



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

**A
SURVEY OF
AGRICULTURAL
ECONOMICS
LITERATURE**

VOLUME 2

*Quantitative Methods
in Agricultural Economics,
1940s to 1970s*

*George G. Judge, Richard H. Day, S. R. Johnson,
Gordon C. Rausser, and Lee R. Martin, editors*

Published by the University of Minnesota Press, Minneapolis,
for the American Agricultural Economics Association

Agricultural Production Function Studies

Roger C. Woodworth
Agricultural Economist
Agricultural Resource Branch
Tennessee Valley Authority

Following World War II, agricultural economists made a sustained effort to improve methodology and develop applications in quantifying agricultural production relationships mathematically and in using this knowledge to determine economic attributes of the production process. These studies involved calculus and incorporated such recent developments in statistics as more efficient design of experiments, multiple regression, and tests of significance. Perhaps more important from the standpoint of applied economics, the work used production principles based on marginal analysis and equilibrium conditions.

In 1939 Sune Carlson in his classic book, *A Study on the Pure Theory of Production*, defined the production function as the relationship between the variable productive services and the output under the assumption that the plant or fixed services remained constant. He said that this relationship could be most conveniently expressed in mathematical form, writing the amount of output as a function of the different variable services. He also defined marginal productivity, the production surface, isoquants, isoclines, ridge lines, the expansion path, isocosts, and other properties with economic implications derived from the production function.

Several contributions to agricultural economics literature synthesized the advances of Carlson [1939], Hicks [1946], and others, relating the theory of the firm to the applied field of agricultural production economics. The well-known text by Black and his associates [1947], *Farm Management*, and

Heady's "Elementary Models in Farm Production Economics Research" [1948] were important contributions in the immediate postwar period. Later, Heady's *Economics of Agricultural Production and Resource Use* [1952a] and Bradford and Johnson's *Farm Management Analysis* [1953] became the basic references for the new orientation in agricultural production economics. During this same period Allen's *Mathematical Analysis for Economists* [1953] and Tintner's *Econometrics* [1952] were widely used texts for mathematical and econometric models and methods.

Earlier work in quantifying production relationships sometimes involved a continuous relationship with or without a mathematical expression of the relationship between input and output. Mitscherlich was perhaps the first to suggest a nonlinear production function relating fertilizer use to crop output [1928]. Spillman [1933] also utilized an exponential yield curve with similar characteristics.

The USDA Technical Bulletin 1277, *Input as Related to Output in Farm Organization and Cost of Production Studies*, by Tolley, Black, and Ezekiel [1924] stimulated much interest in production function analyses of farm enterprises from farm data. Examples of production relationships include a tabular production surface of daily gain for steers as related to daily corn and hay consumption and output of pork showing diminishing marginal feed productivity.

During the World War II period three USDA studies, stimulated by John D. Black of Harvard, were published. They related output of milk, pork, and beef to total feed consumed (Jensen et al. [1942], Atkinson and Klein [1945], and Nelson [1945]).

A variety of applications has been made involving the production function approach with and without reference to agriculture. Of special interest to agricultural economists are those concerning the farming and agribusiness industries, groups of farms, production of specific crops or livestock, and other rural applications. This review primarily relates to production functions of agricultural crop and livestock enterprises from experimentally derived input-output data. Prior to this main area of concentration, however, a brief discussion of other applications of the production function approach in agriculture is presented.

The pre-World War II industry studies (Douglas [1934], Douglas and Gunn [1942]) using cross-section or time series data, provided some of the methodology for more recent work including use of the exponential function generally known as the Cobb-Douglas. Logarithmic transformations have been widely used because of their convenience in interpreting elasticities of production, minimal requirements for degrees of freedom, and its simplicity of computation.

Aggregate Production Functions

A series of whole-farm production function studies have been conducted in which different farms were used to get different levels and combinations of inputs, and farm income was used as the dependent variable. The best-known early applications in the United States are those presented by Tintner and Brownlee [1944] and Heady [1946].

Bradford and Johnson [1953] analyzed TVA test-demonstration farm records in Marshall County, Kentucky. Marginal value productivities were derived for acres of land, months of labor, investment in forage and livestock, and current expense. They concluded that a larger investment in livestock and forage, a lower machinery investment, and a reduced relative labor input would be needed to equate marginal value productivities with costs. Studies by Heady and Shaw [1954] and Heady and Baker [1954] were concerned with productivity in four farming areas in Montana, Alabama, and northern and southern Iowa. Heady [1955] compared resource productivity and imputed shares between landlord and tenant for a sample of rented farms. Heady and Swanson [1952] compared marginal productivities of farm resources for five areas of Iowa. In 1956 Heady, Johnson, and Hardin edited *Resource Productivity, Returns to Scale, and Farm Size*, a collection of studies concerned with a variety of concepts, procedures, and problems of production function analysis using cross-sectional farm data.

Hildebrand [1960] reported results from Kansas using farm record data for different years and with variations in the model used. An important finding of his research was the wide variability of results from year to year and from model to model in spite of the fact that nearly all of the correlation coefficients were significant at the one-percent probability level.

These and later studies note important implications for the allocation and productivity of resources in agriculture. Their major limitations relate to the great heterogeneity of conditions from farm to farm, the complete or relative absence of control or measurement of variables not included in the function, and the real possibility of multicollinearity among variables. The literature in economics journals contains many articles on the limitations and possible sources of bias in production function research with cross-section or time series data. Plaxico [1955] warned against use of this research for making adjustments on individual farms. Griliches [1957] showed how lack of specification of a management variable could bias the productivity estimates for capital upward and returns to scale downward. Similarly, lack of quantification of the quality of labor could increase the elasticity of capital and decrease the elasticity of labor. (See also Bronfenbrenner [1944], Mundlak [1961], and Reder [1943].)

Other studies have used time series or cross-sectional data in a Cobb-

Douglas analysis of various policy issues for the United States. Two examples are D. G. Johnson's analysis [1960] of output implications of a declining farm labor force and Griliches's study [1957] of the sources of productivity growth using sixty-eight regions as observations and including levels of education as an input.

In the latter part of the 1950s researchers began to utilize linear programming as a way of synthesizing production relationships without having to rely on time series or cross-sectional data from existing farms. Early work includes that of McPherson and Faris [1958] to derive milk output as a function of the price of milk. Martin, Coutu, and Singh [1960] analyzed levels of capital and management on small farms. O'Neal [1959] studied resource productivity in north Georgia using data from linear programming to obtain income estimates for different levels and combinations of resources. Several regional projects, such as the Southern Farm Management Research Committee S-42 work, were conducted to determine the effects of alternative prices and programs on farm adjustments and output.

Applications of functional analysis in the agricultural processing and marketing industries developed in the 1940s were also important forerunners to the crop and livestock production function work which followed. Bressler's approach [1945] in synthesizing cost curves for milk plants using budgeting and industrial engineering techniques resulted in great interest within the profession. Nicholls [1948] used weekly time series data from fourteen departments of a midwestern meat packing firm to predict the number of hogs processed as a function of total man-hours and labor per person per week.

Few production function studies are reported relating to rural development. Undoubtedly the difficulty of specifying outputs has inhibited research in this and related fields. While water supply, sewage treatment, and refuse removal can be quantified relatively easily, many other services have no physical unit of measure. One recent example deals with functions for student achievement in rural high schools by Bieker and Anschel [1974a, 1974b]. For a review of applications in the field of public finance, see Shoup [1969] and Hirsch [1970].

The literature in recent years contains numerous articles on alternative or modifying forms for the Cobb-Douglas production equation to change the assumptions on the elasticity of substitution, marginal products, and returns to scale. Examples include Zellner and Revankar [1969], and Dobell [1968]. Halter, Carter, and Hocking [1957] showed how modifications could allow for all three phases of the production relationship.

Production Functions for Crop Production

In about 1950 production economists, inclined toward the new emphasis on

production economics research, started investigations in the economics of fertilizer use. Interest among professional workers developed rapidly, and investigations and assessments were under way at agricultural experiment stations and by the regional farm management research committees, the USDA, the TVA, and private industry.

In assessing the present state of knowledge researchers pointed out that recommendations to farmers traditionally had been the responsibility of physical scientists (Dorner [1954], Hutton [1955]). As a result, criteria of physical response rather than economic response was generally used. Also, experiments were relatively inefficient for quantifying the economic range of the production surface. Examination of agronomic data revealed that rates of application were generally not at high enough levels to permit identification of the economic optimum. The reliance on testing for significant differences in yield for different levels of a fertilizer nutrient typically resulted in research designs where nutrient levels were spaced geometrically, whereas characterization of response as a continuous relationship with treatment levels evenly spaced is more efficient for estimating functional relationships. Reporting only averages of locations and years obscured or concealed economically important variables. Physical and economic interrelationships among nutrients and other important variables were unknown or of uncertain validity. Questions were raised about what effect optimizing N, P, and K simultaneously would have on economic optima compared with determining optimum levels of each nutrient separately with the others at a constant level.

During 1954 formal multidisciplinary studies involving agronomists, economists, and statisticians were under way in several states including Iowa, Kentucky, Michigan, North Carolina, Virginia, Idaho, Indiana, Texas, and Vermont and also at the USDA (National Academy of Sciences [1961]). The fertilizer industry was providing substantial support for projects on the economics of fertilizer use. The TVA was supporting projects and held the first of several annual seminars bringing together production economists, agronomists, and statisticians.

The extent and magnitude of multidisciplinary cooperation which developed was remarkable. Glenn L. Johnson [1957] stated that these "evidences of cooperation on the part of agronomists make it inappropriate to continue the protestations long made by economists, that the design of agronomic experiments does not permit economic interpretation of experimental results."

Ibach and Mendum [1953] wrote a USDA report showing procedures for calculating the most profitable combinations of N, P, and K using the exponential yield curve. At Iowa State Heady and Shrader [1953] delved into the interrelationships of agronomy and economics in research and in making recommendations to farmers. The multifactor experiments at Iowa conducted

by Heady and his associates were reported in a series of journal articles and station bulletins. The initial work on corn in 1952 involved a 9 by 9 incomplete N-P factorial replicated twice in a completely randomized design. This type of design was used to include a wide range of nutrient inputs without making the experiment too large. The wide range of nutrient applications was selected to ensure that the most profitable rates derived from marginal analysis would fall within the limits of the experiment and for efficiency in estimating the production surface.

Data presented by Heady, Pesek, and Brown [1955] include several types of regression equations estimated by least squares regression procedures including the Cobb-Douglas, the quadratic cross-product, and the quadratic square root equations. Isoquants were calculated to estimate combinations of nutrients to produce given yields, and the marginal properties were derived to obtain the least cost combinations of nutrients to produce a given yield and the combinations and levels of nutrients to maximize profits per acre for given sets of prices. A series of experiments followed for different crops and sections of the state and involved other variables such as rotations, initial levels of nutrients, and seeding rates. (See Heady, Doll, and Pesek [1958], Heady, Pesek, and McCarthy [1963], and Heady, Pesek and Rao [1966].)

Several important contributions during this period resulted from projects at North Carolina State University. Initially, work involved alternative procedures for analyzing existing data. These included analysis of alternative continuous functions, use of a price map to simplify presentation of optimum rates for alternative prices, and the development of a discrete model less restrictive than the traditional continuous function but still subject to a diminishing returns restriction. The involvement of statisticians Richard L. Anderson and D. D. Mason resulted in methodological developments over the years. (See P. R. Johnson [1953], Stemberger [1957], and C. G. Hildreth [1954].)

Nine years of cooperative agronomic-economic research in Michigan, conducted by Glenn L. Johnson and his associates, were summarized in Hoffman and Johnson [1966]. This report traces the attempts of researchers to characterize the response from fertilizer use under conditions where response is often obscured by other factors. When experiments involving complete rotations and "conventional" small-plot techniques began in 1954, the results generally showed a high unexplained within-treatment variation. Researchers became increasingly concerned about the universe for which the results would apply. These two problems became the central focus of experimentation.

A system was developed for using large plots on randomly selected farm fields that met selective soil and management conditions. These "controlled survey" experiments had a common check plot on each field so that between-farm differences could be accounted for, and the number of plots on any one

farm was reduced to four, including the check plot. Comparable data were obtained from a farmer survey and from small-plot experiments. The authors concluded that the "controlled survey" technique was a more reliable way of getting input-output data than past efforts and that future investigators should be encouraged to define explicitly the population about which they hoped to make inferences. The authors indicated a belief that this technique should be considered as an approach in developing countries in attempting to get maximum research and extension information from a given outlay of funds.

The work and cost required for multifactor experiments caused research workers to develop and try new designs. A composite design developed by Box [1954] for industrial research was used at North Carolina (Hurst and Mason [1957], Mason [1956, 1957]). This design required a minimum of 15 treatment combinations per replication compared with 125 for the 5^3 complete factorial. Tramel [1957a] developed a modification called the triple cube design, requiring 31 treatment combinations, and C. G. Hildreth [1957] proposed an interlaced factorial design. The designs were compared by B. P. Havlicek, Smith, and J. Havlicek [1962] in a greenhouse experiment using a 5^3 factorial as a standard of comparison. The authors concluded that the composite designs are useful when successfully centered on the point of maximum yield but that miscentering resulted in biased production functions. In agricultural crop studies the location of the maximum varies with moisture conditions and other factors, often resulting in an observed maximum different from a generalized predicted one.

Interest by statisticians in improving methodology has continued. Recently Anderson and Nelson [1975] explored techniques using intersecting straight lines as an alternative to conventional curvilinear forms of curve fitting. Other important contributions are also being made by statisticians overseas (Gomes [1970], Yates [1967]).

Several studies have incorporated water from irrigation as a variable. Moore [1961] dealt with a general analytical framework. Hexem, Heady, and Caglar [1974] derived production functions relating water and nitrogen to yield for corn, wheat, cotton, and sugar beets from seventy experiments in five western and southwestern states.

A significant number of research reports have dealt with the variations of yield curves over time. Involved are variations in weather, an important factor over which farmers have little or no control, and the accumulation or depletion of nutrients in the soil. Economists have used several approaches to this problem. Brown and Oveson [1958] discussed year-to-year variations in the response of spring wheat to nitrogen applications for ten years. Orazem and Herring [1958] analyzed the effects of soil moisture at seeding time and rain-

fall during the growing season as related to nitrogen response by grain sorghum in Kansas. Knetsch and Smallshaw [1958] calculated a response relationship to nitrogen and drought on millet for a Tennessee location. Smith and Parks [1967] extended this analysis by simulating results over many years with a computer simulation technique and long-term weather records. They calculated a probability distribution of different outcomes from using alternative levels of nitrogen.

Swanson, Taylor, and Welch [1973] used three decision models for analyzing the year-to-year variation in corn response to nitrogen for eight locations and for five seasons in Illinois: (1) to maximize the average return, (2) to maximize the minimum return, and (3) to minimize the maximum regret or loss from not choosing the correct rate given the season.

Researchers have given attention to the importance of varying fertilizer use with changing crop-fertilizer price ratios. Hutton and Thorne [1955] pointed out that for the 1953 Iowa corn experiment it would take a substantial change in the ratio to make a difference of \$4 in per-acre income and that the difference would be less than \$1 based on the historical annual price ratios of the 1951-54 period. They also pointed out that using N and P_2O_5 in a one-to-one ratio instead of the optimum ratio would decrease income \$0.11 to \$0.33 per acre for 1951-54 annual prices.

Using North Carolina data, J. Havlicek and Seagraves [1962] found similar net income consequences for corn. The highest cost of a wrong decision with corn prices varying between \$0.75 and \$1.75 and nitrogen at \$0.11 per pound was \$2.90 per acre. Similarly, Knetsch [1961], using Tennessee data for corn, found that nitrogen rates could be varied by fifty or sixty pounds in either direction from the optimum with very small profit losses. Swanson, Taylor, and Welch [1973] came to similar conclusions for corn from nitrogen fertilizer studies at eight locations in Illinois. Taking one location as an example, they concluded that a drop in the corn-nitrogen price ratio from thirty to ten would require less than a twenty-pound decrease per acre in the economically optimum level of nitrogen application.

Other studies have shown a higher economic consequence for use of non-optimum rates. For example, a Georgia study by Woodworth et al. [1957] for Coastal Bermuda hay in an unfavorable season shows a loss of \$7 per acre if the hay value is \$30 per ton when using an optimum rate for \$20 hay. For a favorable season, the loss is \$21 per acre. High economic consequences were also found for Bahiagrass hay (Beaty et al. [1961]).

The problems of determining the population to which a given response function would be applicable and relating results from agricultural experiment stations to given populations of farmers have been especially troublesome to economists concerned with crop production. Ibach, of the USDA

Economic Research Service, developed a generalized response concept and used it to make estimates of responses to fertilizer for major crops by states. The specific estimates were made by researchers in the state experiment stations and published as USDA Agricultural Handbook 68 [1954]. These basic data were used by Ibach and others for examining the outcomes of alternative policy proposals. Ibach [1957] developed estimates of land and fertilizer combinations to produce the United States corn crop. The generalized relationships were revised and published by Ibach and Adams [1968] as USDA Statistical Bulletin 431. The revised publication contains estimates for the agricultural subregions of each state. They represent an interpretation of experimental evidence, farmers' experiences, and also the distribution of the crop by soil type, cropping patterns, and levels of management.

Taylor and Swanson [1974], dealing with the economic effects of imposing per-acre restrictions on nitrogen fertilizer in Illinois, compared results from research on experiment stations with the Ibach-Adams [1968] generalized response functions and with farmers' yields for eight subregions of the state. They concluded that while the Ibach-Adams response functions do not agree exactly with actual average yield, they seem much closer than experimental functions.

Some of the more interesting applications of production function research relate to specialty crops. Eidman, Lingle, and Carter [1963], working with cantaloupe production in California, identified the relationship between fertilizer use as a function of time of ripening and total yield. Nitrogen delayed maturity while phosphate tended to hasten maturity. They developed a procedure for handling multiharvest periods. Many publications have combined production function analyses with budgeting or other techniques. A useful example is the Woolf, Sullivan, and Phillips [1967] study of cotton production, which includes production functions relating to irrigation, fertilizer, and plant population per acre. By budgeting costs, they compared irrigated and nonirrigated net returns.

Several publications summarize aspects of production function research in crop production. Heady and Dillon [1961] present the Iowa research and include a chapter from other countries. Dillon [1968] contains chapters on concepts, procedures, and applications from the United States and other countries.

Publication 918 of the National Academy of Sciences [1961] summarizes basic economic, design, and statistical analysis concepts, with sections on historical development, examples of practical application, and an extensive bibliography.

In two books resulting from TVA-sponsored seminars Baum, Heady, and Blackmore [1956] and Baum, Heady, Pesek, and C. G. Hildreth [1957]

document the principal developments in methodology and application in articles by authors from economics, agronomy, and statistics. They are useful in describing research needs and problems as seen by the authors at that time. A journal article by Munson and Doll [1959] gives an excellent overview of concepts and research experiences from a number of states. For practical applications of economic principles in fertilizer use based on research analysis, see North Central Regional Publication 54 (North Central Farm Management Research Committee [1954]) and Southern Farm Management Extension Publication Number 10 (North Carolina Agricultural Extension Service [1962]).

Production Functions for Animal and Poultry Production

Production functions for animal and poultry production date back to USDA technical bulletins by Jensen et al. [1942], A. G. Nelson [1945], and Atkinson and Klein [1945]. These studies carried out by the USDA and several agricultural experiment stations in collaboration have been widely quoted and stand as landmarks in the field of production economics and farm management. They were concerned primarily with optimum marketing rates rather than the estimation of marginal rates of substitution between feeds, and succeeded in developing interdisciplinary cooperation and data appropriate for some aspects of marginal analysis.

In the early 1950s Heady and Olson [1951, 1952] and (independently) Redman [1952] at Kentucky published exploratory studies estimating iso-product and marginal rate of substitution relationships for grain-forage feed for milk production. Heady and Olson used selected treatment from the Jensen study, and Redman also used existing data.

Each set of authors indicated that a basic purpose in conducting the research was to contribute to the national goals of soil conservation and the interests of many in agriculture to conserve grain and use more forage. Each was concerned with developing appropriate methodology for delineating new knowledge and exploring the interrelationships between the physical and economic implications of grain-forage feeding relationships in milk production.

Redman, in relating his research to the field of feeding standards, concluded: "It was undoubtedly necessary in the earlier stage of development to make such simplifying assumptions as perfect substitutability of feeds and constant returns of milk per unit of feed input in order to derive more useful knowledge about feeding for milk production. However, the time has now arrived for relaxing these assumptions of linear relationships and for incorporating the concept of changing marginal rates of substitution implied by the law of diminishing returns."

These studies were exploratory and uncertainties still remained about the true nature of the response surfaces and substitution rates. Opposing views were expressed in the *Journal of Farm Economics* (Mighell [1953a, 1953b], Heady and Olson [1953]). See H. R. Jensen [1977] for further comment.

In May 1957 a symposium on the nutritional and economic aspects of feed utilization was held at Michigan State University. It was sponsored by that university, the North Central Dairy and Farm Management Research Committees, the Farm Foundation, and the USDA. This meeting brought together research and extension workers in dairy nutrition, production economics, animal breeding, statistics, and agronomy to focus attention on interdisciplinary opportunities for improving knowledge concerning feed utilization by dairy cattle. The proceedings were published as a book edited by Hoglund et al. [1958].

Starting in 1956, a series of research reports document interdisciplinary research at Iowa involving experiments specifically designed to estimate the production surface for milk output. These experiments used four levels of hay-to-concentrate ratios and three levels of intensity of feeding. In the analysis of these data logarithmic, quadratic, and square root functions were derived by least squares regression as alternative means of specifying the production function. Heady, Jacobson, et al. [1964] include an analysis incorporating other variables such as different characteristics of cows (maturity, ability, inbreeding, and weight) and environmental conditions so that optimum feeding ratios and level of milk production can be estimated for more specific conditions of production. Also, point estimates are supplemented by confidence regions.

The existence of diminishing marginal rates of substitution in feeding has been confirmed by other researchers. Coffey and Toussaint [1963] pointed out that from analysis of the Iowa experiments the most profitable rations lie near the stomach capacity limit for most historical prices of hay, grain, and milk and that returns do not vary much over a fairly wide range of feeding levels. Dean [1960] reported on a California experiment in which rations for some treatments were changed after each twenty-eight-day period to measure carry-over effects. Hoover et al. [1967] used Kansas experiments carried out from 1956 to 1961 and concluded that the resulting production surfaces were similar to the Iowa study and that the general forms of the equations of best fit were similar. Paris et al. [1970] reported on dairy production functions where yield was alternatively measured as whole milk, fat, 4-percent milk, and skim milk.

Feeding trials to determine concentrate roughage production relationships in beef feeding have been conducted in Oklahoma (Plaxico and Pope [1959]) and in Iowa (Heady, Carter, and Culbertson [1964]). Plaxico and his associ-

ates concluded that comparisons of the Oklahoma beef study with the Iowa dairy results imply a greater curvature of the isoproduct contours for feedlot beef animals compared with milk production and that the economic incentive to adjust rations to price ratios may be greater. Also, under certain price relationships substantial savings might be made by feeding steers and heifers different rations.

Studies of corn-soybean meal feed substitution relationships for hogs in drylot feeding were reported from Iowa (Heady, Woodworth, et al. [1953, 1954]). The experimental trials included three experiments with treatments ranging from 10-percent to 20-percent protein. The data derived were used to specify: (1) least cost rations for different price relationships; (2) rations to get hogs to market weight in minimum time; (3) maximum profit rations based on historical prices for two weaning dates; and (4) optimum marketing weights. Methodological aspects included alternative equations and the use of three weight intervals as well as the whole-weight range to allow greater flexibility in substitution rates. Least cost rations resulted in a higher net return per pig compared with least time rations in fifteen years of a sixteen-year period for a November 1 weaning date and when marketed at 225 pounds. The difference was \$1.00 or more per pig in five of the years and \$5.82 in one year. A second publication in this series (Heady, Catron, et al. [1958]) reported results from feeding hogs corn and soybean meal for hogs produced on pasture instead of drylot.

A number of studies have been concerned with feed-weight of bird relationships in broiler production. Fellows and Judge [1952] were concerned in a Connecticut bulletin with marginal costs and returns from feeding broilers to different weights. Budgeting was used to relate this to total costs, total returns, and net returns. A special "slide rule" device made it possible for the producer to find maximum profit marketing weights for various prices. In a Washington State University study Baum and Walkup [1953] analyzed the feed-weight of bird relationships for high energy feeds compared with other rations. Heady, Balloun, and McAlexander [1956] analyzed the results of an experiment in which chicks were fed protein levels varying from 16 percent to 26 percent. Data are presented to determine least cost and least time rations as well as optimum marketing weights for specified protein levels. Heady, Balloun, and Dean [1956] published similar data for turkeys.

Assessments

Accomplishments. A wide range of experiments to characterize farm production relationships and to derive economic implications has been conducted since 1950. More studies have been concerned with crop than animal pro-

duction owing in part to higher costs of large-scale animal experiments. These experiments have provided a test of the practical application and importance of principles of production economics and of plant and animal science. The literature contains evidence of a considerable advance in methodology as well as many practical applications. The results serve as a reminder to agricultural workers and farmers that most inputs are not combined in fixed proportions as point recommendations imply, but that combinations and levels can be changed as prices and other conditions warrant. They serve as a conceptual guide for making recommendations to farmers. In one state a single-level recommendation for fertilizer was changed to three levels for alternative management situations based on production function studies. Most states that had agronomic-economic studies in the 1950s and early 1960s changed recommendations as a result of the work. Generally, the change was to increase the level of application.

These studies were a useful source of input-output data for farm adjustment studies, either directly or as a basis for making judgments from all available data, of the most appropriate alternative levels of inputs and associated production. The methodology, or at least the less complex aspects, had a very important application for guiding research and development in increasing food production in developing countries. In this case, the higher costs of inputs such as fertilizer, restrictions in foreign exchange, supply restrictions, and a pressing need for increased food supply multiply the importance of efficient use of scarce resources.

A climate has evolved from this research that demonstrates the potential accomplishments of interdisciplinary approaches to problem solving. Undoubtedly, it has been a crucial factor in developing the awareness on the part of production specialists and administrators that production economists can make important contributions in planning and analysis of production experiments. Physical scientists have become more aware of the need to design experiments using the production function approach, involving design and analysis of experiments for continuous relationships as opposed to discrete responses. In recent years increasing numbers of plant and animal scientists have been conducting their own experiments on this basis—a healthy trend which helps to provide production economists with useful input-output data and at the same time allows more effort in economic analysis compared with time spent in obtaining physical data.

Limitations. Greater difficulties are experienced in quantifying biological response relationships than would be encountered in most industrial processes. Soil variability, weather, insects, diseases, residual fertilizer in the soil, nutrients in the soil, and previous crop history frequently conspire to confound researchers with unpredictable results. Response to applied P and K has been

particularly uneven because of the accumulation from previous fertilization. In multivariate crop experiments in several states responses to these nutrients have not been statistically significant, reducing economic analysis to responses to applied N only. Perhaps these results should reinforce the view that precision crop production is a long way in the future. A second view would be that advances in plant science knowledge in terms of response prediction are needed before full utilization can be made of marginal relationships in crop production.

In the past decade many production economists have expressed the opinion that production function research from controlled experiments was of limited consequence, pointing to the very modest differences in net income associated with a fairly wide range of application rates for selected production functions or along certain isoproduct curves. At the same time interest waned because of continuing crop surpluses and relatively low prices for feed-grain, protein, and fertilizer. Researchers went on to new problems and approaches.

There appears to be adequate evidence that for corn and similar crops the profit consequences of not adjusting fertilizer rates optimally for normal year-to-year changes in prices are small or inconsequential. A finding that the nature of some or many response relationships does not make it worthwhile to change the levels or combinations of inputs from year to year for usual variations in price ratios is important knowledge. Finding the conditions and commodities for which it is worthwhile is also important knowledge. Discussions in the literature on this issue are based on only a few experiments, mostly for corn, and for the price variations arising in the 1950s and early 1960s.

In justifying, planning, and conducting projects economists frequently have overemphasized the importance of economic optima. The factor-product equilibrium is an appropriate guide for economic decision making but cannot be applied with the precision implied by the theoretical model. Discovery of a precise optimum may not in itself be valuable knowledge if a wide range of application rates makes little difference in net income. Rauchenstein [1953] observed that choice of forage production systems and how they fit into the whole-farm business were far more important than feed substitution possibilities to the economic health of dairy farms. (See also Coffey and Toussaint [1963].)

Many of the multinutrient fertilizer experiments were designed without sufficient basic knowledge of yield response patterns. For some of these, no meaningful economic analyses were possible because of nonsignificant or erratic responses. Where meaningful responses were obtained, they varied greatly from year to year because of weather differences. Clearly much more needs to be known about factors which affect response to P and K before elaborate

3-factor experiments to obtain economic optima can be routinely justified. The longer range aspects of N and P buildup in the soil have economic implications for farmers and for society, yet these have received scant attention in the literature on production functions and on the economics of fertilization. The general assumption of a variable resource (fertilizer) applied to a fixed resource (land) does not have universal application. Other models should be considered—for example, when land is not fixed for the individual farmer and idle or rented land can be substituted for fertilizer to produce a given level of output, when risk is an important factor, or when it is appropriate to consider an animal as the fixed plant rather than land.

Future Outlook and Research Needs

The desirability of quantifying production relationships in agriculture will continue in the future. While linear programming has become the dominant methodology for obtaining the most profitable farming systems, partial analyses based on production function studies have merit in analyzing numerous policy and farm-level decisions when interrelationships with other aspects of the farm organization are of secondary importance. In addition to that, the results of production function studies are useful in selecting data for linear programming studies.

Recent events should remind us not only that price ratios do not remain indefinitely within prescribed limits but that restrictions in the supply of fertilizer or feed can occur. Farmers and farm magazines have again raised questions about how much fertilizer to use or how to minimize feed costs. Important policy issues have again been raised about how threatened shortages and higher costs of fertilizer and energy could affect total production and how fertilizer and energy could be used more efficiently to minimize increases in the costs of food to consumers. Production function studies provide useful insights into these and such other national goals as reducing energy requirements and minimizing detrimental effects on the environment.

Additional assessment is needed of the conditions under which it could be desirable to change the rates and levels of inputs as price ratios change. This should be done systematically for a variety of crops. Similarly, additional assessments are needed of the production and income implications of restrictions in the use of inputs which may be scarce or subject to environmental controls.

In the future production function research will be needed to update existing information on production relationships as new technology and other conditions change and to provide new information in several priority areas. Much of the production function research now available in the literature was

conducted a decade or more ago and is probably out of date. One area of needed research involves alternative processes in beef production utilizing forages. There is a general lack of knowledge concerning production relationships needed in selecting a forage system from the many possible combinations, the selection of fertilizer rates, and the effect of these variables on quality of beef and on net income. The rising relative cost of water along with the fact that water costs are being more closely associated with the level of water use in irrigated farming areas will increase the benefits from pinpointing optimum water application rates.

For farmer decision making improvements are needed in specifying the population for which the relationships apply. In crop production this means more research carried out on farms rather than on experiment stations. Also, a variety of economic models and research techniques should be used to provide more useful information and to duplicate better the decision making models most appropriate for farmers in different circumstances.

The production function approach needs to be an integral part of the training of physical scientists with more of the needed projects designed and carried out by physical scientists themselves. At the present time many experiments are being carried out by physical scientists with objectives that suggest the production function approach as the most efficient but that utilize more traditional, less efficient procedures. If the production function approach were used by these researchers in the future, much more data on production processes would be available for economic analysis.

In many developing countries production function studies have a higher value than in the United States economy. Typically, there is a critical need to increase food production, but foreign exchange may be required for the importation of fertilizer and the cost of the fertilizer to farmers may be high. Policy issues involve the provision of adequate incentives for using fertilizer efficiently. In a controlled economy this may require setting the prices of fertilizer and product so that the desired production can be accomplished with a minimum of foreign exchange. If the use of fertilizer is a relatively new technology in the country, neither farmers nor agricultural workers have the historical experience to determine the best rates of use except by costly trial and error. Economic studies to determine optimum rates of fertilizer use can make a major contribution under these circumstances.

Many advances in methodology will probably be made by physical scientists and statisticians or will involve them in some way. In crop production there is a great need for a greater understanding of response relationships and of nontreatment variation. Advances in this direction could lead to improved criteria for selection of functional relations and to the development of new models with the desired characteristics. Advances in knowledge which result

in increased statistical efficiency, lower cost, and greater reliability for decision making could result in much greater utilization in partial or complete farm decision making models and in models for the analysis of related policy issues.

References

- Allen, R. G. D. [1953]. *Mathematical Analysis for Economists*. London: Macmillan.
- Anderson, R. L., and L. A. Nelson [1975]. "A Family of Models Involving Intersecting Straight Lines and Concomitant Experimental Designs Useful in Evaluating Response to Fertilizer Nutrients." *Biometrics* 31:303-318.
- Atkinson, L. J., and J. W. Klein [1945]. *Feed Consumption and Marketing Weight of Hogs*. USDA Technical Bulletin 894.
- Baum, E. L., E. O. Heady, and J. Blackmore, eds. [1956]. *Methodological Procedures in the Economic Analysis of Fertilizer Use Data*. Ames: Iowa State University Press.
- Baum, E. L., E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds. [1957]. *Economic and Technical Analysis of Fertilizer Innovations and Resource Use*. Ames: Iowa State University Press.
- Baum, E. L., and H. G. Walkup [1953]. "Some Economic Implications of Input-Output Relationships in Fryer Production." *J. Farm Econ.* 35:223-235.
- Beaty, E. R., R. C. Woodworth, G. A. Slappey, and J. Powell [1961]. *Response of Pensacola Bahiagrass to Nitrogen*. Georgia Agricultural Experiment Station Bulletin NS 85.
- Beringer, C. [1956]. "Estimating Enterprise Production Functions from Input-Output Data on Multiple Enterprise Farms." *J. Farm Econ.* 38:923-930.
- Berry, R. L. [1956]. *Most Profitable Use of Fertilizer on Corn, Oats, and Wheat in South Dakota*. South Dakota Agricultural Experiment Station, Agricultural Economics Pamphlet 69.
- Bieker, R. F., and K. R. Anshel [1974a]. "Estimating Educational Production Functions for Rural High Schools: Some Findings." *Am. J. Agr. Econ.* 56:515-519.
- [1974b]. "Estimating Educational Production Functions for Rural High Schools: Reply." *Am. J. Agr. Econ.* 56:835-36.
- Black, J. D., M. Clawson, C. R. Sayre, and W. W. Wilcox [1947]. *Farm Management*. New York: Macmillan.
- Box, G. E. P. [1954]. "The Exploration and Exploitation of Response Surfaces: Some General Considerations and Examples." *Biometrics* 10:16-60.
- Bradford, L. A., and G. L. Johnson [1953]. *Farm Management Analysis*. New York: Wiley.
- Bradley, M. [1974]. "Estimating Educational Production Functions for Rural High Schools: Comment." *Am. J. Agr. Econ.* 56:833-834.
- Bressler, R. G., Jr. [1945]. "Research Determination of Economies of Scale." *J. Farm Econ.* 27:526-539.
- Bronfenbrenner, M. [1944]. "Production Functions: Cobb-Douglas, Interfirm, Intra-firm." *Econometrica* 12:35-44.
- Brown, W. G. [1956]. "Free Choice versus Least-Cost Mixed Rations for Hogs." *J. Farm Econ.* 38:863-868.
- Brown, W. G., and G. H. Arscott [1958]. "A Method for Dealing with Time in Determining Optimum Factor Inputs." *J. Farm Econ.* 40:666-673.

- [1960]. "Animal Production Functions and Optimum Ration Specifications." *J. Farm Econ.* 42:69-78.
- Brown, W. G., T. L. Jackson, and R. G. Petersen [1962]. "A Method for Incorporating Soil Test Measurement into Fertilizer Response Functions." *Agron. J.* 54:152-154.
- Brown, W. G., and M. M. Oveson [1958]. "Production Functions from Data over a Series of Years." *J. Farm Econ.* 40:451-457.
- Carlson, S. [1939]. *A Study on the Pure Theory of Production*. London: P. S. King and Son.
- Carter, H. O., and H. O. Hartley [1958]. "A Variance Formula for Marginal Productivity Estimates Using the Cobb-Douglas Function." *Econometrica* 26:306-313.
- Chucka, J. A., A. Hawkins, and B. E. Brown [1943]. *Potato Fertilizer-Rotation Studies on Aroostook Farms—1927-1941*. Maine Agricultural Experiment Station Bulletin 414.
- Coffey, J. D., and W. D. Toussaint [1963]. "Some Economic Aspects of Free-Choice Feeding of Dairy Cows." *J. Farm Econ.* 45:1213-1218.
- Dean, G. W. [1960]. "Consideration of Time and Carryover Effects in Milk Production Functions." *J. Farm Econ.* 42:1512-1514.
- De Janvry, A. [1972]. "Optimal Levels of Fertilization under Risk—the Potential for Corn and Wheat Fertilization under Alternative Price Policies in Argentina." *Am. J. Agr. Econ.* 54:1-10.
- Dillon, J. L. [1968]. *The Analysis of Response in Crop and Livestock Production*. New York: Pergamon Press.
- Dobell, R. [1968]. "A Symposium on CES Production Functions: Extensions and Comments—Introductory Remarks." *Rev. Econ. and Stat.* 50:443-445.
- Doll, J. P. [1958a]. "Evaluation of Alternative Algebraic Forms for Production Functions." Ph.D. dissertation, Iowa State University.
- [1958b]. "A Method of Deriving Fertilizer Nutrient Combinations for Limited Capital Situations." Unpublished paper, Tennessee Valley Authority.
- [1972]. "A Comparison of Annual versus Average Optima for Fertilizer Experiments." *Am. J. Agr. Econ.* 54:226-233.
- [1974]. "On Exact Multicollinearity and the Estimation of the Cobb-Douglas Production Function." *Am. J. Agr. Econ.* 56:556-563.
- Doll, J. P., E. H. Jebe, and R. D. Munson [1960]. "Computation of Variance Estimates for Marginal Physical Products and Marginal Rates of Substitution." *J. Farm Econ.* 42:596-607.
- Dorner, P. [1954]. "Economic Interpretation of Agronomic Data Relating to Fertilizer Usage." Tennessee Agricultural Experiment Station.
- Douglas, P. H. [1934]. *The Theory of Wages*. New York: Macmillan.
- Douglas, P. H., and G. T. Gunn [1942]. "The Production Function for American Manufacturing for 1914." *J. Pol. Econ.* 50:595-602.
- Eidman, V. R., J. C. Lingle, and H. O. Carter [1963]. "Optimum Fertilization Rates for Crops with Multi-Harvest Periods." *J. Farm Econ.* 45:823-830.
- Engelstad, O. P., and W. L. Parks [1971]. "Variability in Optimum N Rates for Corn." *Agron. J.* 63:21-23.
- Engelstad, O. P., and G. L. Terman [1966]. "Fertilizer Nitrogen: Its Role in Determining Crop Yield Levels." *Agron. J.* 58:536-539.
- Faris, J. E. [1960]. *Economics of Replacing Cling Peach Trees*. California Agricultural Experiment Station, Giannini Foundation Report 232.

- Fellows, I. F. [1952]. "The Economics of Grassland Farming in the Northeast." *J. Farm Econ.* 34:759-764.
- Fellows, I. F., and G. G. Judge [1952]. *Economic Decision Making for Broilers*. Connecticut Agricultural Experiment Station Bulletin 302.
- Foreman, W. J., and O. Steanson [1956]. *A Method of Determining Profitable Rates of Fertilizer Use: Nitrogen on Coastal Bermuda for Hay*. Georgia Agricultural Experiment Station NS 22.
- French, B. L. [1956]. "Functional Relationships for Irrigated Corn Response to Nitrogen." *J. Farm Econ.* 38:736-747.
- Fuller, W. A. [1965]. "Stochastic Fertilizer Production Functions for Continuous Corn." *J. Farm Econ.* 47:105-119.
- Gomes, F. P. [1970]. "Use of Polynomial Response Surfaces in the Study of Experiments with Fertilizers." *Proceedings, International Biometrics Conference*. Hannover, Germany.
- Griliches, Z. [1957]. "Specification Bias in Estimating Production Functions." *J. Farm Econ.* 39:8-20.
- [1963]. "Estimates of the Aggregate Agricultural Production Function from Cross-Sectional Data." *J. Farm Econ.* 45:419-428.
- Halter, A., H. O. Carter, and J. Hocking [1957]. "A Note on the Transcendental Production Function." *J. Farm Econ.* 39:966-974.
- Hansen, P. L. [1949]. "Input-Output Relationships in Egg Production." *J. Farm Econ.* 31:687-697.
- Hansen, P. L., and R. L. Mighell [1956]. *Economic Choices in Broiler Production*. USDA Technical Bulletin 1154.
- Havlicek, B. P., W. G. Smith, and J. Havlicek, Jr. [1962]. "On the Choice of Designs for the Estimation of Production Functions." Purdue University Production Economics Paper 6210.
- Havlicek, J., Jr., and J. A. Seagraves [1962]. "The Cost of the Wrong Decision as a Guide in Production Research." *J. Farm Econ.* 44:157-167.
- Heady, E. O. [1946]. "Production Functions from a Random Sample of Farms." *J. Farm Econ.* 28:989-1004.
- [1948]. "Elementary Models in Farm Production Economic Research." *J. Farm Econ.* 30:201-225.
- [1951]. "A Production Function and Marginal Rates of Substitution in the Utilization of Feed Resources by Dairy Cows." *J. Farm Econ.* 33:485-498.
- [1952a]. *Economics of Agricultural Production and Resource Use*. New York: Prentice-Hall.
- [1952b]. "Use and Estimation of Input-Output Relationships or Productivity Coefficients." *J. Farm Econ.* 34:775-786.
- [1954a]. *Resource Productivity and Returns on 160-Acre Farms in North Central Iowa*. Iowa Agricultural Experiment Station Research Bulletin 412.
- [1954b]. "Choice of Functions in Estimating Input-Output Relationships." *Proceedings of the Southern Agricultural Workers Association, 51st annual meeting, Agricultural Economics and Sociology Section*.
- [1955]. "Marginal Resource Productivity and Imputation of Shares for a Sample of Rented Farms." *J. Pol. Econ.* 43:500-511.
- [1957]. "An Econometric Investigation of the Technology of Agricultural Production Functions." *Econometrica* 25:249-268.

- [1963]. "Marginal Rates of Substitution between Technology, Land and Labor." *J. Farm Econ.* 45:137-145.
- Heady, E. O., and C. B. Baker [1954]. "Resource Adjustments to Equate Productivities in Agriculture." *Southern Econ. J.* 21:36-52.
- Heady, E. O., S. Balloun, and G. W. Dean [1956]. *Least-Cost Rations and Optimum Marketing Weights for Turkeys*. Iowa Agricultural Experiment Station Research Bulletin 443.
- Heady, E. O., S. Balloun and R. McAlexander [1956]. *Least-Cost Rations and Optimum Marketing Weights for Broilers*. Iowa Agricultural Experiment Station Research Bulletin 442.
- Heady, E. O., W. G. Brown, J. T. Pesek, and J. Stritzel [1956]. *Production Functions, Isoquants, Isoclines and Economic Optima in Corn Fertilization for Experiments with Two and Three Variable Nutrients*. Iowa Agricultural Experiment Station Research Bulletin 441.
- Heady, E. O., H. O. Carter, and C. C. Culbertson [1964]. "Production Functions and Substitution Coefficients for Beef." *Agricultural Production Functions*, E. O. Heady and J. L. Dillon, eds. Ames: Iowa State University Press. Pp. 452-474.
- Heady, E. O., D. V. Catron, D. E. McKee, G. Ashton, and V. Speer [1958]. *New Procedures in Estimating Feed Substitution Rates and in Determining Economic Efficiency in Pork Production. II, Replacement Rates for Growing-Fattening Swine on Pasture*. Iowa Agricultural Experiment Station Research Bulletin 462.
- Heady, E. O., and J. L. Dillon [1961]. *Agricultural Production Functions*. Ames: Iowa State University Press.
- Heady, E. O., J. P. Doll, and J. T. Pesek [1958]. *Fertilizer Production Functions for Corn and Oats, Including Analysis of Irrigated and Residual Return*. Iowa Agricultural Experiment Station Research Bulletin 463.
- Heady, E. O., N. L. Jacobson, J. P. Madden, and A. E. Freeman [1964]. *Milk Production Functions in Relation to Feed Inputs, Cow Characteristics and Environmental Conditions*. Iowa Agricultural Experiment Station Research Bulletin 529.
- Heady, E. O., G. L. Johnson, and L. S. Hardin, eds. [1956]. *Resource Productivity, Returns to Scale, and Farm Size*. Ames: Iowa State University Press.
- Heady, E. O., J. P. Madden, N. L. Jacobson, and A. E. Freeman [1964]. "Milk Production Functions Incorporating Variables for Cow Characteristics and Environment." *J. Farm Econ.* 46:1-19.
- Heady, E. O., and R. O. Olson [1951]. "Marginal Rates of Substitution and Uncertainty in the Utilization of Feed Resources with Particular Emphasis on Forage Crops." *Iowa State J. Science* 26:49-70.
- [1952]. *Substitution Relationships, Resource Requirements and Income Variability in the Utilization of Forage Crops*. Iowa Agricultural Experiment Station Research Bulletin 390.
- [1953]. "Mighell on Methodology." *J. Farm Econ.* 35:269-276.
- Heady, E. O., and J. T. Pesek [1954]. "A Fertilizer Production Surface with Specification of Economic Optima for Corn Grown on Calcareous Ida Silt Loam." *J. Farm Econ.* 36:466-482.
- Heady, E. O., J. T. Pesek, and W. G. Brown [1955]. *Crop Response Surfaces and Economic Optima in Fertilizer Use*. Iowa Agricultural Experiment Station Research Bulletin 424.
- Heady, E. O., J. T. Pesek, and W. O. McCarthy [1963]. *Production Functions and Meth-*

- ods of Specifying Optimum Fertilizer Use under Various Uncertainty Conditions for Hay*. Iowa Agricultural Experiment Station Research Bulletin 518.
- Heady, E. O., J. T. Pesek, and V. Y. Rao [1966]. *Fertilizer Production Functions from Experimental Data with Associated Supply and Demand Relationships*. Iowa Agricultural Experiment Station Research Bulletin 543.
- Heady, E. O., J. Schnittker, S. Bloom, and N. L. Jacobson [1956]. "Isoquants, Isoclines, and Economic Predictions in Dairy Production." *J. Farm Econ.* 38:763-779.
- Heady, E. O., J. Schnittker, N. L. Jacobson, and S. Bloom [1956]. *Milk Production Functions, Hay/Grain Substitution Rates and Economic Optima in Dairy Cow Rations*. Iowa Agricultural Experiment Station Research Bulletin 444.
- Heady, E. O., and R. Shaw [1954]. *Resource Return and Productivity Coefficients of Selected Farming Areas in Iowa, Alabama, and Montana*. Iowa Agricultural Research Bulletin 427.
- Heady, E. O., and W. D. Shrader [1953]. "The Interrelationships of Agronomy and Economics in Research and Recommendations to Farmers." *Agron. J.* 45:496-502.
- Heady, E. O., and E. R. Swanson [1952]. *Resource Productivity in Iowa Farming*. Agricultural Experiment Station Bulletin 388.
- Heady, E. O., R. C. Woodworth, D. Catron, and G. C. Ashton [1953]. "An Experiment to Derive Productivity and Substitution Coefficients in Pork Output." *J. Farm Econ.* 35:341-355.
- [1954]. *New Procedures in Estimating Feed Substitution Rates in Determining Economic Efficiency in Pork Production. I, Replacement Rates of Corn and Soybean Oilmeal in Fortified Rations for Growing-Fattening Swine*. Iowa Agricultural Experiment Station Research Bulletin 409.
- Hexem, R. W., E. O. Heady, and M. Caglar [1974]. *A Compendium of Experimental Data for Corn, Wheat, Cotton and Sugar Beets Grown at Selected Sites in the Western United States*. Center for Agricultural and Rural Development, Iowa State University Special Research Report.
- Hicks, J. R. [1946]. *Value and Capital*. London: Oxford University Press.
- Hildebrand, J. R. [1960]. "Some Difficulties with Empirical Results from Whole-Farm Cobb-Douglas-Type Production Functions." *J. Farm Econ.* 42:897-904.
- Hildreth, C. G. [1954]. "Point Estimates of Ordinates of Concave Functions." *J. Am. Stat. Assoc.* 49:598-619.
- [1955]. "Economic Implications of Some Cotton Fertilizer Experiments." *Econometrica* 23:88-98.
- [1957]. "Possible Models for Agronomic-Economic Research." In *Economic and Technical Analysis of Fertilizer Innovations and Resource Use*, E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds. Ames: Iowa State University Press. Pp. 176-186.
- Hildreth, R. J. [1957]. "Influence of Rainfall on Fertilizer Profits." *J. Farm Econ.* 39: 522-524.
- Hildreth, R. J., F. L. Fisher, and A. G. Caldwell [1955]. *An Economic Evaluation of Experimental Response to Coastal Bermuda Grass to Nitrogen under Irrigation*. Texas Agricultural Experiment Station Miscellaneous Publication 128.
- [1956]. *Influence of Rainfall on Profits from Fertilizer Applications to East Texas Forage*. Texas Agricultural Experiment Station Miscellaneous Publication 184.
- Hirsch, W. Z. [1970]. *The Economics of State and Local Governments*. New York: McGraw-Hill. Pp. 147-165.
- Hoffman, B. R., and G. L. Johnson [1966]. *Summary and Evaluation of the Cooperative*

- Agronomic-Economic Experimentations at Michigan State University—1955-1963*. Michigan Agricultural Experiment Station Research Bulletin 11.
- Hoglund, C. R., G. L. Johnson, C. A. Lassiter, and L. D. McGilliard, eds. [1958]. *Nutritional and Economic Aspects of Feed Utilization by Dairy Cows*. Ames: Iowa State University Press.
- Hoover, L. M., P. L. Kelley, G. M. Ward, A. M. Feyerherm, and R. Chaddha [1967]. "Economic Relationships of Hay and Concentrate Consumption to Milk Production." *J. Farm Econ.* 49:64-78.
- Hurst, D. C., and D. D. Mason [1957]. "Some Statistical Aspects of the TVA-North Carolina Cooperative Project on Determination of Yield Response Surfaces for Corn." In *Economic and Technical Analysis of Fertilizer Innovations and Resource Use*, E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds. Ames: Iowa State University Press. Pp. 207-216.
- Hutton, R. F. [1955]. *An Appraisal of Research on the Economics of Fertilizer Use*. Tennessee Valley Authority, Agricultural Economics Report T 55-1.
- Hutton, R. F., and D. W. Thorne [1955]. "Review Notes on the Heady-Pesek Fertilizer Production Surface." *J. Farm Econ.* 37:117-119.
- Ibach, D. B. [1953]. "Use of Production Functions in Farm Management Research." *J. Farm Econ.* 35:938-956.
- [1957]. *Substituting Fertilizer for Land in Growing Corn*. USDA Agricultural Research Service, ARS 43-63.
- Ibach, D. B., and J. R. Adams [1968]. *Crop Yield Response to Fertilizer in the United States*. USDA Statistical Bulletin 431.
- Ibach, D. B., and S. W. Mendum [1953]. *Determining Profitable Use of Fertilizer*. USDA, Bureau of Agricultural Economics, FM 105.
- Jacobson, N. L. [1959]. "Problems in Designing Feeding Experiments from a Nutritional Standpoint." In *Nutritional and Economic Aspects of Feed Utilization by Dairy Cows*, C. R. Hoglund, G. L. Johnson, C. A. Lassiter, and L. D. McGilliard, eds. Ames: Iowa State University Press. Pp. 206-212.
- Jensen, E. [1940]. "Determining Input-Output Relationships in Milk Production." *J. Farm Econ.* 22:249-258.
- Jensen, E., J. Klein, E. Rauchenstein, T. E. Woodward, and R. H. Smith [1942]. *Input-Output Relationships in Milk Production*. USDA Technical Bulletin 815.
- Jensen, H. R. [1977]. "Farm Management and Production Economics, 1946-1970." In *A Survey of Agricultural Economics Literature: Volume 1, Traditional Fields of Agricultural Economics, 1940s to 1970s*, Lee R. Martin, ed. Minneapolis: University of Minnesota Press.
- Johnson, D. G. [1960]. "Output and Income Effects of Reducing the Farm Labor Force." *J. Farm Econ.* 42:779-796.
- Johnson, G. L. [1955]. "Results from Production Economic Analysis." *J. Farm Econ.* 37:206-222.
- [1956a]. "A Critical Evaluation of Fertilization Research." In *Farm Management in the West—Problems in Resource Use*. Proceedings of the Western Agricultural Economics Research Council, Farm Management Research Committee. Report 1, pp. 33-40.
- [1956b]. "Interdisciplinary Considerations in Designing Experiments to Study the Profitability of Fertilizer Use." In *Methodological Procedures in the Economic Analysis of Fertilizer Use Data*, E. L. Baum, E. O. Heady, and J. Blackmore, eds. Ames: Iowa State University Press. Pp. 22-36.

- [1957]. "Planning Agronomic-Economic Research in View of Results to Date, and the Role of Management in Planning Farms for Optimum Fertilizer Use. In *Economic and Technical Analysis of Fertilizer Innovations and Resource Use*, E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds. Ames: Iowa State University Press. Pp. 217-225, 261-270.
- [1963]. "Stress on Production Economics." *Australian J. Agr. Econ.* 7:12-26.
- Johnson, P. R. [1953]. "Alternative Functions for Analyzing a Fertilizer-Yield Relationship." *J. Farm Econ.* 35:519-529.
- Judge, G. G., J. S. Plaxico, D. L. Brooks, W. L. McCaslan, R. H. Thayer, G. W. Newell, and K. E. Dunkelgod [1959]. *The Economics and Technical Impact of Floor Space per Bird and Temperature in Broiler Production*. Oklahoma Agricultural Experiment Station Processed Series P-318.
- Knetsch, J. L. [1956]. "Methodological Procedures and Applications for Incorporating Economic Considerations into Fertilizer Recommendations." M.S. thesis, Michigan State University.
- [1959]. "Moisture Uncertainties and Fertility Response Studies." *J. Farm Econ.* 41:70-76.
- [1961]. "Some Possible Implications of Fertilizer Response Studies Conducted over Time." Unpublished manuscript.
- Knetsch, J. L., and W. L. Parks [1958]. *Interpreting Results of Irrigation Experiments—A Progress Report*. Tennessee Valley Authority, Agricultural Economics Report T 59-1.
- Knetsch, J. L., L. S. Robertson, Jr., and W. B. Sundquist [1956]. "Economic Considerations in Soil Fertility Research." *Michigan Agricultural Experiment Station Quarterly* 39:10-16.
- Knetsch, J. L., and J. Smallshaw [1958]. *The Occurrence of Drought in the Tennessee Valley*. Tennessee Valley Authority, Report T58-2 AE.
- Krantz, B. A., and W. V. Chandler [1954]. *Fertilize Corn for Higher Yields*. North Carolina Agricultural Experiment Station Bulletin 366.
- Lorenz, O. A., J. C. Bishop, B. J. Hoyle, M. P. Zobel, P. A. Minges, L. D. Doreen, and A. Ulrich [1954]. *Potato Fertilizer Experiments in California*. California Agricultural Experiment Station Bulletin 744.
- McAlexander, R., and R. Hutton [1957]. "Determining Least-Cost Combinations." *J. Farm Econ.* 39:936-941.
- McPherson, W. W. [1955]. *Some Algebraic Expressions Used in Estimating Input-Output Relationships*. Southern Farm Management Research Committee.
- McPherson, W. W., and J. E. Faris [1958]. "Price Mapping of Optimum Changes in Enterprises." *J. Farm Econ.* 40:821-834.
- Martin, L. R., A. J. Coutu, and H. S. Singh [1960]. "The Effects of Different Levels of Management and Capital on the Incomes of Small Farmers in the South." *J. Farm Econ.* 42:90-102.
- Mason, D. D. [1956]. "Functional Models and Experimental Designs for Characterizing Response Curves and Surfaces." In *Methodological Procedures in the Economic Analysis of Fertilizer Use Data*, E. L. Baum, E. O. Heady, and J. Blackmore, eds. Ames: Iowa State University Press. Pp. 76-98.
- [1957]. "Statistical Problems of Joint Research." *J. Farm Econ.* 39:370-381.
- Mighell, R. L. [1953a]. "What Is the Place of the Equal-Product Function?" *J. Farm Econ.* 35:29-43.

- [1953b]. "A Further Note on the Equal-Product Function." *J. Farm Econ.* 35: 276-280.
- Mitscherlich, E. A. [1928]. "Das Gesetz des Minimums und das Gesetz des abnehmenden Bodenertrages." *Landwirtschaft Jahrbuch* 38:537-552.
- Moore, C. V. [1961]. "A General Analytical Framework for Estimating the Production Function for Crops Using Irrigation Water." *J. Farm Econ.* 43:876-888.
- Mundlak, Y. [1961]. "Empirical Production Function Free of Management Bias." *J. Farm Econ.* 43:44-56.
- Munson, R. D. [1958]. *Some Considerations in the Future Development of Agronomic Economic Research*. Unpublished paper, Tennessee Valley Authority.
- Munson, R. D., and J. P. Doll [1959]. "The Economics of Fertilizer Use in Crop Production." *Advances in Agronomy* 11:133-169.
- National Academy of Sciences [1961]. *Status and Methods of Research in Economic and Agronomic Aspects of Fertilizer Response and Use*. National Research Council Publication 918.
- Nelder, J. A. [1966]. "Inverse Polynomials, a Useful Group of Multi-Factor Response Functions." *Biometrics* 22:128-141.
- Nelson, A. G. [1945]. *Relation of Feed Consumed to Food Products Produced by Fattening Cattle*. USDA Technical Bulletin 900.
- Nelson, M., E. N. Castle, and W. G. Brown [1957]. "Use of the Production Function and Linear Programming in Valuation of Intermediate Products." *Land Economics* 33:257-261.
- Nicholls, W. H. [1948]. *Labor Productivity Functions in Meat Packing*. Chicago: University of Chicago Press.
- North Carolina Agricultural Extension Service [1962]. *The Economics of Fertilizer Use in the South*. Southern Farm Management Extension Publication 10.
- North Central Farm Management Research Committee [1954]. *Profitable Use of Fertilizer in the Midwest*. Wisconsin Agricultural Experiment Station Bulletin 508 (North Central Regional Publication 54).
- O'Neal, W. G. [1959]. "Effects of Different Farm Resource Combinations on the Marginal Value Productivity of Resources on General Commercial Farms in the Limestone Valley Area of Georgia." M.S. thesis, University of Georgia.
- Orazem, F., and R. B. Herring [1958]. "Economic Aspects of the Effects of Fertilizers, Soil Moisture and Rainfall on the Yields of Grain Sorghum in the (Sandy Lands) of Southwest Kansas." *J. Farm Econ.* 40:697-708.
- Oury, B. [1965]. "Allowing for Weather in Crop Production Model Building." *J. Farm Econ.* 47:270-283.
- Paris, Q., F. Malossini, A. Pilla, and A. Romita [1970]. "A Note on Milk Production Functions." *Am. J. Agr. Econ.* 52:594-598.
- Parks, W. L., and J. L. Knetsch [1959]. "Corn Yields as Influenced by Nitrogen Level and Drought Intensity." *Agron. J.* 51:363-364.
- Paschal, J. L. [1953]. *Economic Analysis of Alfalfa Yield Response to Phosphate Fertilizer at Three Locations in the West*. USDA, Bureau of Agricultural Economics, FM 104.
- Paschal, J. L., and B. L. French [1956]. *A Method of Economic Analysis Applied to Nitrogen Fertilizer Rate Experiments on Irrigated Corn*. USDA Technical Bulletin 1141.
- Pesek, J. T., and E. O. Heady [1958]. "Derivation and Application of a Method for De-

- termining Minimum Recommended Rates of Fertilization." *Soil Science Society of America Proceedings* 22:419-423.
- Pesek, J. T., Jr., E. O. Heady, and E. Venezian [1967]. *Fertilizer Production Functions in Relation to Weather, Location, Soil and Crop Variables*. Iowa Agricultural Experiment Station Research Bulletin 554.
- Plaxico, J. S. [1955]. "Problems of Factor-Product Aggregation in Cobb-Douglas Value Productivity Analysis." *J. Farm Econ.* 37:664-675.
- Plaxico, J. S., P. Andrienas, and E. S. Pope [1959]. *Economic Analysis of a Concentrate-Roughage Ratio Experiment*. Oklahoma Agricultural Experiment Station Bulletin P310.
- Rauchenstein, E. [1953]. "Forage-Grain Substitution: Its Importance in the Economics of Milk Production." *J. Farm Econ.* 35:562-571.
- Reder, M. W. [1943]. "An Alternative Interpretation of the Cobb-Douglas Function." *Econometrica* 11:259-264.
- Redman, J. C. [1952]. "Economic Aspects of Feeding for Milk Production." *J. Farm Econ.* 34:333-345.
- Redman, J. C., and S. Q. Allen [1954]. "Some Interrelationships of Economic and Agronomic Concepts." *J. Farm Econ.* 36:453-465.
- Robertson, L. S., G. L. Johnson, and J. F. Davis [1957]. "Problems Involved in the Integration of Agronomic and Economic Methodologies in Economic Optima Experiments." In *Economic and Technical Analysis of Fertilizer Innovations and Resource Use*, E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds. Ames: Iowa State University Press. Pp. 226-240.
- Robinson, J. [1955]. "The Production Function." *Econ. J.* 65:67-71.
- Schechter, M., and E. O. Heady [1970]. "Response Surface Analysis and Simulation Models in Policy Choices." *Am. J. Agr. Econ.* 52:41-50.
- Schultz, T. W. [1958]. "Output-Input Relationships Revisited." *J. Farm Econ.* 40:924-932.
- Shaw, R. H. [1956]. "The Fertilizer Problem: Resource-Enterprise and Tenure Relationships and Criteria for Optima." In *Farm Management in the West-Problems in Resource Use*. Proceedings of the Western Agricultural Economics Research Council, Farm Management Research Committee. Report 1, pp. 9-22.
- Shephard, R. W. [1953]. *Cost and Production Functions*. Princeton: Princeton University Press.
- Shoup, C. S. [1969]. *Public Finance*. Chicago: Aldine.
- Smith, W. G., and W. L. Parks [1967]. "A Method for Incorporating Probability into Fertilizer Recommendations." *J. Farm Econ.* 49:1511-1515.
- Spillman, W. J. [1933]. *Use of the Exponential Yield Curve in Fertilizer Experiments*. USDA Technical Bulletin 348.
- Stauber, S., and F. Miller [1963]. *Corn Yield Response to Nitrogen and Irrigation in Southeast Missouri*. Missouri Agricultural Experiment Station Special Report 39.
- Stemberger, A. P. [1957]. *Economic Implications of Using Alternative Production Functions for Expressing Corn-Nitrogen Production Relationships*. North Carolina Agricultural Experiment Station Technical Bulletin 126.
- Stritzel, J. A. [1958]. "Agronomic and Economic Evaluation of Direct and Residual Fertilizer Nutrients." Ph.D. dissertation, Iowa State University.
- Sullivan, G. D. [1964]. *Profitable Levels of Forage Fertilization*. Louisiana Agricultural Experiment Station, DAE, Research Report 334.
- Sundquist, W. B., and L. S. Robertson [1959]. *An Economic Analysis of Some Con-*

- trolled Fertilizer Input-Output Experiments in Michigan*. Michigan State University Agricultural Experiment Station Technical Bulletin 269.
- Swanson, E. R. [1956]. "Determining Optimum Size of Business from Production Functions." In *Resource Productivity, Returns to Scale, and Farm Size*, E. O. Heady, G. L. Johnson, and L. S. Hardin, eds. Ames: Iowa State University Press. Pp. 133-143.
- Swanson, E. R., C. R. Taylor, and L. F. Welch [1973]. "Economically Optimal Levels of Nitrogen Fertilizer for Corn—An Analysis Based on Experimental Data, 1966-71." *Ill. Agr. Econ.* 13:16-25.
- Swanson, E. R., and F. H. Tyner [1965]. "Influence of Moisture Regime on Optimum Nitrogen and Plant Population for Corn." *Agron. J.* 57:361-364.
- Taylor, C. R., and E. R. Swanson [1973]. "Experimental Nitrogen Response Functions, Actual Farm Experience and Policy Analysis." *Ill. Agr. Econ.* 13:26-32.
- [1974]. "Economic Impact of Imposing per Acre Restrictions on Use of Nitrogen Fertilizer in Illinois." *Ill. Agr. Econ.* 14:1-5.
- Tintner, G. [1944a]. "A Note on the Derivation of Production Functions from Farm Records." *Econometrica* 12:26-34.
- [1944b]. "An Application of the Variate Difference Method to Multiple Regression." *Econometrica* 12:97-113.
- [1952]. *Econometrics*. New York: Wiley.
- Tintner, G., and O. H. Brownlee [1944]. "Production Functions Derived from Farm Records." *J. Farm Econ.* 26:566-571.
- Tolley, H. R., J. D. Black, and M. J. B. Ezekiel [1924]. *Input as Related to Output in Farm Organization and Cost of Production Studies*. USDA Technical Bulletin 1277.
- Tramel, T. E. [1957a]. "Suggested Procedure for Agronomic-Economic Fertilizer Experiments." In *Economic and Technical Analysis of Fertilizer Experiments*, E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds. Ames: Iowa State University Press. Pp. 168-175.
- Tramel, T. E. [1957b]. "Alternative Methods of Using Production Functions for Making Recommendations." *J. Farm Econ.* 39:790-793.
- Ulveling, E., and L. Fletcher [1970]. "A Cobb-Douglas Production Function with Variable Returns to Scale." *Am. J. Agr. Econ.* 52:322-326.
- United States Department of Agriculture [1954]. *Fertilizer Use and Crop Yields in the United States*. USDA Agr. Handbook 68.
- Walker, O., S. Wiggins, and T. Pogue [1962]. *An Economic Analysis of Fertilizer and Seeding Rates for Spinach Production in Eastern Oklahoma*. Oklahoma Agricultural Experiment Station Bulletin 596.
- Woodworth, R. C. [1956]. "Organizing Fertilizer Input-Output Data in Farm Planning." In *Methodological Procedures in the Economic Analysis of Fertilizer Use Data*, E. L. Baum, E. O. Heady, and J. Blackmore, eds. Ames: Iowa State University Press. Pp. 158-170.
- Woodworth, R. C., R. E. Proctor, G. W. Burton, and A. B. Mackie [1957]. *Profitable Use of Fertilizer in the Production of Coastal Bermuda in the Coastal Plain Area of Georgia*. Georgia Agricultural Experiment Station Technical Bulletin 13.
- Woolf, Willard F., G. D. Sullivan, and S. A. Phillips [1967]. *An Economic Analysis of Irrigation, Fertilization, and Seeding Rates for Cotton in the Macon Ridge Area of Louisiana*. Louisiana Agricultural Experiment Station Bulletin 620.
- Yates, F. [1967]. "A Fresh Look at the Basic Principles of the Design and Analysis of

Experiments." In *Proceedings of the Fifth Berkeley Symposium*, vol. IV, *Biology and Problems of Health Sciences*, L. M. Le Cam and Jerzy Neyman, eds. Berkeley: University of California Press. Pp. 777-790.

Zellner, A., and N. S. Revankar [1969]. "Generalized Production Functions." *Rev. Econ. Studies* 36:241-250.

Zulberti, C. A., J. T. Reid, and G. L. Casler [1973]. *The Use of the Daily Production Function to Select the Feeding Program for Growing and Fattening Cattle*. Cornell University, Department of Agricultural Economics, AE Res. 73-14.