AN EXPOSITORY REVIEW OF BERNOULLIAN DECISION THEORY IN AGRICULTURE: IS UTILITY FUTILITY?

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An outline and appraisal is given of Bernoullian decision theory with a view to its potential use in agricultural contexts, both on and off the farm. Despite the existence of a variety of difficulties and unresolved problems, it is argued that Bernoulli's Principle—because of its recognition of the personal nature of decision making in terms of beliefs and preferences—represents the best possible approach to risky choice in agriculture.

1 READERS GUIDE

Literary inelegance aside, is this review worth reading? What chance is there that it's any good. What alternatives do you have? How do they compare? Only *you* can judge. But since some extra information at zero cost must always help, a summary outline may be worthwhile—even if it only enables you to sample snippets more selectively. Our Sections run as follows:

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Probably, for those just wanting a quick rundown on utility and decision theory, Sections 8, 9, and 10 are not too relevant while Sections 5 and 6 would be most relevant. Potential researchers interested in the operational aspects of decision theory may find particular interest in Sections 8 to 11. All, it is hoped, may find the list of references of some value. They are to be found at the end.

2 DEFINITIONS

Risky choice prevails when a decision maker has to choose between alternatives, some or all of which have consequences that are not certain and can only be described in terms of a probability distribution.

Bernoullian decision theory is a normative approach to risky choice based upon the decision maker's personal strengths of belief (or subjective probabilities) about the occurrence of uncertain events and personal valuation (or utility) of potential consequences [37, 120, 237].

Both the above definitions would be controversial to some people—the first because it implies probabilities can be associated with all uncertain consequences; the second because, while Bernoulli was the first to emphasize personal preference, he did not explicitly recognize subjective probabilities.

Neither of the above definitions is specifically agricultural. As a consequence, neither is the bulk of our review. Our emphasis is on principles—though we will keep looking over our shoulder to questions of risky choice in agriculture.

3 BACKGROUND

Some analytic innovations have had an easy run to acceptance and propagation by agricultural economists. Major examples have been production economics theory, production function estimation, linear programming, benefit-cost analysis and, most recently, systems analysis and simulation—all of which have been adopted post-haste around the world. Some of these applications, e.g. [77, 78, 122, 242, 265], have paid their way or would if policy makers and producers had the nous to make use of them. But in general it remains an open question whether these innovations in analysis have been worthwhile to date in a real-world context [182, 279, 315, 361]. Certainly they have not been the panaceas implied by the panache of their promotors. The reason is easily pinpointed: while logically faultless within the various utopias specified by their assumptions, not one of these innovations in itself meets the real-world situation where uncertainty reigns dominant and must be accommodated if economic analysis, either at the farm or policy level, is to carry full weight. In contrast, though it goes direct to the problem of real-world uncertainty, Bernoullian decision theory has not had an easy run with agricultural economists. Only today is it showing signs of possible general acceptance, despite the fact that the case for its use was put to the profession as far back as 1957 in a specifically agricultural context [165] and, in a more general context, had been largely put by an array of authors[1] during the 1950's, [15, 46, 60, 95, 196, 305, 310], followed during the 1960's by a number of agricultural expositions covering part or all of the topic [73, 100, 102, 142, 143, 144, 145, 219, 220, 227, 257, 258, 334, 355].

Three reasons may be adduced for the slow acceptance of Bernoullian decision theory by agricultural economists. For a start, it never had the field of meeting uncertainty in agriculture to itself [82, 103, 183, 186, 263, 335, 336, 337]. Without doubt, much effort was wasted during the 'fifties and 'sixties in checking mechanistic rule-of-thumb procedures based on naive forecast devices, e.g. [67, 187, 316], and games against nature, e.g. [81, 85, 86, 205]. But at least these efforts had a testable normative orientation, which is more than can be said for the efforts spent on Shackle's [318, 319] descriptive but untestable theory of focus gains and losses [16, 40, 41, 204, 256]. More importantly, it seems Bernoullian decision theory was ignored because of its personalistic emphasis based on personal probability assessment and personal utility [82]. Because of these subjective elements, decision theory solutions to problems of risky choice rightly vary from one decision maker to another—a feature which runs against the tidy nature of agricultural economists in tending to seek general rather than individual solutions to problems. Likewise the notion of subjective probability runs against the grain of those raised on objective frequencies, not to mention Knight's [194, p. 19] hard-to-kill classification of non-certainty into "risk" and "uncertainty". Lastly, and not quite in jest, Bernoullian decision theory was probably impeded because Earl Heady never wrote an expository article about it.

With good sense, however, the "seventies should see decision theory blossoming both within and beyond the farm gate; a flowering which will in turn lead to a more fruitful harvest from the vigorous but rather unproductive sproutings of production economics, linear and non-linear programming, benefit-cost analysis and systems" simulation over the last two decades. Justification for such a view is presented here through an outline of the logical bases of the theory, complemented by some illustrative applications.

4 GOALS OF CHOICE

Within the restraints of his available resources, technical knowledge, market information, institutional possibilities, legal opportunities and moral judgements, a rational decision maker aims to achieve as well as possible whatever goals he might have [323]. Choice of goals is his prerogative. Of course, should his selected goals be mutually exclusive, he may end up needing psychiatric treatment. And even if they are

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[1] Today, because of its general relevance, there is a vast and fast growing literature on decision theory, particularly in the fields of economics, engineering, management, mathematics, philosophy, psychology and statistics. Though we will refer to some 377 of these works, they are only a fraction—let's hope the important fraction—of all the work that has been presented.

merely competitive, he may have severe problems in deciding on the best balance of achievement across his goals unless he can establish acceptable trade-off relations between them.

4.1 SURE-CONSEQUENCE EFFECTS

If there is no uncertainty about the outcome of each alternative action and if a decision maker has only a single goal, or has established trade-off relations between his multiple goals, there is little difficulty in choice of action. Both cases imply the existence of a one-dimensional criterion such that choice of consequence (appropriately measured) implies choice of action.

But should a decision maker have multiple goals and, in his view, substitution of achievement between them is not possible, then the situation is a little more difficult. Should he be able to rank his goals in order of preference or priority, this ranking may be used lexicographically as a multidimensional criterion to assess available actions and choose that which is best. For example, suppose a policy maker has two (for him) non-substitutable goals: say, maximizing national farm income and minimizing intra-national transfer payments to agriculture. Further suppose he has four policy alternatives a_1, a_2, a_3, a_4 with the following sure consequences:

	Consequence ($*m*)	
Action	*Farm Income*	*Transfer payment*
a_1	100	20
a_2	110	40
a_3	110	30
a_4	90	20

If our policy maker's first priority is to maximize farm income, lexicographic appraisal indicates a_3 would be his best choice. While both a_2 and a_3 give the same high level of farm income, a_3 is the better of the two in terms of the second-order goal of minimizing transfer payments. Should minimizing transfer payments be his primary goal, a_1 would be optimal. Of course, if a decision maker cannot rank his non-substitutable goals, then he must either dither in indecision or choose at random between seemingly reasonable alternatives.

4.2 UNCERTAIN-CONSEQUENCE EFFECTS

The situation becomes more complex when—as is invariably true—the consequences of actions are uncertain, i.e. when stochastic influences operate so that each action may have a number of potential consequences and can only be described in terms of a probability distribution of outcomes.

Firstly, the presence of uncertainty means that the decision maker has to decide what outcomes are possible and has to specify his relative strengths of belief about the occurrence of these uncertain outcomes. Naturally, in establishing these probabilities the decision maker will take account, to a greater or lesser degree, of historical data and available predictions.

He may rely solely on past frequencies but if so he is making a subjective judgement that the future will be like the past and that no additional relevant information is available.

Secondly, if consequences are uncertain, choice rests not between sure outcomes but between probability distributions of outcomes. This may introduce additional dimensions to the problem of choice. What was a one-dimensional criterion or consequence measure in the non-stochastic case may become multidimensional in the presence of uncertainty. For example, a manager whose only goal under certainty is to maximize net income may, in the stochastic case, wish to maximize expected net income subject to a higher priority goal that actual net income exceeds some minimum level with some specified probability. In similar fashion, multidimensional criteria in non-stochastic situations may change to multi-dimensional criteria when consequences are uncertain.

With the possibility of such complexities, assessment of alternative risky choices might be thought to be so complicated as to be impossible. But if this were so, given the prevalence of uncertainty in the world, multitudes of perplexed, frustrated, dithering, unsuccessful and psychologically disturbed managers would be found. The world provides no such evidence. The implication is that decision makers do manage to cut through the morass of possibilities for complexity and do in general manage to make decisions in line with their strengths of conviction about, and preferences between, probability distributions of uncertain consequences. This is not to say that they make use of all the data available or that they make the best possible decisions relative to their available information and preferences at the time of decision.[2] Rather it suggests that if a normatively sound but relatively simple approach to risky choice can be specified, it is likely to be acceptable to many decision makers. Indeed, it may correspond to what they have been trying to do all the time. There would also be two additional advantages: fuller account could be taken of available information and, if necessary, decision making could be delegated. Such a possibility seems to be well met by Bernoulli's Principle [1, 18, 29, 37, 146, 235, 237, 282].

5 BERNOULLI'S PRINCIPLE

Logic, decision makers' behaviour and introspection all indicate that any adequate procedure for handling choice under uncertainty must involve two components: personal valuation of consequences and personal strengths of belief about the occurrence of uncertain events [28, 37, 214, 287, 290, 305]. Bernoulli's Principle does this beautifully and has the normative justification of being a logical deduction from a small number of postulates or axioms which many people agree are absolutely reasonable and should be met by a person who wishes to be consistent and rational in his workaday decisions.

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[2] Note that, under uncertainty, the "best" decision may turn out to yield a "bad" outcome. If so, the result—not the decision—is judged to be poor.

The relevant axioms differ slightly depending on whether the decision maker (a) has a single goal or (b) has multiple goals between which he can establish acceptable trade-off relations or (c) has multiple goals which are not substitutable. The first two cases lead to a one-dimensional utility measure (i.e. a real number) for each alternative act; the last to a lexicographically-ordered utility vector. We will first consider only the case where the decision maker has a single goal, discussion of multiple goal situations being left to Section 10.

In simple form but sufficient to give their flavour, the postulates underlying Bernoulli's Principle for the case of a single goal are as follows:

(i) *Ordering*. A person's order of preference among alternatives can be represented by an ordering. Thus a person confronted with any two risky prospects[3] G_1 and G_2 either prefers G_1 to G_2, or prefers G_2 to G_1, or is indifferent between them. And for any three risky prospects G_1, G_2, and G_3, if he prefers G_1 to G_2, or is indifferent between them, and if the same is true between G_2 and G_3, then he will prefer G_1 to G_3, or be indifferent between them.

(ii) *Continuity*. If a person prefers G_1 to G_2 to G_3, then there exists a unique probability p such that he is indifferent between G_2 and a gamble with a probability p of yielding G_1 and a probability $1 - p$ of yielding G_3.

(iii) *Independence*. If G_1 is preferred to G_2 and G_3 is some other prospect, then a gamble with G_1 and G_3 as outcomes will be preferred to a gamble with G_2 and G_3 as outcomes if the probability of G_1 and G_2 occurring is the same in both cases.

As has been shown in a number of ingenious ways and with a variety of formulations of the postulates, e.g. [18, 24, 37, 61, 92, 111, 120, 161, 234, 235, 237, 305] these three axioms imply Bernoulli's Principle or, as it is also known, the Expected Utility Theorem.[4]

Expected Utility Theorem. Given a decision maker whose preferences do not violate the axioms of Ordering, Continuity and Independence, there exists a function U, called a utility function, which associates a single real number or utility index with any risky prospect faced by the decision maker. This function has the following properties:

(i) If the risky prospect G_1 is preferred to G_2, then the utility index of G_1 will be greater than the utility index of G_2, i.e. $U(G_1) > U(G_2)$ if G_1 is preferred to G_2. Conversely $U(G_1) > U(G_2)$ implies G_1 is preferred to G_2.

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[3] A risky prospect is any action or choice possibility with a probability distribution of outcomes. It might, for example, be any such risky prospect as a dancing partner, a patent medicine, a second-hand car, a journal subscription or a business investment.

[4] Bernoulli [29] postulated this theorem in 1738. Ramsey [290] proved it in 1926 but his work went unrecognized until a proof was independently rediscovered by von Neumann and Morgenstern [352] in 1944.

(ii) If G is the risky prospect with a set of outcomes $\{g\}$ distributed according to the probability distribution $f(g)$, then the utility of G is equal to the statistically expected utility of G, that is

$$U(G) = EU(G). \tag{1}$$

If $f(g)$ is discrete,

$$EU(G) = \sum_g U(g)f(g), \tag{2a}$$

and if $f(g)$ is continuous,

$$EU(G) = \int_{-\infty}^{\infty} U(g)f(g)dg. \tag{2b}$$

As these equations indicate, only the first moment (i.e. the mean or expected value) of utility is relevant to choice. For a person who accepts the axioms of choice underlying Bernoulli's Principle, the variance or other higher moments *of utility* are irrelevant; the expected value of utility takes full account of all the moments (mean, variance, skewness, etc.) of the probability distribution $f(g)$ of outcomes.

(iii) Uniqueness of the function is only defined up to a positive linear transformation. Given a utility function U, any other function U^* such that

$$U^* = aU + b, \quad a > 0, \tag{3}$$

will serve as well as the original function. Thus utility is measured on an arbitrary scale and is a relative measure analogous, for example, to the various scales used for measuring temperature. Concomitantly, because there is no absolute scale for utility and because a person's utility function reflects his own personal valuations, it is impossible to compare one person's utility indices with another's.[5]

Bernoulli's Principle thus provides a mechanism for ranking risky prospects in order of preference, the most preferred prospect being the one with the highest utility. Hence Bernoullian decision theory implies the maximization of utility which, by the Expected Utility Theorem, is equivalent to maximization of expected utility. Equations (2a) and (2b) provide the empirical basis of application of the theory. Two concepts are involved—degree of preference (or utility) and degree of belief (or probability). These concepts are elaborated in Sections 7 and 8 below. First, however, a simple example of the application of Bernoulli's Principle is given.

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[5] It has been argued that comparable interpersonal utility scales may be established on the basis of equating people's best conceivable situations at the top end and their worst conceivable situations at the bottom end [262]. This assumes that people have the same utility capacity—which seems to be an untestable proposition. The real difficulty with such an approach, however, is that people have no experience of their best and worst conceivable situations so that the approach cannot be made operational.

5.1 EXAMPLE

To illustrate the application of Bernoulli's Principle, consider a farm manager who has to choose between purchasing 1,000, 1,200, or 1,600 store cattle for paddock fattening. The profit from fattening will depend on whether the grazing season is good, fair, or poor. The manager's personal strengths of conviction about these possible states of the world are that there is a 0·4 chance of a good season, a 0·2 chance of a fair season, and a 0·4 chance of a poor season. The consequences for each of his choices, as he has budgeted them out in terms of total net profit on the deal, are shown in table 1. By arranged coincidence the three actions all have the same expected money value of $12,400.

TABLE 1

Cattle Purchase Decision Problem

Type of season	Subjective probability	Action net payoffs		
		Buy 1,000	Buy 1,200	Buy 1,600
		$'000	$'000	$'000
Good	0·4	20	25	34
Fair	0·2	10	12	16
Poor	0·4	6	0	− 11
Expected money value		12·4	12·4	12·4

The Ordering Postulate says that the manager can rank his three available actions in an order of preference (which may involve indifference between one or more of them). This ordering will hinge on his preferences about the possible outcomes and his subjective assessment of their likelihood of occurrence. Suppose the ordering in decreasing order of preference happens to be: Buy 1,000; Buy 1,200; Buy 1,600. The Continuity Postulate says that there exists some probability p such that the manager would be indifferent between the act of Buy 1,200 (his second preference) and a ticket in a lottery which gave a chance p of Buy 1,000 (his first preference) and a chance $1 - p$ of Buy 1,600 (his third preference). The Independence Postulate says that if, for example, he was confronted with a choice between a lottery offering Buy 1,000 and Buy 1,600 as prizes and a lottery offering Buy 1,200 and Buy 1,600 as prizes, so long as the chance of Buy 1,600 was the same in both lotteries, he would prefer the one involving Buy 1,000. That is, the presence of the alternative Buy 1,600 under such *ceteris paribus* conditions does not alter his preference ordering. All these conditions, in terms of workaday managerial choice, are eminently reasonable things to expect of a manager both in terms of what he actually does and of what he should do.

For a farm manager who abides by these postulates, Bernoulli's Principle says that some function U exists which can be used to associate a single real number with each of the available actions and that this number or utility index will, by its size, correctly rank the actions in the order of the manager's preference. Further, the function is such that the utility of an action is equal to its expected utility.

Consider the action Buy 1,000. As shown in table 1, it has possible net profit consequences of \$20,000, \$10,000, or \$6,000. The utility index of each of these consequences is $U(\$20{,}000)$, $U(\$10{,}000)$, and $U(\$6{,}000)$ respectively. In turn, these outcomes are assessed as having a 0·4, 0·2, and 0·4 chance of occurring if Buy 1,000 is the action taken. By the Expected Utility Theorem, therefore, the utility of Buy 1,000 is equal to $[0{\cdot}4U(\$20{,}000) + 0{\cdot}2U(\$10{,}000) + 0{\cdot}4U(\$6{,}000)]$. In such vein, utility indices can be established for the three alternatives as follows:

$$U(\text{Buy } 1{,}000) = 0{\cdot}4U(\$20{,}000) + 0{\cdot}2U(\$10{,}000) + 0{\cdot}4U(\$6{,}000)$$
$$U(\text{Buy } 1{,}200) = 0{\cdot}4U(\$25{,}000) + 0{\cdot}2U(\$12{,}000) + 0{\cdot}4U(\$0)$$
$$U(\text{Buy } 1{,}600) = 0{\cdot}4U(\$34{,}000) + 0{\cdot}2U(\$16{,}000) + 0{\cdot}4U(-\$11{,}000).$$

Ranked in order of size from highest to lowest, these utility indices must, by the inevitability of logical deduction, reflect the manager's preference between the three actions, given his acceptance of the postulates of Ordering, Continuity and Independence. And to emphasize again the essence of Bernoullian Decision Theory, note that these utility indices depend on two highly subjective elements: first, the manager's strengths of conviction or subjective probabilities about the occurrence of the different possible states of the world, and second, his own personal utility index for each consequence if it was to occur.

So far nothing has been said of how the manager's utility function might be derived. That will be done later in Section 8.1. For the moment suppose that his utility function is depicted by the curve in figure 1. Reading from this curve (or making use of the equation of the curve), utility indices for each action are as follows:

$$\begin{aligned} U(\text{Buy } 1{,}000) &= 0{\cdot}4U(\$20{,}000) + 0{\cdot}2U(\$10{,}000) + 0{\cdot}4U(\$6{,}000) \\ &= 0{\cdot}4(37) + 0{\cdot}2(19{\cdot}5) + 0{\cdot}4(11{\cdot}94) \\ &= 23{\cdot}476 \end{aligned}$$

$$\begin{aligned} U(\text{Buy } 1{,}200) &= 0{\cdot}4U(\$25{,}000) + 0{\cdot}2U(\$12{,}000) + 0{\cdot}4U(\$0) \\ &= 0{\cdot}4(45) + 0{\cdot}2(23{\cdot}16) + 0{\cdot}4(0) \\ &= 22{\cdot}632 \end{aligned}$$

$$\begin{aligned} U(\text{Buy } 1{,}600) &= 0{\cdot}4U(\$34{,}000) + 0{\cdot}2U(\$16{,}000) + 0{\cdot}4U(-\$11{,}000) \\ &= 0{\cdot}4(58{\cdot}14) + 0{\cdot}2(30{\cdot}24) + 0{\cdot}4(-30{\cdot}58) \\ &= 17{\cdot}072. \end{aligned}$$

Ranking of these indices corresponds to the manager's preference for Buy 1,000 over Buy 1,200 over Buy 1,600. Another manager who had the same subjective probabilities for good, fair and poor seasons but the utility function of figure 2 would have a preference ordering for Buy 1,200 over Buy 1,600 over Buy 1,000 with utility indices respectively of:

$$\begin{aligned} U(\text{Buy } 1{,}200) &= 0{\cdot}4(17{\cdot}5) + 0{\cdot}2(6{\cdot}84) + 0{\cdot}4(0) \\ &= 8{\cdot}368 \end{aligned}$$

$$U(\text{Buy } 1{,}600) = 0{\cdot}4(26{\cdot}86) + 0{\cdot}2(9{\cdot}76) + 0{\cdot}4(-11{\cdot}3575) = 8{\cdot}153$$

$$U(\text{Buy } 1{,}000) = 0{\cdot}4(13) + 0{\cdot}2(5{\cdot}5) + 0{\cdot}4(3{\cdot}06) = 7{\cdot}524.$$

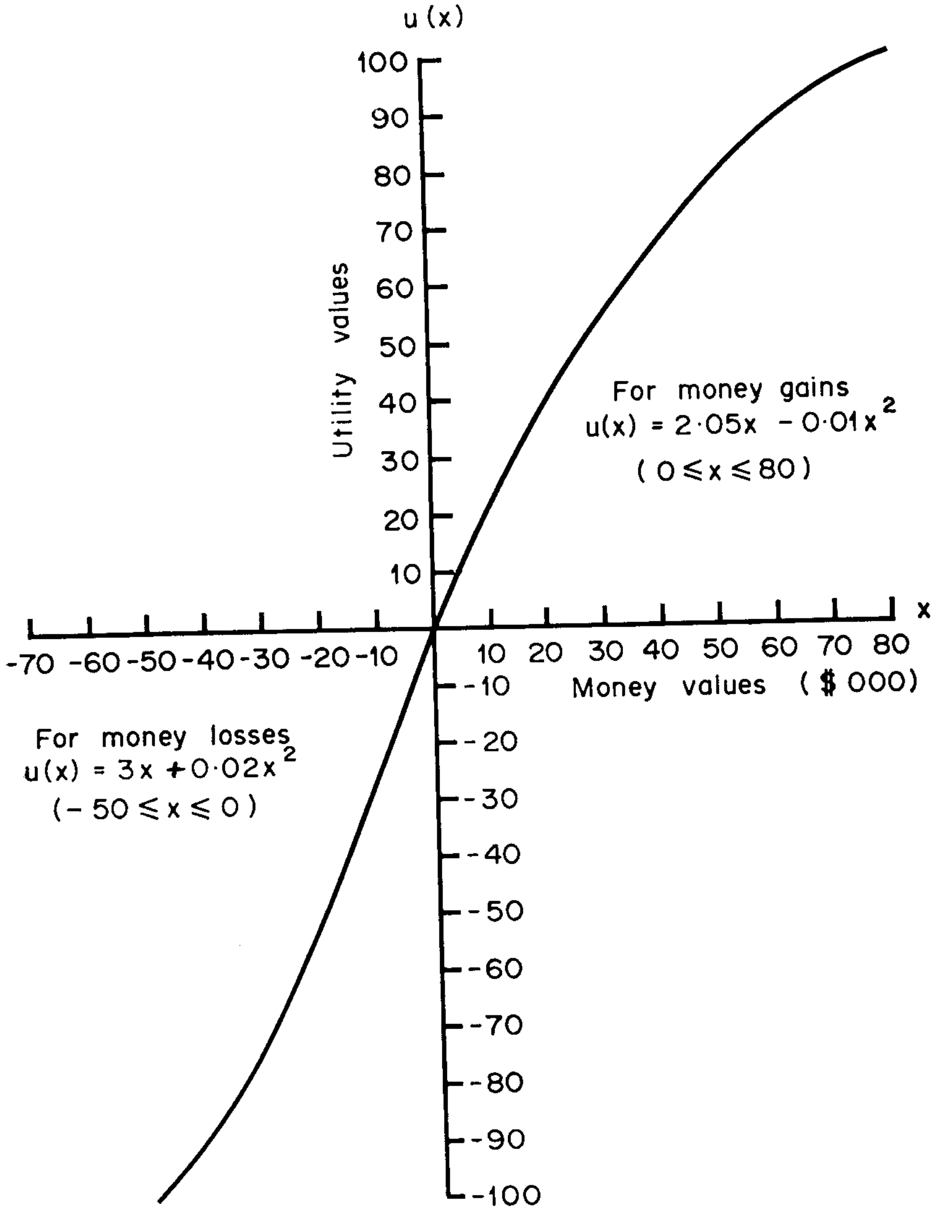

FIGURE 1: *Utility Curve of a Person Exhibiting Risk Aversion for Money Gains and Risk Preference for the Avoidance of Losses*

Thus, as occurs in practice, Bernoulli's Principle reflects the fact that even with the same strengths of conviction about uncertain events, different managers may have different preferences. Of course, since managers will also usually differ in their subjective probabilities, there is all the more reason to expect differences in their preferences between risky prospects.

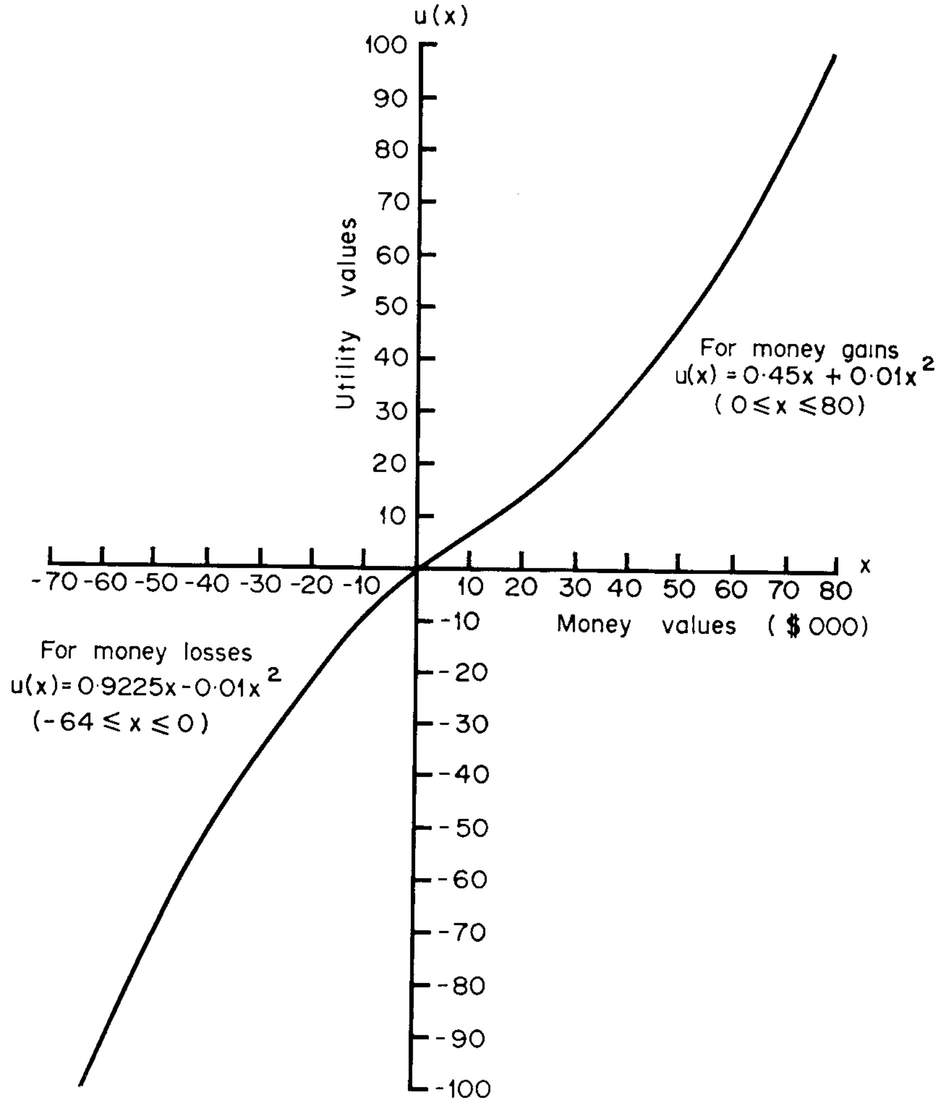

FIGURE 2: *Utility Curve of a Person Exhibiting Risk Preference for Money Gains and Risk Aversion for Money Losses*

The decision problem specified in table 1 is very simple. Most decision makers confronted with such a problem would have no difficulty in deciding which action was their preferred (i.e. optimal) choice. There would be no need, indeed no point, in following through the rigmarole of utility analysis so as to establish preference. However, many decision problems, and often the more important ones, are not so simple. The action alternatives are often so numerous and the distributions of consequences so diverse that intuitive appraisal is infeasible simply because the mass of data is beyond the decision maker's capacity to assimilate and assess. In such cases utility analysis (as sketched above or in one of the equivalent forms outlined later) can be most worthwhile. By applying Bernoulli's Principle, a ranking of preference between alternatives can be established which is fully consistent with the ranking the decision maker would make if he was able to intuitively handle all the complexities involved. In other words, in decision problems where the mass of data or the complexity of the situation are such that sound intuitive appraisal is impossible or thought to be inadequate, the maximization of utility can be used as a criterion for optimal choice, the sole proviso being that the decision maker accepts the postulates of Ordering, Continuity and Independence.

6 VALUE OF ADDITIONAL INFORMATION

Within every risky decision problem there is embedded a further decision problem—that of whether or not to seek additional information before a final decision or terminal action is taken. Such additional information will be worth having if its value is greater than its cost—and it is a most attractive feature of Bernoullian decision theory that it facilitates such economic evaluation. To illustrate the principles involved we will assume that the decision maker whose utility curve is depicted in figure 1 is facing the decision problem of table 1 and that he can purchase additional information at a cost of c thousand dollars. Two situations are possible: first, the information available may be guaranteed perfect; or, second, it may be imperfect. In either case, each of the payoffs of table 1 would be reduced by c thousand dollars. While it is always possible to calculate *a priori* the value of a perfect forecast, this is not so for imperfect forecasts since there are varying degrees of imperfection.

6.1 PERFECT FORECASTS

With perfect forecast information, the correct act may always be chosen. For example, in the cattle decision problem, a perfect forecast would tell us whether the season was in fact going to be good, fair or poor. If it were to be a good season, the best act would be Buy 1,600, giving a money payoff of $\$(34 - c)10^3$; for a fair season, Buy 1,600 with a payoff of $\$(16 - c)10^3$; and for a poor season, Buy 1,000 with a payoff of $\$(6 - c)10^3$.

Before the perfect forecast is purchased, the decision maker's assessment of which type of season will be nominated as the true one must follow the probabilities of 0·4, 0·2, and 0·4 for good, fair, and poor respectively, since these are his beliefs about their chance of occurrence. Thus the utility he can expect with perfect forecast information costing c is:

$$0{\cdot}4U(34 - c) + 0{\cdot}2U(16 - c) + 0{\cdot}4U(6 - c).$$

Without any additional information, as shown in Section 5.1, the decision maker can achieve an expected utility of 23·476. He should therefore only purchase perfect information if the cost c of the forecast is such that

$$0{\cdot}4U(34 - c) + 0{\cdot}2U(16 - c) + 0{\cdot}4U(6 - c) > 23{\cdot}476.$$

For example, if perfect forecast information is available at a cost of \$6,000 (i.e. $c = 6$), expected utility with this information would be

$$\begin{aligned} 0{\cdot}4U(28) + 0{\cdot}2U(10) + 0{\cdot}4U(0) &= 0{\cdot}4(49{\cdot}56) + 0{\cdot}2(19{\cdot}50) + 0{\cdot}4(0) \\ &= 23{\cdot}724, \end{aligned}$$

and it would be economic to purchase the information.

More generally, the above inequality could be solved as an equality to determine the price at which perfect forecast information yields the same utility as is obtainable without it. At and beyond this price, a perfect forecast is not worth buying. In the present example this maximum price is approximately \$6,020.

6.2 IMPERFECT FORECASTS

Since there are varying degrees of imperfection, a particular forecast that is not guaranteed perfect cannot be evaluated until it is known, unless the decision maker is prepared to guess as to what the extra information will be. If it can only be known after purchase, its assessment only has value in judging its source in relation to possible future purchases. If, however, particular forecast information can be obtained on the basis that payment is conditional on its purchase being economic, then evaluation can be of current interest and value. To illustrate both of the above situations, suppose the decision maker obtains a *particular* piece of additional information which convinces him that the probabilities of a good, fair, or poor season are not respectively 0·4, 0·2, and 0·4 as he initially thought, but should be assessed at 0·1, 0·6, and 0·3 respectively. This information is imperfect in the sense that it does not give full or perfect information about which state of nature will prevail.

Based on his prior or initial probabilities our decision maker would have chosen Buy 1,000 which in fact, *given the better information now at his disposal*, would have lead to a utility of:

$$\begin{aligned} 0{\cdot}1U(20) + 0{\cdot}6U(10) + 0{\cdot}3U(6) &= 0{\cdot}1(37) + 0{\cdot}6(19{\cdot}5) + 0{\cdot}3(11{\cdot}94) \\ &= 18{\cdot}982. \end{aligned}$$

Now suppose the additional forecast information, if purchased, has a cost of \$1,000. The expected utility of each alternative, allowing for the cost of information, is:

$$\begin{aligned} U(\text{Buy } 1{,}000) &= 0{\cdot}1U(19) + 0{\cdot}6U(9) + 0{\cdot}3U(5) \\ &= 17{\cdot}118 \\ U(\text{Buy } 1{,}200) &= 0{\cdot}1U(24) + 0{\cdot}6U(11) + 0{\cdot}3U(-1) \\ &= 16{\cdot}254 \\ U(\text{Buy } 1{,}600) &= 0{\cdot}1U(33) + 0{\cdot}6U(15) + 0{\cdot}3U(-12) \\ &= 12{\cdot}840. \end{aligned}$$

The best action based on the additional information is still Buy 1,000 but, because of the cost of the information, the expected utility of the best act is lower than before. The information is therefore not worth buying.

In contrast, if the *particular* piece of new information convinced him that the probabilities were 0·9 for a good season, 0·1 for a fair season, and 0·0 for a poor season, purchase of the extra information for \$1,000 would have been very worthwhile. Without this information, the prior probabilities would have lead to choice of Buy 1,000, giving an actual expected utility of:

$$0{\cdot}9U(20) + 0{\cdot}1U(10) + 0{\cdot}0U(6) = 32{\cdot}25.$$

With the information, the best choice is Buy 1,600 with a utility index of

$$0{\cdot}9U(33) + 0{\cdot}1U(15) + 0{\cdot}0U(-12) = 53{\cdot}934.$$

If need be, the break-even price for this particular piece of extra forecast information can be found by solving the following inequality as an equation to find the break-even value of c:

$$0{\cdot}9U(34 - c) + 0{\cdot}1U(16 - c) + 0{\cdot}0U(-11 - c) > 35{\cdot}25.$$

Making use of the equation of the curve of figure 1, we have

$$35{\cdot}25 = 0{\cdot}9[2{\cdot}05(34 - c) - 0{\cdot}01(34 - c)^2] + 0{\cdot}1[2{\cdot}05(16 - c) - 0{\cdot}01(16 - c)^2]$$

which gives $c = 13{\cdot}078$ or a price of \$13,078 as the maximum amount that could be economically paid for the additional information.

In the statistical decision theory literature, the value of additional information on the probability of occurrence of uncertain events (or, same thing, the relevant states of the world or states of nature with which they are associated) is typically discussed in relation to the extent of sampling (by survey or experiment) to obtain further information [33, 44, 92, 147, 192, 245, 278, 287, 288, 304, 310]. Two types of analysis are developed: "posterior analysis" or the *ex post* evaluation of additional information already to hand; and "preposterior analysis" or the appraisal of the expected value of extra information before it is purchased. The latter, of course, is analogous to the example presented above and is the more relevant type of analysis. The criterion for preposterior analysis is that additional probability information should be acquired so long as its marginal value exceeds its marginal cost. On this basis, procedures have been developed for determining optimal sample size if this must be fixed before sampling begins, and for deciding when to stop sampling if it can be undertaken sequentially [140, 287, 310]. Morgan [245, Chs 5 and 6] has provided a most readable outline of these procedures. In general they are not so pertinent to workaday decision making in agriculture, though they will doubtless become more relevant as computer use expands. Anderson and Dillon [10] have indicated the relevance of preposterior analysis to decisions on investment in agricultural research, both in terms of how much to invest and the direction of investment. A distinguishing feature of their work is that it is couched in a utility context. In general, while the statistical literature pays lip service to the concept of utility, the value of additional information is usually assessed only in financial terms. This can be quite misleading if the decision maker's utility function is non-linear—as most are.

An example in the policy arena of the appraisal of the value of forecast information has been provided in an outstanding piece of work by Byerlee and Anderson [51]. They were concerned with the problem of whether or not a newly developed predictor of annual rainfall trend should be generally promulgated for farmer use. Their analysis indicated release of the predictor would be economically justified. Its value, however, was only some 15 per cent of that of a perfect predictor of growing season rainfall, thus also providing information on the extent to which public funds might reasonably be devoted to further research on rainfall prediction.

A number of other agricultural researchers [49, 74, 102, 146, 285, 286] have also presented appraisals of the value of forecast information derived from experiments, surveys or other outside sources. These analyses have all been oriented to farm production and marketing decisions and, with few exceptions [146, 285, 286], have been couched in financial rather than utility terms.

We will return to the topic of additional information in Section 9.1 when we discuss the general Bernoullian decision theory model.

6.3 REVISION OF THE PAYOFF MATRIX

Discussion above has related only to additional information on the chance of occurrence of the states of nature for a given payoff matrix. A more general problem only slightly touched upon in the literature [119, 236] is that of revision of the payoff matrix itself. This might occur through correction of the payoff elements or by the addition of further acts or by the addition or refinement of the states of nature. Such revision, for example, is the major role of agricultural consultants whether they be working at the farm or national level. Because such revisions can usually only be known after their purchase, preposterior appraisal of their likely value is infeasible except on the basis of guesswork. Posterior analysis, however, is possible and might be used as a guide to the value of future purchases from particular suppliers.

Perhaps of most interest is the addition of new acts. This involves a process of search, typically at some non-zero cost [181, 282]. Suppose $U(X)$ is the utility of the best alternative so far found and that the utility of acts follows the subjective probability distribution $f(x)$. The probability of finding a better alternative each time a search is undertaken is

$$P[U(x) > U(X)] = \int_{x=X}^{\infty} f(x)dx. \tag{4}$$

The expected utility of a better opportunity, if c is the cost of search, is

$$\int_{x=X}^{\infty} U(x-c)f(x|x>X)dx = \left[\int_{x=X}^{\infty} U(x-c)f(x)dx\right] \bigg/ \int_{x=X}^{\infty} f(x)dx. \tag{5}$$

Search should cease when this value falls below $U(X)$.

The above approach assumes persistent opportunities in the sense that, once found, an opportunity remains available. Also it ignores the possibility of learning about $f(x)$ as search proceeds. Search with learning and also transitory opportunities are discussed in a linear utility function context by Morris [246].

7 DEGREES OF BELIEF—PROBABILITY

As equations (2a) and (2b) indicate, the utility of a risky prospect G with a set of possible outcomes $\{g\}$ depends upon the decision maker's utility function U and the probability distribution $f(g)$ associated with the set of possible outcomes. In this section we will consider what is meant by the probability distribution $f(g)$, leaving the concept of utility to Section 8 below.

Like the concept of preference, the concept of probability is a fundamental one and there is a large literature arguing its foundations and interpretation [133, 138, 305, 308]. Only the merest sketch is possible here but enough, it is hoped, to show that real-world decision making can only be based on subjective or personal probabilities and not on the other types of probability that have been posited.

Broadly, three interpretations of probability as a concept have been proposed. They are the frequency (or objective), logical (or necessary), and personal (or subjective or judgmental) approaches [72, 210, 358, 376].

7.1 PROBABILITY AS A FREQUENCY CONCEPT

The objective approach to probability grew out of the assessment of games of chance as developed by, among others, Laplace [209] and Jacob Bernoulli [30]. Their work suffered from the restriction of being based on equally likely events. In its modern axiomatic form, largely associated with the work of Von Mises [350, 351], this restriction is overcome, probability being defined as the limit of a relative frequency. Since a limit implies an infinite set of observations, this is not an operational definition. It is an abstraction that can never be verified. This unverifiability, in itself, is not a difficulty since the probability concept may still be useful—just as the unreal notions of points and lines in Euclidean geometry are useful. However, because the theory relates to infinite sets of observations, there is a logical difficulty in applying it to finite sets of observations. In particular, probability statements about single events (Will the coming season be good, fair, or poor?) are excluded by the frequency approach. Most users of the objective approach get around this difficulty by assuming that a finite set of observations is good enough to estimate the limit. In doing so they make a subjective judgement and so, in fact, are using subjective probabilities. As well, in applying objective frequencies based on finite historical sets of observations to future probabilities, they make a subjective judgement that the structure of the situation has not changed. For these reasons the unqualified use of what are thought to be objective probabilities might be described as the inefficient and ignorant use of subjective probabilities chosen in a lazy mechanical fashion.

7.2 PROBABILITY AS A LOGICAL CONCEPT

In its modern form, the concept of logical or necessary probability is largely associated with the work of Carnap [55, 313] and Jeffreys [177]. Keynes [190] was one of the first developers of the theory.

Logical probability is defined as the logical relationship between a proposition and a body of evidence. It is independent of personal taste and there can only be one logically true probability for a particular proposition on the basis of specified evidence. For example, the logical probability of drawing a red ball from an urn with five black and two red balls may be 2/7. Likewise, the logical probability of rain tomorrow would be 1/5 if this is the logical relationship between the prediction of rain tomorrow (proposition) and the meteorological report (evidence). In both these cases, ascertainment of the logical probabilities implies knowledge of all the relevant physical laws and circumstances and their interaction—an impossible task. There is thus a major problem of applicability with logical probabilities. If they are used, it must be on the basis of a judgement that all the relevant laws of operation and circumstances of the system have been taken into account.

7.3 PROBABILITY AS A PERSONAL CONCEPT

The degree of belief or strength of conviction an individual has in a proposition is the subjective or personal probability of that proposition for that individual [211, 306]. Imagine a risky prospect that yields a utility of u_a if event A occurs and a utility of u_b if event B occurs, u_a being larger than u_b. If I am indifferent between the risky prospect and a sure payment of u, then this implies that my degree of belief or subjective probability p about the occurrence of A is $(u - u_b)/(u_a - u_b)$ since indifference implies $u = pu_a + (1 - p)u_b$. In this sense a person's subjective probability for an uncertain event corresponds to what he regards as fair odds for a bet on the event. In the above example these odds would be $p : (1 - p)$ or $(u - u_b) : (u_a - u)$ for a bet on the occurrence of A. Such personally fair odds must typically be expected to differ from person to person for the same event; and for the same person, they may vary over time.

While some of the classical 18th century founders of statistics recognized subjective probability, Ramsey [290] in the 1920's was the first to axiomatize the concept logically within a theory of consistent preferences. Modern development of the theory rests particularly on the work of de Finetti [70], Good [132], Koopman [195], and Savage [305, 307, 309]. Koopman's [195] approach is based on intuitive feelings about "more probable than" relations. Savage [305] bases his system on the subject's preference between acts, assuming an idealized subject who is always consistent. The ordering among acts implicitly defines a set of probabilities for the relevant states of nature [188]. Because the ordering among acts is subjective, the underlying probabilities are also subjective. More recently, Pratt, Raiffa, and Schlaifer [277] have presented a system in which utility and personal probability are derived simultaneously.

These various approaches to subjective probability leave open the question of how such probabilities should be revised on the basis of experience or additional information. Usually, Bayes' Theorem (see Section 7.4 below) is the procedure used, but—within the logic of subjective probability itself—there is no reason why revision of initial subjective probabilities should not be carried out in a purely personalistic way. Of course, a mechanical procedure such as Bayes' Theorem is desirable since it aids consistency in personal revisions.

For decision making (and what other purpose can probabilities have?) the only valid probability approach is the subjective one. The decision maker bears responsibility for his decisions and should use his own strengths of conviction. Moreover, use of any other probabilities implies a judgement that they are the relevant ones, i.e. they correspond to his own judgement about the situation. As well, compared to objective frequencies, personal probabilities allow the incorporation of intuitive knowledge and recognition that the future may not be like the past [294, 310]. Nor are they restricted, like frequencies, to situations where a series of observations are available; they are just as pertinent to unique non-repeatable situations as to repeated ones. This is especially important since the majority of real-world decisions occur in non-repeatable contexts.

On an operational basis, the difference between logical and subjective probabilities is not so great. Carnap [56] makes a distinction between internal and external induction. Internal induction corresponds to what has been defined as logical probability. External induction is concerned with the occurrence of the event. The degree of belief about the occurrence of the event should be supplied by the logical probability. Although for many decision makers their subjective probability will not coincide with the logical probability, this is argued as due to their inability to assess all the relevant physical laws and circumstances. In a sense, therefore, a person's subjective probability about an event may be regarded as his estimate of the true but unassessable logical probability. At the same time, it must be emphasized that the only "true" subjective probabilities are those that correctly reflect the decision maker's personal degrees of belief or strengths of conviction. Of course, for rationality, these degrees of belief must be coherent with probability calculus in the sense that it is impossible to set up a series of bets against their owner in such a way that he is sure to lose regardless of the outcome of the events being bet on.

Over the last decade, and not without some strong argument [47, 48, 154, 155, 311, 346, 362], the subjective approach has become quite well accepted both in statistics (see e.g. [147, 211, 314]) and in quantitative business analysis (see e.g. [192, 229, 246, 312]). The major difficulty for most people in the acceptance of subjective probability is the loss of "scientific objectivity" [13; 287, p. 278] which may occur since two people facing the same problem with the same data may have different subjective probabilities. There is, however, nothing sacred about "scientific objectivity" especially when it so often leads to what Schailfer

[310, p. 654] has defined as an error of the third kind: "when the statistician delivers a carefully computed solution of the wrong problem".

Two difficulties arise in personal probability assessment: first, they should be coherent with probability calculus in the sense noted above; and, second, they should correspond with the assessor's judgement. To check coherence is a simple mathematical exercise [99, 238], but there is no way of knowing whether the specified probabilities do truly reflect their owner's judgement. One problem here is psychic bias [64] in the form of probability preferences—some people simply have a preference for some probabilities over others [94]. So far as possible such psychic bias has to be recognized and overcome. Too, there is evidence [252, 369] that people tend to overstate low chances and understate high chances. Fellner [104, 111] has argued a theory in which subjective probabilities are slanted or discounted according to the psychic uncertainty with which they are formulated. There are, however, strong arguments against the rationality of such a theory [45, 104, 328].

Questions of psychic bias are something of a refinement. A more fundamental problem is the practical one of getting any sort of estimate at all of many peoples' subjective probabilities. It is by no means self-evident that numerical personal probabilities exist, although as Savage [305] has shown, they are implied by the decisions taken—even though (as is often true with farmers) the decision maker has no formal concept of probability. At the same time, it is a tenet of Bernoullian decision theory (not without psychological support [63]) that subjective probabilities can always be associated with uncertain events. Not surprisingly, the last decade has seen much interest in practical procedures involving both direct and indirect interrogation for the assessment of subjective probabilities and, while some problems remain, much progress has been made [26, 28, 71, 97, 134, 137, 246, 248, 273, 287, 297, 303, 312, 327, 342, 344, 345, 363, 364, 365, 366, 367]. Within the agricultural economics literature, Carlson [54] has reported an apparently successful derivation of farmers' subjective probability distributions as, in much earlier and less detailed work, did Williams [360].

7.4 MANIPULATION OF PROBABILITIES

If they are to be manipulated, personal probabilities are by definition subject to the ordinary rules of probability calculus. One of these mechanical procedures—Bayes' Theorem—is of particular relevance to decision theory[6] since it is the logical procedure for revising probabilities on the basis of additional information [146]. Given an initial or "prior" probability distribution $f(g)$ for the random variable g, the availability of forecast information z whose likelihood or conditional probability relative to g, $P(z|g)$, is known, allows revision of the prior probabilities to obtain the "posterior" probabilities $f(g|z)$. The procedure, known as

[6] For this reason decision theory involving Bayes' Theorem is often referred to as Bayesian statistics. See Anscombe [13].

Bayes' Theorem, is specified as follows:

$$f(g|z) = f(g)P(z|g)/P(z) \tag{6}$$

where $P(z)$ is the marginal probability of forecasts. An example of the procedure is given in Section 9.2 below. Further applications are to be found in the literature, see e.g. [51, 54, 98, 146, 170, 245, 246, 314]. Often the likelihoods $P(z|g)$ arising as direct information from sampling, experimentation or some other predictive mechanism will be accepted at face value. Just as often, however, they will be subjectively adjusted so as to reflect the decision maker's personal assessment of the credibility of forecast information provided from outside sources. Also relative to the use of Bayes' Theorem, note that the posterior probabilities from one revision may be the prior probabilities in a further revision.

In situations where a consultant is acting for a remote client, complications may arise in applying Bayesian procedures. Hildreth [166] discusses several approaches that might be used; and Carlson [54] has illustrated a procedure suggested by Halter and Dean [146] to take account of the prior probabilities and additional information of both the client and the consultant. The procedure, which assumes the two persons' data are stochastically independent, consists of forming the posterior probabilities of each and then applying Bayes' Theorem again to combine the two sets of data.

Studies by psychologists, e.g. [20, 28, 266, 267, 268, 273], have shown many people to be conservative in their intuitive assessment of additional information relative to the revision of their probabilities implied by Bayes' Theorem. Similar conservatism has been found amongst Australian graziers.[7] Such effects suggest people tend to be inefficient learners and to make errors of intuition which could be overcome by mechanical application of Bayes' Theorem [326].

8 DEGREES OF PREFERENCE—UTILITY

Degrees of belief about the likelihood of outcomes are one half of Bernoullian decision theory; degrees of preference about outcomes are the other half. Just as subjective probabilities reflect degrees of belief, degree of preference is reflected or measured by utility [179, 354]. Examples of utility functions have been presented in figures 1 and 2. As the discussion of Section 5.1 showed, a person's utility function expresses the relative value to him of different amounts of gain or loss, and by the Expected Utility Theorem, enables utility indices to be associated with alternative risky prospects. The utility function of figure 1, for example, indicates that its owner obtains diminishing satisfaction (utility) from every extra dollar of gain but increasing satisfaction from every extra dollar reduction in loss. That is, if money gain or loss is denoted by X, dU/dX is decreasing for $X > 0$ and increasing for $X < 0$. The reverse is true for the owner of the curve of figure 2.

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[7] Personal communication from E. Francisco, University of New England.

Suffice to note that different people will have curves of different shape [227]. For financial outcomes it is typically assumed that less money is never preferred to more, so that the utility function is restricted to being non-decreasing from left to right, i.e. it must always have $dU/dX \geqslant 0$.

Looking at the curves of figures 1 and 2 it can be seen that both decision makers have a utility index of 100 for a money gain of $80,000. However, because interpersonal comparisons of utility are impossible, it cannot be said that both derive equal utility from a gain of $80,000. Nor, for example, that the decision maker of figure 1 obtains more utility from a gain of $20,000 than does the decision maker of figure 2. Note also that since the utility function is only defined up to a positive linear transformation, it cannot be said, for example, that the decision maker of figure 1 derives twice as much utility from $80,000 as he does from $28,300.

If desired, a person's utility function can also be used to derive the sure (i.e. non-risky) prospect which for him is equivalent to a given risky prospect. For example, the risky prospect of Buy 1,000 in the decision problem of table 1 has a utility of 23·476 for the decision maker of figure 1. The curve of figure 1 indicates a utility of 23·476 corresponds to a sure gain of $12,174. This sure money amount which has an equivalent utility value to the original risky prospect is known as the certainty equivalent of the risky prospect and remains invariant under linear transformations of the utility function. For the decision maker of figure 1, indifference would exist between an action with a sure gain of $12,174 and the action Buy 1,000 with its specified probability distribution of consequences.

8.1 PLOTTING THE UTILITY FUNCTION

The establishment of a person's utility function is rather analogous to the establishment of his subjective probabilities—it involves the pinning down in quantitative form of subjective feelings which may not have been thought of before in a precise quantitative way [118, 287]. Four approaches may be distinguished: direct measurement; the von Neumann-Morgenstern or standard reference contract method; the modified reference contract approach; and the Ramsey method.

The direct measurement approach, proposed and used by Galanter [129], involves asking a series of questions of the type: "Suppose I were to give you an outright gift of $100. This $100 comes from a foundation whose resources are limitless. How much money would you need to make you twice as happy as the $100 would make you feel?" The answers to a chained series of such questions enables the plotting of a utility curve against whatever arbitrarily chosen utility scale is desired. Galanter used this method with over 100 subjects and found it satisfactory. However, it is a very gross approach and, for many people, cannot be expected to be as precise as other methods.

The standard reference contract or von Neumann-Morgenstern method [146, 287, 312, 352] is based on the concept of certainty equivalence. If outcome x_1 is preferred to x_2, and x_2 is preferred to x_3, then (by the

Continuity Postulate) there exists a probability p such that:

$$pU(x_1) + (1 - p)U(x_3) = U(x_2). \tag{7}$$

For specified values of x_1, x_2, and x_3, the utility of x_2 can be determined by questioning to find the value of p at which x_2 is the certainty equivalent of the risky prospect involving x_1 and x_3, $U(x_1)$ and $U(x_3)$ being given values on an arbitrary scale. For example, if $U(x_1)$ is set at unity and $U(x_3)$ at zero, then $U(x_2) = p$. By finding the values of p corresponding to an array of values of x_2 between x_1 and x_3, the utility curve may be plotted for values of x from x_1 to x_3.

Two criticisms may be made of the standard reference contract procedure [66, 96, 259, 345]. First, if the subject has a liking or disliking for gambling *per se*, his certainty equivalents may be influenced by this bias since the questions posed involve a risky prospect versus a sure prospect. Second, bias may be introduced if the subject has preference for some probabilities *per se* over others, or if he does not understand the concept of probability.

The modified von Neumann-Morgenstern method [83, 227] uses neutral probabilities of $p = 0{\cdot}5 = 1 - p$ to overcome the problem of probability preference. Questions are posed to determine the certainty equivalent x_2 for a fifty-fifty or even chance prospect of x_1 or x_3, arbitrary utilities $U(x_1) > U(x_3)$ again being set for x_1 and x_3. Thus we have

$$0{\cdot}5U(x_1) + 0{\cdot}5U(x_3) = U(x_2). \tag{8}$$

If $U(x_1)$ is set at unity, say, and $U(x_3)$ at zero, say, then $U(x_2) = 0{\cdot}5$. In similar fashion, the certainty equivalent may be established for the fifty-fifty prospect of x_1 and x_2, say x_4, which will have a utility of:

$$U(x_4) = 0{\cdot}5U(x_1) + 0{\cdot}5U(x_2) = 0{\cdot}75; \tag{9}$$

and for the fifty-fifty prospect of x_2 and x_3, say x_5, which will have a utility of:

$$U(x_5) = 0{\cdot}5U(x_2) + 0{\cdot}5U(x_3) = 0{\cdot}25. \tag{10}$$

By further linked questions, additional points on the utility curve may be established. While the modified von Neumann-Morgenstern method overcomes bias due to probability preferences, it is still open to the criticism of involving choice between a risky and a certain outcome.

The Ramsey method aims to overcome both criticisms of the standard reference contract approach. It involves a sequence of choices between linked pairs of fifty-fifty gambles. The procedure was suggested by Ramsey [290] and has been tested by Davidson, Suppes and Siegel [69] and Officer and Halter [259] who give details of the questioning procedure.

The only comparative test of the above three indirect procedures appears to be that of Officer and Halter [259]. They concluded that the modified von Neumann-Morgenstern and Ramsey procedures were better than the standard reference contract approach, most consistent results being obtained by the Ramsey method but at the cost of more complicated questioning. More recent work at the University of New England suggests that the modified von Neumann-Morgenstern method is generally quite satisfactory.

8.2 ALGEBRAIC SPECIFICATION

Given that a decision maker follows the choice psotulates of Ordering, Continuity and Independence, Bernoulli's Principle implies the existence of a utility function U with the properties previously listed. As we are here concerned with financial outcomes, it will also be assumed that $U(X)$ is a non-decreasing function of X. Though it may be linear over some range, $U(X)$ will usually be curved. This implies that $U(kX)$ will generally not be equal to $kU(X)$ so that utility must always be considered not in terms of financial outcomes per technical unit (e.g. $ per acre) but in terms of aggregate net financial outcomes (i.e. total $ gain or loss per act).

Though Bernoulli's Principle implies the existence of $U(X)$, it tells us nothing of its precise form. Nor does the decision maker intuitively know the algebraic form of his utility function. But, as outlined above, the function can be readily plotted. Once plotted, if desired (as is often the case), it may easily be fitted by some suitable functional form using standard estimation procedures, for example by regression or by "eye-ball" techniques [227]; or it may be approximated by a series of linear segments—a procedure that will often be most convenient if further analysis is to proceed via some variety of linear or non-linear programming [285]. If a continuous functional form is required, a regression fit may be made to the observed points on the curve, or a curve of visual best fit may be drawn through the observed points and its functional form estimated by computer. The essential aspect, if algebraic specification is needed, is to obtain an estimate that is (subjectively) judged to fit the plotted curve satisfactorily over the relevant range of gains and losses. A variety of different functional forms may suit—polynomial, logarithmic, exponential, etc.—in which case the simplest to manipulate should be used.[8] Most often this will imply the use of a polynomial of second or third degree, or the use of a series of such polynomials spliced together [128]. Furthermore, the use of a polynomial may often be justified on the grounds of it being a Taylor series approximation to the unknown true utility function for gains and losses over the relevant range. Thus if $U(X)$ has a finite nth derivative $U^n(X)$ for all X and $U^{n-1}(X)$ is continuous everywhere, Taylor's Theorem [14, p. 96] states that, for any X^* and every $X \neq X^*$, there exists a point ξ in the interval joining X and X^* such that $U(X)$ may be approximated within a specified bound of error as

$$(11) \quad U(X) = U(X^*) + \sum_{k=1}^{n-1} [U^k(X^*)(X - X^*)^k/k!] + U^n(\xi)(X - X^*)/n!.$$

Collecting like powers of X, $U(X)$ may thus be approximated as a polynomial

$$(12) \qquad U(X) = \alpha_0 + \alpha_1 X + \alpha_2 X^2 + \alpha_3 X^3 + \ldots$$

[8] A variety of functional forms are noted in [230, 233, 276, 282, 285].

Since the utility function is only defined up to a positive linear transformation, equation (12) may be written as

$$U(X) = X + bX^2 + cX^3 + \ldots \tag{13}$$

Further, by the Expected Utility Theorem, we have

$$\begin{aligned} U(X) &= EU(X) \\ &= E(X + bX^2 + cX^3 + \ldots) \\ &= E(X) + bE(X^2) + cE(X^3) + \ldots \end{aligned} \tag{14}$$

so that utility can be expressed relative to a risky prospect with a consequence X following some probability distribution $f(X)$. For any such random variable, the expression $E(X^n)$ can be expressed in terms of the first n moments about the mean of $f(X)$. For example, if π_i denotes the ith moment $E[X - E(X)]^i$, the first three terms of $U(X)$ can be written as:

$$\begin{aligned} E(X) &= \pi_1 \\ E(X^2) &= \pi_2 + \pi_1{}^2 \\ E(X^3) &= \pi_3 + 3\pi_1\pi_2 + \pi_1{}^3. \end{aligned}$$

Making use of such identities, the expectation of the polynomial may be written as

$$U(X) = \pi_1 + b(\pi_2 + \pi_1{}^2) + c(\pi_3 + 3\pi_1\pi_2 + \pi_1{}^3) + \ldots \tag{15}$$

which expresses the utility of any risky prospect $f(X)$ as a function of its moments.

An alternative approach arises from considering the Taylor series expansion of $U(X)$ about the mean π_1 of $f(X)$. Thus we have

$$U(X) = U(\pi_1) + \sum_{k=1}^{n-1} [U^k(\pi_1)(X - \pi_1)^k/k!] + R_n(X) \tag{16}$$

where $R_n(X)$ is a remainder term whose absolute value declines as more terms are included in the series. By the Expected Utility Theorem, $U(X) = EU(X)$ so that the utility of the risky prospect X is

$$U(X) = U(\pi_1) + \sum_{k=1}^{n-1} U^k(\pi_1)\, E[(X - \pi_1)^k]/k! + E[R_n(X)]. \tag{17}$$

Since $E[(X - \pi_1)^k]$ is the kth moment of $f(X)$ and $E(X - \pi_1) = 0$, the utility function may be expressed as

$$U(X) = U(\pi_1) + (1/2)\,[U^2(\pi_1)]\pi_2 + (1/6)\,[U^3(\pi_1)]\pi_3 + E[R_4(X)] \tag{18}$$

where bounds of error may be placed on the remainder term [169, p. 36]. The utility of a risky prospect may thus be approximated via its first three moments as

$$U(X) \doteq U(\pi_1) + (1/2)\,(d^2U/dX^2)\pi_2 + (1/6)\,(d^3U/dX^3)\pi_3. \tag{19}$$

If $\pi_3 = 0$, as is the case with symmetric distributions, or if $d^3U/dX^3 = 0$ as applies if the utility function is quadratic, the third term in equation (19) is zero and utility may be calculated from the mean and variance of $f(X)$. Note that this procedure implies knowledge of the algebraic form of the utility function $U(X)$ so that the necessary derivatives may be calculated. Halter and Dean [146] give some examples of the procedure.

If a polynomial such as equation (13) is to be fitted (as is typically a good first step to obtain a continuous functional form), the question arises as to how many terms should be included. What degree should the polynomial be? An immediate answer is that as many powers of X should be used as are needed to satisfactorily fit the data. Such a procedure, on a pragmatic basis then specifies how many moments of $f(X)$ are taken into account by the decision maker. Alternatively, though it does not yet seem to have been attempted, the decision maker might be tested in some way to discover which moments are relevant to him. If he only considers mean and variance, the quadratic is relevant; if skewness is also considered, then the cubic is relevant. Of course, it is to be expected that if two risky prospects have the same mean and variance, the one with the more positive skewness will be preferred even if for most of his decisions the decision maker's choices are dominated by consideration of the mean and variance [7, 66]. As an empirical matter, it seems the ith moment is more important than the $(i + 1)$th, and that for most decision makers, moments beyond the third play no great role in choice. Too, experience indicates that if the curve is anyways S-shaped, a pair of spliced quadratic or cubic polynomials provide the best approach to a satisfactory fit. Figures 1 and 2, for example, both involve spliced quadratics.

Because of their ease of fitting and apparently satisfactory goodness of fit, quadratic and cubic polynomials have been the most common type of function used in the (limited) agricultural economics literature [146, 259, 260]. A further advantage of the polynomial form is that it allows risky prospects to be evaluated in terms of the moments of $f(X)$ as in equation (15). Often this will be more convenient than direct use of the probability distribution as implied by equation (2).

8.3 QUADRATIC UTILITY AND (E,V) ANALYSIS

In this and the following section we are mainly concerned with some of the algebra of quadratic and other polynomial utility functions. This makes dull reading and might best be skipped by those not particularly interested.

If the utility function is quadratic, we have

$$U(X) = X + bX^2, \tag{20}$$

and the restriction $dU/dX > 0$ necessitates

$$\begin{aligned} X &> -1/2b \quad \text{if } b > 0, \\ X &< -1/2b \quad \text{if } b < 0. \end{aligned}$$

Within these ranges, X is the certainty equivalent of all risky prospects whose utility is equal to $U(X)$. In a sense, therefore, the function thus encompasses all such risky prospects even though they may have some outcomes whose values fall outside the relevant range of the quadratic. The second derivative of the quadratic shows $b > 0$ implies increasing marginal utility as X increases; and $b < 0$ implies decreasing marginal utility as X increases. If $b = 0$, $U(X)$ is linear and marginal utility is constant as X increases.

If X is a risky prospect, the quadratic may be written

$$(21) \qquad \begin{aligned} U(X) &= E(X) + bE(X^2) \\ &= \pi_1 + b\pi_1^2 + b\pi_2 \end{aligned}$$

where π_1 is the mean and π_2 the variance of X about π_1. For utility to increase as π_1 increases, with π_2 fixed, $\delta U/\delta\pi_1$ must be positive. This will always be true over the relevant range of the quadratic so that if two prospects have the same variance, the one with the higher mean will always be preferred. Likewise, since π_2 is necessarily positive and $\delta U/\delta\pi_2 = b$, increasing marginal utility of money (i.e. $b > 0$) implies variability in X is attractive—the greater is π_2, the greater is $U(X)$ for b positive. Conversely, diminishing marginal utility of money (i.e. $b < 0$) implies variability in X is disliked. These relations are usually referred to in the context of the quadratic by saying that if $b > 0$, the decision maker is a risk preferrer; and if $b < 0$, he is a risk averter. If $b = 0$, he is risk indifferent. For these reasons the coefficient b in the quadratic is often referred to as the coefficient of risk preference or aversion[9] [229, 339].

The expected money value of the risky prospect X is $E(X)$. In the quadratic case, the utility of this expected money value is

$$(22) \qquad \begin{aligned} U[E(X)] &= E(X) + b[E(X)]^2 \\ &= \pi_1 + b\pi_1^2 \end{aligned}$$

which differs from $EU(X)$ by the amount of $b\pi_2$. Thus the utility of a risky prospect is greater than, equal to, or smaller than the utility of its expected monetary value according as the quadratic decision maker is a risk preferrer, risk indifferent or a risk averter. These relations imply that the certainty equivalent of a risky prospect will be greater than, equal to, or smaller than its actuarial money value according as risk preference, indifference or aversion respectively prevail.

With $U(X)$ quadratic, discussion is often presented in terms of mean-variance or (E,V) analysis, e.g. [9, 10, 87, 274, 300, 340, 370]. The function

$$U(X) = \pi_1 + b\pi_1^2 + b\pi_2$$

implies a utility surface in the three dimensions U, π_1, and π_2. For constant values of U, the function can be represented by a series of iso-

[9] Such terminology equates variance with risk and, while perhaps justified in the context of quadratic utility, is not in general justified. As Markowitz [233] and Machol and Lerner [225] have emphasized, risk might be defined by real-world decision makers in a variety of other ways such as the expected value of loss (as is done in statistical decision theory), the probability of loss, the expected absolute standard deviation, the maximum expected loss, the probability of ruin, the semi-variance, or simply as a random variable. Like beauty, risk lies in the eyes of the beholder! From a normative point of view, the relative risk of alternative prospects seems best described by comparisons of their cumulative probability distributions—for example, any prospect will be riskier than another one which has the same consequences but whose cumulative distribution never exceeds that of the former. See [139, 149, 298, 359].

utility contours in mean-variance space. Thus, setting $U(X)$ equal to some constant U^*, rearrangement gives

$$\pi_2 = U^*/b - (1/b)\pi_1 - \pi_1^2 \tag{23}$$

as the (E,V) locus of all mean-variance combinations which yield the same level of utility. Such loci are also known as (E,V) indifference curves since a decision maker with a quadratic utility function would be indifferent between all prospects whose mean and variance lie on the same iso-utility locus.[10] Corresponding to the relevant range of the quadratic, the relevant range of the iso-utility loci is $\pi_1 > -1/2b$ for $b > 0$, and $\pi_1 < -1/2b$ for $b < 0$. Note also that the intercept of an iso-utility curve with the E axis, i.e. where $V = 0$, is the certainty equivalent of all mean-variance combinations on that iso-utility curve.

For a decision maker with a quadratic utility function, the slope of an (E,V) indifference curve measures his trade-off or substitution rate between mean and variance at the specified level of utility U^*. The slope of the iso-utility curve is

$$dE/dV = -b/(1 + 2b\pi_1). \tag{24}$$

Since $(1 + 2bX)$ is dU/dX and must be positive, $(1 + 2b\pi_1)$ must be positive also Hence dE/dV will be positive, zero or negative within the relevant range according as b is negative, zero or positive. As is intuitively obvious, a risk averter will need increases in mean value to compensate for increased variance if his utility is to remain unchanged. The reverse applies to a risk preferrer.

The second derivative of the iso-utility curve is

$$d^2E/dV^2 = [2b^2(1 + 2b\pi_1)^{-2}]\,(d\pi_1/d\pi_2). \tag{25}$$

The term in square brackets is always positive, and $d\pi_1/d\pi_2$ is positive or negative over the relevant range according as b is positive or negative. Hence for a risk averter, the (E,V) indifference curves have increasing slope (i.e. the trade-off rate increases) as V increases; and for a risk preferrer, the (E,V) indifference curves have increasing negative slope as V increases. The greater the degree of risk aversion or preference, i.e. the greater is $|b|$, the steeper the indifference curves.

As has been implicit in the above discussion, available risky prospects from which a choice is to be made may also be pictured in (E,V) space, e.g. see [10, 87]. These prospects may have moments beyond the second but, if a quadratic utility function is used, such higher moments are assumed irrelevant to choice. In an (E,V) context, they cannot influence the decision maker.

Given a decision maker with a quadratic utility function and a set of risky prospects too complex to handle intuitively, how might utility

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[10] If the iso-utility curves are drawn in mean-standard deviation space (i.e. with axes measuring π_1 and $\pi_2^{\frac{1}{2}}$), they will be concentric circles with centre at $\pi_1 = \frac{1}{2}|b|$, $\pi_2^{\frac{1}{2}} = 0$. The relevant range of these indifference curves will be the arcs in the region $0 < \pi_1 \leqslant \frac{1}{2}|b|$ and $\pi_2^{\frac{1}{2}} \geqslant 0$.

analysis be used to optimize choice? If only discrete prospects are pertinent, choice may be made by calculating the utility value of each act and then choosing the act with the highest utility. An example of this procedure for the case where the consequence probabilities are discrete has already been given by the cattle purchase problem of Section 5.1. If the probability distribution of consequences for each act is continuous, then the first two moments of the distributions may be used to assess utility values. For example, consider the following two-act decision problem with each act specified in terms of its moments:

	Act 1	Act 2
First moment, π_1	100	120
Second moment, π_2	3,600	6,400
Third moment, π_3	2,000	$-4,000$

Suppose the decision maker's utility function is

$$U(X) = X - 0{\cdot}001X^2.$$

Being quadratic, this function implies that the third moment is irrelevant and the utility of each act may be calculated as

$$U(X) = \pi_1 - 0{\cdot}001\pi_1{}^2 - 0{\cdot}001\pi_2.$$

This yields utility values of 86·4 and 99·2 for Act 1 and Act 2 respectively so that Act 2 is to be preferred. The same assessment would also result if appraisal were made via the procedure of equation (19). For example, for Act 1 we have

$$\begin{aligned} U(X) &= U(\pi_1) + (1/2)\,(d^2U/dX^2)\pi_2 + (1/6)\,(d^3U/dX^3)\pi_3 \\ &= U(100) + (1/2)\,(-{\cdot}002)\,(3600) + (1/6)\,(0)\,(2000) \\ &= 86{\cdot}4. \end{aligned}$$

Should the array of available acts be continuous rather than discrete, analytical rather than iterative procedures may be used to maximize utility. Consider the following simple example. A farm manager has to decide how much of a fertilizer input N to use per acre on an area of Z acres, yield per acre Y being stochastically related to fertilizer input by the response function

$$Y = d_0 + d_1N + d_2N^2 + e$$

where e is a random variable with variance d_3N and mean zero.[11]

Monetary gain or loss over Z acres from use of the input at a rate of N per acre is:

$$\begin{aligned} X &= (p_yY - p_nN - h)Z \\ &= [p_y(d_0 + d_1N + d_2N^2 + e) - p_nN - h]Z \end{aligned}$$

[11] More realistically, yield variance would also be influenced by climate. For example, d_3 may be some function of rainfall with its associated uncertainty. In such cases it may be possible to obtain predictions of the levels of the uncontrollable factors which can be incorporated into the analysis. Such a procedure has been used by Byerlee and Anderson [51].

where p_y and p_n are the unit prices of Y and N respectively and h is fixed production cost per acre.[12] The mean (π_1) and variance (π_2) of the distribution of net returns are respectively:

$$\pi_1 = [p_y(d_0 + d_1N + d_2N^2) - p_nN - h]Z$$
$$\pi_2 = d_3p_y^2Z^2N.$$

Substituting these expressions into the manager's quadratic utility function,

$$U(X) = \pi_1 + b\pi_1^2 + b\pi_2,$$

gives $U(X)$ as a function of the decision variable N:

$$U(X) = Z[p_y(d_0 + d_1N + d_2N^2) - p_nN - h] + bZ^2[p_y(d_0 + d_1N + d_2N^2) - p_nN - h]^2 + bd_3p_y^2Z^2N.$$

In this equation N is the only independent variable. Setting $dU/dN = 0$ and solving for the value of N at which $d^2U/dN^2 < 0$ gives the level of N per acre which maximizes utility. Had the area Z also been a decision variable, utility would be maximized by setting both $\delta U/\delta N$ and $\delta U/\delta Z$ equal to zero and solving simultaneously for the optimal levels of N and Z. And so on for any number of decision variables.

As the above example tends to indicate, the analytical approach to utility maximization may become quite complicated with even the simplest of functional forms. Computerized iterative appraisal is likely to be more convenient in most cases. Alternatively a non-linear programming approach with $U(X)$ represented by a series of linear segments might be used.

Somewhat analogously to the above example, some recent studies have used utility maximization based on a quadratic utility function instead of profit maximization as the allocative criterion in the broader context of the whole farm firm. Anderson [9] and Sadan [300] have integrated utility analysis with Cobb-Douglas production functions while McArthur and Dillon [217] have applied it in the context of parametric budgeting for a single product firm. Gale [130] has considered utility in the decision context of an agricultural processing firm. Rae [286] has used a programming approach with a linearly segmented representation of the utility function.

8.4 CRITICISMS OF POLYNOMIAL UTILITY FUNCTIONS

Simple polynomial utility functions (quadratic and cubic) have generally proved quite satisfactory in empirical application, especially in agriculture [9, 259]. They have, however, been strongly criticized on theoretical grounds.

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[12] Note that it is necessary to work in terms of total net returns over Z acres rather than on a per acre basis because, as previously noted, in general $U(kX) \neq kU(X)$. Also that it is necessary to include fixed costs in the analysis because in general $U(X - h) \neq U(X) - U(h)$. Only if $U(X)$ is linear will $U(kX) = kU(X)$ and $U(X - h) = U(X) - U(h)$ over the range of X.

By introspection and intuition, it seems reasonable that as a risk averse person's wealth increases, the insurance premium he is prepared to pay against a given risk should decline. This is never true if the utility function is quadratic; and for higher order polynomials is at best true only for part of their relevant range. The criticism has been put formally by Arrow [18] and Pratt [276] in their independent development of the coefficient of (local or absolute) risk aversion r, defined as the negative ratio of the second and first derivatives of the utility of wealth[13] function $U(W)$, that is

$$r = -(d^2U/dW^2)/(dU/dW). \tag{26}$$

This ratio is a pure number and so allows interpersonal comparisons of the degree of risk aversion at particular wealth levels. If multiplied by W, it measures the elasticity of the marginal utility of wealth and is then called the coefficient of relative risk aversion. As Pratt [276] has shown, intuition implies that r should decline as W increases for a risk averse person, i.e. one for whom $d^2U/dW^2 < 0$. However, for the general polynomial

$$U = (k - W)^c, \qquad c > 1, \quad k > W, \tag{27}$$

the coefficient of risk aversion,

$$r = -(c - 1)/(k - W), \tag{28}$$

increases with W since $dr/dW > 0$. By an appropriate linear transformation, any quadratic can be put into the general form with $c = 2$; and many higher order polynomials can be analogously transformed. Moreover, even if r is decreasing over some range of a higher-order polynomial, this will usually only be over part of its relevant range. In general, therefore, polynomials do not meet the condition of decreasing absolute risk aversion when they are used to represent the utility of wealth.

But what if, instead of wealth or initial endowment W, we use gain or loss X as the argument of the utility function? It has been shown [37, 108, 270] that if a person's preferences are to be independent of his wealth, then his utility of wealth function must be either linear or exponential. In turn this implies that the utility function for gains or losses about an initial wealth position must be either linear or exponential. This can be shown as follows. If $U(W)$ is linear, the utility of a new wealth position $(W_0 + X)$, where W_0 is initial wealth and X is the change, is

$$U(W_0 + X) = a(W_0 + X) \tag{29}$$

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[13] In contrast to the operational approach we have used of couching utility in terms of net returns or gains and losses, theoretical discussion of utility is most often posed in the context of wealth, i.e. wealth W is used as the argument of the utility function $U(W)$. See, e.g. [18, 121, 125, 141, 232].

and the utility of X is given by

(30)
$$\begin{aligned} U(X) &= U(W_0 + X) - U(W_0) \\ &= a(W_0 + X) - a(W_0) \\ &= aX \end{aligned}$$

which is linear. If $U(W)$ is exponential, we have

(31)
$$\begin{aligned} U(X) &= U(W_0 + X) - U(W_0) \\ &= e^{a(W_0+X)} - e^{aW_0} \\ &= e^{aW_0}(e^{aX} - 1) \end{aligned}$$

which, for a given level of initial wealth, can be written by a positive linear transformation as the simple exponential

(32)
$$U^*(X) = e^{aX}.$$

Since polynomial functions are not linear or exponential, this implies that the coefficient of risk aversion $r(X)$ for a polynomial function has implications relative to the coefficient of risk aversion $r(W)$ for the corresponding utility of wealth function. In particular,

(33)
$$U^*(X) = X + dX^2, \qquad d < 0,$$

implies

(34)
$$\begin{aligned} U(X) &= (1 + 2bW_1)X + bX^2 \\ &= U(W_1 + X) - U(W_1) \end{aligned}$$

where

(35)
$$U(W) = W + bW^2, \qquad b < 0.$$

Thus a quadratic utility of net returns function implies increasing absolute risk aversion in the utility of wealth function. Analogous results follow for higher-order polynomials. Hence the use of net return (or rate of return [124, 163]) rather than wealth as the argument in the utility function does not overcome the problem of increasing risk aversion. There is, however, a way around the difficulty [9, 250]. For example, in the quadratic case with

(36)
$$U(X) = X + bX^2, \qquad b < 0,$$

the coefficient b may be regarded (hypothesized) not as a constant but as some function of wealth W. This seems a far more realistic and less restrictive assumption than that the coefficient b is invariant with respect to wealth. Anderson [9] has shown, for example, that if

(37)
$$b = k_1 + k_2W^q, \qquad k_1, k_2, q < 0,$$

the Pratt coefficients $r(W)$ and $r(X)$ are diminishing as required by intuition. The cubic case can be handled in a similar manner. The implication of such a hypothesis is that the utility function for gains and losses cannot be derived directly from the utility of wealth function without knowledge of such relations as that between b and W. Operationally this poses no difficulty since $U(X)$ can be easily obtained directly at a particular point in time and wealth for a given decision maker. From a practical point of view, the approach fits in well with the pragmatic approach of regarding a polynomial as an approximation to the unknown true utility function, recognizing that a new utility of net returns function should be assessed whenever the decision maker's situation changes significantly.

The other criticism of polynomial utility functions is that they are restricted in range or shape [39, 109, 149, 150, 341]. The quadratic, for example, is not everywhere monotonically increasing and is only relevant over its range of positive slope. Likewise, if the cubic function

$$U(X) = b_1X + b_2X^2 + b_3X^3 \tag{38}$$

is to be relevant over its entire range, it must have $b_2^2 < 3b_1b_3$, $b_1 > 0$. If these restrictive requirements are met, the shape of the function necessarily involves an initial stage of decreasing marginal utility followed beyond the inflexion point at $X = -b_2/3b_3$ by a final stage of increasing marginal utility [83]. If marginal utility is first increasing and then decreasing, the only cubic functions which can be used will not be everywhere increasing, i.e. their relevant range will be restricted. From an operational point of view, the fact that polynomials are restricted to some relevant range is no great problem. Empirical utility functions are estimated over a particular range of gains and losses and no one would recommend their use beyond that range.

Overall, despite the theoretical criticisms that have been made of quadratic and cubic polynomials as utility functions, they must still be regarded as satisfactory first-steps to practical application of utility analysis.

9 BELIEF AND PREFERENCE COMBINED

Having outlined the concepts of degrees of belief (Section 7), degrees of preference (Section 8) and value of information (Section 6), we may now bring them together in a fairly general model of Bernoullian decision theory. In essence, such a statement consists of no more than an extended version of Bernoulli's Principle as given in equations (2a) and (2b). We will present the discrete probability case corresponding to equation (2a) since the majority of practical decision problems in agricultural production, policy and marketing could be framed most conveniently in this way. The model for situations where the uncertain consequences follow a continuous distribution could be presented in an exactly analogous way, the only requirement being replacement, where relevant, of the discrete summation by integration. Expositions of the general Bernoullian decision model somewhat analogous to that presented below have also been given by Forester [123], Halter and Dean [146], Morgan [245] and Morris [246], among others.

9.1 GENERAL MODEL

Though the symbols used are explained as they occur, we will first list most of them in their order of appearance so as to provide a ready reference to their meaning. They are not as bad as they look.

G_i : ith risky prospect or available action ($i = 1, 2, \ldots, m$).
$\{g_i\}$: set of consequences associated with G_i.
$f(g_i)$: subjective prior probability distribution followed by the uncertain variable g_i.
$U(g_i)$: utility of a particular consequence from $\{g_i\}$.
$U(G_i)$: utility of G_i.
$G_i{}^*$: optimal action with no forecast information.
z_j : a particular forecast relative to the occurrence of $\{g_i\}$.
c : cost of forecast information.
$P(z_j|g_i)$: conditional probability of z_j given g_i.
$f(g_i|z_j)$: revised or posterior probability of g_i given z_j.
$U(G_i|z_j)$: utility of G_i based on the posterior probability associated with z_j.
$G_i{}^{**}$: optimal act based on posterior probabilities.
$U(z_j)$: utility value of the particular forecast z_j.
$g_i{}^*$: a particular consequence of the prior optimal act $G_i{}^*$.
$g_i{}^{**}$: a particular consequence of the posterior optimal act $G_i{}^{**}$.
$\{z_j\}$: set of potential predictions about the occurrence of the relevant n possible states of nature ($j = 1, 2, \ldots, n$).
$G_{ij}{}^{**}$: optimal action given z_j.
$U(\{G_{ij}{}^{**}\})$: utility of the optimal strategy $\{G_{ij}{}^{**}\}$ given $\{z_j\}$.
$P(z_j)$: probability of z_j.
$U(\{z_j\})$: utility value of the predictor generating $\{z_j\}$.
z' : a perfect predictor generating the set of potential perfect predictions $\{z_j'\}$.
$G'ij$: optimal action given the perfect prediction zj'.
$U(\{G_{ij}'\})$: utility of the optimal strategy with a set of perfect predictions $\{z_j'\}$.
$U(\{z_j'\})$: utility value of the perfect predictor generating $\{z_j'\}$.

Suppose choice has to be made between the set of actions or risky prospects $G_1, G_2, \ldots, G_m$ where the ith action G_i has a set of uncertain consequences $\{g_i\}$ following the subjective probability distribution $f(g_i)$. Thus the consequence g_i of G_i is a random variable and a particular consequence g_i has a utility of $U(g_i)$. The consequences $\{g_i\}$ are specified as gains or losses. Both the utility function U and the set of subjective probability distributions $\{f(g_i)\}$ belong personally to the decision maker. The (expected) utility of the ith action is

(39) $$U(G_i) = \sum_{g_i} U(g_i)f(g_i),$$

and the optimal action $G_i{}^*$ will be the one with the greatest utility. The decision procedure is thus to choose the action such that

(40) $$\begin{aligned} U(G_i{}^*) &= \max_i U(G_i) \\ &= \max_i [\sum_{g_i} U(g_i)f(g_i)]. \end{aligned}$$

Further suppose a *particular* piece of forecast information z_j relative to the occurrence of the consequence sets $\{g_i\}$ is available at a cost of c. Typically z_j will be a prediction as to the state of nature and hence which consequence g_i will occur. Given knowledge of $P(z_j|g_i)$, the conditional distribution of z_j relative to each set of consequences $\{g_i\}$, Bayes' Theorem allows the set of prior probability distributions $\{f(g_i)\}$ to be revised by the formula of equation (6) to obtain the set of revised or posterior probability distributions $\{f(g_i|z_j)\}$. Once the additional information z_j is known, the utility of each action G_i given the extra information can be calculated as

(41) $$U(G_i|z_j) = \sum_{g_i} U(g_i - c)f(g_i|z_j).$$

Again the optimal action, say $G_i{}^{**}$, will be the one with the greatest utility, i.e. the action such that

(42) $$\begin{aligned} U(G_i{}^{**}) &= \max U(G_i|z_j) \\ &= \max_i [\sum_{g_i} U(g_i - c)f(g_i|z_j)]. \end{aligned}$$

The value, in utility terms, of the extra information z_j can be evaluated *ex post* as

(43) $$\begin{aligned} U(z_j) &= U(G_i{}^{**}) - U(G_i{}^*|z_j) \\ &= [\sum_{g_i{}^{**}} U(g_i{}^{**} - c)f(g_i{}^{**}|z_j)] - [\sum_{g_i{}^*} U(g_i{}^*)f(g_i{}^*|z_j)], \end{aligned}$$

the second term in square brackets being the utility obtainable with the prior optimal act $G_i{}^*$ under the conditions of the revised probabilities $f(g_i{}^*|z_j)$. If $f(g_i{}^{**}) = f(g_i{}^*)$, as is often the case or as can be arranged by appropriate definition of the states of nature, equation (43) may be simplified to:

(44) $$U(z_j) = \sum_{g_i} [U(g_i{}^{**} - c) - U(g_i{}^*)]f(g_i|z_j).$$

The information z_j will have been worth having only if $U(z_j) > 0$; and the maximum price that should have been paid for the information is given by the value of c for which $U(z_j) = 0$.

The above analysis is in terms of having to hand a particular forecast z_j at a cost c. Often, before he obtains the actual forecast information, the decision maker will know the possible forms it may take. In such cases it is possible to carry out preposterior analysis, i.e. to evaluate the various strategies that might be followed contingent upon the various possible particular forecasts and choose, *a priori*, the best strategy. Suppose there are n possible states of nature and the forecast information z, again at a cost of c, consists of a prediction z_j of the occurrence of the jth

($j = 1, 2, \ldots, n$) state of nature or state of the world. There is thus a set $\{z_j\}$ of n possible predictions. A strategy consists of a policy specifying what action will be taken under each prediction, for example a particular strategy might be: "If the forecast is z_1, then the action G_2 will be chosen; if z_2, then G_m; if z_3 then G_1; . . . if z_n, then G_2". Since there are n possible forecasts and m possible actions, the total number of possible strategies is m^n which can easily be quite a formidable array. For example, with only five actions and four possible forecasts, there are $5^4 = 625$ possible strategies. However, Bayesian calculation procedures of evaluating actions in terms of revised or posterior probabilities ensure quick selection of the optimal strategy.

The essence of the Bayesian computation procedure is to evaluate the utility of each action under the posterior probabilities associated with each forecast, as in equation (41). The action which gives the highest utility for a particular forecast, as in equation (42), is then the strategy component for that forecast. Thus derivation of the optimal strategy corresponds to repeated application of equation (42) relative to each possible prediction. If we denote by G_{ij}^{**} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$) the optimal action for the jth forecast z_j, the optimal strategy may be specified by the n-element vector

$$\{G_{ij}^{**}\} = (G_{i1}^{**}, G_{i2}^{**}, \ldots, G_{ij}^{**}, \ldots, G_{in}^{**}).$$

The probability that each of these actions will be followed is the probability that its associated forecast z_j will occur, i.e. $P(z_j)$ where

(45) $$P(z_j) = \sum_{gi} f(g_i)P(z_j|g_i).$$

The utility $U(\{G_{ij}^{**}\})$ of the optimal strategy can therefore be calculated as the weighted average

(46) $$U(\{G_{ij}^{**}\}) = \sum_j U(G_{ij}^{**})P(z_j).$$

Without considering the possibility of forecast information, the optimal action based on the prior probabilities $f(g_j)$ is that specified by equation (40). The expected value in utility terms of the forecast information (which will be some as yet unknown prediction z_j ($j = 1, 2, \ldots, n$) from the set of possible predictions ($\{z_j\}$) is therefore the difference between the utility expected with the forecast (given by equation (46)) and the utility to be expected if the prior optimal act is used, i.e. G_i^*, but the posterior probabilities prevail. Since there may be n forecasts, there will be n sets of posterior probabilities to consider relative to evaluation of the prior optimal act. As for the optimal strategy, the utility of the prior optimal act must be evaluated as a weighted average relative to the probability of each forecast $P(z_j)$. Thus the utility value of the forecast predictions is given by

(47) $$\begin{aligned} U(\{z_j\}) &= \sum_j U(G_{ij}^{**})P(z_j) - \sum_j [\sum_{gi^*} U(g_i^*)f(g_i^*|z_j)]P(z_j) \\ &= \sum_j [\sum_{gi^{**}} U(g_i^{**} - c)f(g_i^{**}|z_j) - \sum_{gi^*} U(g_i^*)f(g_i^*|z_j)]P(z_j). \end{aligned}$$

In this expression the term in square brackets is the utility value of a particular forecast as derived in equation (43). Thus the preposterior utility value of the possible prediction set $\{z_j\}$ is equal to the weighted average of the value of the individual predictions, the weight for each prediction being its probability of occurrence. Hence we have

(48) $$U(\{z_j\}) = \sum_j U(z_j)P(z_j).$$

If this value is negative, the forecast information is not worth purchasing; and the maximum price that may be economically paid for the forecast is given by the value of c in equation (47) for which $U(\{z_j\}) = 0$.

Again if, as is often the case, $f(g_i^{**}) = f(g_i^*)$, equation (44) may be used to rearrange equation (48) as follows (making use of the fact from Bayes' Theorem that $f(g_i|z_j) = f(g_i)P(z_j|g_i)/P(z_j)$ and $\sum_j P(z_j|g_i) = 1$):

(49) $$\begin{aligned} U(\{z_j\}) &= \sum_j U(z_j)P(z_j) \\ &= \sum_{g_i} [U(g_i^{**} - c) - U(g_i^*)]f(g_i) \\ &= U(\{G_{ij}^{**}\}) - U(G_i^*). \end{aligned}$$

Equation (49) shows that the utility value of the set of possible predictions $\{z_j\}$ is equal to the difference in utility between the optimal strategy and the prior optimal act.

The utility value of a perfect prediction z' available at a cost c may be calculated as follows. Since a perfect predictor is one that is never wrong, it implies a posterior probability of unity for some particular consequence and of zero for all others. Conversely, by rearrangement of Bayes' formula of equation (6), it also implies $f(g_i) = P(z'_j)$. Further, with a perfect forecast, the optimal action may always be chosen. This action will be the one yielding the highest utility under the state of nature that is known to be the one which will occur, i.e. it is the action G'_{ij} such that

(50) $$U(G'_{ij}) = \max_i U(g_i - c|z'_j).$$

Since each perfect forecast will occur with probability $P(z'_j) = f(g_i)$, the expected utility value of a strategy based upon a perfect predicting mechanism is

(51) $$\begin{aligned} U(\{G'_{ij}\}) &= \sum_{g_i} U(G'_{ij})f(g_i) \\ &= \sum_{g_i} [\max_i U(g_i - c|z'_j)]f(g_i). \end{aligned}$$

The utility value of the perfect predictor is given by the difference between the utility expected with the perfect forecast and the utility expected if the prior optimal act G_i^* is used. Thus

(52) $$U(\{z'_j\}) = U(\{G'_{ij}\}) - U(G_i^*)$$

where the right-side parts of the equation are as given respectively by equations (51) and (40). The maximum price that should be paid for the perfect information is given by the value of c in equation (51) which makes $U(\{z'_j\})$ of equation (52) equal to zero.

Finally, having calculated the utility values of both the imperfect forecasts $\{z_j\}$ and of the perfect forecasts $\{z'_j\}$, the percentage efficiency of the predictor can be evaluated as $100\ U(\{z_j\})/U(\{z'_j\})$. Being a ratio, this measure of predictor efficiency is independent of the arbitrary scale used to measure $U(\{z_j\})$ and and $U(\{z'_j\})$.

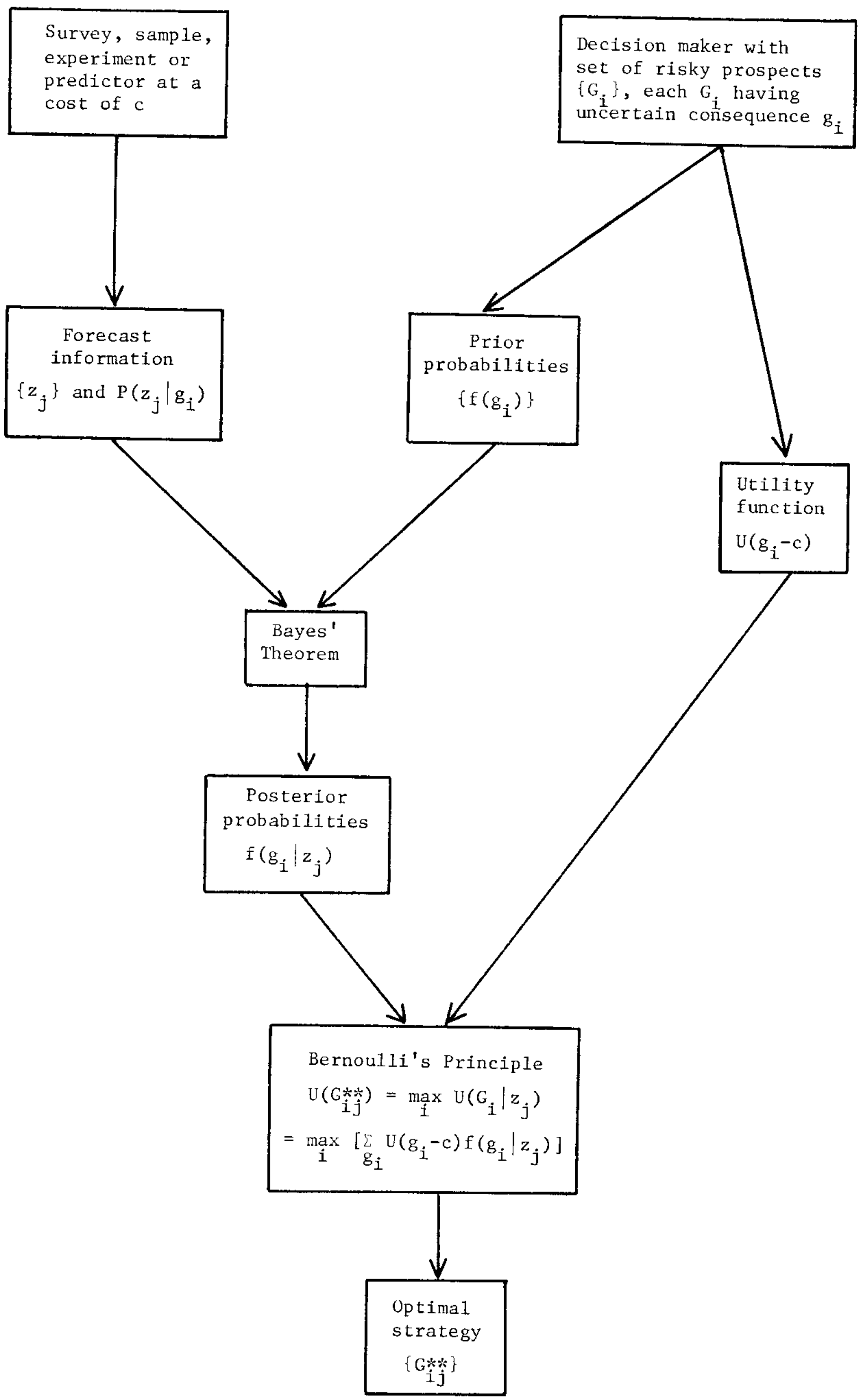


FIGURE 3: *Block Diagram of Bernoulli Decision Theory Procedure*

As a practical matter, in applying the above procedure there may sometimes be a lack of confidence in the likelihoods and prior probabilities used. If so, these may be varied in some systematic fashion to provide an indication of the sensitivity of the optimal strategy to variation in the data [42, 123, 271, 292].

Figure 3 gives a diagrammatic representation of the Bernoulli decision theory procedure for arriving at an optimal strategy. The diagram is summarized by equation (42).

9.2 EXAMPLE

We will again consider the cattle purchase problem of table 1 with a choice to be made by the decision maker whose utility function is shown in figure 1. Suppose that a weather forecast of the coming season is available at a cost of \$200. The forecast of a good season is denoted z_1; of a fair season, z_2; and of a poor season, z_3. The likelihoods $P(z_j|g_i)$ of these various forecasts relative to the possible values of g_i have been estimated by our decision maker on the basis of past experience and subjective judgement. They are shown in the top-left side of table 2 above the utility payoffs $U(g_i - c)$ where G_1 denotes Buy 1,000; G_2 denotes Buy 1,200; and G_3 denotes Buy 1,600. Calculation of the probability of prediction $P(z_j)$ and of the posterior probabilities is shown on the right side of the table above the derivation of the optimal strategy $\{G_{ij}^{**}\} = (G_{31}, G_{12}, G_{13})$ and the expected utility of this strategy.

The utility value of the possible predictions at a cost of \$200 may be calculated as in equation (49). We have

$$\begin{aligned} U(\{z_j\}) &= U(\{G_{ij}^{**}\}) - U(G_i^*) \\ &= 28{\cdot}108 - 23{\cdot}476 \\ &= 4{\cdot}632. \end{aligned}$$

Since this utility value is positive, the forecast is worth having at its price of \$200.

The utility increment that would be gained if a perfect forecast were available at a cost of \$200 can be calculated via equation (52). We have

$$\begin{aligned} U(\{z'_i\}) &= U(\{G'_{ij}\}) - U(G_i^*) \\ &= U(\{G'_{31}, G'_{32}, G'_{13}\}) - U(G_i^*) \\ &= [(57{\cdot}866)\,(0{\cdot}4) + (29{\cdot}894)\,(0{\cdot}2) + (11{\cdot}554)\,(0{\cdot}4)] - 23{\cdot}476 \\ &= 10{\cdot}271. \end{aligned}$$

The efficiency of the predictor generating $\{z_j\}$ relative to a perfect predictor, both being assumed to cost \$200, is thus $(4{\cdot}632/10{\cdot}271)\ 100 = 45{\cdot}1$ per cent. As already calculated in Section 5.1, the maximum price our decision maker could economically pay for a perfect predictor is \$6,020.

Empirical applications analogous to the above example, but assuming a linear utility function, have been presented by Bullock and Logan [49] for cattle feedlot marketing decisions, Byerlee and Anderson [51] for crop fertilizer use, Carlson [54] for crop pesticide use, Dean [73, 74] for ranch stocking rate decisions, Eidman, Dean and Carter [101, 102] for

TABLE 2

Calculation of Optimal Strategy for Cattle Purchase Decision Problem

Data

State of nature	Likelihood $P(z_j \mid g_i)$ z_1	z_2	z_3	Prior probability $f(g_i)$
Good . .	·7	·2	·1	·4
Fair . .	·4	·4	·2	·2
Poor . .	·1	·5	·4	·4

State of nature	Utility payoffs $U(g_i - c)$ G_1	G_2	G_3
Good . .	36·670	44·690	57·866
Fair . .	19·130	22·798	29·894
Poor . .	11·554	−0·599	−33·575
$U(G_i \mid z_j) = \sum_{g_i} U(g_i - c) f(g_i \mid z_j)$ →			
$U(G_{ij}^{**}) = \max_i U(G_{ij} \mid z_j)$ →			
Optimal Strategy =			
Expected Utility of optimal strategy =			

Calculation of Posterior Probabilities and Strategy Values

Forecast z_j =	z_1 Good	z_2 Fair	z_3 Poor
(Likelihood × Prior) =	·28 ·08 ·04	·08 ·08 ·20	·04 ·08 ·16
$P(z_j)$ = sum =	·40	·36	·24
Posterior probabilities	$f(g_i \mid z_1)$	$f(g_i \mid z_2)$	$f(g_j \mid z_3)$
Good ($i = 1$)	28/40 = ·70	8/36 = ·22	4/24 = ·17
Fair ($i = 2$)	8/40 = ·20	8/36 = ·22	4/24 = ·17
Poor ($i = 3$)	4/40 = ·10	20/36 = ·56	16/24 = ·66
$U(G_1 \mid z_j)$ =	30·65	18·75	17·11
$U(G_2 \mid z_j)$ =	35·78	14·51	11·08
$U(G_3 \mid z_j)$ =	43·13	0·50	−7·24
$U(G_{ij}^{**})$ =	43·13	18·75	17·11
G_{ij}^{**} =	G_{31}	G_{12}	G_{13}
$\sum_j U(G_{ij}^{**}) P(z_j)$ =	(43·13) (·4) + (18·75) (·36) + (17·11) (·24) = 28·108		

poultry production decisions, and Halter and Dean [146] for crop enterprise combinations. Of these studies, only that of Byerlee and Anderson [51] has a policy orientation, this being framed in terms of the question whether or not public support should be given to the further development and promulgation of a rainfall predictor. All of these studies are illustrative of the art of incorporating forecast information of either a direct or indirect nature into the analysis. Carlson [54], for example, generates a forecast of crop disease loss based (via regression) on observable measures of fruit maturity, rainfall and disease spore density.

9.3 CERTAINTY EQUIVALENT APPROACH

Use of the general Bernoulli model elaborated above is only likely to be justified for important complicated problems—complicated in the sense that there are many alternatives to be considered, and important in the sense of having significant differences in their consequences. Such problems could typically be classified as those justifying the use of a consultant analyst, if need be with computer backup. Often, however, even for important problems a far more direct and simpler approach is feasible. This is the certainty equivalent or decision tree approach [107, 162, 226, 229, 245, 287, 312, 325]. Whereas the general model of Section 9.1 implies separate assessment by the decision maker of his degrees of preference and of belief, the certainty equivalent approach relies on taking intuitive account simultaneously of both belief and preference.

As a simple example we will again use the cattle purchase decision problem of table 1. Figure 4 shows the information of table 1 in the form of a decision tree of action and event possibilities. For example, if Buy 1,000 is the action chosen and the season turns out to be fair, the net payoff will be $10,000; there is, however, only a chance of 0·2 that the season will be fair. Other possibilities are similarly represented. Any payoff matrix or decision problem can be depicted in such extensive fashion with a sequence of act/event forks and branches. Fork A is the current decision fork; forks B, C, and D are event forks. In a more complicated problem we would have a series of act-event sequences rather than a single act-event sequence as in figure 4. Note that "actions" are at the discretion of the decision maker while "events" are not.

Analysis proceeds by a process of backward induction, working from right to left, each event fork being replaced by its certainty equivalent, i.e. by the sure sum which the decision maker assesses as equivalent to the risky prospect represented by the event fork. For example, our decision maker's certainty equivalent for the risky prospect of a 0·4 chance of $20,000, a 0·2 chance of $10,000, and a 0·4 chance of $6,000 at event fork B of figure 4 might be $12,200. This sum, nominated by the decision maker on the basis of introspective judgement, encompasses both his degrees of preference and degrees of belief about the risky prospect.

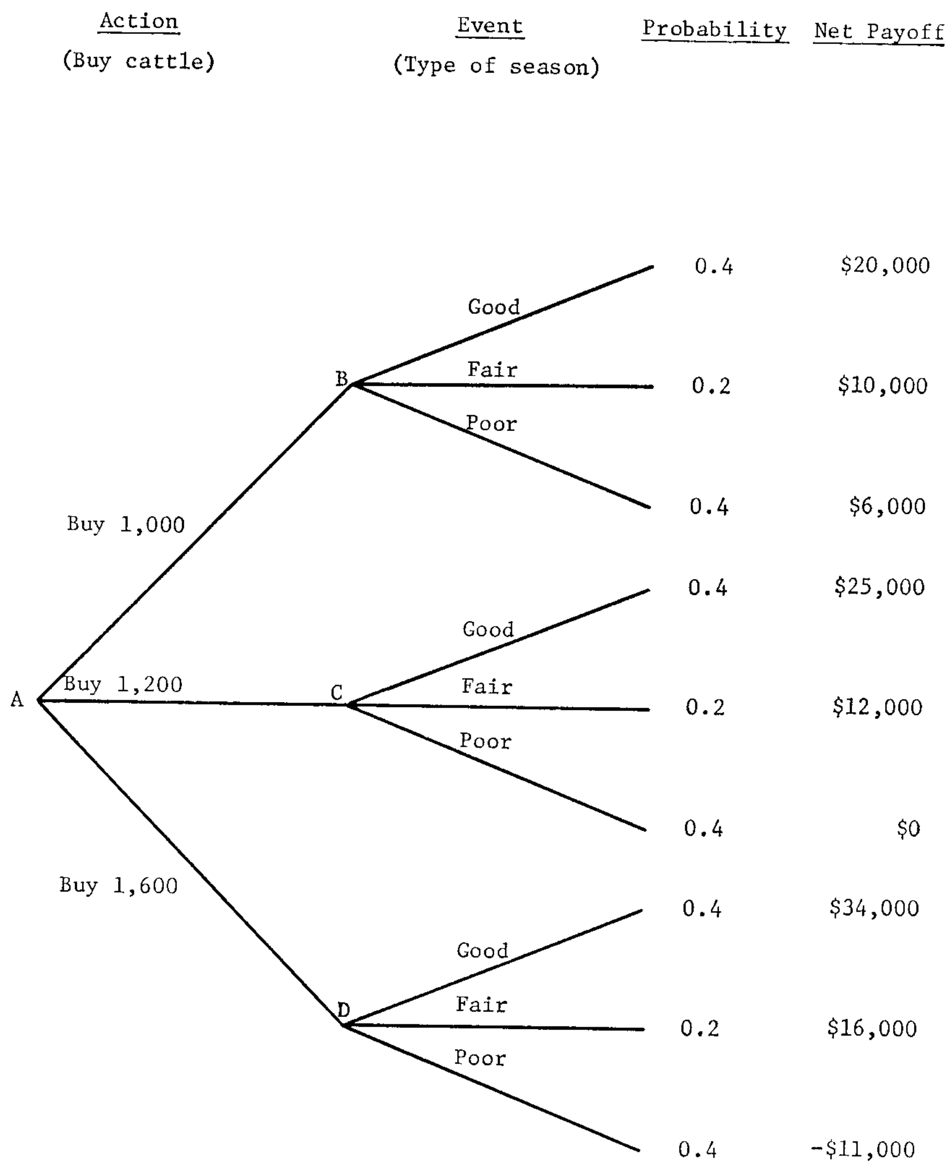
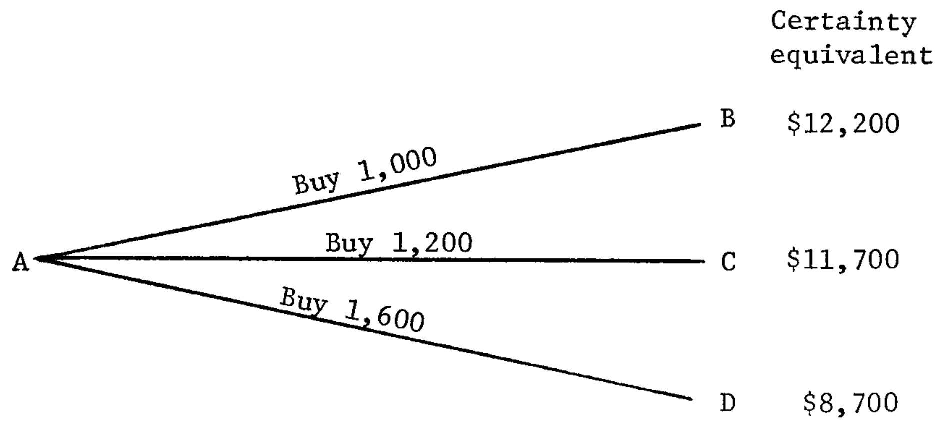


FIGURE 4: *Decision Tree for Cattle Purchase Decision Problem*

Replacement of the risky prospect at each event fork by its certainty equivalent gives a simpler but equivalent decision problem of choice between certainty equivalents. Obviously the highest certainty equivalent would be the preferred choice. For example, if the certainty equivalents of event forks B, C, and D are as shown in the sketch below, the preferred choice would be Buy 1,000.

The decision of whether or not to seek extra information in the form of a prediction from a possible set of predictions $\{z_j\}$ can also be assessed via the certainty equivalent approach. For example, suppose the possible forecast set consists of z_1, z_2, z_3 as specified in table 2. To encompass this further action alternative of purchasing forecast information, the decision tree of figure 4 has to be extended as shown in figure 5. The forecast is assumed to cost $200.



In figure 5, event forks such as J, K, and M must first be replaced by their certainty equivalents. Suppose for J, K, and M these are respectively $16,000, $19,000, and $23,000. This implies that if a forecast is purchased and it turns out to be z_1, then Buy 1,600 with a certainty equivalent of $23,000 is the preferred action, i.e. it is the z_1 component of the optimal strategy. The decision tree to the right of event fork F may thus be replaced by the certainty equivalent value of $23,000. Likewise, suppose the analogous certainty equivalents to be placed at G and H are respectively $9,000 (for Buy 1,000 if z_2) and $8,000 (for Buy 1,000 if z_3). Given the analysis already made for figure 4, figure 5 may now be replaced by the equivalent but simpler decision problem of figure 6. All that remains is to replace event fork E by its certainty equivalent. If this is greater than $12,200 (the certainty equivalent of the optimal act without extra information), the forecast should be purchased and the optimal strategy would be to Buy 1,600 if z_1 occurs, and to Buy 1,000 if z_2 or z_3 is the forecast received.

Though simpler because of its simultaneous consideration of degrees of belief and preference, the certainty equivalent approach is formally equivalent to that of the general Bernoulli model of Section 9.1. A disadvantage of the approach is that it is not conveniently applicable if it is desired to delegate choice. The decision maker has to specify the required certainty equivalents himself. With the utility approach of the general model, however, decision making may be more easily delegated to a subordinate who, making use of his superior's utility function and probabilities, would make exactly those decisions his superior would have made. At the same time, there are advantages *per se* in drawing a decision tree—the disciplined thinking such an exercise demands will generally lead the decision maker or his consultant analyst to a better understanding of the decision problem. An obvious difficulty, however, is to prevent the decision tree from becoming a bushy mess, as Raiffa

[287, p. 240] nicely puts the problem. As acts and events multiply, the tree explodes exponentially—and particularly so if multi-period possibilities are included. Art as well as Science seems necessary to keep the picture comprehensible. A workable procedure appears to be to begin with a rather coarse tree specifying only the major branches, check which of these might be lopped, develop the unlopped branches in further detail, and repeat the cycle. In other words a continuing cyclical process of specification, pruning and extension might be followed to keep the tree in workable shape.

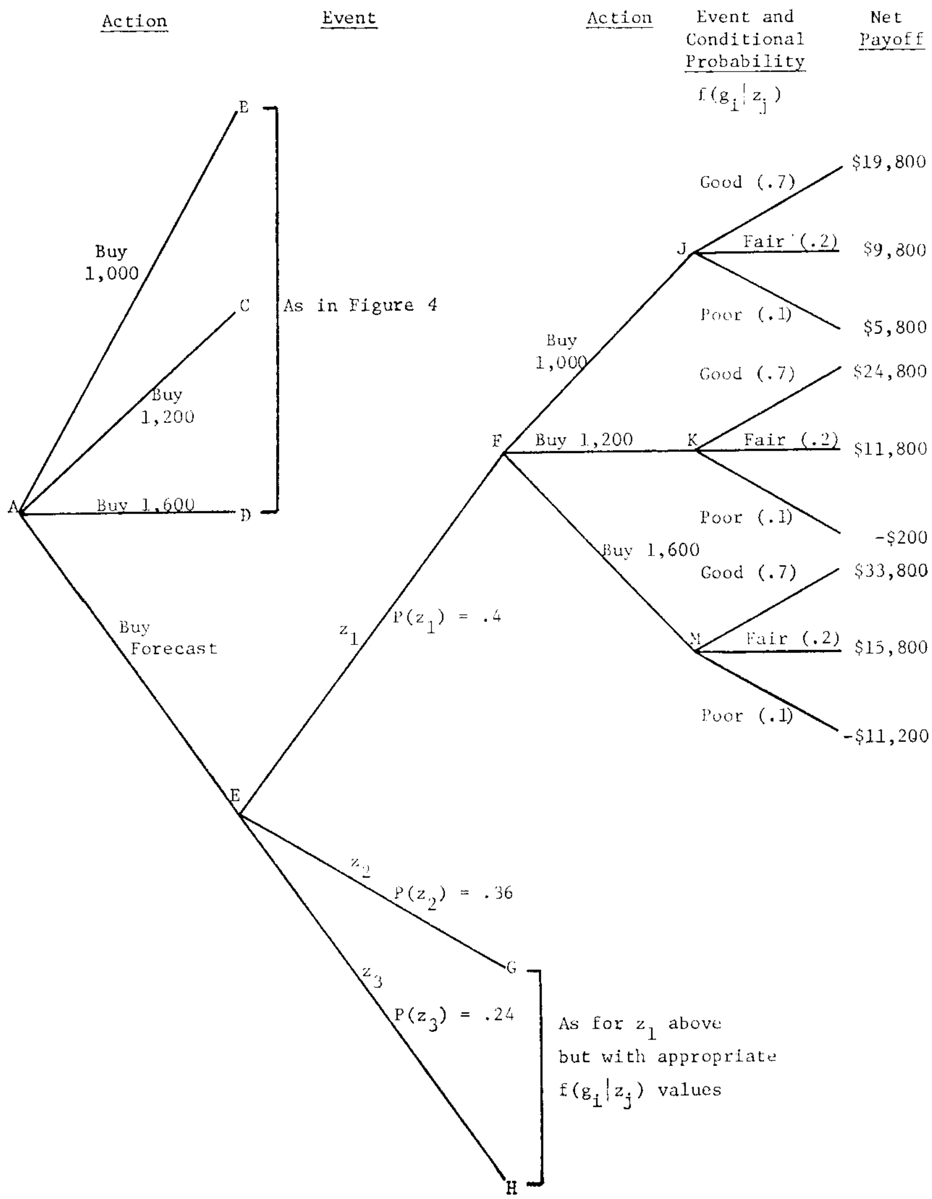


FIGURE 5: *Extension of Figure* 4 *to Encompass Possible Purchase of Additional Information*

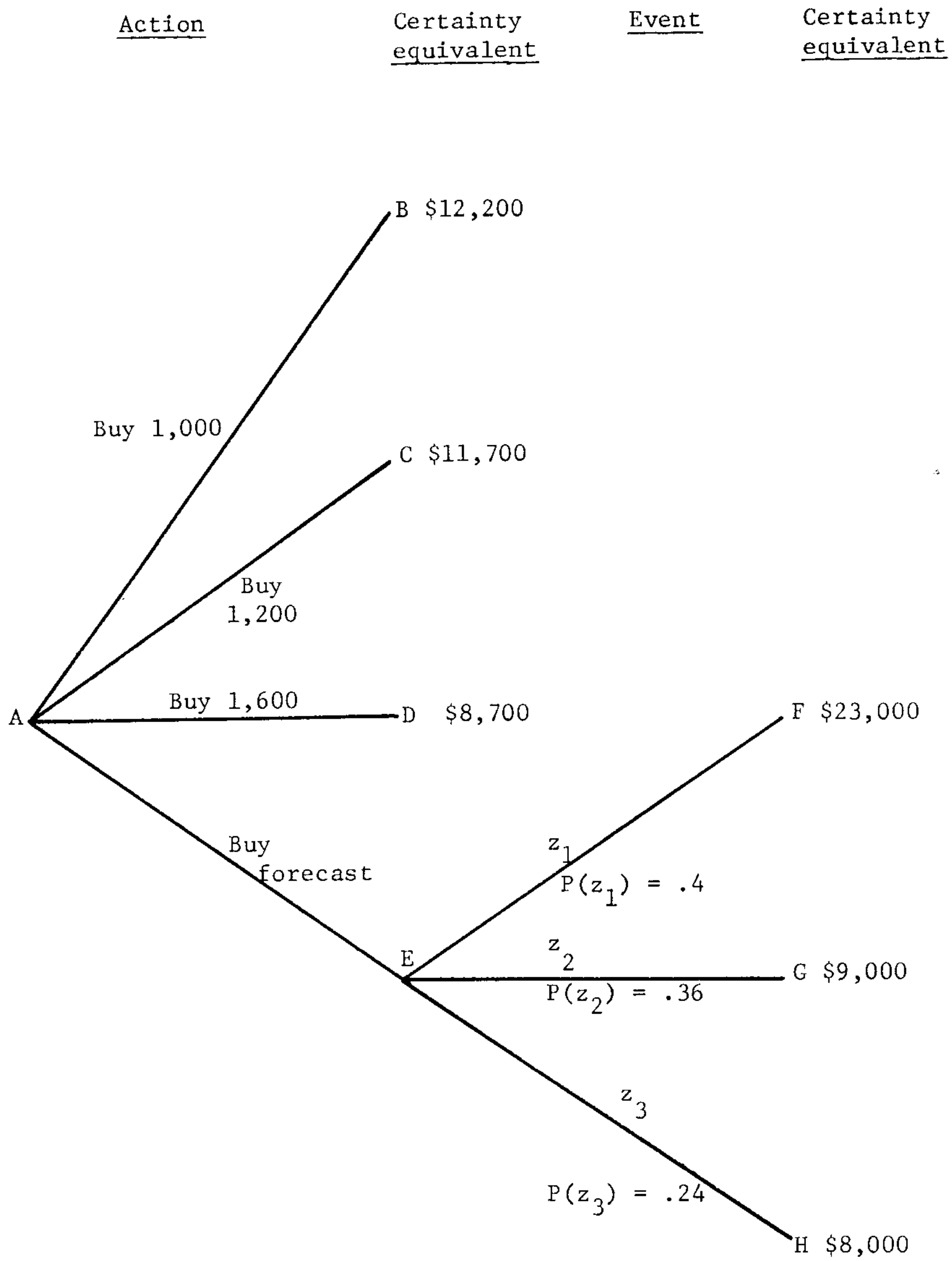


FIGURE 6: *Simpler Decision Problem Equivalent to that of Figure 5*

Fuller treatment of the certainty equivalent approach, including the assessment of sequential sampling for further information, is to be found in Raiffa [287] and Schlaifer [312]. Capital budgeting decisions are considered by Hespos and Strassmann [162]. Dillon and Trebeck [89], Hardaker [153] and Makeham, Halter and Dillon [227] have illustrated some agricultural applications. Rae [285] has used decision trees as a basis for specifying information structures[14] involving the possibility of additional information. His specific concern is the use of stochastic programming for utility appraisal of sequential decision problems in farm management.

9.4 CHOICE OF A PORTFOLIO

So far we have only considered choice situations involving "all of this or all of that" alternatives. It was Buy 1,000 or Buy 1,200 or Buy 1,600 head of cattle with no possibility of making a mixed choice of, say, buying 1,400 head by spending half our capital on Buy 1,200 (giving 600 head) and half on Buy 1,600 (giving 800 head). Situations where such mixtures of risky prospects are feasible constitute the field of portfolio analysis, the aim being to find the portfolio which maximizes the decision maker's utility. The most obvious and most discussed application is to the choice of a stock market portfolio (and the related question of equilibrium in the market) [23, 32, 37, 38, 178, 212, 229, 249, 302, 320, 321, 339, 340]. The principles involved, however, are just as relevant to the selection of an enterprise mix for a farm, a product mix for a region, or a policy mix for a government—though in the latter case there would be data and criterion problems.

If we assume a decision maker with a quadratic utility function or that the returns from the portfolio follow a normal distribution (both of which are restrictive assumptions), the problem of portfolio choice from a set of n risky prospects may be specified as follows. Let

E_i = expected net return per unit of investment in prospect i, $i = 1, 2, \ldots, n$;

V_i = variance of net return from a unit of investment in prospect i;

C_{ij} = the covariance of the per unit net returns from prospects i and j;

Z = total units of investment allowed;

t_i = units of investment in prospect i.

If borrowing and lending are excluded, we must have $t_i \geqslant 0$ and $\Sigma t_i \leqslant Z$, and any specified mixture of the risky prospects will have an expected net return of

$$E = \Sigma t_i E_i; \tag{53}$$

and a variance of net return of

$$V = \Sigma t_i^2 V_i + \sum_{i \neq j}\Sigma\, t_i t_j C_{ij}, \qquad i, j = 1, 2, \ldots n. \tag{54}$$

Substitution of these expressions for E and V into the decision maker's quadratic utility function will give utility as a quadratic function in

[14] By information structure is meant the pattern of information receival in relation to decision dates. Rae [285] distinguishes structures involving: incomplete knowledge of the past; complete knowledge of the past and incomplete knowledge of the present; complete knowledge of both the past and present; and any of the former plus forecast knowledge of the future.

$t_1, t_2, \ldots, t_n$, which, given the constraints noted above, may be solved by quadratic programming [4, 229]. Only if the decision maker is risk (variance) averse will the optimal portfolio involve a mixture of risky prospects. A risk preferrer will always have a portfolio containing only a single risky prospect since, if the prospects are not perfectly correlated, diversification always reduces variability of total returns [340].

Halter and Dean [146] and Rae [286] have illustrated the above procedures relative to the selection of a crop enterprise mix. Rae [285] has also outlined stochastic programming approaches for sequential farm resource allocation problems with a variety of non-linear and linear-segmented utility functions, both uni- and multi-dimensional (see Section 10), under a range of information structures. His analyses also incorporate the purchase and appraisal of forecast information. There have also been a number of other agricultural applications of portfolio analysis basically in the Markowitz [233] style of constructing an (E,V) efficiency locus either by quadratic programming or simulation, choice from the efficient set being left to the decision maker on an inspection basis, e.g. [53, 65, 87, 160, 184, 185, 221, 243, 284, 332, 349, 377].

Because of its consideration of only the mean and variance of the distribution of returns, there has recently been strong theoretical criticism of portfolio analysis [3, 39, 109, 149, 150, 171, 172, 176, 341]. The analysis could be extended to higher moments if required, though there would be a much enlarged job of computation to be done. However, as Samuelson [301] has commented, "in practice, where crude approximations may be better than none, the 2-moment models may be found to have pragmatic usefulness".

9.5 PRODUCTION ECONOMICS AND UTILITY

For decision makers who accept Bernoulli's Principle and have non-linear utility functions, the traditional profit maximizing models of production economics are inappropriate.[15] They need to be rephrased in terms of utility maximization which, *ipso facto*, means incorporating uncertainty as an integral part of production. A start has been made in this direction in the context of activity analysis or programming models by Rae [285, 286], and in the context of continuous production function models by

[15] We assume here that utility has only a single dimension—that of profit. If there is no uncertainty, such utility maximization would be incompatible with long-run survival of the firm under perfect competition—at least theoretically [291]. With uncertainty present, this is no longer true so long as we are prepared to assume computer-like entrepreneurs [37, 38, 283]. At the more real-world level with which we are concerned, there seems no difficulty in assuming no conflict between utility maximization and long-term survival of the firm. There are enough imperfections and dynamic factors in the real-world to counteract all the lemmas of the theory of perfect competition. Somewhat relatedly, it has been shown [207] that long-term survival of the competitive firm is compatible with utility maximization when utility has the two substitutable dimensions of profit and leisure. Using cross-section data and a decision theory approach, Dillon and Anderson [84] have attempted to test (with only limited success) the extent of profit maximizing behaviour by producers in traditional agricultures.

Anderson [9], Byerlee and Anderson [51], Sadan [300] and Turnovsky [343]. For illustration, we will follow Anderson [9] who assumes the single product production function

$$Y = f(Q, C, S) \tag{55}$$

where Y is output, Q a vector of controlled variable inputs, C a vector of uncontrolled stochastic inputs such as climatic factors, and S is a parameter representing size of firm and fixed costs. Expected output is

$$E(Y) = f(Q, E(C), S) \tag{56}$$

with variance

$$V(Y) = g(Q, V(C), S). \tag{57}$$

The profit equation, assuming the stochastic factors are costless, is

$$\pi = p_y Y - pQ - c(S) \tag{58}$$

where p_y denotes product price, p variable input price and $c(S)$ fixed costs. Assuming, as is reasonable for most farm firms, that p_y and Y are stochastically independent, expected profit is

$$E(\pi) = E(p_y)E(Y) - pQ - c(S) \tag{59}$$

with variance

$$V(\pi) = [E(p_y)]^2 V(Y) + [E(Y)]^2 V(p_y) + V(p_y)V(Y). \tag{60}$$

Assuming only the first two moments of the profit distribution are relevant, the functions for $E(\pi)$ and $V(\pi)$ may then be used as per equations (19) or (21) to determine the utility maximizing levels of Q and S. Such an approach may be extended to allow for higher moments of profit if they are relevant to the decision maker.

Byerlee and Anderson [51] have also shown that additional forecast information on the uncontrolled factors will only have economic value if there is interaction between the controlled factors (Q and S) and the uncontrolled factors (C). If there is no interaction, marginal products of the controlled factors will be independent of the uncertain factors and predictions will have no influence on resource use.

Casting production economics theory into a utility maximizing framework has a number of interesting implications relative to profit maximizing analysis. For example, fixed costs must be taken into account and returns must be assessed on a total rather than a technical unit basis since, for a non-linear utility function, $U(k[X - c]) \neq kU(X - C) \neq kU(X) - kU(c)$. Too, if they have different utility functions, producers with identical resource bundles and environments will have different optimal resource allocations. Optimal scale of operation will also be influenced—no longer will increasing returns to scale necessarily imply outward movement on the expansion path. Conversely, decreasing returns to scale might be more than balanced by a producer's increasing marginal utility. Such implications of utility maximization for resource allocation also have policy implications. As McArthur and Dillon [216, 217] have suggested, if producers are generally risk averse, their input use and output levels will be low relative to the allocation implied by a criterion of maximizing expected profit. However, since the State can take a long-term view, it may not be worried about risk and may prefer producers to follow an expected profit maximizing approach [19]. If so, appropriate price policies in the form of input subsidies or output bounties (or perhaps tax policies [110]) would be needed to encourage producers to allocate their resources in the fashion deemed optimal by society.

10 MULTIDIMENSIONAL UTILITY

So far, in discussing Bernoulli's Principle, we have assumed gain or loss to be measured in a single dimension, for example in terms of net profit. Often, however, gain or loss will be measured in several dimensions [181], for example, net profit, hours of work and closeness to home. Or it may be that choice has to be made between acts with quite distinct consequences, for example: brunettes, potatoes, violins, and votes. In such cases multidimensional utility functions are relevant [112, 113, 120]. Two broad classes of multidimensional utility may be distinguished: first, the case where the various dimensions of utility may be amalgamated in some way to give a single overall utility index; and, second, the case where amalgamation is not possible and only a lexicographic ordering of the utility dimensions is possible [105].

10.1 AMALGABLE UTILITIES

If utilities in different dimensions can be amalgamated, then for a prospect K giving utility $U_i(K_i)$ in the ith dimension of n relevant dimensions, we must have

$$U(K) = f[U_1(K_1), U_2(K_2), \ldots, U_n(K_n)] \tag{61}$$

where U_i is the utility function pertinent to the ith preference factor. The restriction on the function of equation (61) must be that it leads to consistent decisions relative to some acceptable set of axioms or postulates. The two simplest forms of function are multiplicative, that is

$$U(K) = \Pi U_i(K_i), \tag{62}$$

and additive (or separable as it is sometimes called), that is

$$U(K) = \Sigma a_i U_i(K_i), \tag{63}$$

where a_i/a_j is the rate of substitution or trade off between $U_i(K_i)$ and $U_j(K_j)$. For example, the utility of three dances K_1, K_2, K_3 with three separate partners may be additive; but with a single partner, utility might be multiplicative.

Doubtless, somewhere someone has considered multiplicative utilities in an axiomatic sense (perhaps in consumer demand theory?) but they do not seem to have been considered in decision theory. In contrast, separable and quasi-separable utilities have been investigated in some depth. Fishburn [114, 117, 118, 120], among others, has shown that if the Bernoulli axioms are extended to include a further axiom of independence between components (i.e. K_i and K_j do not interact), then additive or separable utilities are permissible. Keeney [189] has shown that a weaker extension of the Bernoulli axioms implies a quasi-separable function of the form

$$U(K) = \Sigma U_i(K_i) + \sum_i \sum_j b_{ij} U_i(K_i) U_j(K_j). \tag{64}$$

The utility function $U_i(K_i)$ for the ith factor in additive utility may be estimated by the usual procedures. It is then necessary to establish the trade-off coefficinets between factors. A variety of procedures for determining these weights have been developed [93, 116, 189, 324]. A popular do-it-yourself version concerned with home location in relation to traffic noise, density, fumes and parking convenience was recently presented in *The Economist* [11].

Rae [285] has demonstrated how an additive utility function may be used as the objective function in stochastic programming analyses of sequential resource allocation problems. Shechter and Heady [322] have implicitly used an additive utility function in a simulation study of alternative policy choices relative to the U.S. Feed Grain Program. The dimensions of policy makers' utility were taken as treasury costs, net farm income, income of participating farmers and feed grain stock accumulation. Though it does not include the appraisal of potential forecast information, the analysis is suggestive of the application of Bernoullian decision theory to policy choice. As yet, there seem to be no analyses explicitly involving additive utilities in an agricultural context. Fromm and Taubman [127] have used both multiplicative and additive utilities in a simulation appraisal of macro policy alternatives.

10.2 LEXICOGRAPHIC UTILITIES

Lexicographic utilities prevail where substitution of achievement between goals or trade-off between dimensions of preference is not allowed by the decision maker. In terms of the Bernoulli axioms, this implies weakening or omission of the Continuity (or Archimedian) Postulate. Instead of being measurable as a single real number, multidimensional utilities must then be expressed as a lexicographically-ordered vector [61, 105, 106, 146, 156, 338].

For example, suppose utility in three dimensions K_1, K_2, K_3 is pertinent to choice between two risky prospects G_1 and G_2. If trade-off between K_1, K_2, and K_3 is not allowed, $U(G_i)$ must be lexicographic. If K_1 is more important than K_2, which in turn is more important than K_3, $U(G_i)$ may be expressed as the ordered vector:

$$U(G_i) = [U_1(K_{1i}), U_2(K_{2i}), U_3(K_{3i})], \qquad i = 1, 2. \tag{65}$$

If G_1 is preferred to G_2 then either:

$U_1(K_{11}) > U_1(K_{12})$; or $U_1(K_{11}) = U_1(K_{12})$ and $U_2(K_{21}) > U_2(K_{22})$; or $U_1(K_{11}) = U_1(K_{12})$ and $U_2(K_{21}) = U_2(K_{22})$ and $U_3(K_{11}) > U_3(K_{12})$.

In other words, lexicographic utility implies dominant priorities are attached to factors in some specified order. They are thus closely related to and may be used to represent satisficing behaviour [224, 285].

The incorporation of lexicographic utility into stochastic programming or sequential farm management problems has been demonstrated by Rae [285]. Applications of lexicographic utility functions have also been made for a number of real-world agricultural problems. Halter and Dean [146], Eidman *et al.* [102] and Carlson [54] have respectively evaluated problems of crop enterprise selection, poultry production and crop pesticide use using a lexicographic function of two dimensions: K_1, a "survival" goal specifying that actual payoff must exceed some critical level with some specified probability; and K_2, a profit mazimizing goal. Such an approach is closely related to the concept of ruin in actuarial theory [25, 36, 37, 151, 299] and to the objective function concept in chance-constrained programming [59, 254]. The assumption of a focus-loss orientation by Boussard and Petit [40, 41] is also somewhat related. Wharton [357], in a paper reviewing much of the literature on farmer decision making, has argued a lexicographic criterion of survival as a descriptive theory of subsistence farmers' reaction to new technology.

11 SOME DIFFICULTIES

Four areas of difficulty must be noted relative to operational use of Bernoullian decision theory as a normative choice procedure. These relate to acceptability of the underlying postulates; specification of the utility function and of subjective probabilities; group decisions; and time considerations. None of these difficulties are seen as insuperable in general, although they may imply that the Bernoulli approach is infeasible or inappropriate for some decision makers or decision problems.

11.1 ACCEPTABILITY OF POSTULATES

There is an extensive literature, both theoretical and empirical, on the reasonableness of the Bernoulli axioms. An overview of this material is given in some of the survey articles on Bernoullian utility theory [1, 28, 95, 96, 111, 180].

In principle, most decision makers agree that the Ordering axiom is reasonable. In practice, however, intransitivities (e.g. A preferred to B, B preferred to C, and C preferred to A) are sometimes exhibited. If a decision maker accepts the Ordering axiom, any such intransitivities imply inconsistency and probably arise from the difficulty of comparing complex alternatives [21] or from imperfect discrimination between alternatives.[16] In such cases, utility analysis provides a procedure for achieving consistency. The Continuity or Archimedean assumption is the one most

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[16] Some people, it has been suggested, may exhibit probabilistic patterns of choice leading to apparent intransitivities [27, 68, 213, 264].

likely to be unacceptable to some decision makers as it implies that trade-off between goals is always possible. However, as noted in Section 10.2 above, if lexicographic utility is pertinent, the axiom may be reasonably weakened in a satisfactory way [118, 120]. From an operational point of view, there appears to be no substantial difficulties with the Independence axiom.

Looking at the postulates as a whole (and accepting their necessary amendment for multidimensional utility), it would be inconsistent for a decision maker who accepts them not to accept Bernoulli's Principle as a logical method of assessing choices that were too complex for intuitive appraisal. Should he not accept the postulates, it would be illogical for him to use or accept Bernoullian decision theory.

So far, we have emphasized Bernoulli's Principle as a normative or prescriptive approach to risky choice. An unavoidable hypothesis is that most decision makers are in fact utility maximizers so that the theory is descriptive as well as prescriptive [28, 345]. Some evidence in this regard is available [69, 129, 135, 145, 252, 333, 371]—the work of Officer and Halter [259, 260] being particularly relevant to farmers. It has also been found that many decision makers, if shown to be inconsistent with utility analysis, are apt to change so as to be consistent [37, 215, 238, 259]. As Savage [305, p. 103] commented after having been trapped in an inconsistency by Allais [8, 247], he was pleased to be able to find his error by reference to the Bernoulli axioms and thereby correct his mistake—a sentiment of resipiscence with which many agree [104, 288, 293, 301, 328].

11.2 SPECIFYING PREFERENCE AND BELIEF

A number of difficulties may arise in relation to the questioning procedures used to establish a decision maker's utility function and subjective probabilities [131]. Certainly, if the subject does not understand the concept of chance, interrogative procedures cannot be satisfactory. Hopefully, such difficulties can be overcome by explanation. Another difficulty (see Section 7.3) is psychic bias for particular probabilities. Most serious, however, is the need to rely, particularly for utility assessment, on the answers to questions about hypothetical situations. There must always be some doubt as to how well a subject can succeed in answering hypothetical questions as if they were real-life actualities. In particular this implies that the range of gains and losses used in establishing the utility function must be within the range of relevance and experience of the decision maker. Suffice to note that students at the University of New England over the last four years have generally had no difficulty in adequately specifying decision makers' degrees of belief and preference in a very wide variety of real-world decision contexts. Investigation of the one reported instance of meagre success in estimating farmers' utility functions [218] indicates that the cause was probably due to a combination of inadequate training of interviewers and an overly complicated questioning procedure.

A practical problem may be the extent to which decision makers' utility functions change over time. The factors determining the shape of a person's utility function are his experience and background, current wealth, expectations and aspirations. To the extent that these inevitably change over time, utility must also be expected to change. The little work available to date indicates that, except in unusual circumstances, rapid changes are not to be expected and the utility function once specified is likely to be relevant for some months or even longer [259]. At any rate, it is relatively easy to check the function with the decision maker and make revisions if necessary.

11.3 GROUP DECISIONS

Group decision making in both its prescriptive and descriptive aspects is a relatively unresolved topic. The descriptive side is of no great concern here—it probably relates more to psychology, sociology and politics than economics. Normatively, it would seem that axioms of rational behaviour relevant to an individual should be just as relevant to a group which attempts to act as an individual—for example, a company's board or a government's ministry [244]. The basic problems relate to obtaining a group utility function and a probability consensus. For example, should each member of the group form his own posterior probabilities and then seek consensus, or should consensus be sought on the prior probabilities, assuming all agree on the likelihoods of the information available for revising prior probabilities? [365]. Likewise, should the group confine itself to Pareto-optimal choices?—or is the group entity more important than its individual members so that Pareto optimality is irrelevant? And, of course, we all know groups which have foundered because some member had a lexicographic utility function which made compromise impossible. These and other unresolved problems of group decisions are well discussed by Raiffa [287] and Radner [281], and, though not in such a direct Bernoulli Principle context, by Luce and Raiffa [213].

The only consideration of group decisions in an agricultural context relative to utility analysis appears to be that of Officer *et al.* [260]. This work was concerned with the use of group utility functions as a basis for extension advice to individual members of farmer groups. For the particular group of farmers studied, it was shown that a type of group-average utility function performed better than an expected profit maximizing approach. However no generality attaches to these results. In fact, group decision making is not particularly relevant to farmer decisions. Where it is implicitly relevant is in the field of government policy choice. Fromm and Taubman [127] suggest the only feasible approach is for their advisers to present the policy making group with a series of assessments based on a variety of utility functions, leaving the policy group to make its own choice of a utility function. Though it sounds far-fetched, procedures akin to this already operate in The Netherlands and some East European countries.

11.4 TIME EFFECTS

So far no mention has been made of time effects in relation to Bernoulli's Principle.[17] Once the time dimension is introduced, a number of new preference concepts may become relevant. Fishburn [118] categorizes these as impatience (e.g. only dollars received today have any value), eventual impatience (discounting is applied only to payments beyond some point in the future), time perspective (differences between consequences are judged less important the further off they are), patience or zero discounting (e.g. a dollar has the same value regardless of when it is to be received), persistence or stationarity (e.g. if X is preferred to Y in one period, it will be preferred to Y in any other period), and variety or nonpersistence which might be best described as the Harem Syndrome (e.g. if chops are preferred to sausages tonight, tomorrow night we will prefer sausages to chops). These effects have been considered within the framework of additive utilities by a number of workers [79, 80, 106, 115, 120, 136, 141a, 197, 201, 202, 203, 272, 289, 368].

The additive utility approach is to regard each future time period (e.g. year) as a separate dimension in a multidimensional additive utility function. Thus if an act has an uncertain consequence x_t $(t = 1, 2, \ldots, n)$ at the start of period t, following the probability distribution $f_t(x_t)$, the utility of the act might be most simply specified as

$$(66) \qquad U(x_1, x_2, \ldots, x_n) = \sum_t \alpha_t^{t-1} u_t(x_t), \qquad 0 < \alpha < 1,$$

where $u_t(x_t)$ is the presently held utility function for consequences in period t and α_t is a discount factor pertinent to period t, the corresponding discount rate being $(1 - \alpha_t)/\alpha_t$. Note that x_1 would encompass initial outlay and x_n any asset salvage value associated with the risky prospect. Such a formulation implies aggregate utility of a risky prospect with a time-trace of uncertain consequences is the discounted sum of all future period utilities, and that the utilities of different periods are separate and independent. Further, it is implied that the functions $u_t(x_t)$ are stationary or unchanging as time passes so that choices are independent of when they are made; in other words that preferences once specified, do not change. In general, this will not be true and it must be recognized that a present choice may restrict future opportunities. Such questions have been considered by Koopmans [198] in the additive utility context, and by others in a more general context [193, 251].

Note that the additive aggregate utility approach may involve intra-period utility that is itself multidimensional. Fromm and Taubman [126, 127], for example, in a simulation appraisal of various macro government policies, consider such intra-period utility function forms as the linear

$$(67) \qquad u_t = \sum_i \beta_{it} u_i(x_{it}),$$

[17] Some of the material in this section has been drawn from unpublished material developed with Brian Hardaker in 1968.

the Cobb-Douglas

$$u_t = \prod_i [u_i(x_{it})]^{\beta_{it}}, \tag{68}$$

and the constant elasticity of substitution

$$u_t = \{\sum_i \beta_{it}[u_i(x_{it})]^{\delta}\}^{-\delta}, \tag{69}$$

where $u_i(x_{it})$ is the utility function expressing preference in the ith dimension for period t consequences.

By appropriate specification of the period utility function $u_t(x_t)$, account can be taken of the relevant moments of the probability distribution of intra-period returns. For example, if $u_t(x_t)$ were quadratic, account would be taken of the mean and variance of returns in the tth period. The total utility function of equation (66), however, does not allow for the inter-period variation or pattern of consequences. For example, two acts may have uncertain time-traces of consequences which are very different yet have exactly the same utility value by equation (66). One may have $u_t(x_t)$ declining as t increases and the other the reverse; or one may be fairly steady over time while the other fluctuates wildly. Intuition tells us that the time pattern of consequences is relevant to choice and should be accounted for somehow [37, 353]. One way of accounting for such effects would be through a market where risky prospects with time series payoffs could be traded or mortgaged for an annuity, or covered by insurance. Arrow [17, 18, 19] has discussed such possibilities. In general, however, such market trading or insurance arrangements are not available. This is especially so in agricultural production based on family farms. It is not so true with respect to agricultural policy choices where often the State stands ready to provide support if untoward consequences occur as a result of exogenous effects. Governments, for example, generally do not let their buffer-stock or reserve price schemes collapse overnight.

If inter-period variability cannot be neutralized (at a price) by trading or insurance, it might (should?) be accounted for in the total utility assessment of the risky prospect. Variability, for example, might be encompassed by extending equation (66) to include the variance of $u_t(x_t)$ over time. For an act with consequences over n periods, the variance of $u_t(x_t)$ is given by

$$V(u_t) = \sum_t\{u_t(x_t) - [\sum_t u_t(x_t)/n]\}^2/n, \quad t = 1, 2, \ldots n. \tag{70}$$

The total utility of an act with risky consequences extending over n periods might then be assessed as

$$U(x_1, x_2, \ldots x_n) = \sum_t \alpha_t^{t-1}u_t(x_t) + \beta V(u_t) + \gamma[V(u_t)]^2 \tag{71}$$

with usually $\beta < 0$ and $\gamma > 0$ to allow for the fact that inter-period variation probably has a depressing effect on utility ($\beta < 0$) but that as $u_t(x_t)$ becomes larger, worry about inter-period variation may lessen ($\gamma > 0$). A somewhat similar approach has been used by Fromm and Taubman [127]. As yet no axiomatic basis for such procedures seem to have been developed; nor at the descriptive level, apart from intuition, is there any evidence yet that decision makers do assess risky prospects with time-trace consequences in such fashion. Possibly, a lexicographic specification of the total utility function would be more appropriate for some decision makers. For example, for each period some critical level of utility u_t^* which must be exceeded with some minimum probability P_t might be specified. The total utility function would then be

$$U(x_1, x_2, \ldots x_n) = \{\text{Prob}[u_t(x_t) > u_t^*] \geqslant P_t \text{ for each } t; \sum_t \alpha_t^{t-1} u_t(x_t)\}. \tag{72}$$

The approaches of equations (66), (71), and (72) may each be extended to handle portfolio choice along the lines of equations (53) and (54) of Section 9.4.

From a theoretical point of view, the additive utility approach is attractive because it implies certain desirable characteristics of constancy for the marginal rate of substitution between consequences at different dates [127, 164]. For this reason the additive utility approach has been the basis of a number of models of optimal economic growth, e.g. [136, 199, 200, 228]. In terms of more practical considerations of day to day decision making, however, the additive utility procedure has the disadvantage that the period utility functions $u_t(x_t)$ and the utility discount factors α_t have to be specified. A far more practical procedure, which may also be of descriptive relevance, is to aggregate discounted raw period outcomes back to a present value, and then evaluate this present value in terms of utility. Such an approach has been argued by Hespos and Strassmann [162], Hillier [167, 168, 169] and Murphy [253], and applied by Adelson [2], Mao [229, 230, 231] and Weingartner [356], and, in an agricultural marketing context, by Ikerd and Schupp [175].

To illustrate the utility of present value approach, we will again consider a risky prospect with uncertain consequences x_t following the distribution $f_t(x_t)$ and occurring at the start of period t ($t = 1, 2, \ldots, n$). If μ_t denotes the expected value and σ_t^2 the variance of x_t, the expected present value $E(PV)$ of the time trace $x_1, x_2, \ldots, x_n$ is

$$E(PV) = \Sigma k_t^{t-1} \mu_t \tag{73}$$

where k is the decision maker's discount factor for period t consequences, the corresponding discount rate being $r_t = (1 - k_t)/k_t$. The variance of present value $V(PV)$ is given by

$$V(PV) = \Sigma(k_t^{t-1}\sigma_t)^2 + \sum_{t \neq w}\Sigma k_t^{t-1} k_w^{w-1} \rho \sigma_t \sigma_w, \quad t, w = 1, 2, \ldots n, \tag{74}$$

where ρ is the correlation coefficient between x_t and x_w. If x_t and x_w are perfectly correlated (i.e. $\rho = 1$),

$$V(PV) = [\Sigma k_t^{t-1}\sigma_t]^2. \tag{75}$$

If x_t and x_w are independent (i.e. have zero covariance),

$$V(PV) = \Sigma(k_t^{t-1}\sigma_t)^2. \tag{76}$$

More generally, it may be possible to split the consequence x_t into two components: x_t' and x_t'', where x_t' is a part that varies independently and x_t'' is a part that is perfectly correlated between periods [167]. If such a decomposition of x_t can be made, the variance of present value is given by

$$V(PV) = \Sigma(k_t^{t-1}\sigma'_t)^2 + [\Sigma k_t^{t-1}\sigma_t'']^2 \tag{77}$$

where σ_t' and σ_t'' are the respective standard deviations of x_t' and x_t''. Given $E(PV)$ and $V(PV)$, a decision maker who only considered these two moments in his utility assessments could substitute these values into his utility function to appraise the particular risky prospect being considered. For example, with a quadratic utility function we would have

$$U(PV) = E(PV) + b[E(PV)]^2 + bV(PV). \tag{78}$$

If need be, higher moments of the present value distribution could also be considered.

While the above procedure allows for covariation between consequences in different periods, it does not allow for the time sequencing of consequences. This might be done via a lexicographic procedure such as specifying a utility function $U^*(PV)$

$$U^*(PV) = \{\text{Prob}\,[PV_t > PV_t^*] \geqslant P_t \text{ for each } t;\ U(PV)\} \tag{79}$$

where P_t is some required minimum probability that the present value of the tth period consequence PV_t exceeds some specified level PV_t^*. Alternatively, as was suggested in the additive utility approach, inter-period variation might be accounted for through the variance of the discounted expected period consequences, $V(k_t^{t-1}\mu_t)$. This variance is

$$V(k_t^{t-1}\mu_t) = \Sigma[k_t^{t-1}\mu_t - (\Sigma k_t^{t-1}\mu_t/n)]^2/n, \quad t = 1, 2, \ldots n, \tag{80}$$

and could be brought into the utility function $U(PV)$ analogously to equation (71).

As with the additive utility approach to time-trace consequences, the above utility of present value procedures may be extended to portfolio-type problems [229]. Hillier [169] and others (e.g. [52]) have used, for example, a lexicographic approach based on chance-constrained programming to obtain portfolio solutions for multi-period risky prospects. Rae [285] has given some consideration to such problems in an agricultural production context.

Overall, it must be said that Bernoullian decision theory has as yet a lot of loose ends relative to the assessment of risky prospects involving time considerations [37, 251]. As recently suggested by Hushleifer [374], one possibility for overcoming some of these difficulties may be to use a state preference approach based on certainty equivalents for time-trace distributions of consequences to estimate a utility function reflecting both inter- and intra-period variability.

12 ALTERNATIVES TO BERNOULLI'S PRINCIPLE

Systems analysis and simulation [44, 57, 58, 75, 90, 152, 239, 255, 348, 370], operations research procedures (including the ever increasing variety of programming procedures) [50, 173, 353], control theory [91, 206], and even "games against nature" procedures [5, 208, 261, 296] would have their advocates as the appropriate procedure for handling choice between risky prospects. In essence, none of these procedures are alternatives to Bernoulli's Principle. To the extent that they recognize degrees of belief and degrees of preference in their evaluation, they can be satisfactory. But to this extent also, they simply become algorithms for applying Bernoulli's Principle. An example is given by the spate of recent developments in stochastic and related linear and non-linear programming procedures typified in a general context by, e.g. [21, 22, 43, 62, 174, 254, 317, 329, 330] and in an agricultural context by, e.g. [40, 76, 148, 157, 158, 159, 222, 223, 240, 241, 275, 285, 286, 347]. A few of these works [21, 22, 43, 285] are fully decision theory oriented. Many, however, simply constitute algebraic variations on the simplex theme without any regard to the decision maker's degrees of preference and belief. Instead of starting with the decision maker and finding what his criterion function is like, or postulating some objective function justified on normative grounds, very often a reverse procedure seems to have been followed. The result is a variety of objective function formulations, algebraically tractable and compatible with some new programming twist, but justified neither on normative nor descriptive grounds.

13 STATISTICAL DECISION THEORY

As applied to choice between alternative hypotheses in research, Bernoullian decision theory is known as Bayesian statistics or statistical decision theory. Conceptually, its great advantage over the classical or sampling theory approach is that it uses the economic criterion of maximizing expected utility ("minimizing expected loss") rather than arbitrary significance levels as the basis of choice between alternative hypotheses. A number of introductory expositions of the theory have been presented, e.g. [12, 13, 88, 98, 170] and an increasing number of text presentations are available, e.g. [211, 314, 372, 375]. In general, it might be commented that statistical decision theory is well developed in terms of the use of Bayes' Theorem but not in terms of utility as a criterion. As well, much work remains to be done to develop routine procedures for Bayesian analysis. As yet, in many practical research situations, researchers can be advised to do no more than attempt to consider their problems of choice between hypotheses in a Bayesian framework [13], a major difficulty being the problem of ascribing quantified gains and losses to alternative choices. At least, however, the Bayesian revolution is drawing to the attention of researchers the facts that prior information is not irrelevant, that the classical approach of concentrating on Type I errors via arbitrary and economically irrelevant significance levels is likely to be inadequate, and that economic rules for the purchase of additional information can be formulated.

14 LITERATURE

As the far from complete list of (largely English) references attached to this review indicates, there is a vast literature relevant to Bernoullian decision theory. Though it is an impossible task, the following briefly annotated list represents an attempt to specify a "top fifteen" from the field. The listing reflects a multidimensional utility function with intuitive weightings between mathematical and non-mathematical, theoretical and applied, agricultural and non-agricultural, and literary attributes. As such, it is one man's list. Others can make their own.

(a) General

Alderfer and Bierman [7]: An example of the interrogative procedures required in decision theory research.

Arrow [18]: Short, readable, advanced and original.

Borch [37]: In contrast to this reviewer's Germanic style, a book of literary elegance that reads like a whodunit with Bernoulli as Maigret, covering a wide variety of cases and with a keen eye to economic theory. One for the bedside.

Fishburn [120]: A detailed exposition of utility theory in its multitudinous facets, well organized but rather coldly mathematical in a set theoretic mold. One for those of mathematical bent.

Marschak [237]: Deceptively informal and relatively nontechnical, one of the masters of modern economics shows why you would need to consider carefully before dismissing Bernoulli's Principle.

Morgan [245]: A good little introductory text.

Raiffa [287]: The best—simple, conversational, absorbing and practical, but definitely a pencil and paper job in parts.

Ramsey [290]: A philosopher genius who died in 1930 at the age of 26 and whose decision theory work went unrecognized until the 1940's, argues the case for Bernoulli's Principle as the logical approach to risky choice. With Edgeworth, Ramsey must rank as one of the most interesting figures in economics, as Keynes' [191] irresistible sketches indicate.

Savage [308]: One of the great figures of decision theory—perhaps best described as the St Paul to Ramsey's Gospel—exposes a little of his character with annotations on a reading list.

(b) Agricultural

Anderson and Dillon [10]: A first quantitative utility-based consideration of the economics of agricultural research—when to start and when to stop.

Byerlee and Anderson [51]: An economic evaluation in a Bayesian framework of the value of additional information in production decisions, empirically oriented to climatic prediction.

Carlson [54]: An imaginative application of Bernoullian decision theory to provide meaningful answers to a real-world problem of crop pesticide use.

Makeham *et al.* [227]: A "how to do it", very practically oriented exposition of how to apply Bernoulli's Principle in farm management decisions.

Officer and Halter [259]: Unlike most literature in agricultural economics, this article sets out to test a theory—Bernoulli's Principle—and shows it to be of value.

Porter [274]: A relatively early article, generally overlooked, strongly arguing the relevance of utility considerations in agricultural development policies.

Rae [285]: One for the programming buffs, fully in the Bernoulli mold, elegantly presented and covering a lot of territory.

15 OVERVIEW

Is utility futility? The answer, definitely, is No! As Borch [37, p. 213] aptly puts it, what other decision procedure so well takes account of what we believe, what we know, and what we want. This is not to say that all the various rigmaroles and recipes covered in this review have to or can be followed through in practical application of Bernoulli's Principle. For many problems, indeed the day to day majority of those requiring some consideration, the certainty equivalent approach will suffice with no need for thinking of a utility function. Even sketching the roughest of decision trees will often powerfully augment a decision maker's choices. Only in important problems, where complexities are such that choice is best formalized and extensive calculations are worth paying for, will it be worthwhile attempting to apply the formal Bernoulli model. Even then, the payoff may often only be clearer thinking and guidance rather than outright answers. Nor, if the best decision is taken, is a "good" outcome guaranteed. The fact that formal attempts at application are often worthwhile is evidenced by the increasing use of the model in a variety of fields, for example medicine [6], insurance [34, 35], geology [135], climatology [367], public investment [280] and business, particularly marketing, product control and corporate planning [118, 246, 373]. None the less, as Roberts [295, p. 68] has commented, it is still early days in the application of Bernoulli's Principle and the real "test of usefulness of decision theory lies mainly in the future, and it will be made by those who learn about decision theory early enough in their lives that practical experience will not yet have made them feel that orderly, careful thinking about human decisions is futile".

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