Portfolio selection with growth optimization and downside protection

by

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Abstract

This paper applies growth optimization with downside protection as a portfolio selection technique. The model is based on power-log utility functions that combine portfolio growth maximization with the behavioural tenets of prospect theory. We use three assets (a farm return index, a stock market index, and a Treasury bond index) to illustrate how effective this technique is compared to the standard model of growth maximization.

Keywords: portfolio management, growth optimization

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Introduction

A problem facing many farmers today is the risk-return imbalance often associated with farm asset returns. When considered as independent, individual investments, the returns to farm assets often prove uncompetitive with the return performances of non-farm assets given the level of risk involved (Bjornson and Innes, 1992). The historic variability of returns to farm assets appears set to continue as competition in agriculture increasingly occurs on a global scale and traditional sources of risk remain. Encouraging farmers to use effective investment and portfolio planning methods should improve the risk-return efficiency of farm and total assets as well as force rationality upon current capital allocations in the face of competition from alternative investments.

If investors take a long-time view on their capital, maximization of the rate of growth of available investment opportunities is an operational criterion for multiperiod portfolio selection. Growth maximization means that the investor maximizes the probability of exceeding a certain wealth (capital) within a fixed time (Breiman, 1961). Furthermore, the criterion implies decision rules that are myopic (Hakansson, 1971). Ways to construct portfolios that maximizes long-term growth are well documented (e.g. Hakansson, 1971; Luenberger, 1998; Hunt, 2002). A standard approach in multiperiod portfolio theory is to determine a portfolio that maximizes the expected utility for a specified utility function (Ingersoll, 1987; Moss, Featherstone and Baker, 1987). Results from Luenberger (1993; 1998) shows that it is rational for an investor who considers only long-time performance to evaluate the portfolio "based on its logarithm of single-period return using only the expected value and the variance of this quantity" (Lunberger, 1988, p. 427) given that investment alternatives are independent and identically distributed between periods. The latter requirement becomes intriguing when applying standard growth maximization to a portfolio selection where farm assets are included. Our data set of farm level data on return to farm assets

(ROA) suggests that the return distributions may have significant higher moments and that characteristics of the return distributions varies over time. Besides invalidating the log mean-variance efficiency criterion as suggested by Luenberger (1998), such a situation also invalidates the mean-variance model by Markowitz (1952).

Recently, interest in combining long-term portfolio growth maximization with the behavioral implications of prospect theory has been developed (Kane, 2006). Tversky and Kahneman (1992) defined an S-shaped utility function characterized by reference dependence and loss, as well as, risk aversion. An additional characteristic of the S-shaped utility function is that of diminishing sensitivity to both gains and losses. Kane (2006) introduced growth optimization with downside protection as a portfolio selection technique based on power-log utility functions which is an interesting extension of Tversky and Kahneman's approach if the objective of the investor is to construct portfolios that reflect both desire for high growth over long term and monotonic aversion to losses. Kane's model treats portfolio gains and losses asymmetrically. With positive returns the model is identical to the log-optimal growth portfolio but with negative returns, a power utility function is proposed such that lower (more negative power) values of downside protection represent greater loss aversion, while the utility value of gains remains unchanged. The myopic property of both the log and power utility function allows the optimal portfolio selection problem to be solved as a one-period problem.

Maximizing long-term portfolio growth is a reasonable goal for agricultural investors given the fixity of major farm assets. Considering the importance of expected portfolio growth to farm operators, it is surprising that so few examples of studies that focus on the empirical strategies to maximize farm household portfolio growth exist. This study aims to redress this need by applying the growth optimization with downside protection technique to investigate the possibilities of obtaining risk-return efficiency benefits and gains through holding more optimal combinations of farm and financial assets (common stocks and long-term government bonds). It should be noted that we derive optimal portfolios imposing a no-short sale constraint. Improving and maintaining farm asset return performances may consume the majority of the farmer's capital budget. This leaves limited capital for acquiring the non-farm assets. Moreover, we are not aware of that farmers engage in short-selling assets to a considerable extent. The model in this study represents a way of constructing portfolios that reflect both desire for high growth over long term and aversion to losses. No other studies have been found that have looked at the optimal combination of assets using real data for in a dynamic setting including the behavioral tenets of prospect theory. The model applied is solved by simulation under optimization in order to recognize higher-order moments of the return distribution in the portfolio selection.

Our main results are: (a) the farm asset is not competitive in the growth optimal portfolio without downside protection; (b) if downside protection is introduced as an objective the situation changes so that the farm asset becomes more competitive; and (c) the classification of farms according returns has a significant impact on the portfolio returns as well as on the degree of competitiveness of the farm asset in relation to financial assets.

Methodology

The class of power utility functions $U(X) = (1/\gamma) X^{\gamma}$ with $\gamma \leq 1$, where γ denotes the degree of power is flexible. It includes the logarithmic utility function since $\lim_{\gamma \to 0} \{(1/\gamma) X^{\gamma} - 1/\gamma\} = \ln X$; and it includes the risk neutral linear utility $U(X) = X$ for $\gamma = 1$ (Luenberger, 1998). Selecting the degree of power to zero for portfolio selection purposes will therefore result in the growth optimal portfolio. The power utility function with a more negative value implies an increasingly conservative behavior. Suppose that $\gamma = -2$, and that an investor has two opportunities: (*i*) capital will double with certainty, and (*ii*) with probability 0.5 capital remains constant and with probability 0.5 capital is multiplied by 10 Million. Since - $0.5*2^{-2} > -0.5*0.5 - 0.5*0.5*(10*10^6)^{-2}$, an investor will prefer (i). Similarily, if $\gamma = -20$ and assuming the same possible outcomes, since $-(1/20)*2^{-2} > -0.5*(1/20) - 0.5*(1/20)*(10*10^6)^{-2}$, an investor will prefer (*i*) but now the utility difference in favor of (*i*) is larger. The power utility function thus has higher degrees of risk aversion (related to positive augments) for more negative powers. On the other hand, a more negative power implies greater aversion to losses.

Following the work by Kane (2006) growth optimization with downside protection is based on a two-segment utility function. The utility of gains is modeled by a log utility function while the utility of losses is modeled with a power utility function (with degree of power less than or equal to zero). The model is

$$U = \ln(1 + r_s) \quad \text{for } r_s \geq 0$$

$$= \frac{1}{\gamma}(1 + r_s)^{\gamma} \quad \text{for } r_s < 0 \qquad (1)$$

$$\text{with } r_s = \sum_i w_i r_{i,s}$$

where r_s is portfolio return in state *s*, γ is the degree of power, less than or equal to zero, w_i is portfolio weight of asset i in the portfolio, and $r_{i,s}$ is return on asset *i* in state *s*. It should be noted that these utility functions are continuously differentiable and that the power utility function by a transformation have a value of zero when the return is zero. Applying this model in portfolio selection will allow investors to take benefit from growth maximization if returns turn out positive since the utility function for gains is always the log utility function, while it also allow for a shield against adverse outcomes through the specification of degree of downside protection because the power utility function then penalizes losses.

Figure 1 displays an example of log and power utility functions (degrees of power set at -2 and -20, respectively). The growth optimization with downside protection resembles the behavioral tenets of prospect theory, except for that there is no diminishing loss aversion. Instead, lower values of the degree of power in the power utility function represent greater loss aversion, i.e. an increasing sensitivity to losses as the size of the losses increases.

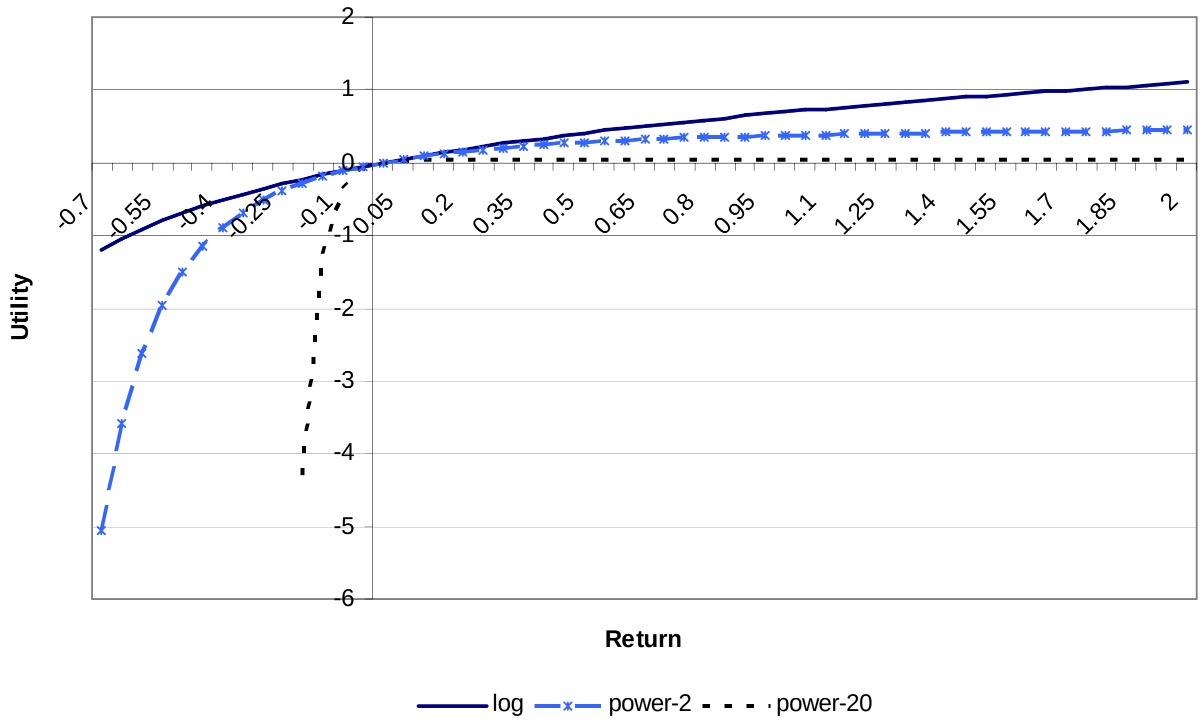

Figure 1. Log and power utility functions. Power utility functions illustrated with degrees of power equal to -2 and -20, respectively.

Data

Returns from three investment alternatives are considered in this study; farm assets, common stocks, and government bonds. Annual data from 1969 through 2001 for common stocks and government bonds are from www.economagic.com. Data series extending further than 2001 was unavailable. Farm data are from the Southwestern Minnesota Farm Business Management Association records. The time series initially collected covers the period 1984 through 2001 only sole proprietors are included. Approximately 220 farms in 12-15 Minnesota counties annually submit firm information with which the data has been developed. The farms from which data was derived are primarily involved in soybean, corn, hogs, cattle, milk, and combinations of these and other activities of lesser importance.

The rates of return to farm assets are the return on farm assets divided with average farm investment, where the return to farm assets equals the net of net farm income plus farm interest minus value of operator's labor and management, and average farm investment equals the average of beginning and ending total farm assets. Farm assets are set at fair market values. Data are available for each individual farm operation as from 1993.

Common stock returns are based upon the Standard and Poor's Composite Index, including 500 of the largest companies in the U.S. Capital appreciation and dividends are included in the returns. The bond return includes coupon payments (interest) and appreciation and is based upon the 10-year-to-maturity U.S. Government Index.

Simulation

For each year included in the analysis the individual farm return data was used to fit a probability distribution by the *chi-square* method using the @Risk software (Palisade). Distribution fitting was performed for two subsets of the farm assets for subsequent use in the simulation process. First, the return distributions were fitted for all farms. Secondly, return distributions were fitted for farms with above median returns. It is noted that the return distribution for the latter subset exhibit significant higher-order moments. Table 1 displays the characteristics of the fitted probability distributions. Furthermore, an ARIMA analysis did not indicated any deviations from the normality assumptions for returns on common stocks and government bonds. Table 2 summarizes the mean and standard deviations for the returns on common stocks and government bonds. For each period *k* the mean and standard deviation is calculated based on the data from the *n* previous periods using 1970 as the first date of observation.

Table 1. Probability distributions fitted to farm asset returns. Southwestern Minnesota Farm Business records 1993 through 2001

Year	*Distributions (All farms)*	*Distributions (Farms with above median returns)*
1993	Logistic(α =0.036044; β = 0.046258)	Pearson5(α =3.2599; β = 0.17703)
1994	Logistic(α =0.037992; β = 0.036252)	Gamma(α =1.0643; β = 0.041451;shift =0.039475
1995	Logistic(α =0.064561; β = 0.038645)	InvGauss(μ =0.055224; λ = 0.035245; shift = 0.060819)
1996	LogLogistic(γ = -0.24251; β = 0.3103; α =8.0095)	Pearson5(α =1.8081; β = 0.074922; shift = 0.05274)
1997	Logistic(α =0.069506; β = 0.039147)	Expon(β = 0.056332; shift = 0.06665)
1998	Logistic(α =0.029373; β = 0.037082)	Expon(β = 0.045449; shift = 0.031066)
1999	Logistic(α =0.053059; β = 0.032380)	ExtValue(a = 0.078696; 0.024474)
2000	LogLogistic(γ = -0.71779; β = 0.79027; α =24.511)	InvGauss(μ =0.062738; λ = 0.093039; shift = 0.058905)
2001	Logistic(α =0.044565; β = 0.032021)	Expon(β = 0.042197; shift = 0.045005)

Table 2. Continous time mean return and associated standard deviations (in parenthesis) of common stocks (Standard and Poor's Composite Total Return Index and the 10-year-to-maturity Total Return Index for U.S. Government Bonds)

Year	Common stocks	Government bonds
1993	0.108517 (0.152173)	0.093987 (0.105983)
1994	0.107996 (0.14899)	0.089487 (0.105973)
1995	0.104201 (0.147159)	0.091393 (0.104251)
1996	0.112463 (0.150098)	0.088688 (0.103117)
1997	0.115953 (0.148363)	0.088339 (0.101205)
1998	0.122094 (0.149144)	0.09022 (0.099861)
1999	0.126552 (0.148436)	0.088641 (0.098479)
2000	0.128699 (0.146398)	0.086122 (0.09777)
2001	0.121468 (0.149364)	0.087177 (0.096353)

To simulate the joint return distribution for the three assets, we first tested if the index series are independently distributed. If investment returns are correlated across time, there is a need of adjusting the subsequent analysis to take into account the contemporaneous correlations. A system of vector-autoregressive (VAR) regressions was estimated. An equation for each asset's rate of return was estimated as a function of all assets previous year's returns using ordinary least squares, following Moss, Featherstone and Baker, 1987. The residuals from the VAR systems were then used to estimate contemporaneous covariance matrices. The data do not reject the null hypothesis that the rates of returns for the three assets are uncorrelated.

We assume that the decision problem is, in the beginning of each period, to successively rebalance the three investment opportunities with a rebalancing period of one year (by data availability). In addition, a short-selling constraint is imposed to make the strategy more realistic. At any period *k*, the following steps are undertaken:

1. The mean and standard deviation of the continuous return data for common stocks and government bonds on the previous *n* periods (years) are employed to fit a normal distribution for each asset type, respectively.
2. Expectations for farm asset returns in period *k* are based on the probability distribution corresponding to period *k-1*.
3. Probability distributions from steps 1 and 2 are used to maximize the mean of the simulated distribution corresponding to Eq.(1) by using the RiskOptimzer software (Palisade). A budget solving method is applied to ensure that the aggregated portfolio weights sums to one, and a non-negativity constraint is imposed on each asset type to ease interpretation. 1,000 simulations with 200 iterations in each simulation are performed for each choice of downside power. Each simulation uses the same random number seed.
4. The time-frame is moved forward one year.

Steps 1 to 5 are repeated until the data set is exhausted.

Results

Table 3 displays the optimal portfolios constructed using power-log utility functions for downside powers of 0 to -20 for each year, respectively. For each year, two subsets of optimal portfolios are simulated. The first part, denoted ‘All’, refers to the case where farm asset returns are represented by the return distribution for all farm operations. The second subset, denoted ‘Best 50%’, refers to the case where the farm asset return distribution is derived from farm operation with above median returns.

The first notable result is associated with the optimal portfolios in the first row of each year represented in Table 3. These portfolios have been constructed with a downside power of 0, which makes them to represent the growth optimal portfolios. These portfolios are, more or less exclusively, based on common stocks. These portfolios are very risky with a wide range of possible returns (given by the difference between the maximum return and the minimum return).

Second, introducing downside protection by increasing the negative power substantially alters the portfolio composition. For the subset of optimal portfolios using all available farm asset returns it is observed that the farm asset is the most competitive for lower (less negative) degrees of downside power. It is then observed that the farm asset portfolio weight decreases as the downside power becomes stronger (more negative). However, for the extreme downside power of -20, the farm asset weight is then again higher. For the subset of portfolios that is based on the above median farm asset returns, the situation is similar. The farm asset is then, however, much more strongly competitive across the included range of downside powers.

Third, optimal portfolios constructed with downside protection reveals substantially different return characteristics compared to the growth optimal portfolios. Typically, the expected return is 2 to 3 percentage units less for a portfolio with downside protection versus a growth optimal portfolio but the standard deviation is, more or less, only one-half, or even lower for the subset of portfolios based on above median farm returns. In addition, the range of possible outcomes also varies substantially between growth optimal portfolios and portfolios with downside protection. The latter portfolios reveal narrower return ranges. This finding is emphasized for the subset of portfolios based on the above median farm returns. Note also that the minimum return for portfolios constructed using subsets of farm asset returns above median is very different from the minimum return from portfolios based on all farm asset returns.

Table 3. Return and portfolio composition for optimal portfolios

1993	downside power	Return mean	standard deviation	min	max	Weights farm weight	stock weight	bond weight
All	0	0.108517	0.154686	-0.30716	0.556132	1.61E-06	0.999994	3.9E-06
	-0.1	0.083471	0.068601	-0.09744	0.245527	0.246666	0.259922	0.493412
	-0.5	0.08675	0.073919	-0.11114	0.277151	0.179991	0.219706	0.600303
	-1	0.093984	0.085835	-0.14929	0.328423	0.050654	0.201763	0.747583
	-2	0.100775	0.09124	-0.13393	0.302312	0.000488	0.46914	0.530371
	-4	0.100844	0.091622	-0.1336	0.300765	0.001214	0.476752	0.522035
	-20	0.09595	0.083202	-0.1209	0.291377	0.064432	0.392045	0.543524

Best 50%	downside power	Return mean	standard deviation	min	max	Weights farm weight	stock weight	bond weight
	0	0.108516	0.15468	-0.30714	0.556113	1.58E-05	0.99996	2.42E-05
	-0.5	0.083704	0.05207	0.00016	0.362741	0.806055	0.160548	0.033398
	-2	0.083693	0.049824	0.000399	0.346545	0.776824	0.128306	0.09487
	-20	0.092429	0.061809	-0.05791	0.263585	0.40898	0.333341	0.25768

1994	downside power	Return mean	standard deviation	min	max	Weights farm	stock	bond
All	0	0.107996	0.150193	-0.27697	0.61779	2.03E-07	1	6.73E-08
	-0.1	0.07732	0.062274	-0.08907	0.222951	0.319562	0.231739	0.448699
	-0.5	0.079479	0.065638	-0.09126	0.241648	0.273289	0.21962	0.507091
	-1	0.075951	0.060679	-0.08815	0.213858	0.345555	0.230049	0.424396
	-2	0.098178	0.090115	-0.13437	0.298712	4.68E-05	0.469665	0.530289
	-4	0.098109	0.090008	-0.13413	0.298408	0.00116	0.469078	0.529762
	-20	0.090864	0.078814	-0.11045	0.268976	0.115943	0.396965	0.487091

Best 50%	downside power	Return mean	standard deviation	min	max	Weights farm	stock	bond
	0	0.107996	0.149514	-0.29898	0.479874	4.73E-07	0.999992	7.63E-06
	-0.5	0.088905	0.043826	0.000981	0.254585	0.775848	0.21569	0.008463
	-2	0.088855	0.042831	0.000159	0.250128	0.761019	0.208256	0.030725
	-20	0.091108	0.050655	-0.02113	0.246541	0.655932	0.296497	0.047572

Table 3 (continued). Return and portfolio composition for optimal portfolios

1995	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
All	0	0.104201	0.149587	-0.29777	0.537061	8.11E-06	0.999982	9.6E-06
	-0.1	0.079521	0.054287	-0.06866	0.215161	0.553984	0.233648	0.212367
	-0.5	0.08434	0.058698	-0.07612	0.198369	0.391375	0.269225	0.339399
	-1	0.084488	0.058928	-0.07626	0.199282	0.387204	0.272047	0.340749
	-2	0.083987	0.058271	-0.07523	0.194853	0.406355	0.273092	0.320553
	-4	0.087952	0.066455	-0.08433	0.25036	0.24625	0.247209	0.506541
	-20	0.089111	0.067779	-0.08102	0.230846	0.252517	0.350855	0.396629

	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
Best 50%	0	0.104201	0.147676	-0.29777	0.471508	1.3E-05	0.999987	2.99E-07
	-0.5	0.116042	0.065054	0.06469	0.459986	0.999969	9.73E-07	3.05E-05
	-2	0.116043	0.065056	0.06469	0.45999	0.99999	9.28E-06	1.1E-06
	-20	0.11604	0.065056	0.06469	0.45999	0.99999	3.5E-06	3.46E-06

1996	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
All	0	0.11201	0.149619	-0.29043	0.54292	0.00014	0.98085	0.01901
	-0.1	0.08852	0.058559	-0.05452	0.26921	0.54103	0.28405	0.17492
	-0.5	0.08767	0.056562	-0.05591	0.26115	0.53906	0.24734	0.21359
	-1	0.09009	0.060065	-0.05977	0.25086	0.46072	0.30711	0.23216
	-2	0.08858	0.058716	-0.05445	0.26965	0.54082	0.28649	0.17268
	-4	0.08872	0.058949	-0.05459	0.26942	0.53722	0.29046	0.17233
	-20	0.09291	0.066735	-0.07194	0.24593	0.38299	0.38358	0.23342

	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
Best 50%	0	0.11201	0.149314	-0.28958	0.54190	0.00089	0.97888	0.02023
	-0.5	0.14545	0.121364	0.06327	0.46284	0.99990	7.67E-05	2.23E-05
	-2	0.14545	0.121376	0.06326	0.46291	0.99999	4.77E-06	1.65E-06
	-20	0.14545	0.121375	0.06327	0.46290	0.99999	6.96E-06	5.85E-06

Table 3 (continued). Return and portfolio composition for optimal portfolios

1997	downside power	Return mean	Return standard deviation	Return min	Return max	Weights farm	Weights stock	Weights bond
All	0	0.115953	0.150813	-0.28931	0.552362	2.93E06	0.999997	1.4E-07
	-0.1	0.088012	0.058077	-0.06706	0.212467	0.45008	0.295129	0.25479
	-0.5	0.08801	0.058077	-0.06706	0.21247	0.45008	0.29513	0.25479
	-1	0.08806	0.058037	-0.06748	0.21062	0.44389	0.29255	0.26355
	-2	0.08681	0.056601	-0.06594	0.21578	0.48493	0.27524	0.23983
	-4	0.08781	0.058034	-0.06619	0.21611	0.46471	0.29772	0.23758
	-20	0.09546	0.073213	-0.08411	0.26670	0.27253	0.44388	0.28359

	downside power	Return mean	Return standard deviation	Return min	Return max	Weights farm	Weights stock	Weights bond
Best50%	0	0.11595	0.150813	-0.28931	0.55236	3.24E06	0.99999	1.86E06
	-0.5	0.12298	0.055017	0.066917	0.35346	0.99999	8.8E-06	6.12E07
	-2	0.12298	0.055016	0.06692	0.35346	0.99997	3.22E-05	2.04E07
	-20	0.12298	0.055017	0.06692	0.35346	0.99998	1.66E-05	3.89E07

1998	downside power	Return mean	Return standard deviation	Return min	Return max	Weights farm	Weights stock	Weights bond
All	0	0.12209	0.151607	-0.28531	0.56080	2.7E-08	1	1E-07
	-0.1	0.07879	0.060829	-0.08402	0.21948	0.30988	0.23298	0.45714
	-0.5	0.07980	0.061839	-0.08432	0.22571	0.29269	0.23191	0.4754
	-1	0.10229	0.084216	-0.12282	0.30921	0.00024	0.37919	0.62057
	-2	0.10226	0.084198	-0.12288	0.30941	8.72E05	0.37805	0.62186
	-4	0.10234	0.084259	-0.12276	0.30897	0.00027	0.38088	0.61885
	-20	0.10336	0.085336	-0.1213	0.30341	0.00228	0.41659	0.58113

	downside power	Return mean	Return standard deviation	Return min	Return max	Weights farm	Weights stock	Weights bond
Best50%	0	0.12209	0.151607	-0.28531	0.56080	8.85E07	0.99999	7.23E10
	-0.5	0.08555	0.044195	0.00123	0.26287	0.79879	0.19715	0.00406
	-2	0.08553	0.043912	0.00116	0.26175	0.79554	0.19494	0.00952
	-20	0.10046	0.073231	-0.07304	0.28456	0.33138	0.46387	0.20475

Table 3 (continued). Return and portfolio composition for optimal portfolios

1999	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
All	0	0.126552	0.148959	-0.27891	0.49705	3.66E07	0.999997	2.15E06
	-0.1	0.08346	0.04906	-0.04851	0.198469	0.44459	0.28065	0.27475
	-0.5	0.08076	0.046984	-0.04418	0.19196	0.50372	0.26479	0.23149
	-1	0.08235	0.048386	-0.04606	0.19866	0.47629	0.28100	0.24270
	-2	0.09949	0.070886	-0.07258	0.24704	0.11215	0.39137	0.49647
	-4	0.10489	0.080225	-0.0834	0.26505	3.44E05	0.42856	0.57140
	-20	0.10492	0.080248	-0.08361	0.26535	0.00062	0.43001	0.56937

	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
Best50%	0	0.12655	0.150888	-0.27892	0.56317	8.61E09	1	8.76E08
	-0.5	0.10289	0.04777	0.001023	0.24072	0.69949	0.29862	0.00188
	-2	0.10278	0.047458	0.00022	0.23862	0.69108	0.29679	0.01212
	-20	0.10778	0.067226	-0.05034	0.30753	0.55646	0.44353	7.79E-06

2000	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
All	0	0.12869	0.148816	-0.2712	0.55933	1.7E-10	1	1.49E10
	-0.1	0.09350	0.055677	-0.03992	0.23257	0.54128	0.31923	0.13949
	-0.5	0.09341	0.05664	-0.03699	0.24187	0.57912	0.32724	0.09363
	-1	0.09363	0.056193	-0.03901	0.23565	0.55075	0.32467	0.12458
	-2	0.09370	0.055353	-0.04318	0.22371	0.50299	0.31366	0.18335
	-4	0.09370	0.055567	-0.0419	0.22735	0.51709	0.31741	0.16550
	-20	0.10416	0.077091	-0.07934	0.30350	0.28367	0.5	0.21633

	downside power	Return: mean	standard deviation	min	max	Weights: farm	stock	bond
Best50%	0	0.12869	0.148816	-0.2712	0.55933	3.64E08	1	6.71E09
	-0.5	0.12415	0.059477	0.00211	0.32089	0.64346	0.35653	1.22E06
	-2	0.12418	0.059782	0.00144	0.32067	0.64049	0.35949	8.49E06
	-20	0.12420	0.060156	0.00061	0.32040	0.63691	0.36309	8.31E08

Table 3 (continued). Return and portfolio composition for optimal portfolios

2001	downside power	Return mean	standard deviation	min	max	Weights farm	stock	bond
All	0	0.12147	0.151831	-0.28653	0.56082	3.1E-07	0.99999	2.44E07
	-0.1	0.07611	0.052376	-0.0664	0.17874	0.46019	0.24919	0.29062
	-0.5	0.07611	0.052456	-0.06614	0.18004	0.46337	0.25304	0.28359
	-1	0.07451	0.050957	-0.06634	0.17682	0.47941	0.22642	0.29417
	-2	0.09912	0.081232	-0.11747	0.29814	0.01468	0.36659	0.61873
	-4	0.09982	0.082294	-0.11998	0.30161	0.00207	0.37141	0.62652
	-20	0.09954	0.081285	-0.10851	0.27920	0.05120	0.42424	0.52455

	downside power	Return mean	standard deviation	min	max	Weights farm	stock	bond
Best50%	0	0.12148	0.150073	-0.22625	0.56089	2.36E06	0.99999	4.38E06
	-0.5	0.09725	0.049757	0.00112	0.26233	0.70643	0.29331	0.00016
	-2	0.09707	0.049097	0.00073	0.26108	0.70556	0.28821	0.00623
	-20	0.10185	0.065486	-0.02761	0.29333	0.57321	0.42671	7.84E06

Conclusion and discussion

Many developed countries consistently expend a tremendous amount of resources in assisting farmers with the reduction and management of risk to enhance of farm incomes. An underlying goal of such policies is to improve farm asset risk-return imbalances. However, using policy mechanisms to stabilize and enhance farm asset returns has met with mixed results and often times proved counter productive (Clark, Klein, and Tompson, 1993; Kalaitzandonakes, 1994; Featherstone, Moss, Baker, and Preckel, 1988). In fact, the existence of such policies often acts as an additional source of uncertainty and risk as farmers are not guaranteed the continuance or consistency of the existing policies (Lagerkvist, 2005). Encouraging alternative means of decreasing the volatility of farm cash flows and improving the performance of farmer's total assets could be beneficial to all concerned parties.

This paper employs an optimal growth portfolio model with downside protection and simulation under optimization approach to investigate the possibilities of obtaining risk-return efficiency benefits and gains through holding more optimal combinations of farm and financial assets. Our main results are: (a) the farm asset is not competitive in the growth optimal portfolio without downside protection; (b) if downside protection is introduced as an objective the situation changes so that the farm asset becomes more competitive; and (c) the classification of farms according returns has a significant impact on the portfolio returns as well as on the degree of competitiveness of the farm asset in relation to financial assets. The farm asset is highly competitive in the downside protection portfolios when data is taken from farms with return on assets above median. Substantial risk reduction is obtained from portfolios with downside protection.

For many farms, improving and maintaining farm asset return performances at competitive levels may consume the majority of the farmer's capital budget. This leaves limited capital for acquiring the non-farm assets that optimal portfolio solutions suggest are in order. However, this does not dismiss the fact that risk-reduction benefits may be realized for all classes of farms by incorporating other assets into the farmers' total portfolios.

Classes of farms that exhibited strength in the portfolio setting were found to perform well as independent investments. Therefore, the results in this respect may serve to underscore the benefits of holding certain sizes and types of farm operations or assets. There would, therefore, seem to be incentives for farmers to expand the farm operation in order to achieve the scale economies and accompanying return performances that farm assets must yield to effectively compete with alternative assets.

Many farmers will continue to appear to engage in irrational investments. These farmers may gain from securing even limited diversification measures, where such optimal strategies are developed by adjusting the model, model inputs, and constraint set to fit the individual farm and decision maker.

The poor performance of farm assets in the analysis may not be an accurate reflection of the risk-return efficiency position of these farms. Farms generally take the form of family farm economic units, where on small farms off-farm labor income can often supercede income from farm assets in importance. The potential interdependence between off-farm income cash flows and farm-based cash flows may significantly improve the risk-return efficiency stance of many farms and should be recognized in drawing conclusions from the present study.

Finally, to provide a perspective on the results developed here future work should be directed to a comparison of our results with optimal portfolios using mean-variance analysis to compare our results with matched expected returns. We also would like to exert effort in comparing our results to previous notable studies (e.g. Moss, Featherstone, and Baker, 1986).

Further distinguishing the classification of farms based upon financial leverage levels, or type of farm specialization, for example may provide interesting results. Highly leveraged farms should display more variable return cash flows and therefore prove less competitive in the portfolio environment. Classifying farms in this way will allow us to analyze the impact of capital structure on farm asset risk-return efficiency.

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