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A Detailed Hydro-Economic Model for Assessing the Effects of Surface Water and Groundwater Policies: A Demonstration Model from Brazil

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Abstract

Policymakers, managers of water use associations, and many others in developing countries are considering policy actions that will directly or indirectly change the costs and availability of groundwater and surface water for agricultural users. While in many cases such actions may bring about welcomed increases in water use efficiency, little is known about the likely effects of changes in irrigation costs or water access on farmer behavior, or on farmer incomes in the short or long runs, and virtually nothing is known about the detailed immediate or knock-on effects on water resources that such policy actions might cause. This paper reports the preliminary results of research aiming to fill these large scientific gaps by developing a detailed hydrologic model and a detailed economic model of agriculture in the context of the Buriti Vermelho (BV) sub-catchment area of the São Francisco River Basin in Brazil. A spatially explicit, farm-level, positive mathematical programming model capable of accommodating a broad array of farm sizes and farm/farmer characteristics is being developed to predict the effects of alternative water policies and neighbors' water use patterns on agricultural production. Special attention is given to precisely defining and estimating the distinct variable costs (including labor and electrical energy costs) and capital costs of surface water and groundwater, which are considered perfect substitutes for irrigation. Shadow values for non-marketed inputs (land, family labor, and water) are estimated in the first step of the modeling process. A high-resolution, spatially distributed hydrologic model (MOD-HMS) is being developed to simulate three-dimensional, variably-saturated subsurface flow and solute transport. Subsurface flow is simulated using the three-dimensional Richards equation while accounting for a) application of water at the surface, b) precipitation, c) soil evaporation and crop transpiration, and d) agricultural pumping. Demonstration versions of both models are presented and tested: the economic model assesses the effects of increasing water scarcity on cultivated area, crop mix, input mix and farm profits; the hydrologic model uses two irrigation water use scenarios to demonstrate the effects of each on surface water flows and storage, and on groundwater storage and well depth. The models are not currently linked, but a detailed plan to do so is presented and discussed. The paper concludes by discussing next steps in research and policy simulations.

Part 1 – Introduction

The São Francisco River (see Figure 1) provides about 70% of the surface water in Northeast Brazil and like much of Brazil the basin includes communities characterized by a broad range of incomes and persistent poverty (ANA 2004, Brito and Gichuki 2003, Federal University of Viçosa 2003, Embrapa 2001, CODEVASF undated, CNPq undated, SEPLAN undated, Embrapa and IWMI 2004, OAS 2004). The basin's agricultural systems cover a similar range between capitalized export-focused enterprises and subsistence farms. Major corporations and cottage industries comprise the industrial water use sector while cities and towns tap the basin for municipal supplies. The basin also hosts several important water-dependent ecological zones. Increasingly, the complex web linking water availability, water quality, water productivity, economic growth, poverty alleviation and community and ecosystem health is coming into focus. Conflict for water among various water user communities and sectors is becoming common, often with negative consequences for resource-poor stakeholders. Surface water shortfalls in some areas have increased groundwater utilization which may lead to soil salination.

Figure 1 – The São Francisco River Basin in Brazil and the Buriti Vermelho Research Site

Brazil’s Federal Law 9.433 (Federal Government of Brazil 1997) was implemented to promote and guide public-sector involvement in water management so as to integrate across the connections defined by the flow of water to improve overall social welfare. More specifically, the Law clearly places hydrological resources in the public domain (Article 1) and charges policymakers with the wise and sustainable management of these resources (Article 3) via the use of water price policy and other policy instruments (Article 5), some of which remain to be developed. However, formidable challenges confront the Law’s implementation. Two challenges this research seeks to address in the context of the São Francisco River Basin (SFRB) are (Bassoi et al. 2006):

- incomplete understanding of how water use decisions are taken by important water use groups, and once taken, how these decisions affect the water use options available in other parts of the basin, now and in the future; and

- incomplete information for assessing scale-dependent, freshwater dynamics and using these dynamics to predict the effects of alternative water policies designed to promote the increased water productivity, and livelihood and environmental enhancement.

Key Policy Issues

More specifically, the following more specific policy questions loom large in the SFRB:

- Regarding the Agricultural Sector
  - How much *surface water* <u>should</u> be diverted for agriculture, when and where?
  - How much *groundwater* <u>should</u> be pumped, when and where?
  - What public policy action (if any) is required to improve overall water use efficiency?
  - What might be the effects of alternative water policy actions on cultivated area, crop mix, and production technology choice?

- Regarding Poverty
  - How is water productivity or access to water linked to rural poverty?
  - If linked, how much water should be diverted to poor farmers to reduce poverty?
  - What might be the effects of water policy action (e.g., implementation of water pricing schemes) on poverty?

With these policy issues and the abovementioned knowledge gaps as a backdrop, our specific research objectives are:

- develop and calibrate scale-dependent agricultural production and hydrologic models;
- use these combined modeling systems to quantify the economic and environmental impacts of alternative water and agricultural policy, with particular focus on short-term trade-offs among poverty and environmental objectives; and
- derive policy implications from research results.

This paper presents the preliminary results of this interdisciplinary research effort for one small area within the SFRB[1], the Buriti Vermelho sub-catchment (identified in Figure 1 and characterized below).

Our Two-Model Strategy

In order to provide policy guidance on the issues identified above, a deeper understanding of both biophysical processes and human behavior, and the interaction between the two, is required. This is particularly important in situations in which some of the important components of the biophysical processes are not 'seen' (e.g., groundwater stocks and flows) and hence tend to be overlooked in policymaking.

Regarding human behavior, we model the decisions of agriculturalists who seek to maximize profits seasonally, subject to an array of socioeconomic and biophysical constraints. These farmers take decisions on total cultivated area, crop mix, input mix (including the amount of water applied to each crop) and production technology (including irrigation technology), and derive income from farming activities alone.

Regarding biophysical processes, the Buriti Vermelho basin is the laboratory in which a detailed exploration of the hydrologic processes is underway. The high level of detail reachable for this basin allows a more exhaustive monitoring of the evolution of the water reserves, giving deeper insights into the impact that different water uses have on the usable water stock. This detailed approach needs to be simulated with a comprehensive model able to handle the different and complex mechanisms of water transfer within and out of the basin. For this task, a state-of-the-art, physics-based, fully coupled distributed surface-subsurface hydrologic model will be used.

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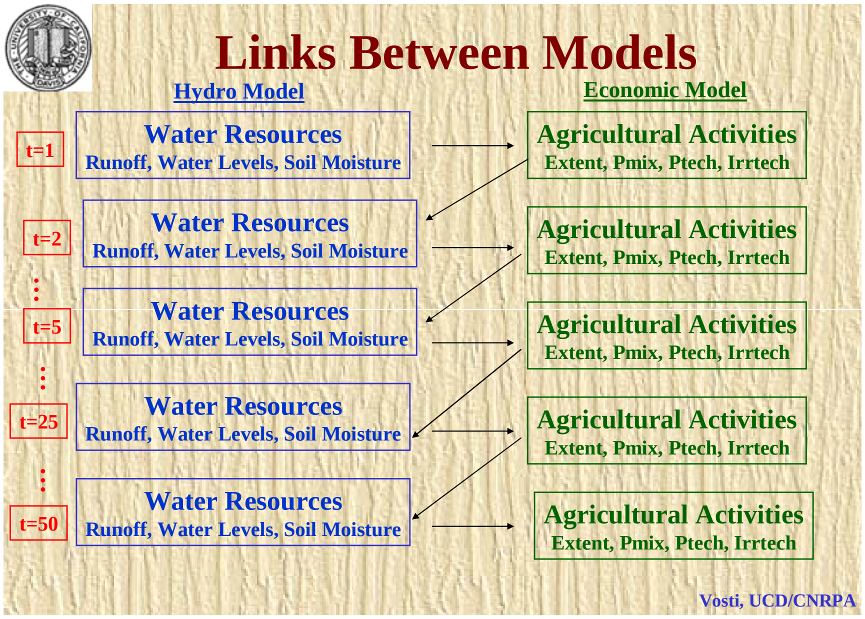
[1] For details of research being undertaken at different spatial scales, see Bassoi et al. 2006.

Figure 2 – Interactions between the Hydrologic and Economic Models



Links Between Models

Hydro Model

Economic Model

t=1

Water Resources
Runoff, Water Levels, Soil Moisture

Agricultural Activities
Extent, Pmix, Ptech, Irrtech

t=2

Water Resources
Runoff, Water Levels, Soil Moisture

Agricultural Activities
Extent, Pmix, Ptech, Irrtech

⋮

t=5

Water Resources
Runoff, Water Levels, Soil Moisture

Agricultural Activities
Extent, Pmix, Ptech, Irrtech

⋮

t=25

Water Resources
Runoff, Water Levels, Soil Moisture

Agricultural Activities
Extent, Pmix, Ptech, Irrtech

⋮

t=50

Water Resources
Runoff, Water Levels, Soil Moisture

Agricultural Activities
Extent, Pmix, Ptech, Irrtech

Vosti, UCD/CNRPA

Once developed and tested, these models will ‘interact’ in ways depicted in Figure 2. Initially, the hydrologic model will identify the stocks and flows of water resources within the BV over the entire year, and ‘inform’ the economic model of the availability of surface water and the depth of groundwater for each farmer. The economic model will then identify the optimal land uses (more specifically, among other things, cultivated area {Extent}, crop mix {Pmix}, production technology {Ptech}, and irrigation technology {Irrtech}) given available surface water and groundwater. Information on cropping patterns is then ‘fed back into’ the hydrologic model to ensure that selected land uses are indeed hydrologically feasible. The long-term sustainability of any collection of land uses can be assessed by identifying the effects on water stocks/flows of selected land uses over time. The models will ultimately be used to identify the effects of changes in agricultural policies on water resources (via the effects of policy changes on land uses), and the effects of water policies on agriculture and farm income).

What Follows

The remainder of the paper flows as follows. Part 2 briefly describes the Buriti Vermelho research site. In Part 3 we set out in some detail the economic model of agriculture. Part 4 makes use of the economic model to predict the effects on farm decisions of successively more scarce surface water. Part 5 provides an overview of the hydrologic model used to simulate the stocks and flows of surface water and groundwater. Part 6 uses the hydrologic model to simulate the effects of two irrigation water use scenarios in the BV; one scenario allows for the conjunctive use of surface water and groundwater for agriculture, while the other scenario makes exclusive use of surface water for irrigation. The two models are so far not linked, but Part 7 identifies a strategy for doing so, with special attention paid to issues of spatial and temporal aggregation. Part 8 provides preliminary conclusions and discusses next steps in research.

Part 2 – Research Site

The Buriti Vermelho sub-catchment area, located near Brasília, is a first-order basin of about 9,407,100 m^2. Its climate is tropical with a clearly defined dry season during the Brazilian winter months of June through September. Total annual rainfall and potential evapotranspiration are about 1300 mm. Topographically, the basin contains gentle slopes with a single water channel flowing northward that drains the entire watershed. The soils are generally clayey tropical red latosols. Despite high clay content, soils tend to agglomerate in stable spherical aggregates that improve drainage and generate higher values of hydraulic conductivity than in other clayey formations. The soils are approximately 16.5 meters deep, forming a single-layered, unconfined aquifer.

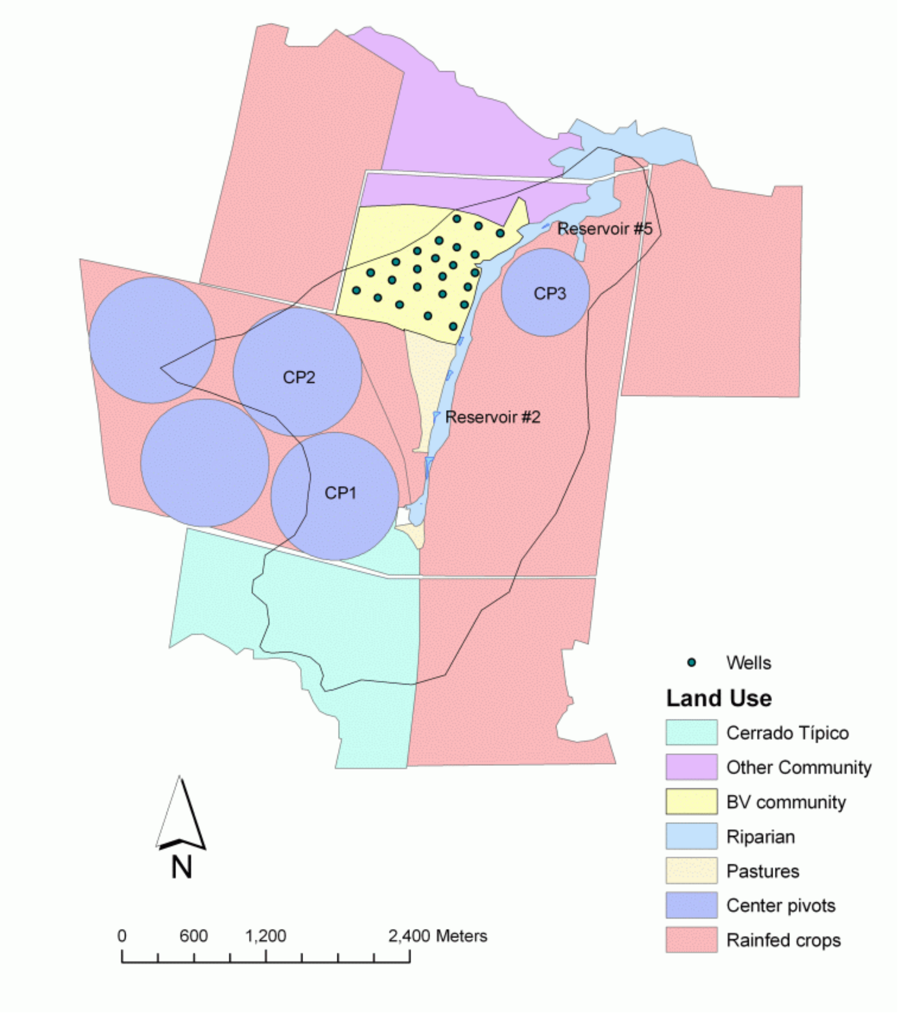
The hydrologic model assumes the following technical parameters: soils with 52% porosity, 24% volumetric residual water content, 37% volumetric soil moisture at field capacity, and 29% volumetric soil moisture at wilting point. The horizontal hydraulic conductivity assigned was 0.037 $m\ h^{-1}$ and the vertical hydraulic conductivity was 0.0037 $m\ h^{-1}$.

The channel was assumed to be 4 meters wide and 2.5 meters deep. Five reservoirs built at different points are used to supply water for different uses, primarily irrigation. Reservoirs #1 and #2 (which are located in the upper watershed) each have a maximum storage capacity of about 2400 m^3 and reservoirs #3 to #5 each have maximum capacities of about 1800 m^3.

Regarding socioeconomic characteristics, the BV sub-catchment area is a small watershed comprised of several types and scales of farming activities. Figure 3 depicts the BV site; the precise boundaries of the sub-catchment area are given by the thin black line. Water emerges from about the south-central part of the site, just outside the green patch of *cerrado típico* (savanna forest) and flows from south to north. Blue circles identify the location and size of capital-intensive center-pivot irrigation schemes, while yellow rectangles identify small-scale farming operations. Large patches of rainfed agriculture remain in the BV and appear as pink in Figure 3. For future reference, the tube well field operated by small-scale farmers appears as green dots, and the reservoirs from which all farmers withdraw surface water have been labeled.

The BV is located about 60 kilometers from Brasilia, one of Brazils major urban and market centers, hence access to markets is relatively cheap and easy, and information related to input/output prices is assumed to spread quickly and cheaply. Farmers (of all scales of operation) in the BV are assumed to be price takers. Markets for all inputs (especially labor) function well. There is currently no market for water.

Figure 3 –Farms and Farming in the Buriti Vermelho Sub-Catchment Area

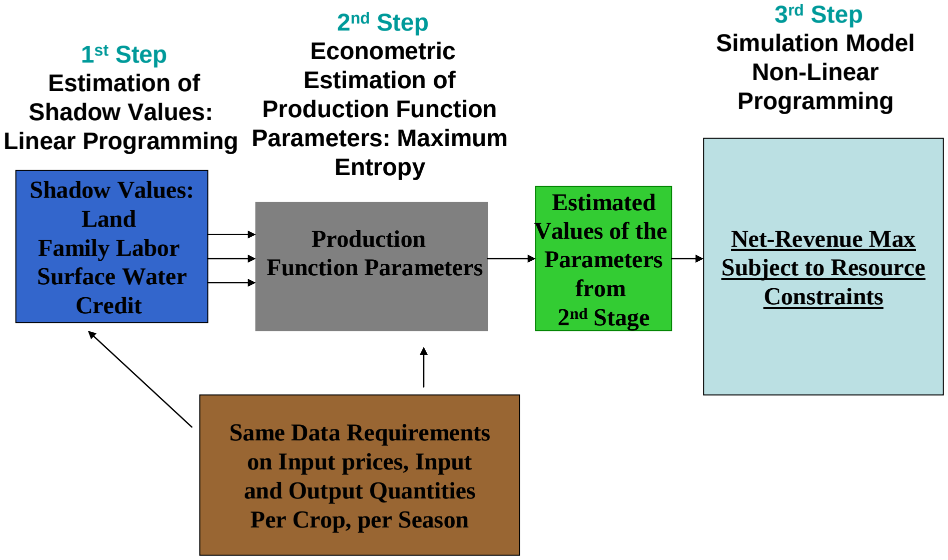


Part 3 – An Economic Model of Agriculture for Buriti Vermelho

The economic model proposed for Buriti Vermelho (BV) is based on a class of models called Positive Mathematical Programming (PMP) models. This methodology is described in detail in Howitt (1995, 2005), and has been widely used in applied research and policy analysis by Howitt and Gardner (1986), House (1987), Kasnakoglu and Bauer (1988), Arfini and Paris (1995), Chattergee, Howitt and Sexton (1998), Lance and Miller (1998), Heckelei and Britz (2000) and Helming et al. (2000).

The PMP approach follows 3 basic steps as shown in Figure 4; each step and its application in the case of the BV will be described in detail below. First, the shadow values associated with non-marketed (and often fixed) resources are calculated using a linear programming model of land allocation in which there are two sets of constraints: 1) resource constraints, and 2) calibration constraints. The first set of constraints assures that the optimal use of each of the non-marketed (fixed) resources cannot exceed their on-farm supply. The second set of constraints prevents corner solutions and crop specialization in cases in which on-farm diversification of crops is observed.

Figure 4 – PMP Modeling Steps



The shadow values associated with the elements of the two sets of constraints in step 1 are then used in a second step, also know as model calibration. This step involves the estimation of the production function parameters based on farm-level input/output data, input/output prices, and the shadow values estimated in step 1. In a third step, the estimated values of production function parameters are introduced into a production function that is nested within a net-income maximization algorithm that is subject to the farm-level resource constraints; this algorithm is then used for policy simulations.

3.1. The Objective Function: Our Conceptual and Analytical Point of Departure

In the context of their farming operations, farmers throughout the worlds seek to maximize the discounted stream of net benefits derived from their farming operations. These same farmers are constrained in the land allocation and input use decisions by a whole host of environmental, agronomic, market, and non-market constraints; this is especially true of resource-poor farmers in developing countries. Therefore, the backbone of our analytical work is an objective function that explicitly sets out what farmers aim to do (maximize profits) and the constraints they face in doing so.

In the BV area, three groups of farmers can be identified: small-scale farmers with operational holding of up to 4 hectares; medium-scale farmers with operational holdings of up to 15 hectares, and large-scale farmers with operational holdings of more than 100 hectares. The model considers farmers as economic agents who manage multi-output, multi-input production operations (perennial and annual crops, and pasture/livestock) during two seasons each year: the 'dry' season, which runs from May through the end of September, and the 'wet' season, which runs from October through April and when it rains, at a historical average, 1175mm or 92% of the annual precipitation. Although precipitation during the wet season is generally sufficient for current farming practices, farmers may still need to occasionally irrigate crops during this period since it is not uncommon to experience extended dry spells in January and in March. In each season, the specific objective of each farm is to maximizing net income subject to an array of biophysical and socioeconomic constraints. Farmers produce a specific crop using irrigation (if relevant) and non-irrigation inputs. Irrigation inputs include the per-unit costs of water (if any), labor costs of irrigation management, and the capital and energy costs associated with on-farm water conveyance, which depend on the irrigation technology chosen to convey water from surface water or groundwater sources to each crop.

Livestock production (cattle, in the case of the BV) is also included in the model; inputs include the land (measured in terms of the carrying capacity of established pastures), labor and purchased inputs for pasture and herd management, and beef (sold or consumed at home) is the only output. Average profitability of land dedicated to cattle production is defined as the total revenue derived from livestock off-take divided by average herd size.

In modeling perennial crop production, we follow the method used in Chatterjee, Howitt and Sexton (1998), in which perennial crop supply is based on 'average' production over trees of different ages. Also, no lags between observed price changes and their realized impacts are explicitly included, and decision-making process is neither designed under uncertainty nor based on expectation formation. Impacts from changes in relative output and input prices, on land allocation toward perennials, on their yields and input use are then based on the assumption that farmers can change the land allocation to perennial crops as quick as any other annual crop and that they look at observed rather than expected output/input prices.[2]

More specifically, our model assumes a net revenue equation for a given farmer in a given season to be:

$$(1) \quad \sum_i p_i q_i(\mathbf{x}_{nirr}, ew(\mathbf{x}_{irr})) - \sum_i w_j x_{nirr_{ij}} - c_{ew} \,;$$

where p_i and $q_i(\mathbf{x}_{niir}, ew(\mathbf{x}_{irr}))$ are, respectively, the output prices of perennial and annual crops and livestock products ($i = 1, \ldots, I$), each of which is produced according to a production function (q_i) that makes use of non-irrigation inputs ($\mathbf{x}_{nirr}$) such as land, fertilizers, pesticides, seeds, hired labor, family labor and machinery, and an (unobserved) amount of effective water (*ew*) that is delivered by the farmer to the plant's root zone. Farmers can increase the effectiveness with which water reaches a crop's root zone by adjusting the amount of irrigation inputs in the vector $\mathbf{x}_{irr}$: applied water, labor for irrigation management, capital, and electricity. That is, to increase *ew*, farmers can apply more water (*aw*) from two sources (surface water, groundwater, or some combination of the two), change the amounts of other irrigation inputs used (irrigation labor, irrigation capital, irrigation electricity), or both.

The total cost of agricultural production is divided into two parts: the cost of non-irrigation inputs, $\sum_i w_j x_{nirr_{ij}}$, where w_j is the price of non-irrigation input *j*, and c_{ew} is the cost of effective water, which we define as the per-unit cost of water (which in most instances will be zero) plus the sum of all water conveyance and irrigation management costs, that is,

$$(2) \qquad c_{ew} = C_{aw} + c_{ilb} \sum_i lb_i + C_{ik} + c_{ie} \sum_i ie_i$$

where C_{aw} is the cost of water, c_{ilb} is the wage paid to irrigation labor, lb_i is the quantity of labor used in the i^{th} irrigation process, C_{ik_i} is the expenditures associated with irrigation capital; c_{ie} is

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[2] A preferred and more realistic approach, as highlighted by Alston et al. (1995), is to consider supply response depending on two elements: yield per bearing acre and the number of bearing acres. These are particularly important for tree crops, for which adjustments to bearing acreage should also take into account life and production cycles. Although our analysis currently does not contain such detail, improvements along these lines for modeling perennial tree crops are being concerned.

the per-unit price of electrical energy, and ie_i is the quantity of electricity used in the irrigation process.

The cost with applied water, C_{aw}, is defined as the cost with surface water plus the cost of groundwater (see equation 3). For the sake of simplicity, it is assumed that farmers have to pay a fee p_{sw} per unit of of surface water used *sw* used; no per-unit fee is paid for groundwater.[3] Farmers must incur in pumping and fixed costs associated with groundwater use. More specifically, the total cost with surface water is defined as:

$$(2') \qquad C_{sw} = p_{sw} sw .$$

where p_{sw} is the per-unit water fee and *sw* and the amount of surface water used. The cost of groundwater use is defined by the following function:

$$(2'') \qquad C_{gw} = \alpha + \beta * gw * depth ,$$

where α represents the fixed cost associated with establishing an irrigation well on the farm (expenses with buying the pump or building the well), β reflects the electrical energy and other marginal costs associated with extracting water from a well, *gw* is the amount of groundwater pumped, and *depth* is the distance from the surface to the water table in the well. First derivatives of C_{gw} with respect to all elements in the equation are positive.

In this context, the price faced by farmers per unit of applied water can be thought as the weighted average of the prices of surface water and the marginal cost associated with groundwater extraction, that is,

$$(2''') \quad p_{aw} = p_{sw} * \frac{sw}{aw} + C'_{gw} * \frac{gw}{aw} ,$$

where C'_{gw} is the marginal cost of groundwater, which given (2'') is defined as $\beta * depth$, *aw* is the total quantity of applied water which is defined as the sum of total surface water (*sw*) and groundwater (*gw*) used by the farmer.

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[3] This assumption does not affect or restrict the ability of the model to predict how the demand for surface water versus groundwater would change if the relative prices of water from these two sources were to change. A per-unit fee for groundwater can be easily accommodated into the model.

In this way, although the price of surface water is exogenous to the farmer, the price of applied water is endogenous since it depends on the amount of surface water and groundwater chosen. Since there are no major quality differences between the surface water and groundwater, they are assumed to be perfect substitutes. This assumption plus the linear relationships assumed in (2') and (2'') imply that farmers will completely switch to the cheapest source if that source can meet all if their irrigation needs. Conjunctive use of ground and surface water can be observed in the event of surface water shortages.

3.2 The Nested CES Production Function

Total output in each season *s* (subscript omitted for clarity) from each cropping activity $i, q_i(\mathbf{x}_{niir}, ew(\mathbf{x}_{irr}))$, is defined by a nested CES production function with eleven categories of inputs:[4]

$$(3) \quad q_i = C_i \left\{ \beta o_i \left[Co_i \left(\sum_{j=1}^{7} b_{ij} x_{ij}^{\gamma_{o_i}} \right)^{\frac{1}{\gamma_{o_i}}} \right]^{\gamma_i} + \beta w_i \left[Cw_i \left(\sum_{j=8}^{11} b_{ij} x_{ij}^{\gamma_{w_i}} \right)^{\frac{1}{\gamma_{w_i}}} \right]^{\gamma_i} \right\}^{\frac{\varepsilon_i}{\gamma_i}} .$$

This functional form consists of two nests and each nest is a CES function in itself. The first set of brackets on the left represents the first nest and includes seven categories of non-irrigation inputs: land, fertilizers, pesticides, seeds, hired labor, family labor, and machinery. The second set of brackets represents the second nest and includes the irrigation inputs: applied water, irrigation labor, irrigation capital, and electrical energy. This nested version is more flexible than a regular CES because it allows the elasticity of substitution among the non-irrigation inputs to be different from the elasticity of substitution among irrigation inputs.

The scalar C_i is the top-nest scale parameter, and βo_i and βw_i are the top-nest share parameters for non-irrigation and irrigation inputs, respectively. Moving to the lower nests,

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[4] The label CES refers to Constant Elasticity of Substitution and the term 'elasticity of substitution' reflects the ease with which inputs can be substituted for one another without reducing output, or, the 'curviness' of the isoquant line. In Leontieff technologies, the elasticity of substitution is zero, that is, the input proportions are fixed. In Cobb Douglas technologies the elasticity of substitution is 1, and in the case of perfectly substitutable inputs in production processes, the elasticity of substitution is infinite. The CES encompasses all these possibilities and allows the data to determine the potential for input substitution.

Co_i and Cw_i are scale parameters for non-irrigation and irrigation inputs input nests, respectively. b_{ij} is the share parameter of the *jth* input. The coefficient $\gamma_i = \frac{s_i - 1}{s_i}$, where s_i is the top-nest elasticity of substitution coefficient. $\gamma_{o_i} = \frac{s_{o_i} - 1}{s_{o_i}}$ and $\gamma_{w_i} = \frac{s_{w_i} - 1}{s_{w_i}}$, where s_{o_i} and s_{w_i} are the elasticity of substitution among non-irrigation inputs and the elasticity of substitution among irrigation inputs, respectively. Finally, ε_i is the returns-to-scale parameter which is bounded between 0 and 1.

3.3. Calculating Shadow Values

In the case of inputs with limited supplies (some of which are not traded on markets), such as family labor, surface water, and land, the marginal cost of an input is represented by the sum of its market prices (w_j) plus its shadow values, λ_j. The shadow values for each non-traded or limited input is calculated using a linear programming model, which has as its explicit objective the maximization of net income (4), subject to two sets of constraints represented by (5) and (6) below. That is,

$$(4) \qquad \max_{land_i} \sum_i p_i \bar{y}_i land_i - \sum_i w_j a_{ji} land_i ,$$

subject to farm-level resource constraints

$$(5) \qquad \begin{cases} Land : \sum_i land_i \leq B_l , \\ Surface\,Water : \sum_i a_{sw_i} land_i \leq B_{sw} , \\ Family\,labor : \sum_i a_{fl_i} land_{si} \leq B_{fl} , \end{cases}$$

an applied water use constraint

$$(5') \qquad \sum_i a_{aw_i} land_i = \sum_i a_{sw_i} land_i + \sum_i a_{gw_i} land_i ,$$

and a model calibration constraint

$$(6)\, land_i \leq \overbrace{land}_i$$

where p_i is the market price of the product of crop i, $\bar{y}_i$ is the yield per hectare of land dedicated to crop i ($land_i$), w_j is the unit cost of input j, used in the production of crop i, and a_{ji} are technical coefficients linking inputs to outputs. In (5), a_{sw_i} and a_{fl_i} are the amounts of surface water and family labor, respectively, that are used per hectare of land dedicated to crop i; and B_l, B_{sw} and B_{fl} reflect the total availability of land, surface water, and family labor. (5') assures that the sum of applied water to all crops not exceed the combined amount of water extracted from each water source. In (6), $\overbrace{land_i}$ is the total amount of land allocated to crop i that is *observed* by researchers; this constraint preserves observed crop allocation patterns while estimating shadow values of limited or non-marketed inputs.[5]

3.4. Production Function Parameter Estimation and Model Calibration

The goal here is to estimate the parameters of the nested CES production function (3). Estimation of the full set of parameters of the production function in each of two seasons with 11 inputs per crop requires that each crop be parameterized in terms of 20 parameters: 11 for the shares parameters b; 5 for the scale parameters $C, \beta o, Co, \beta w, Cw$; 3 for the elasticity of substitution parameters $\gamma_o, \gamma_w, \gamma$; and one for the return to scale parameter ε .

Data for this estimation procedure are drawn from the Buriti Vermelho sub-catchment area. Within the BV area there are a total of 45 farmers: 32 small-scale (less than 10 hectares), 10 medium-scale (between 10 and 50 hectares) and 3 large-scale (larger than 50 hectares). Alternative approaches to data management and estimation can be followed here. One option is to treat all farmers as homogeneous production units making use of the *same* technology to produce crop i. In this case, there would be 45 observations and only one production function for the entire BV area. Another approach would be to consider farms of different operational scales as heterogeneous production units, each with a potentially different production function governing the conversion of inputs to outputs. In this case, 3 production functions would be estimated, with the number of observations in each case determined by the number of farms in each farm size category.

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[5] A credit constraint (following Finan et al. 2005) will be added to the next version of the model.

Estimating three separate production functions is preferable to estimating a single production function, especially in this case, where crop-specific production technologies are observed to depend on scale of operation. However, almost regardless of the estimation approach adopted, difficulties associated with the small sample size quickly emerge. The single production function case generates limited degrees of freedom; the production for the large-scale farming operations generates negative degrees of freedom. These small or negative degrees of freedom pose problems for econometric estimation. In general, econometric models rely on the formula $\beta = (X'X)^{-1}X'Y$, to estimate β, the vector of parameters. X is a matrix of values on the explanatory variables and Y is a vector of values on the dependent variable. If the number of observations is less than the number of parameters to be estimated, the matrix *(X'X)* is of less than full rank and cannot be inverted, so β cannot be estimated.

One solution is to use maximum entropy techniques (Shannon, 1948; Jaynes, 1957; Paris and Howitt, 1998; Golan, Judge and Miller, 1996; Mittelhammer et al. 2000; Heckelei and Wolf, 2003). Under this approach, a maximum entropy maximization problem (ME) is set up and subject to optimality and data consistency conditions. For example, the ME problem and constraint set for our production function estimation would be:

$$(7)\qquad \max_{\substack{\mathbf{p}_C,\mathbf{p}_{\beta o},\mathbf{p}_{Co},\mathbf{p}_b,\mathbf{p}_{\gamma_o},\mathbf{p}_{\beta w},\\ \mathbf{p}_{Cw},\mathbf{p}_{\gamma_w},\mathbf{p}_{\gamma},\mathbf{p}_{\varepsilon},\mathbf{p}_{v_u},\mathbf{p}_{v_c},\mathbf{p}_e}} H(\mathbf{p}_C,\mathbf{p}_{\beta o},\mathbf{p}_{Co},\mathbf{p}_b,\mathbf{p}_{\gamma_o},\mathbf{p}_{\beta w},\mathbf{p}_{Cw},\mathbf{p}_{\gamma_w},\mathbf{p}_{\gamma},\mathbf{p}_{\varepsilon},\mathbf{p}_{vu},\mathbf{p}_{v_c},\mathbf{p}_e),$$

subject to a data consistency constraint,

$$(8)\qquad q_i = C_i\left\{\beta o_i\left[Co_i\left(\sum_{j=1}^{7} b_{ij}x_{ij|}^{\gamma_{o_i}}\right)^{\frac{1}{\gamma_{o_i}}}\right]^{\gamma_i} + \beta w_i\left[Cw_i\left(\sum_{j=8}^{11} b_{ij}x_{ij}^{\gamma_{w_i}}\right)^{\frac{1}{\gamma_{w_i}}}\right]^{\gamma_i}\right\}^{\frac{\varepsilon_i}{\gamma_i}};$$

and to economic optimality constraints on unconstrained inputs,

$$(9)\qquad p_i\frac{\partial q_i}{\partial x_{ij}} = w_j + v_j, \text{ for non-irrigation inputs,}$$

$$(9')\qquad p_i\frac{\partial q_i}{\partial x_{ij}} = c_j + v_j, \text{ for electrical energy and labor used in irrigation;}$$

and to economic optimality constraints on constrained inputs,

(10) $$p_i \frac{\partial q_i}{\partial x_{ij}} = w_j + \lambda_j + v_c$$, for land and family labor;

and to the economic optimality constraint on applied water;

(10') $$p_i \frac{\partial q_i}{\partial aw_i} = p_{aw} + \lambda_{sw} + v_{aw}$$

Equations (9) through (10') represent the economic optimality conditions, which assure that each input us used up to the point at which the value of the marginal product of that input is equal to its marginal cost plus an error term *v*. The marginal costs of the unconstrained inputs are defined as their market prices w_j, and c_j. In the case of the constrained inputs, the marginal cost is the sum of the market prices, w_j, and their respective shadow values, λ_j. For applied water the marginal cost is defined as the sum of the price of applied water, p_{aw}, as defined in (2'''), plus the shadow value of surface water, λ_{sw}. All shadow values are calculated in step 1 using a linear programming model.[6]

In the GME production model, for each parameter, a vector of supporting values (feasible values for the parameters estimates) is constructed. For each vector of supporting values, there is a feasible set of probabilities (**p**'s in bold inside the parenthesis of (7)), which, when multiplied by the supporting values, yield expected values for the parameters with which conditions (8) through (10') are satisfied. The maximum entropy problem finds the vector of probabilities that is most likely to have generated q_i given x_{ij} and the optimality conditions.[7] With the probabilities that solve the maximization problem and the assumed supporting values, the expected values for the parameters of the nested CES are then estimated.

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[6] Future versions of the model will allow shadow values to be simultaneously estimated in conjunction with the CES production function parameters using GME. For more the advantages of this approach see Heckeley and Wolff (2003).

[7] See Paris and Caputo (2001), Paris (2001) and Mittelhammer (2001), for a discussion on consistency, sensitivity to support values, and multicollinearity associated with ME estimators.

3.5. Developing the Economic Simulation Model

Now, the estimated production function parameters of (3) from step 2, $\hat{C}, \hat{\beta}o, \hat{C}o, \hat{b}, \hat{\gamma}_o, \hat{\beta}w, \hat{C}w, \hat{\gamma}_w, \hat{\gamma}$, are re-introduced into the nested CES production function (3) and a new net-revenue-maximization problem subject to resource and water availability constraints is set up, and solved using non-linear programming techniques. More specifically;

$$(11) \qquad \max_{\mathbf{x}_{nirr}, \mathbf{x}_{irr}} \sum_i p_i \hat{q}_i(\mathbf{x}_{nirr}, ew_i(\mathbf{x}_{irr})) - \sum_i w_j x_{ij} - c_{ew} \, ,$$

where

$$\hat{q}_i(\mathbf{x}_{nirr}, ew_i(\mathbf{x}_{irr}) = \hat{C}_i \left\{ \hat{\beta}o_i \left[\hat{C}o_i \left(\sum_{j=1}^{7} \hat{b}_{ij} x_{ij}^{\hat{\gamma}_{o_i}} \right)^{\frac{1}{\hat{\gamma}_{o_i}}} \right]^{\hat{\gamma}_i} + \hat{\beta}w_i \left[\hat{C}w_i \left(\sum_{j=8}^{11} \hat{b}_{ij} x_{ij}^{\hat{\gamma}_{w_i}} \right)^{\frac{1}{\hat{\gamma}_{w_i}}} \right]^{\hat{\gamma}_i} \right\}^{\frac{\hat{\varepsilon}_i}{\hat{\gamma}_i}} .$$

Subject to resource constraints on

$$(12) \qquad \begin{cases} Land : \sum_i land_i \le B_l, \\ Surface\,Water : sw \le B_{sw} \, , \\ Family\ labor : \sum_i fl_i \le B_{fl}, \end{cases}$$

and subject to a water source constraint,

$$(13) \quad \sum_i aw_i = sw + gw \, .$$

A couple of points merit mention. First, comparing the set of resource constraints (12) with those appearing in (5) in step 1, one sees that there are no input coefficients, indicating that the assumption of fixed input proportions has been relaxed; in other words, the simulation model is ‘free’ to choose the profit-maximizing production technology. Also, the optimality conditions are distinguished according to whether the inputs are constrained or unconstrained. Unlike the constrained inputs, the optimality conditions for the unconstrained inputs involve their shadow values λ_j which were calculated in step 1. Another important aspect is that the nested CES functional form specified in (3) allows farmers to operate at decreasing and constant returns to scale, since the returns-to-scale parameter ε_i is bounded between 0 and 1.

Finally, equations (12) and (13), as did equations (5) and (5'), indicate that farmers are not only subject to a resource constraint on surface water, but also to a source constraint (13) which assures that the total amount of applied water must equal the combined amounts of surface water (sw) and groundwater (gw) used. Note also that surface water and groundwater uses (*sw* and *gw*) do not enter directly into the production function, but do so through (13). The maximization algorithm first chooses the optimal *total* amount of water (regardless of source) to be applied to each crop based on the marginal benefits of irrigation and the weighted average cost of applied water. Given the optimal amount of total applied water for all crops, the model then identifies the least-cost source (or sources, in the case of constrained surface water availability) of that irrigation water. Recall that the shadow value of water resources are included in these benefit/cost comparisons.

3.6. Data Requirements for the Economic Model of Agriculture

In order to calculate the shadow values in stage 1 and to estimate the production function (3) in stage 2, crop-specific data on irrigation inputs and non-irrigation inputs for each farm, for each of the two production seasons that comprise the agricultural calendar in the BV area. In addition, crop- and season-specific data on outputs are needed. Farmgate prices for inputs and outputs are needed; shadow values for limited or non-marketed inputs are provided in stage 1 by the linear programming model.[8]

Data on applied water are difficult to collect (though efforts are underway to do so), so an alternative approach was followed to calculate the amount of applied water per crop, farm, and season based on the standard formula

$$(14) \qquad aw_{is} = \frac{E_s - P_s}{I_{effi}},$$

where

- aw_{is} is the amount of water used for irrigation in season s on crop i,
- E_{is} is the evapotranspiration associated with crop i in season s. It is calculated using the Penman-Monteith equation for reference evapotranspiration (Etp_{i_t}) and suitable crop coefficients K_{i_t} for each growth period t in season s: $E_s=\sum(Etp_{i\_t} \cdot K_{i\_t})$

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[8] Future versions of the model will also contain a credit constraint. To do so, additional data on initial wealth, the value of farm and household capital, off-farm wage income, and interest rates on borrowed funds will be required.

- P_s is the amount of water available from precipitation for season *s*,
- I_{effi} is the irrigation efficiency associated with the type of irrigation technology used in the production of crop *i*.

Another approach to calculating applied water would be to directly calculate it using the formula:

$$(15) \qquad Aw_{si} = CP_{si} * N_{si} * DUR_{si} * D_{si},$$

where *CP* is the capacity of the pump in M^3/second used to irrigate crop *i* in season *s*; N_{si} is the number of times per day the pump was switched on to irrigate crop *i* in season *s*; DUR_{si} is the amount of time the pump remained on per day to irrigate crop *i* in season *s*; and D_{si} is the number of days in season *s* that crop *i* was irrigated. (Field data to perform these calculations are currently being collected.)

Part 4 – Using the Demonstration Economic Model to Predict Farmer Responses to Changes in Surface Water Availability

We now present some of the preliminary results of the application of demonstration economic model of agriculture to the case of dry-season farming activities in the BV as practiced by each of four archetypical farmers, one for each for the small- and medium-sized operational scales, and two for the large farm enterprises. In this demonstration model, the CES production function is not nested, so there is only one (common) elasticity of substitution for irrigation and non-irrigation inputs, which is assumed (ad-hoc) to be 0.8. We also assume constant returns to scale, which allows us to calculate analytically the parameters of the production function using the optimality conditions for each input ($VMP_j = Price_j$), and the fact that constant returns to scale assumption assures that $\sum_j B_j = 1$.

We leave the more demanding maximum entropy approach to be applied after field data are collected in the BV area.[9] Step 1 remains exactly the same as described in section 3.3. The objective function changes to accommodate the assumption of constant returns to scale; to avoid monocultures, we add a quadratic cost term to the net-revenue function to reflect the increasing costs associated with land allocation to a particular crop.

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[9] Testing of the field survey instrument is underway. Data collection is expected to begin in June of 2007.

We identify four archetypical farms (see Table 1). Farms 1 and 2 are large-scale grain farms using center pivot irrigation technology. Farm 3 can be viewed as a medium-scale operation with a diverse product mix. Farm 4 is a collection of homogeneous small-scale operations, each comprised of multiple crops grown on about 3.84 hectares. With the exception of farm size and crop allocation information, all data used in this exploratory exercise are for demonstration purposes only. Descriptive statistics appear in Table 1.

Table 1 – Archetypical Farm Characteristics, 2006

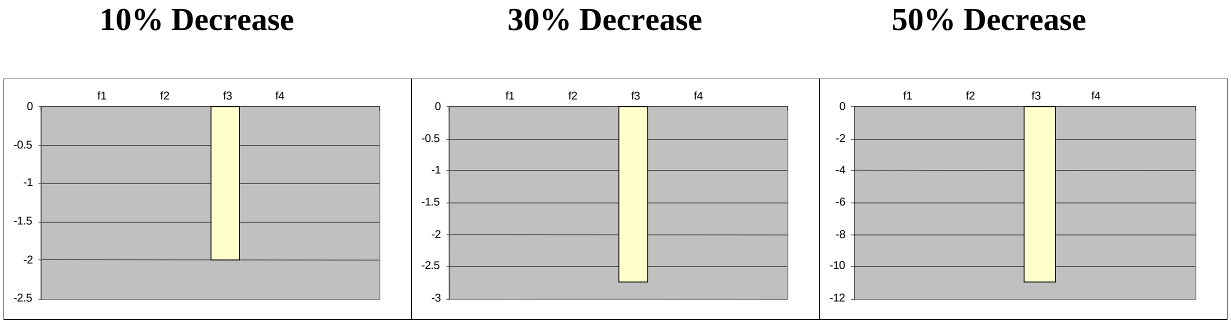
Farms and Agricultural Activities	**Crop Area** (Hectares)	**Crop Allocation** (%)	**Value of Production** (2006 Reais)	**Water Use** (m^3/hectare)		**Irrigation Technology**
				Surface	Groundwater	
Farm Type 1						
Corn	149.2	50.0	296,998	6744	0	Center Pivot
Beans	67.5	22.6	102,708	3852	0	Center Pivot
Soybeans	81.7	27.4	257,029	3496	0	Center Pivot
Total	298.3	100	656,735	14092	0	
Farm Type 2						
Corn	40.4	50.0	102,050	6744	0	Center Pivot
Soybeans	40.4	50.0	113,734	3496	0	Center Pivot
Total	80.8	100	215,784	10240	0	
Farm Type 3						
Corn	2.8	4.6	8,336	7970	0	Furrow
Beans	5.8	9.6	14,616	4553	0	Furrow
Soybeans	3.7	6.1	11,603	3030	0	Sprinklers
Limes	8.8	14.6	27,737	3306	0	Micro sprinklers
Horticulture	14.6	24.1	49,990	3681	0	Drip
Orchards	8.8	14.6	97,627	4775	0	Drip
Vegetables	10.0	16.5	149,860	6358	0	Furrow
Pasture	6.0	9.9	7,200	5400	0	Sprinklers
Total	60.5	100	366,969	39073	0	
Farm Type 4						
Limes	1.09	29.9	2,358	74	0	Micro sprinklers
Horticulture	0.69	19.0	1,249	141	0	Furrow
Orchards	0.69	19.0	6,813	134.4	0	Sprinklers
Vegetables	0.78	21.4	8,160	141.3	0	Furrow
Pasture	0.39	10.7	286	142.2	0	Sprinklers
Total	3.64	100	18,866	632.7	0	

Source: UCD/Embrapa field data

We now use the demonstration model (and the initial conditions, some of which are included in Table 1) to simulate the effects of changes in surface water availability on farm activities and farm income.[10] More specifically, we use the model to assess the effects of the reduction in the amount of surface water available on total of cultivated area, crop mix, the amount of water used on particular crops, and farm profits. The results are presented in the following series of figures.

The first response to be examined is the reduction in cultivated area; Figure 5 depicts the effects on total cultivated area of successively more scarce surface water (10%, 30% and 50% reductions in the amount of water used to irrigate in the unconstrained case). Only Farm type 3 reduced cultivated area in response to reductions in the availability of surface water for irrigation; 50% reductions in water availability led to over a 10% reduction in cropped area. The large-scale operations using center pivot irrigation did not alter cultivated area (not surprising), but neither did the small-scale farmers who shifted product mix rather than let farmland go fallow.

Figure 5 – Percent Change in Cultivated Area Associated with Reductions in Surface Water Availability, by Farm Type


10% Decrease **30% Decrease** **50% Decrease**

Source: Economic model simulations by Torres
Note: f1= Farm Type 1, etc.

A reduction in surface water availability will induce farmers to reallocation their land across crops, reducing the number of hectares allocated to crops that make (at the margin) less profitable use of water and increasing the area dedicated to those making more profitable use of

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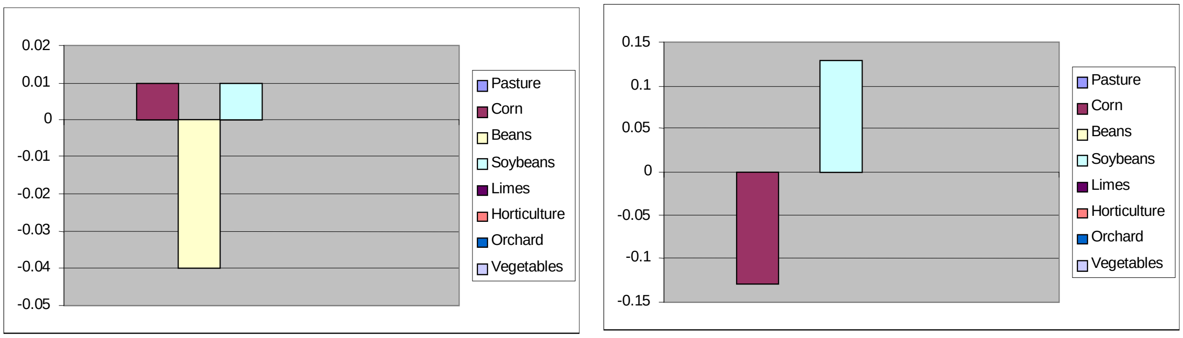
[10] See Torres et al. (2007) for the effects of changes in surface water prices on farm behavior.

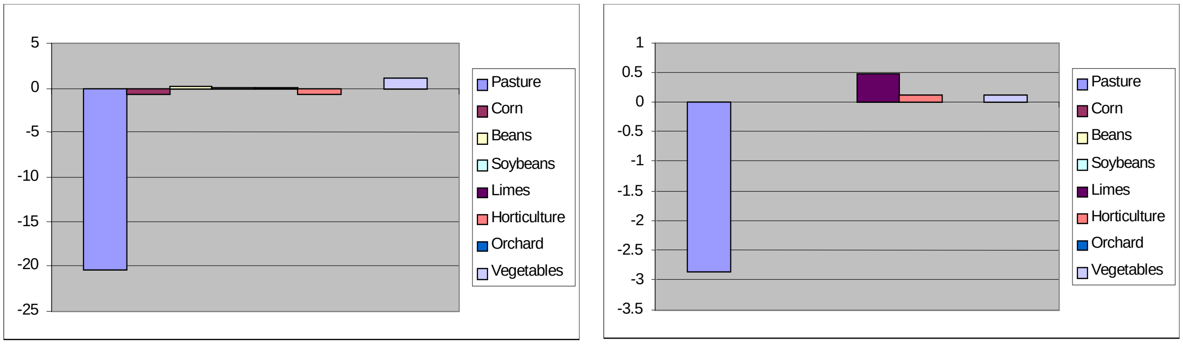
water. Figure 6 depicts the changes in crop mix brought about by successive reductions in the availability of surface water – one panel is dedicated to each of the reductions, and results are reported by farm type. Area dedicated to particular crops on Farm types 1 and 2 are scarcely affected by reductions in surface water availability; even a 50% reduction in available water will not alter cropping patterns for these large farm operations. Farm types 3 and 4, on the other hand, do react to water shortages, primarily by reducing (and eventually eliminating, in the case of Farm type 3) area dedicated to pasture and reallocating that land to alternative uses.

Figure 6 – Percent Change in Crop Mix Associated with Reductions in Surface Water Availability

Panel A -- 10% Reduction



Farm Type 1 **Farm Type 2**

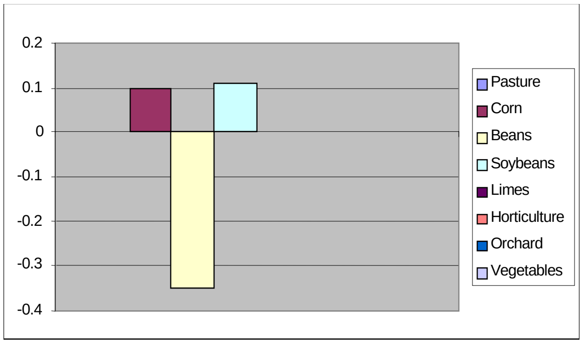


Farm Type 3 **Farm Type 4**

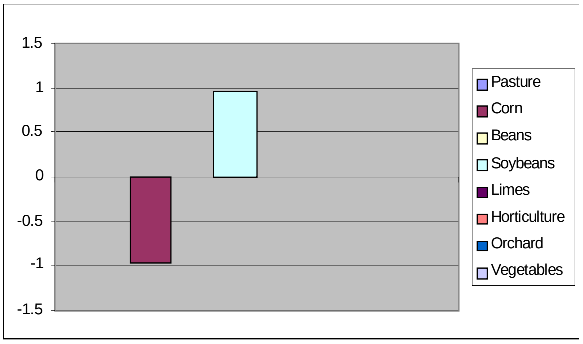
Figure 6 – Continued

Panel B -- 30% Reduction

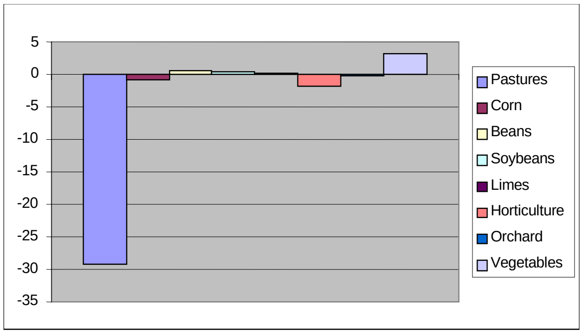
Farm Type 1



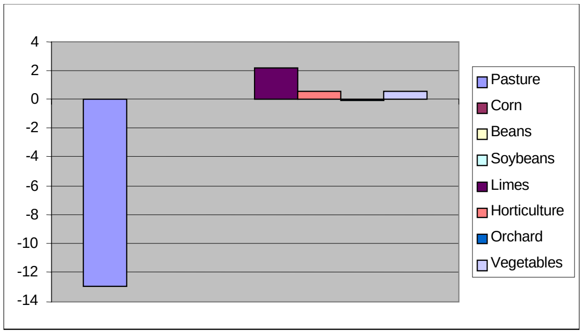
Farm Type 2



Farm Type 3

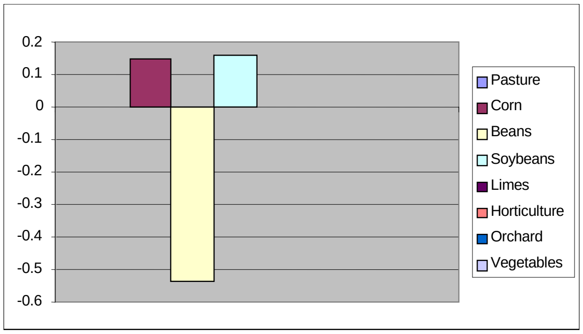


Farm Type 4

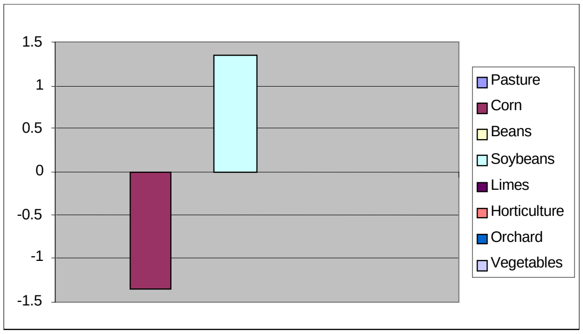


Panel C -- 50% Reduction

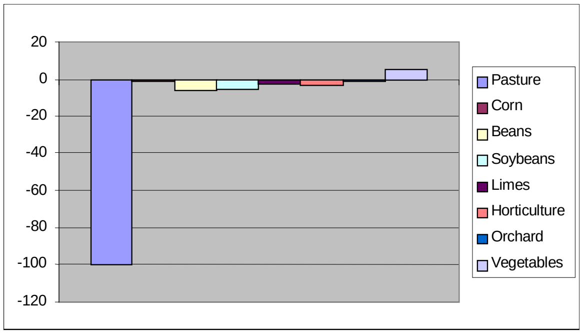
Farm Type 1



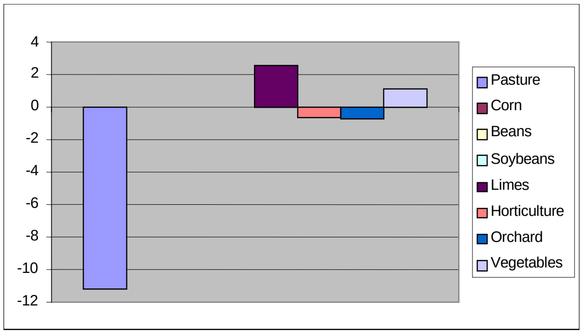
Farm Type 2



Farm Type 3



Farm Type 4

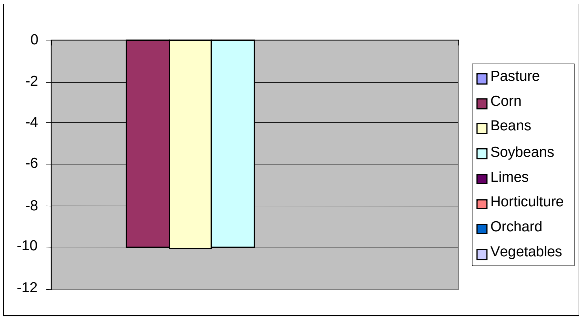


Unlike the case of modifications to crop mix, large-scale farms did modify the amount of water applied to crops in response to shortages in surface water, and did so more or less uniformly across all crops, with systematic reductions in applied water as shortages became more acute. Small-scale and medium-scale farm also reduced applied water as surface water became increasingly scarce. Figure 7 (again, in three panels, one each for successive surface water reductions) reports changes in per-hectare applied water by farm type.

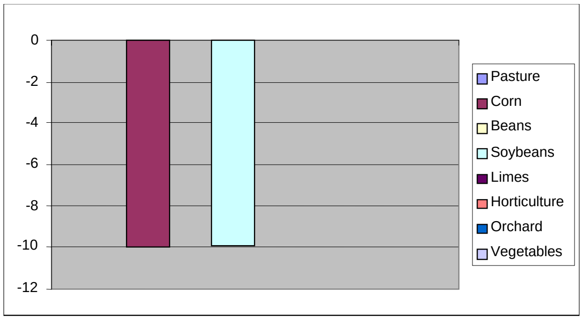
Figure 7 – Percent Changes in Applied Water Per Hectare Associated with Reductions in Surface Water Availability

Panel A -- 10% Reduction

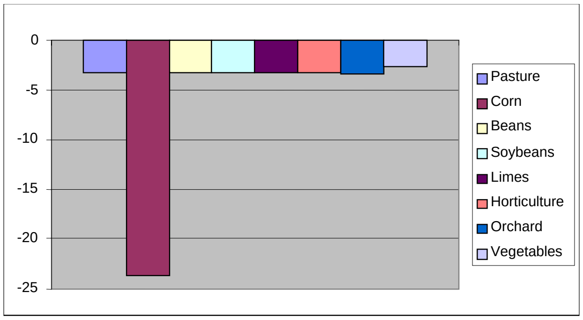
Farm Type 1



Farm Type 2



Farm Type 3



Farm Type 4

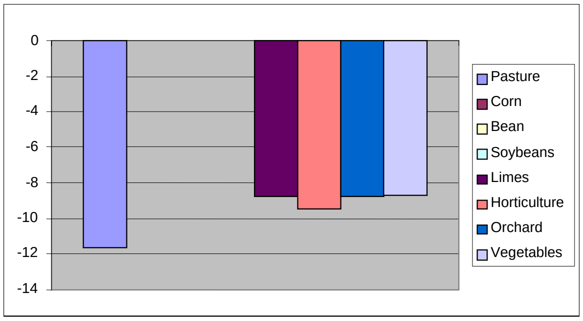
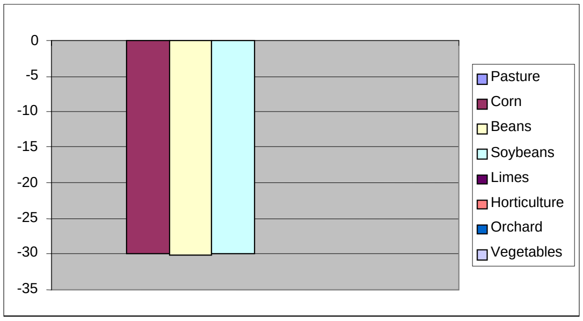


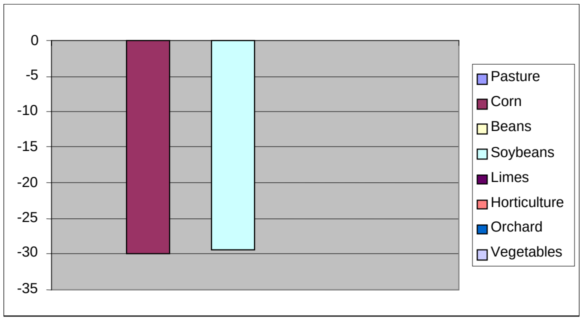
Figure 7 – Continued

Panel B -- 30% Reduction

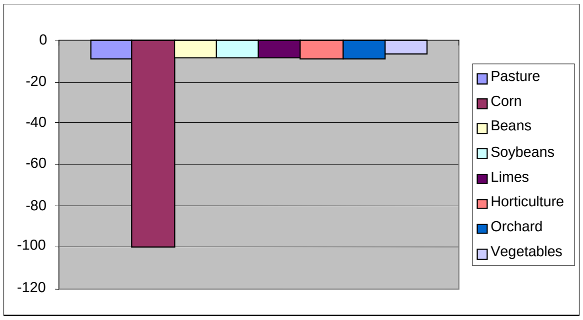
Farm Type 1



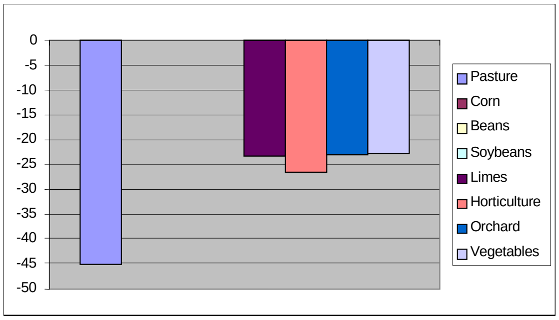
Farm Type 2



Farm Type 3

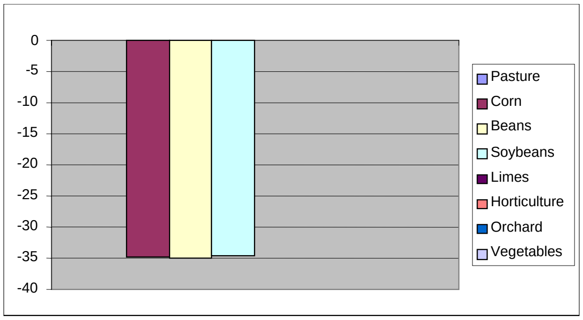


Farm Type 4

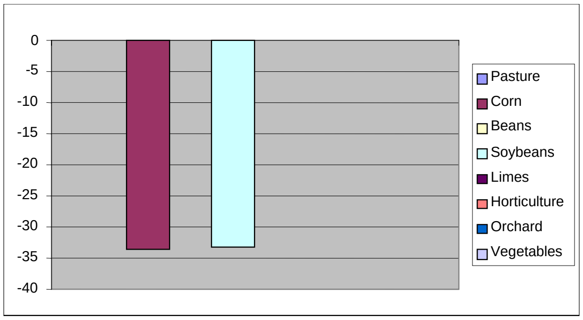


Panel C -- 50% Reduction

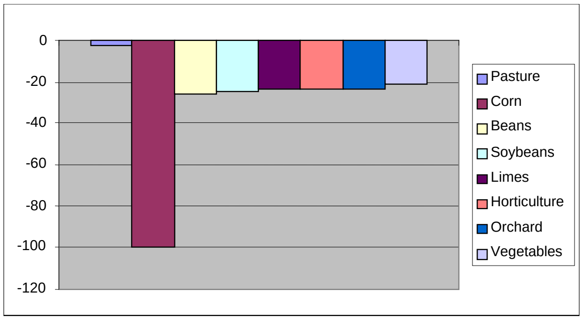
Farm Type1



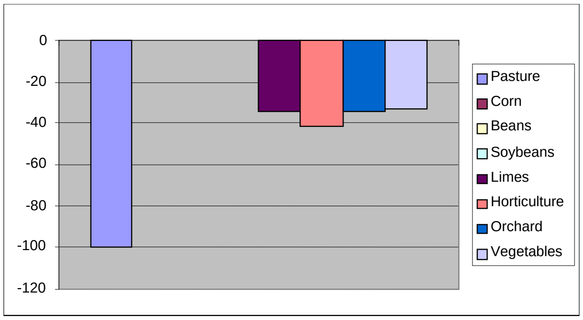
Farm Type 2



Farm Type 3

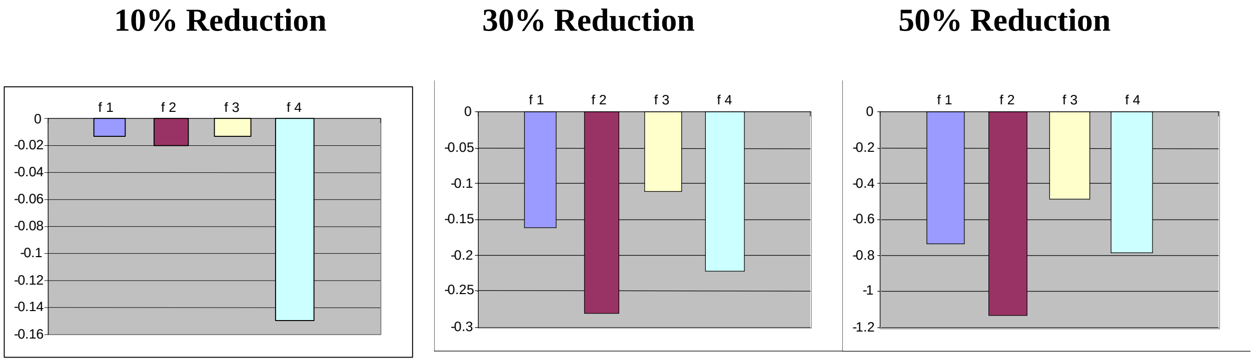


Farm Type 4



Finally, reduced availability of surface water will affect farm profits. However, the effects on profits are generally small (even in the face of large reductions in available surface water), because farmers adjust cultivated area, crop mix, the amount of applied water (and water sources, when permitted to do so) to minimize the effects of surface water shortfalls on farm profits. Figure 8 reports these effects. It is noteworthy, though, that the profits of small-scale farms (Farm type 4) tended to be the most adversely affected.

Figure 8 – Percent Change in Total Profits Associated with Surface Water Availability Reductions



10% Reduction **30% Reduction** **50% Reduction**

Part 5 – A Spatially Distributed Hydrologic Model for the BV

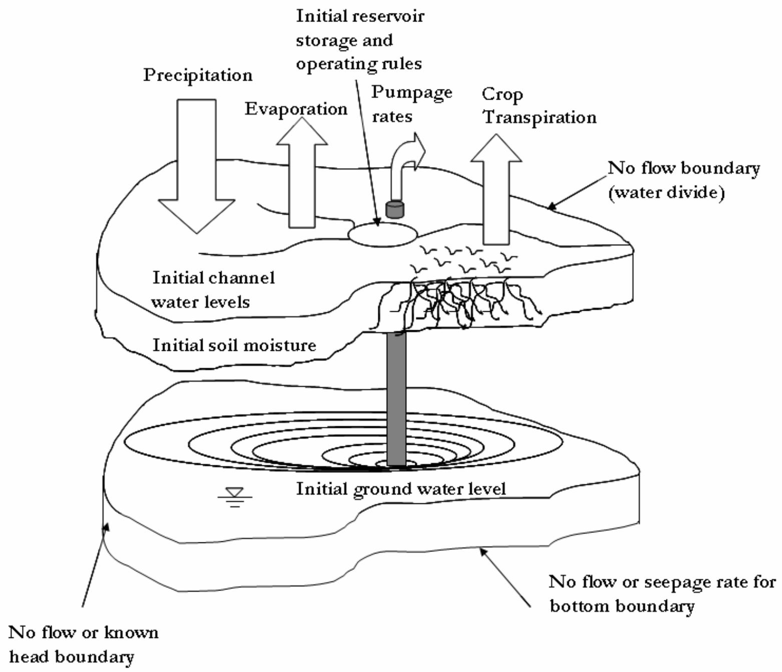
To evaluate the spatial and temporal impacts on water resources of the different agricultural scenarios a comprehensive and physics-based hydrologic model is needed. The model must simulate the main storage elements in a watershed (e.g., soil, channels, ponds, atmosphere, etc.) and the fluxes transferring water through and between them (e.g., rainfall, evapotranspiration, channel routing, seepage, infiltration, etc.). It should also contain mechanisms that capture water storages and transfers associated with agricultural activities (e.g., groundwater pumpage, irrigation).

MOD-HMS (HydroGeoLogic Inc, 1996; Panday and Huyakorn, 2004) is a 3D, spatially distributed watershed model used to simulate the variably-saturated subsurface flow, overland and channel flows, and their interactions in space and in time. Subsurface flows are simulated using the 3-D Richards equation in variably-saturated porous media, what allows the solution of a single set of equations for both the unsaturated and saturated soil layers. Overland and channel flows are simulated using a 2D and 1D solution of the diffusion wave, respectively. The equations that govern the flows in the channel, surface and subsurface are coupled and solved simultaneously, which permits a more robust and efficient treatment of the interaction between the water resource in the three domains. In addition, this fully-coupled solution of surface and subsurface systems solves the problem of having to explicitly specify the recharge to the groundwater system, and accounts in a natural manner for the impacts of water reallocation within the system (e.g., by agricultural pumping and drainage). The model calculates actual evapotranspiration for the different soil covers using an extended version of the Kristensen and Jensen model (Kristensen and Jensen, 1975). It can also accommodate small reservoirs and gates in channels, among other obstacles to water flows.

The models are discretized into 30mX30m field-scale grid cells. Vertical discretization is variable, ranging from submeter near the surface to 5 meters at greater depth. The model uses hourly or daily boundary conditions for irrigation, rain, evapotranspiration, and pumping. Aquifer and soil hydraulic properties are assigned based on a combination of soil survey and well log texture data (data collection is underway). The simulation domain is defined by the limits of the watershed extracted from a DEM and the depth to the bottom limit of the aquifer. The model uses user-based time-step boundary conditions for irrigation, rain, evapotranspiration, and pumping.

A scheme of the boundary and initial conditions supplied to MODHMS are depicted in Figure 9. Some of the boundary conditions are the 'links' between the hydrology and economic models.

Figure 9 – Boundary and Initial Conditions for MOD-HMS Hydrologic Model



The primary boundary conditions of the model include flow conditions at the bottom and border limits of the watershed, precipitation, reference evapotranspiration, groundwater pumpage rates and operating/extraction rules for the reservoirs; the final two are provided by the economic model and are subject to policy action.

Field data will be relied upon to provide the required detailed information on agriculture. Specifically, we are in the process of collecting the following information; spatial distribution of soil variability before crop emergence; spatial distribution of crop types and a classified map of crop distributions; spatial distribution of canopy cover and development over the growing season; spatial distribution of areas of poor crop growth and yield; and some semi-quantitative estimates of relative crop yield.

Part 6 – Using the Hydrologic Model to Predict the Effects of Policy Action

We now use the hydrologic model to illustrate its capability for assessing the effects of different irrigation policies on surface water availability and groundwater. The irrigated areas in BV are the three center pivots shown in Figure 3 and small orchards, groves and gardens in the community. Irrigation demands for each crop for a given cropping scenario was calculated from the type of crop using suitable crop coefficients for the area, the extension of the area cropped, its growth stage and the atmospheric conditions. The total demand supplied by the different water sources will be presented along with the results of the models for the scenarios considered.

Water Use Scenarios

We begin by defining two water use scenarios for the farmers in Buriti Vermelho. Recall (from Figure 3) that the sub-catchment area includes three center pivot (CP) irrigation schemes, and a number of small-scale and medium-scale farms that can draw water either from wells or from reservoirs #2 or #5. Center pivots 1 and 2 import water from the adjacent basin so do not add to the water demand within the BV.

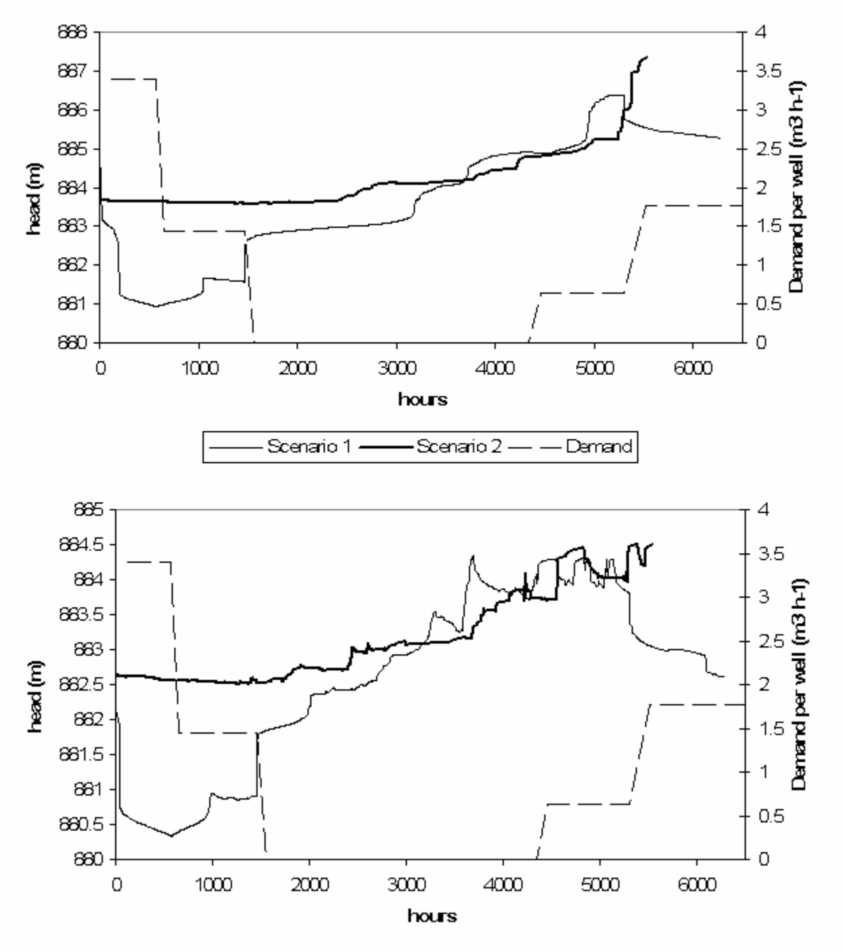
In the Scenario 1, center pivot 3 draws water from reservoir #5. The community satisfies half of the irrigation demand from a field of 24 wells spread throughout the community and half from reservoir #2. The wells are 0.5 meters in diameter and are assumed to fully penetrate the aquifer.

In Scenario 2, we assume that groundwater pumping is not allowed (perhaps by policy action) so farms must satisfy all irrigation needs from reservoir #2, while CP 3 continues to draw water from reservoir #5.

Hydrological Results

The impacts on the spatial availability of surface water and groundwater of the two scenarios are very different. Figure 10 and Figure 11 report the results for groundwater to illustrate the temporal and spatial effects of groundwater use. Figure 10 depicts the elevation of water (above sea level) in time for two arbitrary wells from the field of 24 wells shown in Figure 3. Groundwater head for Scenario 1 is reported by the thin line and for Scenario 2 the same data are reported by the thicker line; groundwater demand for Scenario 1 is reported with the dashed line. Because no pumpage occurs in Scenario 2, the net effect of pumping on well depth can be seen in the vertical distance between the thin and the thick lines; this distance can be over 2.5 meters but it converges quickly as pumpage ceases.

Figure 10 – Head at the Bottom of Wells 4 (top panel) and 10 (bottom panel) for Scenarios 1 and 2

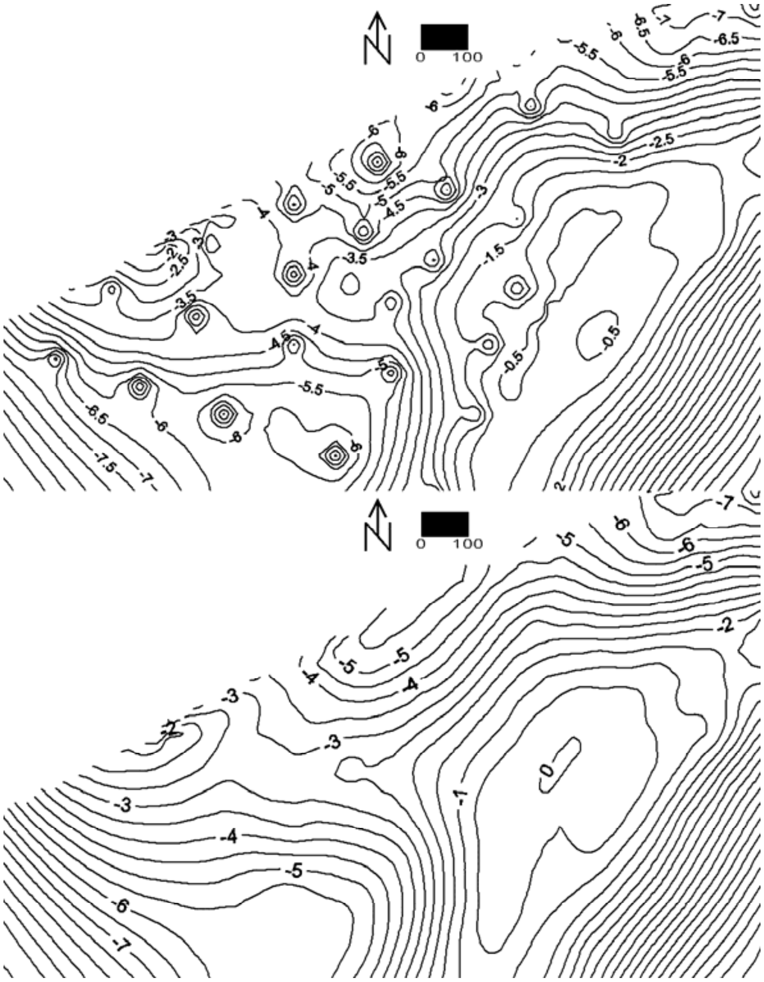


Source: MOD-HMS simulation by Maneta

Note: The horizontal axis is measured in hours and begins in early September; hour 120 is approximately September 3rd.

To illustrate the spatial impact of groundwater extraction, Figure 11 displays the contour lines indicating the depth (in meters) of the water table from the soil surface at approximately the peak of groundwater extraction in scenario 1. It occurs 600 hours after the beginning of the simulation (see groundwater demand in Figure 10); the lower panel of Figure 11 displays the situation of the water table at the same time (600 hours from the beginning of the simulation) for Scenario 2, in which no pumping occurs. Note the local 'depressions' (some of which are quite deep) that will cause increases in pumping costs. Note also that the well depth drawdowns propagate beyond the close neighborhood of the well, which may affect the water table level of adjacent properties; pumpage by one farmer can affect pumping costs and groundwater availability of his/her neighbors.

Figure 11 – Calculated depth to Water Table in Community Well Field at time '600 hours' for Scenario 1 (Upper Panel) and Scenario 2 (Lower Panel)

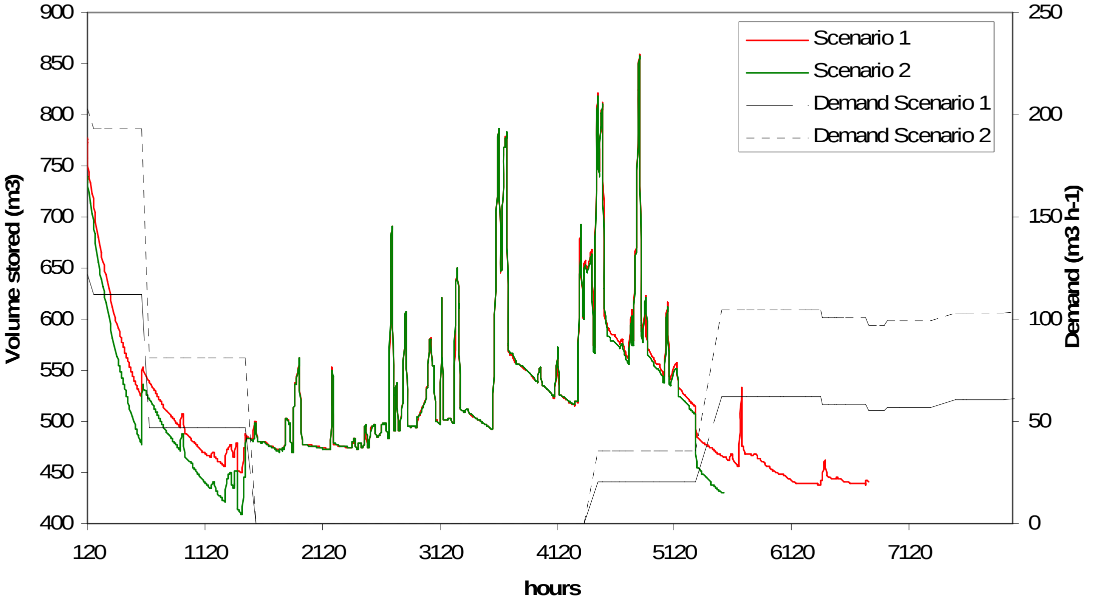


Source: MOD-HMS simulation by Maneta
Note: Contours are equidistant and set at 0.5 meters.

Fortunately, for the inhabitants of BV, the water table level tends to recover quickly (as was observed in Figure 10) and Scenario 1 reaches similar levels as in Scenario 2 once pumping declines with the onset of the rainy season.

The scenario-specific impacts on surface water flows and on water storage is also significant, and varies over space and time. Figure 12 depicts the evolution of surface water demand from, and water storage in, reservoir #2 for both scenarios. Several points merit mention. First, the relatively higher load on the reservoir storage under Scenario 2 keeps the reservoir storage at a lower level than in Scenario 1 (vertical distance between the red and green lines) but as soon as the demand ceases during the wet (Brazilian) summer months the reservoir quickly recovers to the levels of Scenario 1. Second, the level of reservoir #2 never falls below that required to provide water to farmers practicing the base-line agricultural activities, i.e., surface water is not constrained. Third, the dramatic oscillations in reservoir storage during hours 1,500 and 5,200 (roughly October through April) are attributable to precipitation.

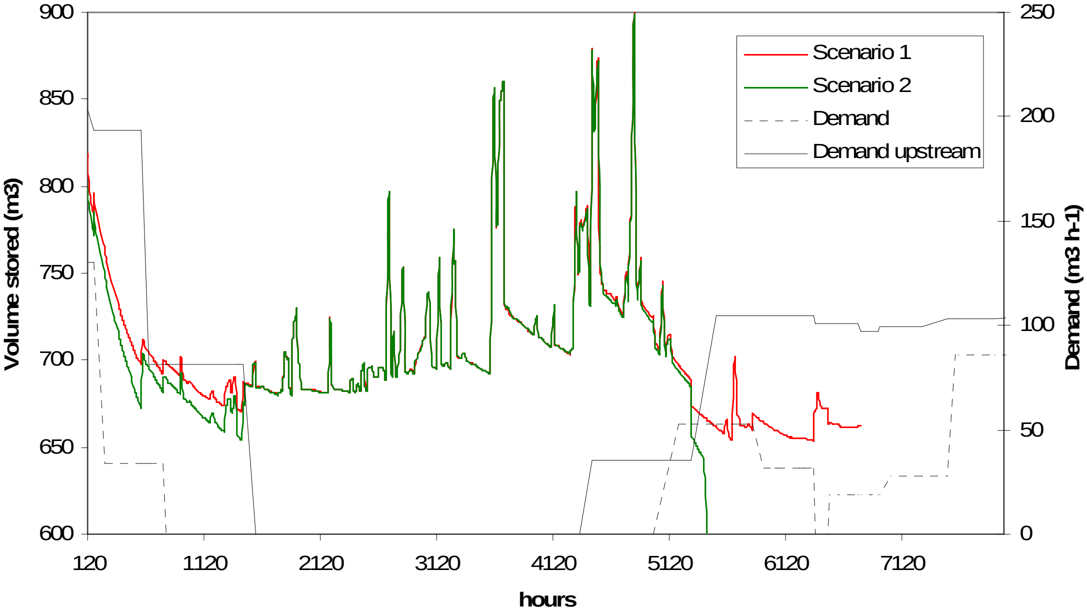
Figure 12 – Evolution of Water Storage in Reservoir #2



Source: MOD-HMS simulation by Maneta
Note: The horizontal axis is measured in hours and begins in early September; hour 120 is approximately September 3rd.

Figure 13 shows water demand and storage for reservoir #5. Because this reservoir is the last in the series of reservoirs in the BV, it is highly affected by what occurs upstream. To meet irrigation demands, all the reservoirs depend in the incoming flows. Under Scenario 1, the volume stored in the reservoir satisfies the demands of center pivot 3, the only 'user' of this storage facility. Under Scenario 2, however, upstream off-take by smallholders from reservoir #2 is substantially increased (due to groundwater pumping restrictions), and this reduces the inflow into reservoir #5, which, by about hour 5120 (around the month of May), is unable to meet irrigation water needs and eventually dries up completely. Without enough upstream recharge, the approximately 670 m^3 stored in the reservoir after the rainy season are not enough to meet the demands of the center pivot 3.

Figure 13 – Evolution of Water Storage in Reservoir #5



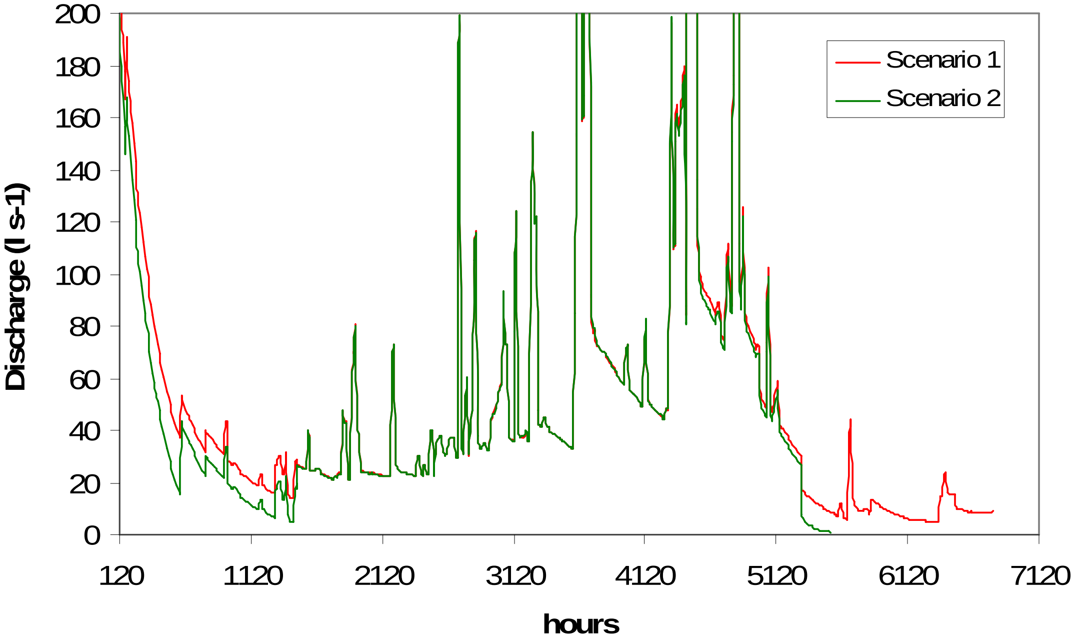
Source: MOD-HMS simulation by Maneta

Note: The horizontal axis is measured in hours and begins in early September; hour 120 is approximately September 3rd.

Finally, the two scenarios envisioned in this analysis also affect the flow of water out of the Buriti Vermelho sub-catchment area, and these flows could have significant environmental value. Figure 14 depicts the flows of water (measured on the vertical axis in liters per second) out of the BV; once again, time is measured on the horizontal axis in hours, beginning with

September 3rd. As one would expect, out-flows are substantial during the rainy season and are essentially unaffected by the irrigation schemes associated with either of our scenarios. However, during the dry season the differences are quite noticeable across scenarios. Of potentially critical importance is that out-flow goes to zero in about late-May under Scenario 2; at that point in time central pivot 3 uses all available surface water from reservoir #5, the final storage in the BV water system. If the downstream areas depend on a continuous out-flow from the BV (either for ecological or economical reasons), the effects of the water policy under Scenario 2 (prohibition of groundwater extraction) may have severe negative implications.

Figure 14 – Discharge at the BV Basin Outlet



Source: MOD-HMS simulation by Maneta
Note: The horizontal axis is measured in hours and begins in early September; hour 120 is approximately September 3rd.

Part 7 – A Strategy for 'Linking' the Economic and Hydrological Models

The hydrologic and economic models must 'meet' in time and space in ways that are theoretically and computationally acceptable, and in ways that are useful for policy analysis. The hydrological model divides the BV into 30 x 30 meter grids and functions on a 5-minute time step. The economic model divides the BV into operational holdings (farms) and functions on a seasonal time step (wet-season production and dry-season production). Data from the hydrological model will aggregated spatially to 'match' the boundaries of the farms that occupy the BV sub-catchment area (see Figure 3), and temporally to 'match' the seasonal crop mix and technology choice decisions to be predicted by the farm model.

While the two models will be explicitly connected, they will *not* be solved simultaneously with feedback terms; model interaction happens sequentially at the end of each model run. The economic model provides optimal product mix, input use, irrigated area, and irrigation technology which collectively will provide an estimate of the derived water demand. The hydrologic model will output water stocks and the spatial impact of water reallocation – these stocks will be compared with the spatially explicit demand for water that emerges from the economic model.

The basic interaction time step is one season. Information on typical planting dates, the length of the crop-specific growth cycles, and crop-specific irrigation coefficients for each agricultural scenario will be used to calculate applied water. Given the annual time series of rainfall and potential evapotranspiration (spatially distributed or not), the required applied water is approximated for each crop as follows:

$$aw_{i\_t} = \frac{Etp \cdot K_{i\_t} - P}{I_{eff}} \tag{16}$$

where aw_{i_t} is the applied water for crop i at growth stage t [LT^{-1}], Etp is potential evapotranspiration [[LT^{-1}]], K_{i_t} is the crop coefficient for crop i at growth stage t, P is precipitation [LT^{-1}] and I_{eff} is the irrigation efficiency coefficient [$0<I_{eff}\leq 1$] used to simulate the irrigation technology.

The decision of the source of water to use (ground or surface water, or some combination of the two) will be made according to the cost and availability. In the case of groundwater, the hydrologic model will provide information on the depth of the water table at the location of the well to calculate the cost of pumping (c_{gw}) as a function g of $depth$:

$$c_{gw} = g(depth) \tag{17}$$

Groundwater will be pumped for irrigation up to the point at which the marginal cost of pumping is just equal to marginal benefit of applied water:

$$p_{si}\frac{\partial q_{si}}{\partial aw_{si}} = \frac{\partial g}{\partial gw_s} + v_{gw} \tag{18}$$

where p_{si} is the price of crop i at season s, q_{si} is the production function for crop i at season s, gw_s is the expected average groundwater level at season s and v_{gw} is an error term to bring to zero to indicate optimal groundwater use.

The availability of stored surface water (reservoirs) will be provided by the hydrologic model and will be used by the economic model as a constraint – farmers can never use more surface water than is available for all crops:

$$\sum aw_{sw\_t\_i}land_{s\_i} \leq B_{sw_s} \tag{19}$$

Finally, the value of the marginal product of surface water will depend on its scarcity:

$$p_{si}\frac{\partial q_{si}}{\partial aw_{si}} = c_{sw_s} + \lambda_{sw\,s} + v_{sw} \tag{20}$$

where $aw_{sw\_t\_i}$ is the applied water for crop i at growth stage t from coming from surface sources, $land_{s\_i}$ is the amount of land allocated to crop i in season s, and B_{sw} is the total usable surface water in season s. In the last equation, c_{sw} is cost of surface water, λ_{sw} is the farm-specific shadow value of surface water, and v_{sw} is the error term to ensure that the optimality conditions for surface water use hold.

Part 7 – Conclusions and Next Steps

Policymakers at national, regional, state and local levels are considering a broad array of policy actions (e.g., introduction of water pricing schemes, reservoir construction, restrictions on the drilling and use of wells) in throughout Brazil, but are doing so without the benefit of any scientific input related to the hydrological or socioeconomic consequences of such policy actions in the short or the long runs.

Filling this knowledge gap requires detailed knowledge of the stocks and flows of surface water and groundwater, and how these will be affected by the reallocation of water in space and time. However, while knowing how the stocks and flows of surface water and groundwater will be affected by alternative water use scenarios is *necessary* for policy guidance, it is not sufficient. Surface water and ground water have different values to different farmers, and even for given farmers, water has different values depending on the time of year, the value and productivity of non-water inputs, and relative input and output prices – all of which influence farmers' choices of total cultivated area, crop mix, and input mix.

An economic model of agriculture is required to predict the behavior of farmers (of different types) who manage farms (of different types). But the economic model, in isolation, may also fail to provide proper policy guidance – for example, predicted farmer behavior (at farm level) may be infeasible given available water resources, which the economic model cannot predict, or, predicted farmer behavior (in the aggregate) may not be sustainable if water resources in the catchment area are depleted over time.

Therefore, spatially explicit and linked hydrologic and economic models are needed to predict the effects of policy action on agriculture and on water resources. No linked, hydro-economic models of the types presented here are currently available in developing countries, in part because their data requirements are difficult to meet. We are developing a pair of such detailed models for the Buriti Vermelho sub-catchment area in the São Francisco River Basin in Brazil. In this paper we present demonstration versions of the economic model and of the hydrologic model, and 'test drive' each model separately. Four distinct farm types (from large-scale farms using highly sophisticated irrigation technologies to small-scale farms using rudimentary irrigation practices) included in the economic demonstration model are shown to react differently to reductions in surface water availability as regards total cultivated area, crop mix, crop-specific applied water, all of which helped buffer farm profits, but not completely so, especially for small- and medium-scale farms. The hydrologic model demonstrated the catchment-wide effects of changes in water use patterns by farmers; the externality effects within and beyond the BV were clear.

Next steps in research include: a) the complete calibration of the hydrologic model using field data (currently being collected); b) estimation of the complete economic model of agriculture (also using field data currently being collected); c) refining and implementing the links between the two models; and d) using the linked models to assess the effects of alternative water and agricultural policies on water resources, agricultural practices and farm income.

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