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# **Optimal Detection Strategies for an Established Invasive Forest Pest**

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## **Introduction**

When it comes to invasive species management, economists have focused on the tradeoff between prevention of potential invasions and management of established populations. The intermediate step—detection of established populations on the landscape so that management can commence—has only recently received attention in the economics literature. A recent paper (Mehta et al., 2007) explores how biological and economic parameters affect optimal detection spending, recognizing that greater expenditures on detection can lead to smaller and more manageable population sizes upon detection because populations are discovered early.

We build upon this framework by considering the optimal spatial allocation of detection effort when it is impossible to stop the advance of the main front of an invasive species, yet it is beneficial to detect and control sub-populations of the species that erupt ahead of the front. This is true in the example of the gypsy moth, where managers have given up hopes of eradication yet still detect and treat sub-populations that establish ahead of the front due to the transport of eggs on vehicles. One paper (Sharov and Liebhold, 1998) solves for the optimal spatial pattern of traps in this context, and finds that it is best to have the highest intensity of traps closest to the front, with diminishing intensity as the distance from the front increases.

In this paper, we use an alternative approach to solve for the optimal pattern of detection intensity ahead of an advancing front. Our approach recognizes that the duration of management of sub-populations is constrained by the amount of time remaining before the main front arrives. Locations close to the front have less time remaining than locations that are more distant. These differences imply different levels of potential benefit from early detection; in particular, shorter

management horizons translate into lower benefits from intervention. The optimal intensity of detection effort varies over space along with this variation in the benefits from management.

## **Model Development**

We envision a situation where an invader is spreading throughout a landscape. The front of the advance moves according to a spread coefficient, and this spread is inevitable. The main population throws off sub-populations ahead of the front. These are transported by humans through, for example, transport of eggs or larvae on vehicles. Although the spread of the front is inevitable, treatment of sub-populations is possible. Sub-populations can only be treated once they are detected, and the intensity of detection determines the date at which management can start.

### *Optimal Management of a Sub-Population*

We start with a model of optimal management given some starting population ( $x(\tau)$ ) at date  $\tau$  (time of detection), and ending date  $T_{\max}$ .  $T_{\max}$  is the date that the main population will catch up to the sub population. Damages are linear in the stock of the pest ( $x$ ). Control costs are quadratic in the removal level ( $R$ ). The unmanaged stock grows exponentially, and management modifies the growth of the population by removing pests. The management problem at date  $\tau$  is to minimize the stream of discounted control and damage costs:

$$\min \int_{\tau}^T e^{-r(t-\tau)} (px(t) + cR(t)^2) dt$$

$$\text{subject to: } \dot{x} = ax - R$$

$$x(\tau) = x_{\tau}$$

$$T \leq T_{\max}$$

$$x(T) \geq 0$$

The parameters are: damages caused by the pest ( $p$ ), management costs ( $c$ ), the interest rate ( $r$ ), and the exponential growth rate ( $a$ ).

We solve this problem using optimal control theory, finding the optimal paths of removals ( $R^*(t)$ ) and stock ( $x^*(t)$ ). We insert these into the integral to get an expression for the optimized cost—including damage plus removal costs. This value function is a function of the starting stock level ( $x(\tau)$ ) and the amount of time in the horizon ( $T_{\max}-\tau$ ):

$$V(x(\tau), (T_{\max} - \tau)) = \int_{\tau}^{T^*} e^{-r(t-\tau)} (px^*(t) + cR^*(t)^2) dt.$$

We have inequality constraints on the ending time and the ending state, so the solution procedure is to solve first with the transversality conditions with a free ending time and ending state:  $\lambda(T)=0$  and  $H(T)=0$ . If either constraint is violated, we impose the constraints one by one, evaluating the value functions with a free ending state and a constrained ending time ( $\lambda(T)=0$  and  $T=T_{\max}$ ), and with a constrained ending state and a free ending time ( $X(T)=0$  and  $H(T)=0$ ). If the terminal constraints are satisfied in both cases, we choose the case with the smallest overall cost. If constraints are violated in both cases, we impose both constraints to determine the overall cost.

### *Optimal Search*

The next step in our model development is to look at search. In this setting, search ( $s$ ) affects the date of detection ( $\tau$ ) in a deterministic manner. We adapt the probabilistic model of Mehta *et al.* by using the expected date of detection so that

$$\tau=1/(k s),$$

where  $k$  is the “detectability” coefficient. A higher  $k$  and a higher  $s$  lead to earlier date of detection and a smaller stock size when detection occurs.

The management problem, then, is to choose search intensity to minimize total costs. There are three components to total costs: search costs, damages caused by the pest before detection occurs, and optimized costs as summarized in the value function. The first cost component is the cost of search. Instantaneous search costs are equal to  $bs^2$ , so that the stream of search costs until detection occurs is:

$$C_1(s)= bs^2(1-e^{-r\tau(s)}).$$

The second cost term is the stream of damages caused by the pest before detection occurs:

$$C_2(s)= px_0(e^{(a-r)\tau(s)}-1),$$

where  $x_0$  is the stock level at the beginning of the time horizon. The third cost term is the cost once the pest is detected as summarized by the value function found above, discounted back from the date of detection  $\tau$  to 0:

$$C_3(s)=e^{-r\tau} V(x(\tau(s)), (T_{\max}-\tau(s))).$$

These terms are all functions of the search level through the date of detection. The total cost with search (TCS) is equal to the sum of these three terms:

$$TCS=C_1+C_2+C_3.$$

To find the optimal search level, we find the level of search that maximizes benefits of search, defined as the difference between doing nothing undertaking search. The costs without search (TCNS) are the damages caused by the pest from time 0 to the time the front arrives ( $T_{\max}$ ):

$$TCNS=px_0(e^{(a-r)T_{\max}}-1).$$

Figure 1 is a graphical depiction of the population of the pest at a given distance from the front, from the perspective of time 0. Different search intensities correspond with different dates of detection, with earlier dates of detection like  $\tau_1$  associated with high search intensities and late dates of detection like  $\tau_3$  associated with low search intensities. The dynamically optimal paths of the pest once detection occurs are represented by the lines that depart from the no-management stock path at the alternative dates of detection. These paths are summarized in the value functions that depend on detection intensity through the date of detection,  $\tau(s)$ , and the stock level at the time of detection,  $x(\tau(s))$ .

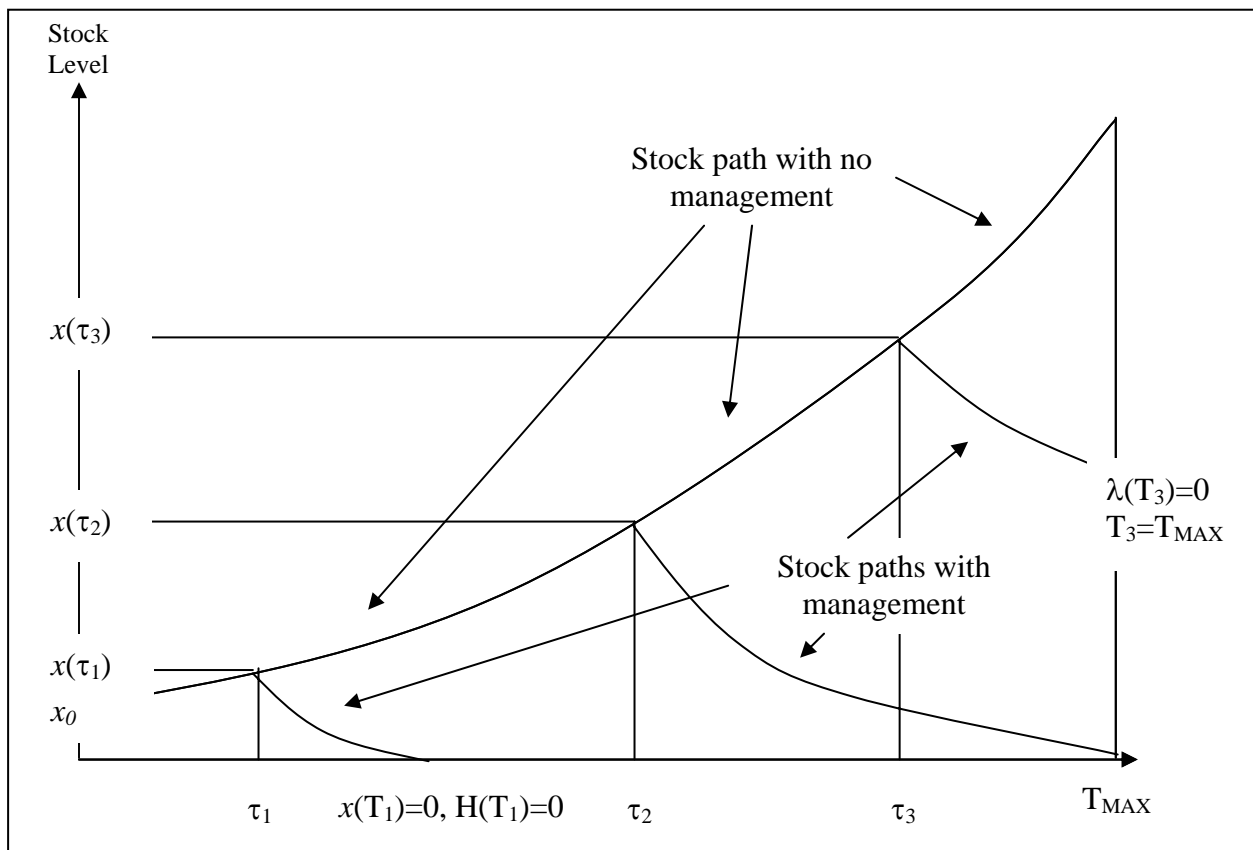


Figure 1: Stock paths with alternative dates of detection. For early detection, stocks are optimally driven to zero. With late detection, ending stock levels can be positive.

Different distances from the main front are characterized by different dates at which the front will arrive. The spatial component is incorporated with a diffusion model, with which we can calculate the spread rate of the species as a function of the diffusion coefficient and the growth rate. The spread rate can then be used to calculate the date of arrival ( $T_{\max}$ ) at any distance from the front. To find detection intensities for different distances from the front, we re-solve the problem for the alternative levels of  $T_{\max}$ .

We observe that the optimal ending date of the management horizon may easily occur before the front arrives at  $T_{\max}$ . For any given starting stock level, the optimal path may be identical with different levels of  $T_{\max}$  because the  $H(T)=0$  transversality condition will imply the same optimal management horizon. However, the optimally chosen detection level may easily differ with distance, as the benefits to management can be significantly higher with a larger distance from the front. This is because the maximum size of the unmanaged population is greater with greater distances, leading to higher benefits to management, tipping the balance towards earlier detection even if that means higher expenditures for search activities.

### **Gypsy Moth (*Lymantria dispar*) Management**

The gypsy moth is an invasive pest that defoliates a wide variety of host trees. It was introduced accidentally from Europe to Massachusetts in 1869, and is now found throughout the Northeast United States. It is currently spreading south and west toward the Midwest and the Southeast United States. Efforts to eradicate the gypsy moth have failed, and the spread of the species to its suitable host range is considered inevitable. Though adult female gypsy moths fly only very short distances, larvae are carried by wind currents to new locations. Human travel can assist



dispersal of gypsy moths: egg masses can be deposited on vehicles at, for example, infested camp grounds. When these vehicles move, sub-populations of gypsy moths can emerge beyond the existing range of the population, speeding the natural dispersal of the pest.

Management of gypsy moth populations starts with detection through the placement of pheromone traps—tent-shaped boxes containing female pheromones and a sticky trapping substance. These traps are monitored to see if male gypsy moths have entered. Once the moths are detected, pesticides or mating disruption methods are used for control. Pesticides are effective, but only recommended for small infestations due to collateral impacts on other species. Table 1 shows the parameter values we will use in our simulations to show how optimal density varies with distance from the advancing front.

Parameter	Value	Source
$a$ : growth rate	4.6	Liebhold, Halverson, Elmes 1992
$b$ : cost of detection	\$54.38 per trap	USDA, 2005
$c$ : cost of treatment	\$6,200 per square kilometer	Sharov and Liebhold, 1998
$r$ : discount rate	0.04 0.10	
$p$ : damages	\$380/square kilometer/year	Sharov and Liebhold, 1998
natural spread rate	2.5 km/yr	Liebhold, Halverson, Elmes 1992

Table 1. Preliminary parameter values for gypsy moth detection model.

## **Discussion**

While large portions of budgets are spent on detection activities (NISC, 2006; MDA 2005), very little economic analysis has been devoted to finding out how these funds should be allocated. To our knowledge, only one paper examines the optimal spatial distribution of detection efforts. Our paper yields very different conclusions about how the intensity of detection should vary over space. In particular, we show that optimal detection intensities increase with distance from the front due to increased benefits from management.

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