# AJAE Appendix for

# Cross-subsidization Due to Infra-Marginal Support in Agriculture: A General Theory and Empirical Evidence[1]

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## Appendix A

This appendix provides an exhaustive description of the potential farm behaviors in both the long and short run under the unrestrictive target price, and the policy conditions required for each. We note the cases appearing in Results 1 and 2 of the paper as they are discussed. Let the target price be given by $T$, and the market price by $P < T$. Thus the farms profit function under the infra-marginal policy can be written

$$\pi_T(Q) = \begin{cases} P(Q-B)+TB-C(Q)-FC & \text{if} \quad Q>B \\ TQ-C(Q)-FC & \text{if} \quad 0<Q\leq B, \\ -FC & \text{if} \quad Q=0. \end{cases}$$

where $B$ is the base production level to which the subsidy is applied, $Q$ is the quantity produced, $C(Q)$ is the total variable cost defined only for positive production with $C'(Q), C''(Q) > 0$ for all positive $Q$,[1] and $FC$ is the costs that are fixed in the short run. Define the output level $Q_T^*$ implicitly as $\pi'(Q_T^*) = 0$, which must be unique if it exists. The set of possible optima are given by $\{0, Q_T^*, B\}$. Alternatively, profits without the subsidy are given by

$$\pi(Q) = \begin{cases} PQ-C(Q)-FC & \text{if} \quad Q>0 \\ -FC & \text{if} \quad Q=0 \end{cases},$$

with $Q^*$ defined implicitly by $\pi'(Q^*) = 0$. Under no subsidy the possible optima are $\{0, Q^*\}$. Note that if $Q^* > B$, then $Q_T^* = Q^*$, and if $Q^* < B$ then $Q_T^* \leq B$ or does not exist.

[1] It need not be the case that $\lim_{Q\downarrow 0} C(Q) = 0$. This requirement would eliminate the possibility of shutdown for positive $Q^*$.

Also note that at every production level $Q$, profits are weakly greater under the infra-marginal payment, with equality at $Q=0$.

**Proposition 1:** Let $P>C(Q^*)/Q^*$ so that the firm produces $Q^*$ in the short run without the infra-marginal payment. Then, under the infra-marginal payment, the firm will produce:

(1) $Q_T^*=Q^*$ if $B<Q^*$, resulting in no production effect,

(2) $Q_T^*>Q^*$ if $B>Q^*$ and $C'(B)>T$, resulting in greater production,

(3) $B>Q^*$ if $B>Q^*$ and $T>C'(B)$, resulting in greater production.

**Proof:** Because $P>C(Q^*)/Q^*$, profit without the subsidy is maximized by $Q^*$, and is given by $\pi(Q)=PQ^*-C(Q^*)-FC>-FC$. Because the firm obtains positive rents from positive production without the payment, profits cannot be maximized by 0 production with the subsidy. Thus, the firm will either produce $Q_T^*$ or *B*.

(1) If $B<Q^*$ then the difference between profit at $Q_T^*$ and $B$ is given by

$$\pi(Q_T^*)-\pi(B)=P(Q_T^*-B)+TB-C(Q_T^*)-FC-TB+C(B)+FC$$

$=P(Q_T^*-B)-C(Q_T^*)+C(B)>0.$ Thus the firm will produce $Q_T^*$. The inequality follows because $C''>0,$ implying that $P=MC(Q_T^*)>\dfrac{C(Q_T^*)-C(B)}{Q_T^*-B}$. And, $Q_T^*=Q^*$ because $B<Q^*$.

(2) If $B>Q^*$ and $C'(B)>T$, then the difference between profit at $Q_T^*$ and $B$ is given by $\pi(Q_T^*)-\pi(B)=TQ_T^*-C(Q_T^*)-FC-TB+C(B)+FC$

$= T\ \ Q_T^* - B\ \ - C\ \ Q_T^*\ \ + C\ \ B\ \ > 0.$ Thus the firm will produce $Q_T^*$. The inequality

follows because $C'' > 0,$ implying that $C'\ \ Q_T^*\ \ = T < \dfrac{C\ \ Q_T^*\ \ - C\ \ B}{Q_T^* - B}$. And, $Q_T^* > Q^*$

because $B > Q^*$.

(3) If $B > Q^*$ and $T > C'\ \ B$ , then $Q_T^*$ does not exist, and the firm produces at $B > Q^*$. $\blacksquare$

Proposition 1 shows that, for firms that would produce without the subsidy, production is only enhanced if the base production level is set above the optimal production level that obtains without a payment. Thus, for this set of firms, production is only affected if the program behaves as a fully coupled subsidy.

**Proposition 2:** Let $P < C\ \ Q^*\ \ /Q^*$, thus profit without the subsidy is maximized when the firm produces nothing in the short run. Then, under the infra-marginal payment, the firm will produce

(1) 0 if $B < Q^*$ and $-\left[PQ^* - C\ \ Q^*\ \ \right] > \ T - P\ \ B$, resulting in no production effect,

(2) $Q_T^*$ if $B < Q^*$ and $-\left[PQ^* - C\ \ Q^*\ \ \right] < \ T - P\ \ B$, resulting in an increase in production (Results 1 and 2).

(3) 0 if $B > Q^*$, $T < \dfrac{C\ \ Q_T^*}{Q_T^*}$ and $T < \dfrac{C\ \ B}{B}$, resulting in no production effect,

(4) $Q_T^*$ if $B > Q^*$, $T < C'\ \ B\ \ ,\ T > \dfrac{C\ \ Q_T^*}{Q_T^*}$, resulting in an increase in production,

(5) $B$ if $B > Q^*$ and $T > C'\ \ B$ , resulting in an increase in production.

**Proof:** With the subsidy, the firm could either produce 0, $Q_T^*$ or *B*.

(1) If $B < Q^*$ and $-\left[PQ^* - C\ \ Q^*\ \ \right] > \ T - P\ \ B$, then the difference in profit between $Q_T^*$ and 0 is given by $\pi\ \ Q_T^*\ \ -\pi\ \ 0\ \ =\ \ P\ \ Q_T^* - B\ \ + TB - C\ \ Q_T^*\ \ - FC + FC$ $= P\ \ Q_T^* - B\ \ + TB - C\ \ Q_T^*\ \ < 0$. The inequality follows directly. The difference in profit between $B$ and 0 is given by

$\pi\ \ B\ \ -\pi\ \ 0\ \ = TB - C\ \ B\ \ - FC + FC = TB - C\ \ B\ \ < 0$. To see this note that

$-\left[PQ^* - C\ \ Q^*\ \ \right] > \ T - P\ \ B$ implies that $\dfrac{C\ \ Q_T^*\ \ - TB}{Q_T^* - B} > P = C'\ \ Q_T^*\ \ $. This and convexity of the cost function implies that $TB < C\ \ B\ \ $. Thus the firm produces 0.

(2) If $B < Q^*$ and $-\left[PQ^* - C\ \ Q^*\ \ \right] < \ T - P\ \ B$, then the difference in profit between $Q_T^*$ and 0 is given by $\pi\ \ Q_T^*\ \ -\pi\ \ 0\ \ =\ \ P\ \ Q_T^* - B\ \ + TB - C\ \ Q_T^*\ \ - FC + FC$

$= P\ \ Q_T^* - B\ \ + TB - C\ \ Q_T^*\ \ > 0$. The inequality follows directly. The difference in profit between $Q_T^*$ and $B$ is given by

$\pi\ \ Q_T^*\ \ -\pi\ \ B\ \ = TQ_T^* - C\ \ Q_T^*\ \ - FC - TB + C\ \ B\ \ + FC$

$= T\ \ Q_T^* - B\ \ - C\ \ Q_T^*\ \ + C\ \ B\ \ > 0$. This latter inequality follows due to convexity of the cost function, and $P = C'\ \ Q_T^*\ \ > \dfrac{C\ \ Q_T^*\ \ - C\ \ B}{Q_T^* - B}$. Thus the firm produces $Q_T^*$.

(3) If $B > Q^*$, $T < \dfrac{C\ \ Q_T^*}{Q_T^*}$ and $T < \dfrac{C\ \ B}{B}$, then variable costs are never covered by positive production.

(4) If $B > Q^*$, $T < C'(B)$, and $T > \frac{C(Q_T^*)}{Q_T^*}$ then the difference in profit between $Q_T^*$ and 0 is given by $\pi(Q_T^*) - \pi(0) = (TQ_T^* - C(Q_T^*) - FC + FC = TQ_T^* - C(Q_T^*)) > 0$. The difference in profit between $Q_T^*$ and $B$ is given by

$$\pi(Q_T^*) - \pi(B) = TQ_T^* - C(Q_T^*) - FC - TB + C(B) + FC$$
$= TQ_T^* - C(Q_T^*) - TB + C(B) > 0$. This last inequality follows from the convexity of the cost function, implying $C'(Q_T^*) = T < \frac{C(Q_T^*) - C(B)}{Q_T^* - B}$. Thus, the firm produces $Q_T^*$.

(5) If $B > Q^*$, $T > C'(B)$ and $T > \frac{C(B)}{B}$, then $Q_T^*$ does not exist, and the firm will either produce 0 or $B$. The difference in profit between $B$ and 0 are

$\pi(B) - \pi(0) = TB - C(B) - FC + FC = TB - C(B) > 0$. Thus the firm produces $B$. $\blacksquare$

Proposition 2 shows a scenario whereby production can be affected without the infra-marginal payment becoming fully coupled (2). This result will obtain if the base production level is set below the production level equating marginal cost and the market price, and if the target price is high enough.

The results in the long run differ only by the necessity of covering fixed costs.

**Proposition 3:** Let $P > \frac{C(Q^*) + FC}{Q^*}$ so that the firm produces $Q^*$ without the infra-marginal payment in the long run. Then, under the decoupled payment, the firm will produce:

(1) $Q_T^* = Q^*$ if $B < Q^*$, resulting in no production effect,

(2) $Q_T^* > Q^*$ if $B > Q^*$ and $C'\ \ B\ \ > T$, resulting in greater production,

(3) $B > Q^*$ if $B > Q^*$ and $T > C'\ \ B$ , resulting in greater production.

**Proof:** This follows directly from Proposition 1 and the fact that profits must be weakly greater under the infra-marginal payments.

Proposition 3 is nearly identical to Proposition 1, again excluding the possibility of increasing production without the policy behaving as if fully coupled.

**Proposition 4:** Let $P < \dfrac{C\ \ Q^*\ \ + FC}{Q^*}$, thus profit without the subsidy is maximized when the firm shuts down in the long run. Then, under the infra-marginal payment, the firm will produce

(1) 0 if $B < Q^*$ and $-\left[PQ^* - C\ \ Q^*\ \ - FC\right] >\ \ T - P\ \ B$, resulting in no production effect,

(2) $Q_T^*$ if $B < Q^*$ and $-\left[PQ^* - C\ \ Q^*\ \ - FC\right] <\ \ T - P\ \ B$, resulting in an increase in production (long run analogs for Results 1 and 2).

(3) 0 if $B > Q^*$ and $T < \dfrac{C\ \ Q_T^*\ \ + FC}{Q_T^*}$, resulting in no production effect,

(4) $Q_T^*$ if $B > Q^*$, $\dfrac{C\ \ Q_T^*\ \ + FC}{Q_T^*} < T < C'\ \ B$ , resulting in an increase in production,

(5) $B$ if $B > Q^*$ and $T > C'\ \ B$ , $\dfrac{C\ \ Q_T^*\ \ + FC}{Q_T^*}$, resulting in an increase in production.

**Proof:** With the subsidy, the firm could either produce 0, $Q_T^*$ or $B$.

(1) If $B < Q^*$ and $-\left[PQ^* - C(Q^*) - FC\right] > (T - P)B$, then the difference in long run profit between $Q_T^*$ and 0 is given by $\pi(Q_T^*) = P(Q_T^* - B) + TB - C(Q_T^*) - FC < 0$. The inequality follows directly. The difference in profit between $B$ and 0 is given by $\pi(B) = TB - C(B) - FC < 0$. To see this note that

$$-\left[PQ^* - C(Q^*) - FC\right] > (T - P)B \text{ implies that } \frac{C(Q^*) + FC - TB}{Q^* - B} > P = C'(Q^*).$$

This and convexity of the cost function implies that $TB < C(B) + FC$. Thus the firm shuts down in the long run.

(2) If $B < Q^*$ and $-\left[PQ^* - C(Q^*) - FC\right] < (T - P)B$, then the difference in profit between $Q_T^*$ and 0 is given by $\pi(Q_T^*) - \pi(0) = P(Q_T^* - B) + TB - C(Q_T^*) - FC > 0$. This inequality follows directly. The difference in profit between $Q_T^*$ and $B$ is given by $\pi(Q_T^*) - \pi(B) = TQ_T^* - C(Q_T^*) - FC - TB + C(B) + FC$

$= T(Q_T^* - B) - C(Q_T^*) + C(B) > 0$. This latter inequality follows due to convexity of the cost function, and $P = C'(Q_T^*) > \dfrac{C(Q_T^*) - C(B)}{Q_T^* - B}$. Thus the firm produces $Q_T^*$.

(3) If $B > Q^*$ and $T < \dfrac{C(Q_T^*) + FC}{Q_T^*}$, then the difference in profit between $Q_T^*$ and 0 is given by $\pi(Q_T^*) - \pi(0) = TQ_T^* - C(Q_T^*) - FC < 0$. The difference in profit between $B$

and 0 is given by $\pi(B)-\pi(0)=TB-C(B)-FC<0$. The latter inequality follows because $TB-C(B)<TQ_T^*-C(Q_T^*)$.

(4) If $B>Q^*$ and $\frac{C(Q_T^*)+FC}{Q_T^*}<T<C'(B)$, then the difference in profit between $Q_T^*$ and 0 is given by $\pi(Q_T^*)-\pi(0)=TQ_T^*-C(Q_T^*)-FC>0$. The difference in profit between $Q_T^*$ and $B$ is given by

$$\pi(Q_T^*)-\pi(B)=TQ_T^*-C(Q_T^*)-FC-TB+C(B)+FC$$
$=TQ_T^*-C(Q_T^*)-TB+C(B)>0$. This last inequality follows from the convexity of the cost function, implying $C'(Q_T^*)=T<\frac{C(Q_T^*)-C(B)}{Q_T^*-B}$. Thus, the firm produces $Q_T^*$.

(5) If $B>Q^*$, $T>C'(B),\frac{C(Q_T^*)+FC}{Q_T^*}$, then $Q_T^*$ does not exist, and the firm will either produce 0 or $B$. The difference in profits between $B$ and 0 are $\pi(B)-\pi(0)=TB-C(B)-FC>0$. Thus the firm produces $B$.

Proposition 4 is very similar to Proposition 2. Again one scenario allows for an increase in production without the subsidy behaving as if fully coupled. This will occur if the base level of production is below the level where marginal cost is equal to the market price, and if the target price is high enough. In this case, the target price must be high enough to cover the total cost, not just the variable cost.

## Appendix B

In this appendix we present a simple example illustrating of Result 1. Consider a quadratic cost function (also used in the empirical analysis) $TC = \gamma_0 + \gamma_1 Q + \gamma_2 Q^2$. In this case, $Q^* = (p - \gamma_1)/2\gamma_2$. We need to prove that the area $(ATC_B - ATC^*)B$, area *a* in Figure 1, is always greater than the area $(ATC^* - p)(Q^* - B)$, areas *b* + *c* in Figure 1.

We find that

(1)
$$\begin{aligned} a = (ATC_B - ATC^*)B &= \left( \frac{\gamma_0 + \gamma_1 B + \gamma_2 B^2}{B} - \frac{4\gamma_0\gamma_2 - \gamma_1^2 + p^2}{2(p - \gamma_1)} \right) B \\ &= (2\gamma_0 + \gamma_1 B - Bp)\frac{(\gamma_1 + 2\gamma_2 B - p)}{2(\gamma_1 - p)}, \end{aligned}$$

and

(2)
$$\begin{aligned} b + c = (ATC^* - p)(Q^* - B) &= \left( \frac{4\gamma_0\gamma_2 - \gamma_1^2 + p^2}{2(p - \gamma_1)} - p \right)\left( \frac{p - \gamma_1}{2\gamma_2} - B \right) \\ &= (4\gamma_0\gamma_2 - \gamma_1^2 + 2\gamma_1 p - p^2)\frac{(\gamma_1 + 2\gamma_2 B - p)}{4\gamma_2(p - \gamma_1)}. \end{aligned}$$

Subtracting (2) from (1) obtains

$$\begin{aligned} a - b - c &= \frac{\left[ 2\gamma_2(2\gamma_0 + \gamma_1 B - Bp) - (4\gamma_0\gamma_2 - \gamma_1^2 + 2\gamma_1 p - p^2) \right](\gamma_1 + 2\gamma_2 B - p)}{4\gamma_2(\gamma_1 - p)} \\ &= \frac{(\gamma_1 + 2\gamma_2 B - p)^2}{4\gamma_2}. \end{aligned}$$

Because the difference in areas reduces to a perfect square, it follows that area *a* in Figure 1 is always greater than areas *b* + *c* if $\gamma_2 > 0$, implying a convex cost function.