more elastic, narrowing price-cost margins and increasing sales. In a multi-product supermarket environment, however, excise taxes also deter the number of products provided. This is important, because the efficiency advantage of ad valorem taxes over specific taxes in output terms is offset in the multi-product retailer case by a disadvantage in product variety: narrower price-cost margins facilitate output, but discourage the introduction of new varieties. In the short-run, ad valorem taxes are welfare superior to equal-yield unit taxes, but it turns out that ad valorem and specific excise taxes are welfare-neutral under conditions of free-entry.

Our observations are based on a retail sector characterized by differentiated product oligopoly competition between multi-product firms ("supermarkets"). Product

differentiation is generally treated one of two ways in the oligopoly literature, either through location models following Hotelling (1929) and Salop (1979), or through representative consumer models following Chamberlin (1933), Spence (1976), and Dixit and Stiglitz (1977). This paper synthesizes the two approaches. Consumers make discrete choices on where to shop among a potentially large number of supermarkets; however, once consumers select their preferred supermarket, all available products within the store compete simultaneously for each consumer.

Some of the key findings of this paper are as follows. First, unlike in Anderson and de Palma (1992) and Ottaviano and Thisse (1999), an increase in product differentiation need not increase the equilibrium length of the product lines. When consumers make discrete choices over retailers, an increase in product differentiation increases the number of products offered by firms when the retailers are not highly differentiated but narrows the product range when retailers are sufficiently differentiated. Second, under both no-entry and free-entry oligopoly conditions, product variety is undersupplied relative to the socially optimal resource allocation. Third, an (exogenous) increase in the number of retailers in the oligopoly equilibrium increases the quantity supplied of each variety, but narrows the product range. Fourth, retailer heterogeneity and product heterogeneity can have very different effects on retailer profits: profits are increasing in the degree of retailer heterogeneity, but may be decreasing in the degree of product heterogeneity. Thus, it may be optimal for strategic reasons to introduce private labels to facilitate interretailer brand identity, while at the same time positioning private labels nearby national brands in product space. Finally, excise taxes affect product variety and prices in different ways. While both unit and ad valorem excise taxes reduce retail sales and raise consumer prices, unit taxes do so by reducing output per product while ad valorem taxes do so by reducing the number of products available in the market.

The remainder of the paper is organized as follows. In the next section, the symmetric no-entry and free-entry oligopoly equilibria are characterized for the special case in which consumers have logarithmic subutility functions over products. The

logarithmic case is presented here, because it yields closed-form solutions for equilibrium prices and the variety range, which clearly illustrates the main ideas of the model.[1] The comparative statics effects of the model and the welfare implications of various market practices are outlined in Section 3, and, in Section 4, some comments are provided on the outcome of the model under more general specifications of utility.

2 The Model

To date, there has been little analysis of multiproduct oligopoly competition. An important reason for this is that the problem is analytically challenging.[2] The goal of this section is to develop a model of multiproduct oligopoly competition that is general enough to encompass some reasonable features of the retail industry, yet sufficiently focused to generate a tractable set of comparative statics effects. To do so, we assume varieties are symmetric in the sense of Spence (1976) and Dixit and Stiglitz (1977) and consider a competitive wholesale market in which retailers purchase products at a unit cost of w for each product.

Retail competition is localized in the sense that supermarkets differ in terms of their proximity to consumers. Following Salop (1979), each retailer is represented as a point on a circle of unit length. The strategic rivalry between retailers is to acquire customers, which is measured continuously as the number of consumers who choose to visit a particular retailer, given the product breadth and prices available at each of the various retailers in the market.

The market equilibrium studied is a non-cooperative Nash equilibrium in product ranges, m, and prices, $p_i, i \in (0, m]$. We consider both no-entry and free-entry oligopoly retail market equilibria. The no-entry oligopoly equilibrium takes as given the number of firms, while the free-entry oligopoly equilibrium endogenously determines the number of firms according to a zero profit constraint. For analytical convenience, the number of products is treated in both cases as a continuous variable

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[1]The more general case of CES subutility produces qualitatively similar results. Results from the CES case are available from the authors on request.

[2]For a good discussion of this issue, see Anderson and de Palma (2006).

and only symmetric equilibria are examined. In such an equilibrium, each retailer takes as given the prices $\overline{p}$ and varieties $\overline{m}$ chosen by the other retailers and selects its own p and m to maximize profits. The problem of retail location choice is suppressed as well. Following Salop (1979), it is assumed that whatever the number of retailers happens to be, they are equally spaced around the circle.

Consumers in the model are identified by points on the circle corresponding to their most preferred set of retail market characteristics. Consumers are uniformly distributed around the circle with a constant density per unit length and this density is assumed to be sufficiently large that a set of consumers with different tastes may be aggregated into a representative consumer at each location.[3] This aggregate utility function is separable between the retail products, $x_i, i \in (0, m]$, and a numeraire good, x_0, and is symmetric in the sense of Spence (1976) and Dixit and Stiglitz (1977). Specifically,

$$U(x, m, x_0) = u \left(\int_{i \in m} x_i^\theta \right) + x_0, \tag{1}$$

where $0 < \theta < 1$ measures the degree of substitutability between products. Product diversity is more valuable to consumers for smaller values of θ, and the products are perfect substitutes (i.e., indifference contours are hyperplanes) in the limit as $\theta \to 1$. Notice that the utility structure in (1) implies a love of variety: for fixed consumption levels of each product, the sub-utility function, $u \left(\int_{i \in m} x_i^\theta \right)$ is increasing in the number of product variants, m. The sub-utility function is assumed to be strictly concave, and this requires $0 < E$ and $1 - \theta + \theta E > 0$, where $E = -(u''/u') \int_{i \in m} x_i^\theta$ denotes (in absolute terms) the elasticity of the marginal subutility function. To ensure profits for the retailer are quasiconcave in p and m, and that profits decline with retailer entry in the free-entry case, we assume $E \leq 1$ and $E > (1 - \theta)/(2 - \theta)$. These conditions are all satisfied when the sub-utility function is logarithmic, and the logarithmic case is pursued from here forward to illustrate the main ideas more

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[3]That a representative consumer whose utility is increasing in product variety can stand for a heterogeneous population of consumers has been shown by Anderson, dePalma and Thisse (1992, Chs. 3-4).

clearly. Specifically

$$U(x, m, x_0) = \ln \left(\int_{i \in m} x_i^\theta \right) + x_0, \tag{2}$$

with $0 < \theta \leq 1$.

For each product, inverse demand is given by

$$p_i = \frac{\theta x_i^{\theta-1}}{\int_{i \in m} x_i^\theta}, \tag{3}$$

where p_i is the retail price of product i. Equations (3) can be inverted to recover the demand functions (Mathematica will do this for you),

$$x_i(p, m) = \frac{\theta p_i^{\frac{-1}{1-\theta}}}{\int_{i \in m} p_i^{\frac{-\theta}{1-\theta}}}. \tag{4}$$

Letting y denote income of the representative consumer, indirect utility is

$$v(p, m, y) = \ln \left[\theta^\theta \left(\int_{i \in m} p_i^{\frac{-\theta}{1-\theta}} \right)^{1-\theta} \right] - \theta + y. \tag{5}$$

Aggregate demand facing each multi-product retailer depends on the decision made by the representative consumer at each point on the line segment regarding where to shop. Let t denote consumer transportation cost per unit distance. A consumer at a distance of $\delta \in (0, 1)$ from the representative retailer could achieve surplus of $v(p, m, y) - \delta t$ by purchasing from that retailer. If there are n retailers located about the circle, for consumers located on the interval $0 \leq \delta \leq 1/n$ between a retailer and his nearest neighbor, the surplus available by purchasing from the best alternative to the retailer is $v(\overline{p}, \overline{m}, y) - t(1/n - \delta)$, where $v(\overline{p}, \overline{m}, y)$ is indirect utility evaluated at the prices and product varieties of the retailer's $(n-1)$ rivals. Let δ^* denote the location of the consumer who is indifferent between these two alternatives. Specifically, δ^* solves

$$v(p, m, y) - \delta t = v(\overline{p}, \overline{m}, y) - t(1/n - \delta).$$

Substituting (5) into this expression and solving for δ^* yields

$$\delta^*(p, m; \overline{p}, \overline{m}) = \frac{1}{2n} + \frac{1}{2t} \left[\ln \left(\int_{i \in m} p_i^{\frac{-\theta}{1-\theta}} \right)^{1-\theta} - \ln \left(\int_{i \in \overline{m}} \overline{p}_i^{\frac{-\theta}{1-\theta}} \right)^{1-\theta} \right]. \tag{6}$$

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Notice that the decision on where to shop in (6) depends on the entire range of retail goods available at each retailer. This implies that retailers can steal business from rivals by setting lower prices and by expanding the range of product variety available in the supermarket.

The aggregate demand for product i sold by the representative retailer, counting consumers on both sides, is

$$X_i(p, m) = 2\delta^*(p, m; \overline{p}, \overline{m})x_i(p, m),$$

where $x_i(p, m)$ is given by (4). Accordingly, aggregate demand facing the representative retailer for his entire range of products is $X(p, m; \overline{p}, \overline{m}) = 2\delta^*(p, m; \overline{p}, \overline{m}) \int_{i \in m} x_i(p, m)di$.

Suppose the retailer pays a fixed set-up cost, f, to market an individual product. Product-specific fixed costs may include promotional expenses, inventory costs, staffing expenses, and the opportunity cost of allocating scarce shelf-space.[4] The total cost to the retailer of selling product i in the supermarket when the product is procured at a farm price of w is $c(p, m) = wX_i(p, m) + f$, and the total cost of selling m products is $C(p, m) = 2\delta^*(p, m; \overline{p}, \overline{m})w \int_{i \in m} x_i(p, m) + mf$.[5]

It is helpful in the analysis to follow to consider profit per customer. Let α denote the ad valorem tax rate and τ denote the specific tax rate. Profit per customer for the representative retailer is

$$\pi^m(p, m) = \int_{i \in m} ((1 - \alpha)p_i - w - \tau)x_i(p, m).$$

Notice that profit per customer for each retailer does not depend on rival retailer prices: consumers decide where to shop based on relative prices and varieties across retailers, but are influenced only by prices within the preferred supermarket when

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[4]In general, these fixed marketing costs could be modeled as $f(m)$. Here, this function is assumed to be linear, so that the promotional cost for each brand is constant. In the case of a binding constraint on retail shelf space, the opportunity cost of allocating shelf space to a given brand will generally increase in the short-run with the number of brands, $f''(m) > 0$. The results in this case would not differ qualitatively from those presented below.

[5]Fixed costs may also exist independent of the product range; however, these fixed costs play no role in the analysis and are suppressed. Results from this case are available from the authors upon request.

shopping. Substituting the demand functions (4) into variable profit and differentiating the resulting expression with respect to p_i and m respectively gives

$$\frac{\partial \pi^m}{\partial p_i} = \frac{(w + \tau)x_i}{(1 - \theta)} \left(\frac{1}{p_i} - \int_{i \in m} x_i \right),$$

(7)

$$\frac{\partial \pi^m}{\partial m} = p_m x_m (w + \tau) \left(\frac{1}{p_m} - \frac{1}{\theta} \int_{i \in m} x_i \right).$$

(8)

With a uniform distribution of consumers, total profit for the representative retailer is

$$\Pi(p, m; \overline{p}, \overline{m}) = 2\delta^*(p, m; \overline{p}, \overline{m})\pi^m(p, m) - mf.$$

(9)

The first-order necessary condition with respect to p_i is

$$\frac{-x_i}{t}\pi^m + 2\delta^* \left(\frac{\partial \pi^m}{\partial p_i} \right) = 0, \qquad i \in (0, m],$$

(10)

where $\partial \delta^*/\partial p_i = -x_i/2t$ follows from equation (6). This condition has an intuitive interpretation. Fixing for the moment the varieties of the representative retailer, $2\delta^*$ represents the number of consumers captured by the retailer given the prevailing menu of prices and varieties offered by his rivals. The term $\partial \pi^m/\partial p_i$ is the marginal value of a change in price i to these consumers, the effect of the price change on profit on the intraretailer margin. A multi-product monopoly retailer would set this term equal to zero for each product in his product line. Under oligopoly, this term is positive –prices are set below the monopoly price level– because the first term on the left-hand side of (10) is negative. This term is the interretailer margin of the oligopoly market. Multi-product monopoly prices, which serve to extract maximal rent from existing consumers, now serve also to drive customers away. A small increase in the price of product i augments profits on the intraretailer margin by $2\delta^*\partial \pi^m/\partial p_i$, but also shifts customers to rival retailers, $2(\partial \delta^*/\partial p_j) = -x_i/t$, which decreases sales of all products and reduces the retailer's profits by $-x_i \pi^m/t$ units.

The remaining first-order necessary condition for a profit maximum is

$$\frac{p_m x_m}{t} \left(\frac{1 - \theta}{\theta} \right) \pi^m + 2\delta^* \left(\frac{\partial \pi^m}{\partial m} \right) - f = 0.$$

(11)

where $\partial\delta^*/\partial m = p_m x_m (1-\theta)/2\theta t$ follows from differentiation of equation (6). This condition has the following interpretation. A small increase in product variety generates additional customers of $p_m x_m (1-\theta)/\theta t$ for the retailer, which augments retail profits on the interretailer margin by $(1-\theta)\,p_m x_m \pi^m/\theta t$. The additional variant also (potentially) increases sales among existing customers on the intraretailer margin. For a change in varieties of ∂m units, this augments retail profit among existing customers by $2\delta^*\partial\pi^m$ units. The sum of these interretailer and intraretailer effects of the new variant is the marginal private gain from introducing an additional product. Equation (11) states that this marginal private gain is equal to the marginal private cost of new product introduction, f.

In the symmetric equilibrium, $p_i = p = \overline{p}$ and $m = \overline{m}$, equations (10) and (11) can be simplified as follows. Under symmetry, the price and variety effects on profit per customer in (7) and (8) reduce to

$$
\left.\frac{\partial\pi^m}{\partial p_i}\right|_{p_i=p} = \frac{(w+\tau)x}{p},
$$
$$
\left.\frac{\partial\pi^m}{\partial m}\right|_{p_i=p} = 0,
$$

respectively, where $x_i = x$ denotes the symmetric output per brand.[6] On substitution of terms, (10) and (11) reduce to

$$
(w+\tau)\,t = np\left[\theta(1-\alpha) - mx(w+\tau)\right], \tag{12}
$$

$$
(1-\theta)px\left[\theta(1-\alpha) - mx(w+\tau)\right] = \theta f t. \tag{13}
$$

2.1 No-Entry Oligopoly Equilibrium

The equilibrium price per product, p_e, and equilibrium variety range, m_e, in the symmetric oligopoly equilibrium is determined by the simultaneous solution of equations

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[6]This result is purely an artifact of the logarithmic sub-utility function; in general,

$$
\left.\frac{\partial\pi^m}{\partial m}\right|_{p_i=p} = xp(w+\tau)(1-E) \geq 0,
$$

where the additional sales generated by a new variety among existing customers is proportional to $1-E$. Under logarithmic sub-utility, $E = 1$, aso that new varieties cannibalize perfectly on existing varieties without stimulating total sales.

(12) and (13) at the symmetric demand level defined by equation (4),

$$x = \frac{\theta}{mp} \tag{14}$$

Substituting equation (4) into equations (12) and (13) yields

$$p_e = \frac{(w + \tau)(\theta n + t)}{\theta(1 - \alpha)n}, \tag{15}$$

$$m_e = \frac{\theta(1 - \theta)(1 - \alpha)}{(\theta n + t)f}. \tag{16}$$

Equilibrium output per product, x_e, can then be recovered from equation (4):

$$x_e = \frac{\theta fn}{(1 - \theta)(w + \tau)}, \tag{17}$$

Total sales in the symmetric oligopoly equilibrium, $\chi_e = m_e x_e$, is given by (17) and (16) as

$$\chi_e = \frac{\theta^2 n(1 - \alpha)}{(\theta n + t)(w + \tau)}. \tag{18}$$

and equilibrium profits are given by

$$\Pi_e = \frac{\theta(1 - \alpha)(t - (1 - \theta)n)}{n(\theta n + t)}. \tag{19}$$

Four notable observations follow these results. First, ad valorem and specific taxes produce starkly different results on output per product and the equilibrium number of products: specific taxes reduce output per product, but do not alter the range of product variety available in the market, whereas ad valorem taxes have no effect on output per product, but curtail the variety range. This is important, because conventional models of tax incidence fix the number of products (usually at $m = 1$), which biases the welfare calculation in the favor of ad valorem instruments.

Second, polices that influence farm prices (w) have no effect on the equilibrium product variety range (m_e) or on firm profits (Π_e). This implies that the multi-product structure of retail markets has little bearing for making calculations on the pass-through of agricultural policies into consumer prices.

Third, the degree of heterogeneity across retailers (t) and the degree of hetero-geneity among products (θ^{-1}) interact with both prices and variety provision. In

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Anderson and dePalma (1992), prices rise with the degree of heterogeneity across firms, but do not depend on product differentiation, and the number of varieties increases with product differentiation, but is independent of firm heterogeneity. The difference here is due to the outside good. An increase in retailer heterogeneity (i.e., larger t) raises equilibrium prices, and this shifts consumption away from the retail product category towards the outside alternative. Retailers consequently provide fewer products. Similarly, increased product heterogeneity (i.e., smaller θ) raises retail prices by drawing consumption into the product category from the outside good. Product heterogeneity has an ambiguous effect on the range of products offered by retailers. In Anderson and dePalma (1992) and Ottaviano and Thisse (1999), retailers respond to an increase in product differentiation by increasing the range of retail products. This is true for highly substitutable products; however, for sufficiently differentiated products ($\theta < -t/n + \sqrt{t(n+t)/n^2}$), the number of products offered by each retailer decreases in response to greater product heterogeneity. The reason is that greater product differentiation makes consumer demand per product less price elastic, which raises equilibrium prices. This increases the return to attracting custom by adding new variants to the product line, but also raises the opportunity cost of product cannibalization. Because the latter effect occurs entirely on the intraretailer margin, it becomes dominant for sufficiently large t.

Finally, changes in retail sales and changes in product variety are positively correlated for changes that occur in ad valorem tax rates (α), transportation cost (t), or the degree of product differentiation (θ), but negatively correlated for changes in the number of firms (for instance due to horizontal merger). Moreover, business practices that produce joint parameter changes in the retail market can also create negative correlation between product variety and retail sales. A decrease in retail fixed costs increases the equilibrium range of product variety and reduces output per product, but has no effect on sales, whereas an increase in farm prices does not influence variety, but harms sales; hence, a policy such as slotting allowances that passes retail fixed costs backwards onto suppliers raises product variety and reduces retail

sales when farm prices increase to compensate for the cost of slotting allowances.

2.2   Free-Entry Oligopoly Equilibrium

The symmetric, free-entry oligopoly equilibrium (p_c, m_c, n_c) is determined by (12), (13), and the free-entry condition, which states that profits in (19) are zero. The latter restriction implies an equilibrium number of firms given by

$$n_c = \frac{t}{1-\theta}. \tag{20}$$

Next, (20) can be substituted into (15)-(17) to get

$$
\begin{aligned}
p_c &= \frac{w+\tau}{\theta(1-\alpha)}, \\
m_c &= \frac{\theta(1-\alpha)(1-\theta)^2}{ft}, \\
x_c &= \frac{\theta ft}{(1-\theta)^2(w+\tau)}, \\
\chi_c &= \frac{\theta^2(1-\alpha)}{w+\tau}
\end{aligned}
$$

Under free-entry oligopoly, changes in each of the various parameters lead to qualitatively similar effects on product variety and sales as those which occur under no-entry oligopoly. Increased product differentiation (i.e., smaller values of θ) decreases retail sales, but, depending on the initial degree of product differentiation, smaller values of θ can narrow the product range (when $\theta \leq 1/3$) or widen the product range (when $\theta > 1/3$).

3   Welfare Analysis

Aggregate welfare in the economy is taken to be the sum of consumer surplus and producer surplus. Letting $\phi = (\alpha, \tau)$ denote the vector of policy variables, welfare is given by

$$W(\phi) = \ln\left(\int_{i \in m(\phi)} x_i(\phi)^\theta\right) - w \int_{i \in m(\phi)} x_i(\phi) - nm(\phi)f - \frac{t}{4n}.$$

The first term is consumer utility gross of transportation costs. The second and third terms are wholesale costs and total product set-up costs, respectively, and the

final term is consumer transportation costs (the average traveling distance between consumers and products is $\overline{\delta} = 1/4n$). In the symmetric case, the welfare expression reduces to

$$W = \ln(m) + \theta \ln(x) - m(wx + nf) - \frac{t}{4n}. \tag{21}$$

The socially optimal x, n, and m are defined by the three first-order conditions,

$$W_x: \qquad \frac{\theta}{x} - mw = 0, \tag{22}$$

$$W_m: \qquad \frac{1}{m} - wx - nf = 0, \tag{23}$$

$$W_n: \qquad \frac{t}{4n^2} - mf = 0. \tag{24}$$

In (22), the quantity of each product satisfies $x = \theta/mw$, which implies $p = w$ given the symmetric demand conditions in (14). In (23), the marginal contribution of an additional product to consumer utility is equal to the cost of retailing it, which is the sum of the wholesale cost and the fixed set-up cost of providing the product at each of the n retail stores. Equation (24) defines the optimal number of retailers. An additional retailer on the circle reduces consumer transportation costs, which provides the marginal social benefit of $t/4n^2$, and this must equal the marginal social cost the retailer must incur to stock the social product range, mf.

In general, three distinct effects cause the market equilibrium for product variety to diverge from the social optimum. First, an individual retailer does not account for the profit reduction at other retailers as his range of product varieties increases. This is the standard business-stealing externality in the domain of variety choices. The marginal private benefit of introducing an additional retail product is comprised of rents earned from consumers on the intraretailer margin (greater sales per consumer) and rents earned from consumers on the interretailer margin (attracting customers from rivals). Each retailer fails to account for the lost profit of his rivals when seeking to attract customers to the store, and the resulting zero-sum game leads retailers to excessively proliferate retail products in the market. Second, the marginal private benefit of an additional retail product to existing consumers on the intraretailer

margin generally differs from the marginal social benefit of a new product. In their product introduction decisions, retailers consider only the marginal contribution of the additional sales to profit, or the gross retail margin, $p - w$. Because providing an additional product generates consumer surplus of $p(1 - \theta)/\theta$ units, the marginal social benefit of product variety, the sum of producer and consumer rents, $p/\theta - w$, exceeds the marginal private benefit, and this causes variety to be undersupplied in the market equilibrium. Finally, new variants cannibalize at least partially on sales of existing products. This effect is fully internalized by each retailer on the intraretailer margin, but product cannibalization introduces an additional externality between retailers in seeking to attract customers. Introducing an additional product causes rival retailers to respond by reducing their retail prices on *all* products, and this intensifies retail price competition. Retailers ignore this socially beneficial effect of product introduction, which causes product variety to be undersupplied in the market.

In the case of logarithmic sub-utility, there is no distortion created in the market by product cannibalization, so that only the first two effects exist. Solving equations (22)-(24) simultaneously, the socially optimal resource allocation is defined by

$$
\begin{aligned}
x^* &= \frac{\theta ft}{4(1 - \theta)^2 w}, \\
m^* &= \frac{4(1 - \theta)^2}{ft}, \\
n^* &= \frac{t}{4(1 - \theta)}, \\
\chi^* &= \frac{\theta}{w}.
\end{aligned}
$$

3.1 No-Entry Oligopoly

In this subsection, we compare the oligopoly equilibrium with the socially optimal resource allocation, given a fixed number of retailers. Suppose the number of firms in the market equilibrium happens to be the socially optimal number, $n = n^*$. In this case, the quantity supplied of each product in the market equilibrium (17) is identical to that in the social optimum, but product variety is under-supplied. To

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see this, substitute n^* into equation (16) to get

$$m_e(n^*) = \frac{4\theta(1 - \theta)^2}{(4 - 3\theta)ft}$$

which satisfies $m_e \leq m^*$. Accordingly, with an insufficient number of varieties transacted in the market at optimal output levels, the total sales level is inefficiently low in the market equilibrium.

What are the welfare effects of public policy? Public policies that reduce farm prices proportionally adjust private and social output levels, but the social return is lower than in a competitive retail sector, because the gains from product innovations in crop production are spread across too few products in the market. Excise taxes exacerbate the market distortion in sales. An increase in the specific tax rate reduces output per brand below the welfare maximizing level, while an increase in the ad valorem tax rate reduces product variety.

It is well-known in the literature on tax incidence that ad valorem taxes welfare-dominate specific taxes in single-product oligopoly markets. In the multi-product case, consider the welfare implications of a revenue-neutral shift in taxation from specific taxes to ad valorem taxes at an initial zero tax position ($\alpha = \tau = 0$). This corresponds with the standard welfare prescription in imperfectly competitive markets of combining the use of ad valorem taxes to rotate demand horizontally and unit subsidies to shift the level of demand upward to equate with marginal cost. Making use of equations (17) and (16), excise taxes affect output per brand and the variety range according to $dx_e = -(x_e/w)d\tau$ and $dm_e = -m_e d\alpha$ at the zero tax position. Totally differentiating welfare expression (21) and making use of these results gives

$$dW_e = \left(\frac{\theta}{x_e} - m_e w\right)\left(\frac{-x_e}{w}\right)d\tau - \left(\frac{1}{m_e} - wx_e - nf\right)m_e d\alpha. \tag{25}$$

Next, consider a tax-neutral shift between excise tax instruments. The total tax yield is

$$T = (\alpha p + \tau)mx.$$

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which, upon making substitutions from equations (15)-(17), reduces to

$$T = \theta \left[\alpha + \frac{\theta n(1-\alpha)\tau}{(\theta n + t)(w + \tau)} \right]$$

Differentiating this expression and evaluating terms at the zero tax position, a revenue-neutral shift from unit taxes to ad valorem taxes satisfies

$$d\tau = - \left(\frac{(\theta n + t)w}{\theta n} \right) d\alpha. \tag{26}$$

Making use of equation (26) in (25) and combining terms, a revenue-neutral shift towards ad valorem taxes affects welfare as

$$\left. \frac{dW_e}{d\alpha} \right|_{T=\overline{T}} = \frac{t \left(t - (1 - \theta)n \right)}{n(\theta n + t)} = \frac{t}{\theta} \Pi_e > 0, \tag{27}$$

where the substitution has been made for Π_e in equation (19) at the zero tax position.

Notice that a revenue-neutral shift towards ad valorem taxes increases welfare in proportion to the level of retailer profit. Ad valorem taxes welfare dominate specific taxes in multi-product oligopoly settings due to the superior ability of ad valorem taxes to transfer profits to tax revenue.

What are the implications of nonlinear procurement practices at supermarkets for welfare? Business practices such as slotting allowances that employ nonlinear pricing techniques to reduce retail fixed costs in exchange for higher farm prices produce offsetting welfare effects. A reduction in f increases variety provision in equation (16), which improves market efficiency; however, an increase in w reduces output per product, which harms social welfare. Suppose the nonlinear contracts between retailers and food manufacturers are cost-neutral in the sense that the decline in per product fixed costs is offset exactly by a proportional increase in wholesale prices, so that $dC(w, f) = (\partial c / \partial w)dw + (\partial c / \partial f)df = 0$. This would be the case if the food chain was perfectly competitive up to its penultimate link in the retail sector.

Making use of equations (17) and (16), changes in w and f affect output per brand and the variety range according to $dx_e = \frac{x_e}{f}df - \frac{x_e}{w}dw$ and $dm_e = -\frac{m_e}{f}df$. Totally differentiating welfare expression (21) with respect to w and f and making

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use of these results gives

$$dW_e = \left(\frac{\theta}{x_e} - m_e w\right)\left(\frac{x_e}{f}df - \frac{x_e}{w}dw\right) - \left(\frac{1}{m_e} - wx_e - nf\right)\frac{m_e}{f}df. \qquad (28)$$

Consider a first-order cost-neutral shift in cost that restructures the contracts of existing products to maintain per-product cost constant at $C/m = wx/n + f$. Differentiating this expression with respect to w and f, a first-order cost-neutral shift per product between fixed and unit procurement cost components entails

$$df = -\frac{x}{n}dw. \qquad (29)$$

Making use of equation (29) in (28) and combining terms, a first–order cost-neutral shift between fixed and unit costs in retailer procurement contracts satisfies

$$\left.\frac{dW_e}{dw}\right|_{dc(w,f)=0} = 0.$$

Slotting allowances and other non-linear pricing arrangements have no welfare implications in the special case of logarithmic sub-utility. (See concluding remarks for how this outcome changes in the general case.)

3.2   Free-Entry Oligopoly

Under conditions of free-entry, the oligopoly retail market equilibrium compares with the socially optimal resource allocation as follows. By inspection, the number of firms in the free-entry oligopoly equilibrium is excessive $(n_c > n^*)$. This, in turn, implies an excessive quantity supplied for each product in the market equilibrium $(x_c > x^*)$ and insufficient product breadth per retailer $(m_c < m^*)$. Unlike the case of single-product retailers, output per product is oversupplied in the oligopoly equilibrium, although it remains the case that retail sales overall are inefficiently low $(\chi_c < \chi^*)$.

The intuition for this result is straightforward. As in Salop (1979), the equilibrium price under monopolistic competition exceeds the social price, which provides an incentive for retail entry. Retail entry in turn places downward pressure on market prices, so that prices in excess of unit costs, $p = w/\theta$, can be maintained either by

reducing the quantity supplied of each product or by reducing the range of product variety. By (17), the quantity supplied per retailer per product, x/n, is constant, and it follows that entry must increase the quantity supplied of each product in the market and the range of products offered by each retailer must fall.

Next, consider the effect of parameter changes on the long-run welfare outcome. By inspection of the free-entry oligopoly equilibrium number of firms, public policy and nonlinear pricing practices have no implications for the equilibrium number of firms. Proceeding as before, a first–order cost-neutral shift between fixed and unit costs in retailer procurement contracts has no implications for welfare in the case of logarithmic sub-utility.

Interestingly, public policies that reduce farm prices exacerbate the problem of excessive output per product. Nonetheless, because the social return to an increase in output in equation (22) depends both on output and on the number of products retailers offer, a policy that reduces the farm price increases welfare

$$\frac{dW_c}{dw} = -\theta(1-\theta)w.$$

Now consider the long-run effect of excise tax reforms. Under conditions of free-entry, a revenue-neutral shift away from specific taxes and towards ad valorem taxes has no effect on welfare. To see this, substitute the equilibrium number of firms into equation (27). The intuition is that ad valorem taxes welfare dominate specific taxes in multi-product oligopoly settings due to the superior ability of ad valorem taxes to transfer profits to tax revenue. As profit in the industry tends to zero, the welfare gain arising from a revenue-neutral shift towards ad valorem taxes evaporates. It follows that ad valorem taxes and specific taxes are welfare-neutral in the long-run.

4   Conclusion

This paper has considered multi-product retail oligopoly competition with localized competition between retailers for custom and the potential for multiple purchases for each consumer. In single product retail environments, oligopoly firms fail to account

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for the detrimental effect of price discounts on the profits of rival firms. Multi-product retailing introduces a second dimension to this familiar business stealing externality, as business stealing can occur both through price reductions and through the provision of a larger range of retail goods.

This paper has considered the case of logarithmic sub-utility. How do the outcomes in this special case compare to the outcomes under more general specifications of utility, such as CES? Table I provides a description of comparative statics effects for the general case, where the results reduce to those presented here when the elasticity of the sub-utility function satisfies $E = 1$. An increase in either the unit procurement price of retail goods (w) or the fixed marketing costs per product (f) raises prices, reduces product variety, and decreases total retail sales per product retailing costs leads to higher prices, fewer varieties, and a decrease in industry sales. In the long-run, retailer entry nonetheless occurs in response to either type of cost increase.

Table I
Comparative statics results
Endogenous variables

		No-Entry Oligopoly			Free-Entry Oligopoly			
		p	m	χ	p	m	n	χ
Exogenous	w	$+$	$(-,0)^1$	$-$	$+$	$(-,0)^1$	$(0,+)^1$	$-$
variables	f	$(0,+)^1$	$-$	$(-,0)^1$	0	$-$	$(0,+)^1$	$(-,0)^1$
	t	$+$	$-$	$-$	0	$-$	$+$	$(-,0)^1$
	n	$-$	$-$	$(-,+)^2$	——	——	——	——

Notes:
1. zero if $E = 1$
2. positive for E sufficiently close to one.

The entry implications of the multi-product retail model in the general case are worth discussing. Somewhat perversely, higher retailing costs stimulate entry. The reason is that firms provide narrower product variety ranges in response to higher costs, which both softens price competition and reduces the fixed costs associated with multi-product entry. Given that the number of firms is excessive in the free-entry case, these entry effects have important implications for welfare. For instance,

excise taxes stimulate entry, exacerbating market distortions in the multi-product case relative to the outcome in single-product markets. (Among other things, this causes taxes to be shifted to a greater degree into market prices.)

The main results on tax incidence go through unaltered in the general case. Ad valorem taxes are welfare superior to specific taxes levied on multi-product firms when the number of firms is fixed at a level generating positive economic profits, but ad valorem and specific taxes are welfare-equivalent under conditions of free-entry.

The effect of contracts with nonlinear pricing is modified significantly by entry in the general case. When retailers receive lump-sum payments in exchange for shelf space or receive payments in-kind through marketing activities, product display, and category management, the overall implication of these practices on retail costs is unclear and depends on the degree to which manufacturers are able to pass these costs back to retailers through higher wholesale prices. To the extent that changes in the structure of wholesale transactions are cost-neutral in the retail industry, the long-run effect of a practice that decreases per-product retail fixed costs in exchange for higher wholesale prices is to raise retail prices, decrease retail sales, and induce retailer entry. The range of varieties stocked by each retailer rises when the degree of product cannibalization is sufficiently large (as in the logarithmic case), and otherwise falls. In general, such practices have unambiguously negative implications for social welfare and the result presented here with no welfare implications arises only in the logarithmic case. The more general outcome suggests careful scrutiny is warranted under prevailing anti-trust laws. Indeed, cost-neutral shifts designed to enhance long-run efficiency in the retail sector would do just the opposite in the general case, absorbing fixed manufacturing costs into the retail market in exchange for wholesale price reductions.

5   References

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