AN APPRECIATION OF DECISION ANALYSIS IN MANAGEMENT

J. R. Anderson and J. B. Hardaker*

Decision analysis, the practice of Bernoullian decision theory and Bayesian statistics, is reviewed in relation to its application in management. Aspects of the scaling of beliefs (probabilities) and preferences (utilities) are discussed, focussing on practical problems. It is concluded that the solution of these difficulties may best be sought through more widespread analysis of real-world decisions.

1 INTRODUCTION

The task we have set ourselves here is to examine the relevance of decision analysis in management and to point to some of the unresolved difficulties in practice. The application of decision analysis to a decision problem involves at least the following steps [9]: (a) a relevant decision problem worthy of systematic analysis is identified; (b) the decision problem is structured to identify available acts, relevant uncertain events and contingent consequences; (c) the manager scales his beliefs about the events and his preference for possible consequences; (d) the expected utility (preference) of each act (or sequence of acts) is computed and the one yielding the highest expected utility is identified; (e) the optimal (utility maximizing) policy is implemented.

To facilitate discussion we have sought to identify two separate topics within the general field of decision analysis—Bernoullian decision theory and Bayesian statistics. Although the two overlap to a considerable degree, there are sufficient differences of emphasis between them to justify some separate discussion.

Bernoullian decision theory is a prescriptive theory of choice that applies to any risky decision making, focusing on the beliefs and preferences of the decision maker. The operational validity of the theory depends first on the acceptability of the underlying axioms which have been exposited by Dillon [9]. We believe that most people will find the axioms

---

*Department of Agricultural Economics and Business Management, University of New England. We are indebted to John L. Dillon, Onko T. Kingma and Warren F. Musgrave for comments on the earlier draft of this paper which was presented at the Annual Conference of the Australian Agricultural Economics Society, Sydney, February, 1972.

reasonable in the sense that they would wish to make at least important decisions in a manner consistent with the axioms. However, for those who, like Menges [35], still have doubts, Pratt, Raiffa and Schlaifer [47] provide a somewhat different and perhaps more convincing axiomatic treatment in which concepts relating to scalings of both beliefs (personal probabilities) and preferences (utilities) are derived together.

Bayesian statistics applies to decisions involving statistical inference and focuses on revision of probability distributions and formalizing beliefs. The Bayesian approach to scaling beliefs involves (a) formal assessment of present information in the form of prior distributions and (b) updating beliefs through use of Bayes' theorem.

We do not attempt to give a unified exposition of either of these aspects of decision analysis, or to give a balanced review of the controversies in the discipline—a task competently handled by Raiffa [49, ch. 10]. As converts to the Bernoullian and Bayesian views, we choose to concentrate discussion on problems that persist. In short, we can find no fault with the ideology but we feel uncomfortable with some aspects of (present-day) methodology.

2 PROBABILITY AND BAYESIAN INFERENCE

The specification and manipulation of probability distributions are central features of the Bayesian approach to decision analysis [49]. In constrast to the sampling theory approach, these distributions may be, and indeed will usually be, subjectively derived [68]. The introduction of explicit subjectivity into decision analysis is most important but raises difficulties. In spite of the ready availability of good reference material on the Bayesian approach to statistical and managerial decision making, application, even to essentially managerial questions, appears to have been rather slow. Doubtless, there are many reasons for this connected with various practical difficulties, some of which are now discussed.

2.1 DERIVATION OF PRIOR AND LIKELIHOOD PROBABILITIES IN DECISION PROBLEMS

Assessment of subjective prior distributions has received considerable attention by psychologists (notably Edwards [12, 13]) and Bayesian statisticians [18, 52, 53, 54, 58, 64]. The general findings are that most people can readily specify personal probabilities. A variety of methods for assessing prior distributions has been used but, unfortunately, the assessed distribution depends to some extent on the method adopted. The methods can be classified according to whether a distribution is assessed directly, either as a cumulative distribution function (CDF) or probability density (mass) function (PDF), or indirectly as an equivalent (past or hypothetical future) sample [64]. The direct methods find most general use in decision analysis whereas in some statistically oriented analyses the indirect methods may be used to advantage.

To date, insufficient experience has been recorded to provide good guidance on the relative merits of alternative methods. Direct assessment of PDF's is the simplest method to use for discrete distributions, particularly for subjects not well-versed in probability calculus. Francisco and Anderson [16] found that farmers could readily specify discrete PDF's with the aid of counters. Direct assessment of CDF's is also quite straightforward (e.g., Raiffa [49, pp. 161–168]) and has some computational advantage if, as will usually be the case, it is desired to smooth out slight irregularities in the assessed distribution. It would be reasonable to assess independently both the CDF and PDF and adaptively correct any inconsistencies between the two methods.[1]

Personal probability assessments exist only in the mind of the assessor so there is no way of knowing whether stated beliefs and true beliefs coincide. However, attempts have been made to develop "scoring rules" to reward or penalize an assessor so as to encourage him to make his stated probabilities conform with his beliefs [39, 53, 58, 66, 67]. When the decision maker bears responsibility for his decision, the decision consequences act as a scoring rule but when an adviser's judgements are to be quantified, scoring rules may be useful in making his probability assessments honest.

Much less work has been done on assessment of likelihood probabilities than priors. Perhaps this reflects the situation that most Bayesian developments have been in statistical inference rather than in decision analysis. In most statistical work the likelihood distribution is unambiguously determined by the nature of the sampled process—for example, binomial sampling or sampling from an assumed normal population.

In many managerial applications the derivation of likelihoods must necessarily be a very subjective business, although this does not seem to have been generally recognized. An example is afforded by the purchase of seismic information in oil search. It is simple enough for a geologist to assess a prior distribution for the productivity of a hole before it is drilled but for a decision analysis of the value of buying a seismic test of the site, he needs conditional probabilities (likelihoods) of the form P (test indication given productivity). The priors and likelihoods can then be combined in Bayes' theorem to yield posterior probabilities of the form P (productivity given test indication).

We believe that it is possible to assess such likelihood probabilities in the same personal manner as for priors although it seems relatively more difficult to do. However, it is unavoidable if one is to be consistent with the philosophy of laying out all the components of a decision analysis in a logical and explicit form. The tendency for authors in decision analysis to snatch likelihood probabilities "out of the air" (e.g., [19, 21])

---

[1] We should note here that a useful step in applied decisions will be to conduct some form of sensitivity analysis (e.g. [44]) on the Bayesian procedures with respect to the prior distributions (and likelihoods if also personal). The results of such sytematic perturbation of these components will indicate to the analyst the degree of accuracy that should be sought in probability assessment.

has given a false impression of both the difficulty and the subjectivity of this phase.

2.2 JUDGEMENTAL JOINT PROBABILITIES

The assessment of multivariate distributions has received less attention and is intrinsically more difficult than assessment of univariate distributions. In considering distributions of several variables the first step is to establish any stochastic independence between variables. We believe that this can usually be done by careful consideration and introspection without direct recourse to numerical enumeration. If this is true, it means that full specification of a joint distribution need only be attempted when dependence is judged relevant. As we shall see, assessing judgemental joint distributions is rather irksome and is to be avoided whenever possible.

A possible test of intuitively assessed independence can be made by simply applying the definition of independence. That is, suppose marginal distributions have been assessed for two discrete random variables A and B, then several products $P(A_i)$ $P(B_j)$ can be compared with independent assessments of the joint probabilities $P(A_i, B_j)$ and if they correspond fairly closely, independence can reasonably be assumed. Such a test procedure can be applied to combinations of random variables to identify independent sets of dependent variables. As a practical matter, it is to be hoped that the sets are small.

The Harvard decision analysts [48, ch. 22; 49, ch. 9; 55, ch. 9] are the only authors who appear to have made serious study of problems associated with assessing joint distributions. They suggest that encumbent problems may be minimized by: (a) using transformations of variables to achieve independence and (b) relating jointly dependent variables to a hierarchical system of explanatory variables to achieve conditional independence. Of these, the former does not seem likely to be used widely and the latter, while of obvious merit, will usually demand some sophisticated analyses.

To some extent, the methods discussed earlier for direct assessment can be extended to the multivariate case. Of most use for distributions which can be handled in discrete form is the visual impact method of directly assessing PDF's. Consider first the simplest case of a pair of variables each composed of n discrete intervals. The fastest method of assessment is to draw up a two-way table with n x n compartments and allocate counters according to the direct assessment of the joint PDF. For the portfolio and other mean-variance problems, this method extends adequately to the general case of k variables by taking $k(k - 1)/2$ pairwise bivariate distributions and thus determining the covariances. However, this generally does not give the full joint distribution which may be required in some problems.

The complete distribution has to be approached by assessing marginal and conditional distributions separately. For the pair of variables with n intervals, one marginal distribution (say B) is assessed and then, for each of several intervals of this variable, the conditional distribution of the other is assessed. Then the joint probabilities are found as $P(A_i, B_j) = P(A_i \mid B_j)\ P\ (B_j)$. The amount of assessment explodes rapidly as the number of variables (k) increases. For the full joint distribution, one (an arbitrary) marginal distribution and $\sum_{i=1}^{k-1} n^i$ progressively more complex conditional distributions must be assessed. Joint assessments are greatly simplified (but are still not easy [48]) in the case of the multivariate normal distribution with its k marginal means and $k(k + 1)/2$ variances and covariances. More work is needed to assess the feasibility of alternative methods of estimating joint distributions, to determine how adequate the normal assumption can be for non-normal distributions and to explore the conditions under which only partial assessment can be sufficient.

2.3 INCORPORATING "OBJECTIVE" EVIDENCE

"Objective" evidence is the term chosen here to describe data that are to be incorporated into a decision analysis by a route that is not entirely subjective and intuitive. It should be noted, however, that the conscious inclusion of any data judged to be pertinent is necessarily another subjective step in the analysis. Entry can be via either the prior or likelihood distributions according to the purpose and convenience of the analysis.

In the more formal analyses of Bayesian statistics (e.g., [59]) the route chosen for incorporating objective data has typically been via the likelihood in association with a diffuse prior distribution which represents an essentially informationless initial state of knowledge. Through the mechanics of Bayes' theorem, a diffuse prior has virtually no influence on the posterior distribution—the net effect is to produce a result very similar to that arising from a conventional classical (sampling theory) analysis and the procedure has consequently been dubbed "scientifically objective" [23]. We believe, however, that to disregard any pertinent subjective or historical knowledge in the formulation of prior distributions is at odds with the spirit of the Bayesian approach. A completely informationless prior must surely be a rare animal! In our view, all available information should contribute to the prior assessment.

The most useful guides to the handling of "objective" data such as a series of historical observations have been set down by Schlaifer [54] and consist of a blend of statistical pragmatism and common sense. Schlaifer argues that most distributions are smooth and unimodal and those that are not probably reflect the influence of some assignable cause which can be explicitly accounted for in the analysis. He considers that assessment of historical data must be based on understanding of the real phenomena rather than on statistical theory. Thus he suggests that pertinent data should be arrayed graphically in the form of either a PDF or CDF and smoothed by hand or by means of an appropriate computer program [6, 55] to iron out irregularities to which causes cannot be assigned. If the CDF is graphed on normal-probability paper, an immediate assessment can be made of the closeness of a normal approximation. The normal distribution permits simple analysis in many problems.

Application of Schlaifer's [54, p. 104] non-parametric fractile rule provides an easy and efficient means of obtaining a preliminary estimate of a CDF (even with very sparse data) which can then be smoothed according to one's knowledge of the phenomena involved. We have found this approach also to be useful in risky response analysis involving sparse data [1].

2.4 BIASES IN PROBABILITY JUDGEMENTS

A manager must guard against biases that can creep into his probability judgements. For example, he should take care that his assessed probabilities are coherent with probability calculus. Any device or procedure which helps the manager to introspect more effectively is worthwhile. For example, it has been shown [16, 33, 43] that people are generally poor information processors in the sense that they fail to extract the maximum amount of certainty that is embodied in a set of data. It is therefore sensible to employ Bayes' formula to ensure that one correctly accounts for any new evidence that comes to hand. The formula can also be used to show the consequences, in terms of revision of prior to posterior probabilities, of observing a range of possible evidential outcomes. On inspecting the results of such a calculation a manager may decide that the derived posterior probabilities are unreasonable for him. In that case, he both can and should revise his prior and/or his likelihoods until he arrives at values which are both consistent with his true beliefs and coherent with the probability calculus embodied in Bayes' formula [50, p. 62].

Other types of bias that have been reported in probability judgements include probability preference [10] and variance preference [11, 25]. Both are "irrational" in the sense that they involve either a lack of adequate introspection and/or a failure to conform with the assumption of independence between beliefs and preferences which is a fundamental axiom of decision theory. The risk of such biases occurring in probability judgements can be minimized by first, recognizing and being on one's guard against it, and second, where appropriate, using such probability smoothing techniques as already described.

2.5 OPERATIONAL LIMITATIONS OF THE BAYESIAN APPROACH

The Bayesian approach is more demanding of analytical resources than is the pre-Bayesian conventional approach—a necessary consequence of deploying all rather than some of the relevant information. A fair question then is "Is it worthwhile?" Such a question must be posed for each decision problem encountered and the answer will depend on the size of payoffs, the quality of information at hand, the skill of the analyst and his access to computational assistance [40].

The computational burden in Bayesian analyses—particularly those involving arbitrarily specified continuous probability distributions and non-linear loss of functions—can be very heavy, although some of the operational limitations of the Bayesian approach we have mentioned may evaporate with further empirical experience. Often the most difficult part is the integration of the denominator in the continuous form of Bayes' theorem, namely

$$\text{posterior density} = \frac{(\text{prior density})\ (\text{likelihood function})}{\int [(\text{prior density})\ (\text{likelihood function})]}$$

To minimize resorting to relatively inconvenient numerical integration procedures, Bayesian statisticians have developed the idea of conjugate prior distributions [50, ch. 3]. Briefly, once certain assumptions are made about the nature of a statistical sampling process, the likelihood function is uniquely determined. A conjugate distribution may then be defined, relative to this likelihood, with the properties of being analytically tractable for computing the posterior distribution, rich enough to describe prior information adequately and yet still readily interpretable. A further desirable feature of tractability is that the posterior distribution should be a member of the same family as the conjugate prior so that sequential application of Bayes' theorem is facilitated. Fairly adequate conjugate distributions have been found for most of the common sampling distributions although the integrations are not always easy [70].

3 INDIVIDUAL PREFERENCE AND DECISION ANALYSIS

3.1 CONSISTENT INTROSPECTION

The procedures for deriving preference functions, at least for money outcomes, are now reasonably well established and tested [15, 41, 55]. It remains an open question, however, just how good a representation of real preferences such functions are. If a manager is to have confidence in his preference scalings they should be internally consistent and reasonably "repeatable." Shifts in preference would normally be expected to occur only with important changes in the status of the individual, such as changes in wealth or health.

The establishment of consistent preference scalings clearly requires considerable introspective ability and may well be beyond the capabilities of some people. The evidence, however, is reasonably encouraging in that there are a number of reports of reasonably successful attempts at deriving preference functions for a variety of subjects [e.g., 16, 19, 57]. If, despite such evidence, we adopt a pessimistic view of the practicability of establishing consistent preference scalings, we must consider the alternatives. Adoption of any of the obvious alternatives to expected utility as the maximand for analysis of a decision problem under uncertainty can be held to imply the existence of a preference function. For example, maximization of expected money value implies linear preference for money.

Some progress in decision analysis can be made by using the fact that many decision makers have risk attitudes of a qualitative nature that are more easily identified than are the quantitative attitudes that must be elicited to derive a preference function. Qualitative risk attitudes, such as risk aversion and decreasing risk aversion, may permit a partitioning of risky prospects into " efficient" and "inefficient" sets. For example, after initial development by Markowitz [34], mean-variance

(E-V) analysis has been widely used, primarily in portfolio selection [e.g., 30, 56, 60]. A portfolio is E-V efficient if no other portfolio with the same or larger mean yield has a smaller variance and no other portfolio with the same or smaller variance has a larger mean. E-V efficiency implies either normally distributed outcomes or a quadratic preference function with (increasing) risk aversion [14, 22, 46].

A more general method of identifying an efficient subset of strategies under risk is provided by use of the principles of stochastic dominance [20, 45, 61]. In its simplest form, stochastic dominance means that if the cumulative density function of the payoff of strategy A is always to the right of the cumulative function for strategy B, then A is first degree stochastically dominant over B and A will be preferred to B by any decisionmaker who has a preference function for the payoff that is non-decreasing over the relevant range. This concept has been extended to second and third degree stochastic dominance, requiring more restrictive assumptions about the form of the preference function.

Segregation of an efficient subset of strategies in these ways leads directly to a decision only when there is but one strategy in the efficient set. In other cases, where the full preference function has not been established, direct intuitive assessment by the decision maker must be used to select the best strategy. For most managers and most decision problems we believe that this approach will not result in a more rational decision than that reached by decision analysis.

Specification of an appropriate preference function is assisted when the decision maker is able to make qualitative statements about his attitudes to payoffs and risks. Thus it has been shown [4] that if the decision maker is uniformly risk averse, his preference function is of the form $U = 1 - e^{-cx}$ where c is a measure of risk aversion [46]. Meyer and Pratt [36] describe how both quantitative and qualitative statements by a decision maker about his feelings towards payoffs, risks and size of risks [69] can be jointly taken into account in estimating the appropriate preference function.

3.2 PREFERENCE FOR MULTIDIMENSIONAL CONSEQUENCES

The consequences of acts and events can seldom be described adequately in terms of a single numerical measure of outcome such as dollars. Often consequences will be multidimensional, incorporating both quantitative and qualitative components. Much of the empirical work in decision analysis has been confined to cases where differences in all but the monetary outcomes are regarded as negligible across the range of possible consequences, which greatly facilitates analysis. Where such an assumption is not valid, however, the basic concepts of preference scaling remain relevant. That is, reference consequences c^+ and c^- should be defined, such that c^+ is at least as good as the best possible consequence in the real problem and c^- is at least as bad as the worst possible real consequence. A utility index $U(c)$ for any given consequence c can then be found from the indifference relationship $c \sim \{U(c)c^+ + (1 - U(c))\, c^-\}$. But if the principles remain the same, the practice of preference scaling for multidimensional consequences is more difficult, especially when some components are qualitative. Qualitative outcomes are difficult to describe unambiguously,[2] while the preference scalings obtained may not be amenable to graphical or algebraic representation. The only practical approach appears to be to confine analysis to a small number of defined points on what might be a continum of possible qualitative consequences.[3]

To indicate how preference scaling might be extended to include multi-dimensional consequences we consider (following Raiffa [49]) first the case where consequences can be described by two attributes, measured by x and y. That is, consequence c_i is equivalent to the pair (x_i, y_i), where, for example, x might be a measure of annual profit and y a measure of rate of capital gain. We now select some "bench-mark" value for one of the components, say $y = y^*$ and establish for the manager a a substitution relationship such that $(x_i, y_i) \sim (x_i^*, y^*)$, where $x_i^* = x_i - \lambda(y_i - y^*)$. The rate of substitution of y for x, λ, may not be constant over the range of possible values of x and y. Where it is constant, we have iso-preference (indifference) curves in the xy place that are parallel straight lines. Otherwise it may be possible either to obtain a sufficient number of indifference relationships to be able to estimate the indifference map,[4] or to convert each individual (x_i, y_i)-pair in the problem to the equivalent (x_i^*, y^*). Finally, a preference scaling for x_i^* can be obtained in the conventional way, keeping the value of y^* in mind. Thus it is possible to associate a utility value with each possible (x_i, y_i)-pair, and so to proceed to resolve the problem in terms of expected utilities.

As we move from two to several components in the vector of consequence, the same basic approach can be applied. Consider the 4-tuple of consequences (x_1, x_2, x_3, x_4). We define bench-mark values for x_2, x_3 and x_4 of x_2^*, x_3^* and x_4^* respectively. It is then necessary to establish the sequence of indifference relationships: $(x_1, x_2, x_3, x_4) \sim (x_1', x_2, x_3, x_4^*) \sim (x_i'', x_2, x_3^*, x_4^*) \sim (x_1''', x_2^*, x_3^*, x_4^*)$. Then a utility function for x_1''' can be derived keeping in mind the values of x_2^*, x_3^* and x_4^*.

Keeney [27, 28] describes how the assessment of preference scalings for multiattributed consequences can be reduced in complexity by exploiting general properties, such as monotonicity and concavity, which the decision maker is prepared to specify for his preference function. Such properties imply that the function will be of restricted form and may be specified completely by assessing only parameters of a general family of functions exhibiting the desired characteristics, or by assessing only "conditional utility functions", defined over subsets of the various

---

[2] For a novel approach to the representation of qualitative consequences in preference analysis see Hoinville and Berthoud [26].

[3] An example of the application of these methods of scaling consequences in medical diagnosis and treatment is given by Ginsberg and Offensend [17].

[4] For a report of some recent work on an experimental approach to the determination of indifference curves, see MacCrimmon and Toda [32].

attributes and thus of smaller dimensionality. Klee [29] presents an operational procedure based on a linear scoring model for multiattributed consequences that minimizes the number of decisions required of the decisionmaker.

3.3 INTERTEMPORAL PREFERENCE

For an important class of investment-type decision problems, consequences may be measured by a vector $(x_1, x_2, \ldots, x_n)$ where x_i is the net cash flow in year i of an n-year planning period. Such intertemporal consequences are no different in principle from other multidimensional consequences and the evaluation procedures described above can be applied directly. It must be admitted, however, that the introspective capacity required to scale preferences for a large number of sets of outcomes over, say, a 20-year planning horizon is enough to daunt most managers. For some managers, a practical route to the derivation of an intertemporal utility function might be found in a "trial and error" approach. For example, it might be plausible to assume an "ordinal utility function" [24] of the form $U = \underset{i}{\pi} x_i$ to describe the intertemporal indifference map. Using such an ordinal function it is possible to deduce an indifference relationship between the actual intertemporal stream of net cash flows and, say, a fixed amount of x^* in the first $n - 1$ years of the planning period, coupled with a compensating amount x_n^* in the final year. A preference scaling for x_n^* given x^*, called a "cardinal utility function" by Hirshleifer [24], could then be obtained in the usual way. Use of such an arbitrary ordinal function might yield results judged by a particular manager to be an adequate reflection of his preferences. By way of illustration, suppose a decision maker faces a stream of after-tax cash incomes over the next three years of \$3,000, \$8,000 and \$6,000, in that order. If an arbitrary "reasonable" after-tax income in years 1 and 2 is taken to be \$4,000, the ordinal utility function given above implies the indifference relationship[5] (3,000, 8,000, 6,000) $\sim$ (4,000, 4,000, 9,000) i.e., $3 \times 8 \times 6 = 4 \times 4 \times 9$. A "cardinal" preference scaling for the prospect could then be derived in terms of the year-3 equivalent income, given an income of \$4,000 in each of the first two years. Note that an ordinal function alone allows the ranking of alternative streams of cash flows, but that the cardinal function is necessary to account for individual preference under risk.

---

[5] Logically one should first ensure that all available borrowing and saving opportunities have been fully exploited to obtain the highest possible value for x_n^*. Assuming a perfect capital market, this will result in a stream of net cash flows each one i per cent greater than the one in the previous year, where i is the interest rate. In our example, if $i = 0{\cdot}1$, we obtain (3,000, 8,000, 6,000) $\equiv$ (5,077, 5,585, 6,143) $\sim$ (4,000, 4,000, 10,887), where $\equiv$ implies equivalence in terms of the capital market and $\sim$ implies indifference in terms of the ordinal utility function.

4 DECISION ANALYSIS IN MANAGERIAL PRACTICE

4.1 INTERPERSONAL ASPECTS OF DECISION MAKING

Most of the literature on decision analysis, including our own presentation thus far, deals with individual choice in which one person is assumed to be responsible for a decision and to be affected by its consequences. Since this is but one, albeit important, class of decision making, we now turn to a brief examination of the case where a group of people, such as a committee or board of directors share in a decision.

If the group holds or agrees upon common beliefs and preferences, then the principles of rational choice are applicable as for an individual. Groups seldom attain such unanimity and it would appear logical to argue that if the members of a group disagree in their assessments of beliefs and preferences relative to a particular decision, they ought to thrash out these differences in order to arrive at compromise scalings which may be used to determine a rational group decision. Various ways of combining probability and preference judgements have been proposed (e.g., [37, 52, 65]) but, of course, any rule for arriving at a consensus must be acceptable to the group. Moreover, many such rules are open to abuse in that a group member might deliberately bias his scalings of beliefs and preferences.

Unfortunately, such extension of decision analysis to group decision making can result in decisions inconsistent with the principle of Pareto optimality [2], opening prospects of coalitions forming within a group to press their common interest, together with bargaining and the exchange of promises and threats. While much research has been done in the application of game theory and related techniques to these types of group decision situations, many issues remain unresolved [4, 8, 31, 42, 51, 62, 63].

Although group decision making presents the decision analyst with some difficulties, there are also advantages in using decision analysis in a large organization (including large farm organizations [38]). In particular, if appropriate beliefs and preferences *can* be identified, the approach offers *the important potential benefit of delegated decision making within the organization*. Arrow [3] notes that decentralized decision making in large organizations has been looked on with much more favour in recent years and that the operating rules for junior managers under a decentralized system take the form "do whatever is necessary to maximize a certain objective function". The problems of specifying an objective function in a large organization, especially to account for corporate rather than individual risk attitudes, might be solved by adoption of a corporate utility function. Within a decision-theoretic framework, some authors (e.g., Wilson [63]) have proposed various schemes for executive sharing of corporate risk in order to sharpen the involvement and responsibility of decision makers.

4.2 CONCLUDING REMARKS

Bernoullian decision theory stands or falls, first, on the "reasonableness" of the basic axioms and on the validity of the logic by which the operational theory is deduced from these axioms. We find it hard to suppose that many people will find either axioms or logic unacceptable. Secondly, the practical value of the theory in decision analysis depends upon the feasibility of convincing real-world managers of the merits of a systematic approach to their decision problems and on their assessments of the relative costs and benefits of systematic versus intuitive decision making.

The auguries for wider adoption of decision analysis are good although, as for any new technology, its introduction to a company must be carefully planned [7]. The approach is given major emphasis in the management education programmes of some of the best-known business schools (notably Harvard), while Brown [5] found a substantial amount of applied work in a small sample of companies in the U.S.A. The number of users has expanded rapidly in the past few years, and applications can be expected to increase markedly in the future. Brown reported that several large companies have arranged orientation courses for their executives, while every year brings reports in the literature of applications to an expanding range of problems.

We have indicated some areas which require further clarification. As far as personal probability is concerned, the main difficulties for managers familiar with the concept lie in assessing likelihood and multivariate distributions. More generally, practising managers might profit from training in the concepts and use of personal probabilities [7]. Developments of both a theoretical and applied nature may lead to reduction in the difficulties currently faced in processing data along Bayesian lines. We have discussed how decision analysis can sometimes proceed without the need to assess preferences, but more often an explicit preference function will be required. This can be elicited by taking account of both quantitative and qualitative attitudes of the decision maker. Extensions to preference for multidimensional consequences requires further development. There is a need for further research relating to group decision making. Despite such residual difficulties, decision analysis is now sufficiently well developed to warrant its use in many more managerial decision situations.

REFERENCES

[1] Anderson, J. R., "Sparse Data, Climatic Variability and Yield Uncertainty in Response Analysis", *American Journal of Agricultural Economics*, (in press).

[2] Arrow, K. J., *Social Choice and Individual Values*, (New York: Wiley, 1951).

[3] Arrow, K. J., "Control in Large Organizations", *Management Science*, Vol. 10, No. 3 (April, 1964), pp. 397–408.

[4] Borch, K. H., *The Economics of Uncertainty*, (Princeton: Princeton University Press, 1968).

[5] Brown, R. V., "Do Managers Find Decision Theory Useful?", *Harvard Business Review*, Vol. 48, No. 3 (May–June, 1970), pp. 78–89.

[6] BUCHIN, S. I., *Smoothing a Cumulative Distribution Through a Set of Specified Points*, (Harvard Business School: Computer Programs for Managerial Economics, 1965).

[7] CARTER, E. E., "What are the Risks in Risk Analysis?", *Harvard Business Review*, Vol. 50, No. 4 (July–August, 1972), pp. 72–82.

[8] COLEMAN, J. S., "The Possibility of a Social Welfare Function", *American Economic Review*, Vol. 56, No. 5 (December, 1966), pp. 1105–1122.

[9] DILLON, J. L., "An Expository Review of Bernoullian Decision Theory in Agriculture: Is Utility Futility?", this *Review*, Vol. 39, No. 1 (March, 1971), pp. 3–80.

[10] EDWARDS, W., "Probability Preferences amongst Bets with Differing Expected Values", *American Journal of Psychology*, Vol. 67, No. 1 (March, 1954), pp. 56–67.

[11] EDWARDS, W., "Variance Preferences in Gambling Decisions", *American Journal of Psychology*, Vol. 67, No. 3 (September, 1954), pp. 441–452.

[12] EDWARDS, W., "Subjective Probabilities Inferred from Decisions", *Psychological Review*, Vol. 69, No. 2 (March, 1962), pp. 109–135.

[13] EDWARDS, W., L. D. PHILLIPS, W. L. HAYS and B. G. GOODMAN, "Probabilistic Information Processing Systems: Design and Evaluation", *IEEE Transactions on Systems Science and Cybernetics*, Vol. SSC–4, No. 3 (September, 1968), pp. 248–265.

[14] FELDSTEIN, M. S., "Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection", *Review of Economic Studies*, Vol. 36, No. 1 (January, 1969), pp. 5–11.

[15] FISHBURN, P. C., "Utility Theory", *Management Science*, Vol. 14, No. 5 (January, 1968), pp. 335–378.

[16] FRANCISCO, E. M. and J. R. ANDERSON, "Chance and Choice West of the Darling", *Australian Journal of Agricultural Economics*, Vol. 16, No. 2 (August, 1972), pp. 82–114.

[17] GINSBERG, A. S. and F. L. OFFENSEND, "An Application of Decision Theory to a Medical Diagnosis—Treatment Problem", *IEEE Transactions on Systems Science and Cybernetics*, Vol. SSC–4, No. 3 (September, 1968), pp. 355–362.

[18] GOOD, I. J., *Estimation of Probabilities: An Essay on Modern Bayesian Methods*, (Cambridge: M.I.T. Press, 1965).

[19] GRAYSON, C. J., *Decisions Under Uncertainty: Drilling Decisions by Oil and Gas Operators*, (Boston: Graduate School of Business Administration, Harvard University, 1960).

[20] HADAR, J. and W. R. RUSSELL, "Rules of Ordering Uncertain Prospects", *American Economic Review*, Vol 59, No. 1 (March, 1969), pp. 25–34.

[21] HALTER, A. N. and G. W. DEAN, *Decisions under Uncertainty with Research Applications*, (Cincinatti: South Western, 1971).

[22] HANOCK, G. and H. LEVY, "The Efficiency Analysis of Choices Involving Risk", *Review of Economic Studies*, Vol. 36, No. 3 (July, 1969), pp. 335–346.

[23] HAYS, W. L. and R. L. WINKLER, *Statistics: Probability, Inference and Decision*, Vol. 1, (New York: Holt, Rinehart and Winston, 1970).

[24] HIRSHLEIFER, J., *Investment, Interest and Capital*, (Englewood Cliffs: Prentice-Hall, 1970).

[25] HOCHANET, B., "Decision-Making in Roulette", *Acta Psychologica*, Vol. 34, No. 2/3 (December, 1970), pp. 357–366.

[26] HOINVILLE, G. and R. BERTHOUD, *Identifying and Evaluating Trade-Off Preferences*, (London: Social and Community Planning Research), 1970.

[27] KEENEY, R. L., "Utility Independence and Preferences for Multiattribute Consequences", *Operations Research*, Vol. 19, No. 4 (July-August, 1971), pp. 875–893.

[28] KEENEY, R. L., "Utility Functions for Multiattributed Consequences", *Management Science*, Vol. 18, No. 5 (January, 1972), pp. 276–287.

[29] KLEE, A. J., "The Role of Decision Models in the Evaluation of Competing Environmental Health Alternatives", *Management Science*, Vol. 18, No. 2 (October, 1971), pp. B52–B67.

[30] LINTNER, J., "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", *Review of Economics and Statistics*, Vol. 47, No. 1 (February, 1965), pp. 13–37.

[31] LUCE, R. D. and H. RAIFFA, *Games and Decisions*, (New York: Wiley, 1957).

[32] MACCRIMMON, K. R. and M. TODA, "The Experimental Determination of Indifference Curves", *The Review of Economic Studies*, Vol. 36(4), No. 108 (October, 1969), pp. 433–451.

[33] MANZ, W., "Experiments on Probabilistic Information Processing", *Acta Psychologica*, Vol. 34, No. 2/3 (December, 1970), pp. 184–200.

[34] MARKOWITZ, H. M., *Portfolio Selection—Efficient Diversification of Investments*, (New York: Wiley, 1959).

[35] MENGES, G., "On Subjective Probability and Related Problems", *Theory and Decision*, Vol. 1, No. 2 (October, 1970), pp. 40–59.

[36] MEYER, R. F. and J. W. PRATT, "The Consistent Assessment and Fairing of Preference Functions", *IEEE Transactions on Systems Science and Cybernetics*, Vol. SSC-4, No. 3 (September, 1968), pp. 270–278.

[37] MINAS, J. S. and R. L. ACKOFF, "Individual and Collective Value Judgements", in M. W. Shelly and G. L. Bryan, eds, *Human Judgements and Optimality*, (New York: Wiley, 1964), pp. 351–359.

[38] MOORE, C. V. and G. W. DEAN, "Industrialized Farming", in A. G. Ball and E. O. Heady, eds, *Size Structure and Future of Farms*, (Ames: Iowa State University Press, 1972).

[39] MURPHY, A. H. and R. L. WINKLER, "Scoring Rules in Probability Assessment and Evaluation", *Acta Psychologica*, Vol. 34, No. 2/3 (December, 1970), pp. 273–286.

[40] NEWMAN, J. W., *Management Applications of Decision Theory*, (New York: Harper and Row, 1971).

[41] OFFICER, R. R. and A. N. HALTER, "Utility Analysis in a Practical Setting", *American Journal of Agricultural Economics*, Vol. 50, No. 2 (May, 1968), pp. 257–277.

[42] PATTANAIK, P. K., *Voting and Collective Choice*, (Cambridge: Cambridge University Press, 1971).

[43] PHILLIPS, L. and W. EDWARDS, "Conservatism in a Simple Probability Inference Task", *Journal of Experimental Psychology*, Vol. 37, No. 3 (1966), pp. 346–354.

[44] PIERCE, D. A. and J. L. FOLKS, "Sensitivity of Bayes Procedures to the Prior Distribution", *Operations Research*, Vol. 17, No. 2 (March-April, 1969), pp. 344–350.

[45] PORTER, R. B. and J. E. GAUMNITZ, "Stochastic Dominance vs. Mean-Variation Portfolio Analysis: An Empirical Evaluation", *American Economic Review*, Vol. 62, No. 3 (June, 1972), pp. 438–446.

[46] PRATT, J. W., "Risk Aversion in the Small and in the Large", *Econometrica*, Vol. 32, No. 1–2 (January–April, 1964), pp. 122–136.

[47] PRATT, J. W., H. RAIFFA and R. SCHLAIFER, "The Foundations of Decision under Uncertainty: An Elementary Exposition", *Journal of the American Statistical Association*, Vol. 59, No. 306 (June, 1964), pp. 353–375.

[48] PRATT, J. W., H. RAIFFA and R. SCHLAIFER, *Introduction to Statistical Decision Theory*, preliminary edition, (New York: McGraw–Hill, 1965).

[49] RAIFFA, H., *Decision Analysis*, (Reading: Addison–Wesley, 1968).

[50] RAIFFA, H. and R. SCHLAIFER, *Applied Statistical Decision Theory*, (Boston: Graduate School of Business Administration, Harvard University, 1961).

[51] ROSING, J., "The Formation of Groups for Co-operative Decision Making under Uncertainty", *Econometrica*, Vol. 38, No. 3 (May, 1970), pp. 430–448.

[52] SAVAGE, L. J., *Foundations of Statistics*, (New York: Wiley, 1954).

[53] SAVAGE, L. J., "Elicitation of Personal Probabilities and Expectations", *Journal of the American Statistical Association*, Vol. 66, No. 336 (December, 1971), pp. 783–801.

[54] SCHLAIFER, R., *Probability and Statistics for Business Decisions*, (New York: McGraw-Hill, 1959).

[55] SCHLAIFER, R., *Analysis of Decisions under Uncertainty*, (New York: McGraw-Hill, 1969).

[56] SHARPE, W. F., *Portfolio Theory and Capital Markets*, (New York: McGraw-Hill, 1970).

[57] SPETZLER, C. S., "The Development of a Corporate Risk Policy for Capital Investment Decisions", *IEEE Transactions on Systems Science and Cybernetics*, Vol. SSC-4, No. 3 (September, 1968), pp. 279–299.

[58] STAEL, VON HOLSTEIN, C-A, S., *Assessment and Evaluation of Subjective Probability Distributions*, (Stockholm: Economic Research Institute, 1970).

[59] TIAO, G. C. and A. ZELLNER, "Bayes' Theorem and the Use of Prior Knowledge in Regression Analysis", *Biometrika*, Vol. 51, Nos 1 and 2 (June, 1964), pp. 219–230.

[60] TOBIN, J., "The Theory of Portfolio Selection", in F. H. Hahn and F. P. R. Brechling (eds), *Theory of Interest Rates*, (New York: St Martins Press, 1965), pp. 3–51.

[61] WHITMORE, G. A., "Third-Degree Stochastic Dominance", *American Economic Review*, Vol. 60, No. 3 (June, 1970), pp. 457–459.

[62] WILSON, R., "The Theory of Syndicates", *Econometrica*, Vol. 36, No. 1 (January, 1968), pp. 119–132.

[63] WILSON, R. B., "Decision Analysis in a Corporation", *IEEE Transactions on Systems Science and Cybernetics*, Vol. SSC-4, No. 3 (September, 1968), pp. 220–226.

[64] WINKLER, R. L., "The Assessment of Prior Distributions in Bayesian Analysis", *Journal of the American Statistical Association*, Vol. 62, No. 320 (September, 1967), pp. 776–800.

[65] WINKLER, R. L., "The Consensus of Subjective Probability Distributions", *Management Science*, Vol. 15, No. 2 (October, 1968), pp. B61–B75.

[66] WINKLER, R. L., "Scoring Rules and the Evaluation of Probability Assessors". *Journal of the American Statistical Association*, Vol. 64, No. 327 (September, 1969), pp. 1073–1078.

[67] WINKLER, R. L., "Probabilistic Prediction: Some Experimental Results", *Journal of American Statistical Association*, Vol. 66, No. 336 (December, 1971), pp. 675–685.

[68] WINKLER, R. L., *Introduction to Bayesian Inference and Decision*, (New York: Holt, Rinehart and Winston, 1972).

[69] ZECKHAUSER, R. and E. KEELER, "Another Type of Risk Aversion", *Econometrica*, Vol. 38, No. 5 (September, 1970), pp. 661–665.

[70] ZELLNER, A., *Introduction to Bayesian Inference in Econometrics*, (New York: Wiley, 1971).