CERTAINTY EQUIVALENCE AND BIAS IN THE MANAGEMENT OF PRODUCTION

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The study uses Theil's concept of certainty equivalence and bias to analyse the production decisions of the purely competitive firm when some of its inputs are controlled and others are non-controlled and subject to random influences. Conditions are outlined under which a surrogate procedure based upon the expected values of the non-controlled variables leads to certainty equivalence and others under which it leads to bias in the controlled variables. General factors which determine the direction of the bias are discussed. The managerial optimality of the surrogate procedure is investigated and also its welfare implications. The analysis is of particular relevance to agriculture because of the prevalence there of random non-controlled inputs in production.

1 BACKGROUND

Optimal current economic decisions are frequently altered by random elements even when decision-makers have preferences which are risk neutral. The best values for the controlled variables often depend not just on the expected values of the non-controlled variables but also on characteristics of the spread of the probability distribution of the non-controlled variables. As early as 1942, Hart [5] stressed that where a firm's actual profit depends on some uncontrolled random elements and the firm wishes to maximise expected profit, the procedure of replacing the non-controlled variables in the profit function by their expectations and selecting the values of the controlled variables by maximising the resultant expression does not as a rule lead to optimality.

Hart's warning about the likely illegitimacy of compounding probabilities [5] is of importance in agricultural management. According to the type of product which he supplies, a farmer's profit, or more generally his utility, depends upon random prices and random inputs such as the volume of rainfall and temperature, degree and type of pest infestation

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and disease and fertility of plants and of animals. As a rule a farmer's maximum expected profit, and more obviously maximum expected utility, depends upon the mean values of such random variables *as well as* higher moments of these variables and this must be allowed for in decision-making. The theory of certainty equivalance and bias enables us to determine the type of allowance, if any, which needs to be made.

Recently, Simon [12], Theil [13] and Malinvaud [10] distinguished some conditions under which optimal values of the controlled variables can be obtained by the above procedure and other conditions under which they cannot. Theil in particular has generalized and extended this subject and has applied his results to macroeconomic and microeconomic management. The concepts of certainty equivalence and certainty bias are used extensively by Theil in his work.

Certainty equivalence has two meanings in the economic literature but in this paper I shall only be concerned with certainty equivalence and bias in Theil's sense[1]. Theil states that certainty equivalence occurs if the procedure mentioned above results in optimal values for the controlled variables; otherwise certainty bias exists. To outline this concept more generally, consider a decision the aim of which is to maximise the expected value of an objective function which depends on controlled and (some random) non-controlled variables. Certainty equivalence exists if by replacing the non-controlled variables in the objective function by their expectations and maximising the resultant expression, the values obtained for the controlled variables are the same as those which maximise the expected value of the objective function. If these values for the controlled variables do not maximise the expected value of the objective function, certainty bias occurs and then the shortcut of replacing the random variables by their expected values and maximizing does not achieve the main aim.

In a multiperiod model, first-period certainty equivalence arises when the above procedure (the surrogate procedure) results in values of the controlled variables for the first period which are the same as those required to maximise the principle function[2]. However, if first-period certainty equivalence occurs, the values of the controlled variables for later periods derived by the surrogate procedure may not maximise the principle function and may need to be freshly determined as new information comes to hand.

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[1] For an outline of the other concept of certainty equivalence see for example Mishan [7, ch. 38] and Dillon [2].

[2] The function representing the expected value of the objective function will be called *the principle function.* The objective function in which the non-controlled variables are merely replaced by their expected values will be called *the surrogate function.* Thus if $U = U(x,y)$ is the objective function after all constraints are taken into account and x is a controlled variable and y is non-controlled, the principle function is $E[U] = E[U(x,y)]$ and the surrogate function is $U(x, E[y])$.

Because a firm's principle function may depend on many characteristics of the probability distribution of the uncontrolled variables, the firm may find it costly to specify this function. In comparison to the alternative of using the surrogate function, the extra gain from specifying and using the principle function may fail to cover the extra cost[3]. It is important from a managerial point of view to determine when this is so.

Clearly, if certainty equivalence exists, the surrogate function is just as efficient as the principle function for determining the optimal values of the controlled variables. If the firm's sole aim is to determine the optimal value of its controlled variables, nothing is gained by specifying the principle function and indeed there may be an extra cost. If there is an extra cost, the surrogate procedure is preferable. However, the surrogate procedure may still be optimal even when it leads to bias in the controlled variables, that is, to values which differ from those which maximize the principle function. The extra gain from specifying and using the principle function may not compensate for the extra cost. In various circumstances, the surrogate procedure is an optimal rule-of-thumb in the same way as Baumol and Quandt [1] show various other rules-of-thumb to be optimal. The conditions under which the surrogate procedure is optimal, is likely to be optimal, or would be optimal if slightly modified, affect the best way of reaching managerial decisions.

Besides applications to managerial economics, the analysis has prescriptive social application. For example let us suppose that most firms do use the surrogate procedure. This can result in a consistent collective (industry-wide) bias in production which might be corrected to social advantage by either taxing or subsidising production, depending upon the circumstances.

In order to make these points concrete and to examine circumstances which tend to make the surrogate procedure optimal, I shall concentrate on the production decisions of a purely competitive firm which aims to maximise its expected profit. In section 2 a simple model is analysed and in section 3 some generalizations are considered in order to isolate the crucial factors which influence bias from the surrogate procedure.

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[3]Where π' represents the firm's profit after deducting decision cost D_c, $\pi' = \pi - D_c$. Hence, where $\bar{L}$ represents the optimal value of the controlled variable obtained from the principle procedure and $\hat{L}$ that from the surrogate procedure, the principle procedure is only preferable if

$$\pi(\bar{L}) - D_c(\bar{L}) > \pi(\hat{L}) - D_c(\hat{L}).$$

As a rule, D_c is larger for the principle procedure. In practice, values of the variables in the above inequality are not likely to be perfectly known by the decision-maker and thus expected values may need to be used.

2 A SIMPLE ILLUSTRATION FROM PRODUCTION THEORY OF CERTAINTY EQUIVALENCE AND BIAS

Consider a purely competitive firm which wishes to maximise its expected profit and suppose that its profit depends upon a controlled variable, L, and a non-controlled variable, ε. The uncontrolled variable may enter into the profit function directly through the production function or it may come in indirectly. In the direct case, the random factor influencing production may for example in agriculture be the amount of rainfall, sunlight, warmth or the degree of infestation of a crop by pests.

The firm's output in any period is assumed to depend on its chosen value of L and the value of an uncontrolled variable ε. Thus the firm's production function is

(1) $$x = H(L,\varepsilon)$$

and its profit is

(2) $$\pi = px - wL,$$

where p is the price per unit of the product and w the price per unit of the factor. Hence, the firm's profit function (its objective function according to the above usage) is

(3) $$\pi = p\, H(L,\varepsilon) - wL.$$

Assume that the firm knows p and w when L must be determined, but not ε. The firm's expected profit function, its principle function, is then

(4) $$E[\pi] = p\, E[H(L,\varepsilon)] - wL.$$

When does the value of L which maximises the surrogate function,

(5) $$R = p\, H(L, E[\varepsilon]) - wL$$

also maximise the value of the principle function?

If the contributions of L and ε to output are *independent*, certainty equivalence clearly occurs. For example, of the firm's production function is

(6) $$H(L,\varepsilon) = f(L) + T(\varepsilon),$$

$E[\pi] - R = p\{E[T(\varepsilon)] - T(E[\varepsilon])\}$, so that the difference is constant and independent of L. But if the contributions of the controlled variable, L, and the non-controlled variable ε to output are interdependent, certainty equivalence also occurs if their *joint interdependent contribution to output depends linearly on* ε. Thus for example certainty equivalence occurs if

(7) $$H(L,\varepsilon) = f(L) + \psi(L)\,(\beta_1 + \beta_2\varepsilon) + T(\varepsilon).$$

The second term in the R.H.S. of (7) shows that output is jointly influenced by L and ε, but ε enters only linearly in *this* term. Thus the functions $E[\pi]$ and R once again only differ by the constant $p\{E[T(\varepsilon)] - T(E[\varepsilon])\}$ and certainty equivalence is maintained.

Function (7) allows for a wide variety of production relationships. For example ψ' and ψ'' might be of any sign, and consequently the cross marginal physical productivity of L can be of any sign and may be a diminishing, increasing or constant function of L. Furthermore, $T(\varepsilon)$ can be of any nature. Nevertheless the linearity in ε of the cross production relationship might well be regarded as restrictive. The cross contribution to output might in reality be a diminishing marginal value of ε or an increasing one, rather than a constant marginal value. In these circumstances, certainty bias is *likely* because the optimal value of L will depend upon characteristics of the distribution of ε besides its mean.

Take a very simple example. Assume that the firm's production function is

$$(8) \qquad x = -aL^2 + bL - \lambda L\varepsilon^2.$$

The firm's profit function is

$$(9) \qquad \pi = p(-aL^2 + bL - \lambda L\varepsilon^2) - wL$$

and ε enters non-linearly into the cross production term involving L and ε.

To evaluate the bias introduced by the surrogate procedure, consider the principle function and then the surrogate function. Noting that the expectation of ε^2, that is $E[\varepsilon^2]$, equals $[E[\varepsilon]]^2$ + var ε, the principle function is

$$(10) \qquad E[\pi] = p(-aL^2 + bL - \lambda LE[\varepsilon^2]) - wL$$

$$(11) \qquad = p(-aL^2 + bL - \lambda L\{[E[\varepsilon]]^2 + \text{var } \varepsilon\}) - wL.$$

The necessary condition for a maximum of this function is that

$$(12) \qquad \frac{dE[\pi]}{dL} = p(-2aL + b - \lambda\{[E[\varepsilon]]^2 + \text{var } \varepsilon\}) - w = 0,$$

and this is satisfied if

$$(13) \qquad \bar{L} = \frac{1}{2a}\left(-\frac{w}{p} + b - \lambda\{[E[\varepsilon]]^2 + \text{var } \varepsilon\}\right).$$

Hence, the optimal value of L varies inversely with the variance of the non-controlled variable, ε. For example, ε may be the amount of precipitation or the deviation of annual rainfall from some ideal amount for the growing of a crop.

The surrogate function is

$$(14) \qquad R = p(-aL^2 + bL - \lambda L[E[\varepsilon]]^2) - wL.$$

The requirement for a maximum of this function is that

$$(15) \qquad \hat{L} = \frac{1}{2a}\left(-\frac{w}{p} + b - \lambda[E[\varepsilon]]^2\right).$$

Thus the surrogate procedure leads to a value for $\hat{L}$ which is in excess of $\bar{L}$. In fact, it leads to the selection of a value for L which is biased above the optimal value by $\frac{1}{2a}$ var ε.

Figure 1 illustrates the type of relationship which exists between functions (11) and (14) if a given probability distribution for ε is assumed. The difference between the two functions equals $p\lambda L$ var ε. The surrogate procedure overestimates expected gains and involves a loss (in terms of the alternative expected profit foregone) of *AB*.

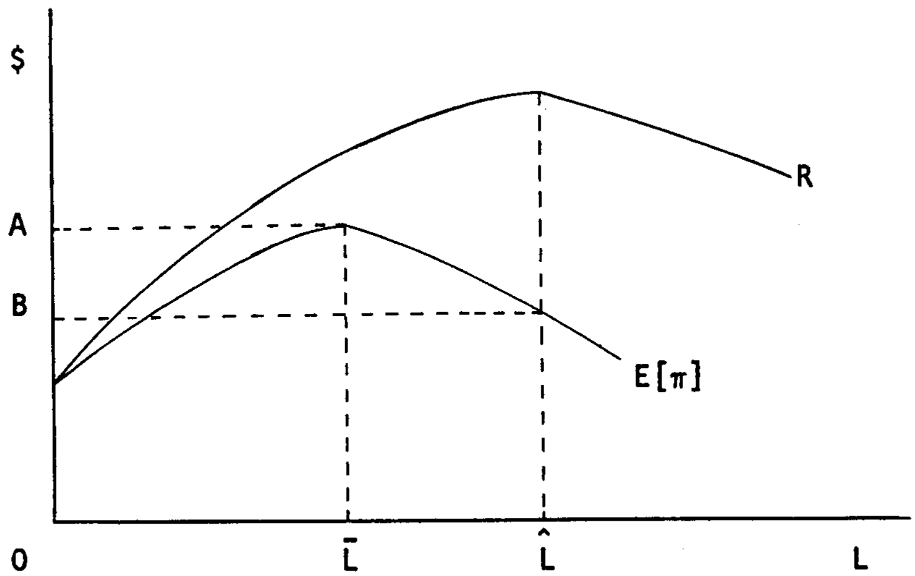

FIGURE 1: *Principle and surrogate profit functions*

Other things equal, the expected profit foregone by adopting the surrogate procedure is less the greater is $2a$. Expected profit foregone varies inversely with the rate of decline in the marginal productivity of L in producing x. This can be shown algebraically but is also easy to illustrate diagramatically.

In figure 2 the surrogate marginal expected productivity of L is shown by curve *BD*. The surrogate production function, as can be seen from equation (8) is

$$x = -aL^2 + bL - \lambda L[E[\varepsilon]]^2, \tag{16}$$

so that the surrogate marginal expected productivity of L is

$$\frac{dx}{dL} = 2aL + b - \lambda[E[\varepsilon]]^2. \tag{17}$$

However, the true expected production is

$$x = -aL^2 + bL - \lambda L[E[\varepsilon]]^2 - \lambda L \text{ var } \varepsilon \tag{18}$$

and thus the true marginal expected productivity of L is

$$\frac{dx}{dL} = -2aL + b - \lambda[E[\varepsilon]]^2 - \lambda \text{ var } \varepsilon. \tag{19}$$

This function differs from (17) by the constant $-\lambda$ var ε and for the hypothetical case illustrated in figure 2 might be represented by curve *CK*, given that the surrogate marginal expected productivity curve of L is *BD*.

If expected profit is to be at a maximum as can be seen from rearranging (12), L must be such that the true marginal expected productivity of L equals $\frac{w}{p}$. As is apparent from the first-order condition for a maximum of (14), the maximum of the surrogate profit function requires that the surrogate marginal expected productivity of L be equated to $\frac{w}{p}$. This occurs in figure 2 for the value of L at which CK intersects the line $\frac{w}{p}$. Thus, given that CK is the true marginal expected productivity of L and BD the surrogate, real expected profit equivalent to the hatched triangle is foregone by using the surrogate procedure.

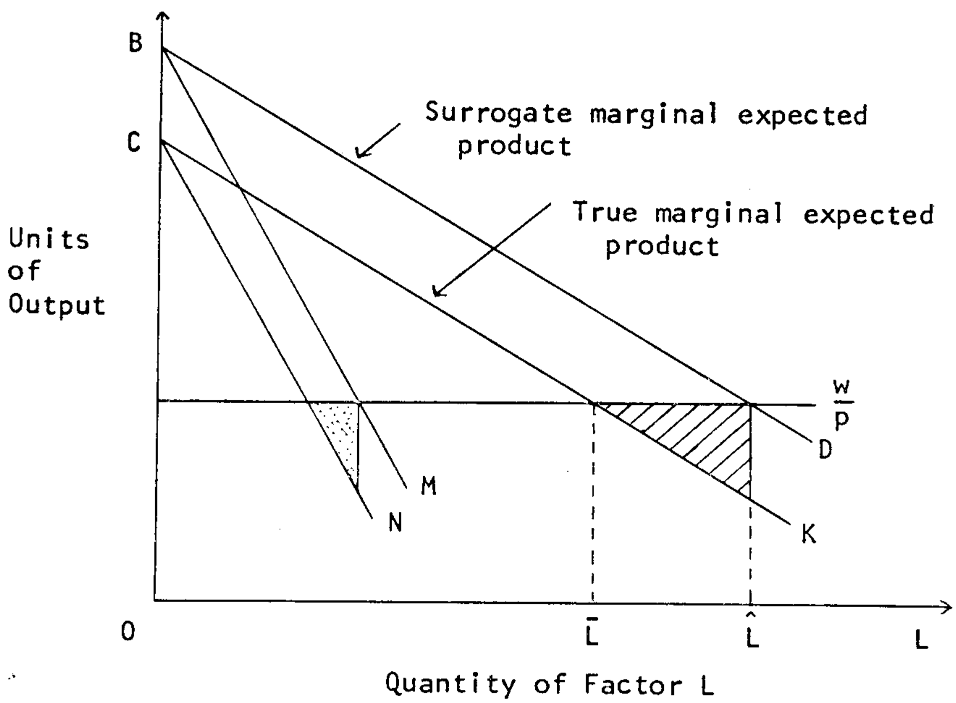

FIGURE 2: *Principle and surrogate marginal product functions*

The profit foregone by using the surrogate procedure becomes smaller (other things unchanged) as output becomes less responsive to changes in L i.e., the greater is the rate of decline of $\frac{dx}{dL}$. As $2a$ becomes greater, both the surrogate and true marginal expected productivity curves become steeper about their respective fixed points C and B, but the difference between the curves remains a constant. In the case shown in figure 2, $2a$ may increase so that the surrogate marginal expected productivity curve is BM and the true one is CN. As $2a$ increases, other things equal, the intercept of the true marginal and corresponding surrogate curve with the line $\frac{w}{p}$ becomes smaller. Hence, the base of the triangle representing expected profit foregone becomes smaller. However, the difference between the curves remains constant (i.e. equal to λ var ε) so that the height of these triangles remains constant. Thus, by the rule for finding the area of triangles, the area of the relevant triangles becomes smaller as $2a$ increases and the real expected profit foregone by adopting the surrogate procedure becomes less. Given steeply diminishing marginal productivity of the controlled variable, the extra information contained in the principle procedure adds little to expected profit and may not cover the extra cost of collecting and using it. This situation parallels similar ones which have been considered by Nelson [8] and Tisdell [15].

This example has indicated some of the managerial implications of certainty bias and can illustrate some of its consequences for economic welfare. Let us assume that all firms use the surrogate procedure to determine their production. Consequently, output of particular commodities may be above or below a socially optimal level. I shall take it that Paretian optimality requires the value of the true marginal expected output from each factor of production to be equal to its price (cost) per unit. If the random elements influencing the production of each firm tend to be independent between firms, then by the law of large numbers, industry output will show little random variation and the above condition seems necessary for Paretian optimality.

For simplicity, assume within the economy that only the production of product x involves any uncertainty.[4] Suppose that a given number of firms produce x and that they all adopt the surrogate procedure and have the same (or similar) production function as in (8) with ε entering as the only random element. The result is that a greater than optimal amount of L is allocated to the production of product x. Each firm employs $\hat{L}$ of L so that, as figure 2 indicates, the value of true marginal expected output of L in producing x is below the price or cost per unit of L. This price also measures the value of output forgone elsewhere. There is over-production of product x from an efficiency point of view. This could be corrected by suitably taxing the use of L in producing x, or in this case by taxing the output of x. The required per unit real tax on the employment of L is λ var ε which is equivalent to BC in figure 2.

The larger is the variance of ε, other things equal, the greater is the over-allocation of L to producing x and the higher the required per-unit tax. Firms operating in districts in which for example, (assuming that ε is an indicator of rainfall) rainfall is very variable, may overexpand their use of L by a much greater amount than those in districts with less variable rainfall. In order to achieve the necessary condition for Paretian optimality, a higher per-unit tax would be required on producers in areas subject to more variability of rainfall.

[4] We could also suppose that there is uncertainty elsewhere but that the optimal rule is applied in producing products other than x. The results below would then apply. However, a difficulty can arise if uncertainty occurs elsewhere and the surrogate procedure is used universally. The uncertainty can then become Paretian irrelevant in the same was as Kahn [6] indicates that universal monopoly or universal externalities *can* become Paretian irrelevant.

Of course, this does assume that all firms use the surrogate procedure. It is a matter for empirical investigation to find out how widely it is used. But if it is widely used, social implications follow, some of which may be of particular relevance to Australian agricultural districts that have variable weather patterns which affect production.

If the sign of the term $\lambda L\varepsilon^2$ in the production function, (8), is changed from negative to positive the direction of the bias introduced by the surrogate procedure is altered. Thus from the point of view of the firm's management the surrogate procedure leads to a less than optimal employment of factor L. However, it is still true that, other things equal, the extent of this error and the expected profit forgone varies inversely with the steepness with which the marginal productivity of L declines. On the social plane, and assuming the collective conditions mentioned above, too little of resource L is devoted to the production of product x if the surrogate procedure is widely used. This could be corrected by a per-unit subsidy on the employment of L in the industry. The real per-unit subsidy to each firm needs to be λ var ε per unit of L employed in producing x. Thus firms experiencing the greatest variance in ε would need to be paid the greatest per-unit subsidy. The policy conclusions are therefore, reversed in this case. It is, of course, an empirical matter to find out the type of interrelationships in production which are really relevant.

In order to facilitate application of the analysis, it is helpful to generalize it. To generalize it to cases involving quadratics in ε in the interdependent production terms is straight-forward but cumbersome. Rather than do this, let us proceed more generally for the type of production function considered above.

3 TOWARDS GENERALIZATION OF THE DIRECTION OF BIAS INTRODUCED BY THE SURROGATE PROCEDURE

While the direction of bias introduced into the controlled variable by the surrogate procedure is obvious in the previous case, the direction of the bias is not immediately apparent in the more general case. Nevertheless, we can set out the factors which determine this bias. By knowing these, we can predict the qualitative correction to the value of the controlled variable which will increase expected profit. Furthermore, we can determine (given certain conditions and the use of the surrogate procedure by firms in the industry) whether taxation *or* subsidization of the employment of the controlled variable in the industry will promote economic efficiency and whether greater or smaller marginal fiscal payments are required when the value of the non-controlled elements is more variable.

Assume that the firm's production function is of the form

$$x = f(L) + h(L)g(\varepsilon) + T(\varepsilon) \tag{20}$$

The middle term on the R.H.S. of (20) indicates the interdependence between L and ε in production involves separability and could for example be of the Cobb-Douglas type. Since only optimal values for

L are to be considered, nothing is lost in generality if the last term on the R.H.S. of (20) is dropped. Thus the relevant production function becomes

$$x = f(L) + h(L)g(\varepsilon). \tag{21}$$

In view of this, the firm's profit function is

$$\pi = p(f(L) + h(L)g(\varepsilon)) - wL, \tag{22}$$

and hence the firm's principle function is

$$E[\pi] = p(f(L) + h(L)E[g(\varepsilon)]) - wL, \tag{23}$$

and its surrogate function is

$$R = p(f(L) + h(L)g(E[\varepsilon])) - wL. \tag{24}$$

The principle and surrogate functions are distinguished by the difference between the terms $E[g(\varepsilon)]$ and $g(E[\varepsilon])$.

If $E[\pi]$ is to be at a maximum with respect to L, L must be such that

$$f'(L) + h'(L)E[g(\varepsilon)] = \frac{w}{p}. \tag{25}$$

The true marginal expected productivity of L must be equal to the real price of L. By contrast, the necessary condition for a maximum R with respect to L is that L be such that

$$f'(L) + h'(L)g(E[\varepsilon]) = \frac{w}{p}. \tag{26}$$

Let $\bar{L}$ and $\hat{L}$ represent the values of L which respectively satisfy equations (25) and (26) and also meet the second-order conditions for a maximum of $E[\pi]$ and R respectively. These results follow:

The difference between $\hat{L}$ and $\bar{L}$, the direction of the bias if any, depends upon the sign of $h'(L)g(\varepsilon)$ and on whether $g(\varepsilon)$ is strictly concave, strictly convex or linear. If either $h'(L)g(\varepsilon)$ is zero or $g(\varepsilon)$ is linear, then certainty equivalence occurs. In the latter case, $E[g(\varepsilon)] = g(E[\varepsilon])$.

Consider cases in which ε has a cross effect on production and $g(\varepsilon)$ is not linear.

(i) *Take the case where $h'(L)g(\varepsilon) > 0$ for all ε which are probable and assume that $g(\varepsilon)$ is strictly concave, i.e., $g'' < 0$.* Two possibilities can be distinguished. If $g(\varepsilon) > 0$ for ε all which are probable, then $h'(L) > 0$ and $\hat{L} > \bar{L}$. The surrogate value of L exceeds the optimal value. On the other hand, if $g(\varepsilon) < 0$ for all ε which are probable, then $h'(L) < 0$ and $\hat{L} < \bar{L}$. The surrogate result is less than the optimal value for L.

Proof: In the possibility where $g(\varepsilon) > 0$ for all probable ε, $h'(L)$ is positive. In view of the strict concavity of $g(\varepsilon)$, it follows from Polya's theorem [4, p. 74] that $g(E[\varepsilon]) > E[g(\varepsilon)]$ and so $h'(L)g(E[\varepsilon]) > h'(L)E[g(\varepsilon)]$. Thus the curve representing the surrogate marginal expected productivity of L is higher than the curve representing the true marginal expected productivity of L. With $\frac{w}{p}$ constant, it follows that $\hat{L} > \bar{L}$. This is illustrated by the example in figure 2.

In the possibility where $g(\varepsilon) < 0$ *for all probable* ε, $h'(L)$ is negative. Since $g'' < 0$, it is still true that $g(E[\varepsilon]) > E[g(\varepsilon)]$. However, the inequality of the absolute values of these negative quantities is reversed. If $g(\varepsilon) < 0$, $|g(E[\varepsilon])| < |E[g(\varepsilon)]|$. Thus given the double negative in the cross expression for the marginal product of L, $h'(L)g(E[\varepsilon]) < h'(L)E[g(\varepsilon)]$. Hence, in this case the surrogate marginal expected product curve for L lies below the true marginal expected product curve of L. Therefore, $\hat{L} < \bar{L}$. The surrogate value of L is biased downward so an upward correction would be called for if the decision-maker wished to get closer to optimality.

(ii) *Take the case where* $h'(L)g(\varepsilon) < 0$ *for all* ε *which are probable and assume that* $g(\varepsilon)$ *is strictly concave, i.e.,* $g'' < 0$. Then *if* $g(\varepsilon) > 0$ for all probable ε, $\hat{L} < \bar{L}$ and the surrogate procedure biases the value of L downward. Corrective subsidies can improve economic efficiency. On the other hand, if $g(\varepsilon) < 0$, $\hat{L} > \bar{L}$, the surrogate procedure leads to upward bias in L and corrective taxes can increase economic efficiency. The proof follows in a similar way to that stated in (i) above.

(iii) *If* $g(\varepsilon)$ *is strictly convex,* i.e. $g'' > 0$, each of the above propositions is reversed, for if $g(\varepsilon)$ is strictly convex rather than strictly concave $g(E[\varepsilon]) < E[g(\varepsilon)]$. Thus the relative positions of the surrogate and true marginal expected productivity curves are reversed and the policy conclusions correspondingly changed.

It is impossible to summarize the full range of these relationships briefly in a non-technical fashion. However, note that when the cross marginal physical product of L is positive and the random non-controlled input adds positively to output through the cross production relationship, the surrogate procedure causes the value of L to be biased downward if $g(\varepsilon)$ is strictly convex, and to be biased upward if $g(\varepsilon)$ is strictly concave. When $g(\varepsilon)$ is strictly convex the marginal variation in g for a change in ε (and also the consequent change of output) is at a decreasing rate, whereas the opposite is so when $g(\varepsilon)$ is strictly concave.

4 CONCLUDING DISCUSSION

The above production functions involve only one controlled variable (a variable input) and one random non-controlled variable (another input). Because of the multiplicative term in the production function the cross contributions of the two factors to production are separable. The discussion can be extended to include a number of variable controlled factors and non-controlled factors in the production function. For example, this is easy if say K is a vector of factors and the production function is

(27) $$x = f(K,L) + h(L)g(\varepsilon) + T(\varepsilon).$$

More complicated relationships could be cited. However, unless a particular problem needs to be solved, there seems little point in outlining these possibilities. The simple case of one controlled and one non-controlled factor indicates the type of relationships (such as convexity or otherwise of particular functions) which are liable to have an influence in the more general case and the variety of results which are possible.

It is also apparent that the degree of flexibility which it is optimal to incorporate into the productive process can be easily analysed by using production functions with separable cross terms. To decide between alternative techniques if the firm wishes to maximize expected profit, it is necessary (given one controlled and one non-controlled factor) to examine the size of the area between the line $\frac{w}{p}$ and the alternative true marginal expected productivity curves of L which correspond to the different techniques. The analysis is relevant whether ε enters in a direct way into production or indirectly through its adjustment to another random variable, such as a price, the value of which becomes known or less uncertain with the passage of time. Because it specifically identifies the source of the possible alteration of output, this approach to considering flexibility has an advantage over those which merely concentrate on the cost of changing planned output [14, 16].

In the above analysis, it is supposed that utility is a linear function of profit. The extent to which this is a restrictive assumption in practice is debatable. Nevertheless there is evidence that farmers' utility functions are on the whole non-linear. For example, Officer, Halter and Dillon [11] found from their early study in the Armidale region that farmers tend to be risk averse. Yet this aversion is surprisingly small if one judges by the coefficients which they give in their table 1 [11, p. 173]. Stronger evidence of non-linearity of utility functions comes from a study by Francisco and Anderson [3] of pastoralists' decisions in the West Darling region. In further extension of the present analysis it does seem desirable to relax the non-linearity assumption. Even when the assumption is relaxed, the surrogate procedure can be shown to be an optimal rule-of-thumb under a range of conditions, and important managerial implications still follow.

Yet analysis in terms of expected profit remains significant. It sometimes can be used to indicate optimal choices from a social welfare point of view. Secondly, the analysis demonstrates that uncertainty may rationally cause a firm which is maximizing its expected profit to bias its controlled variables away from those values which are optimal when the non-controlled variables are equal to their certainty equivalents. Thus bias of the controlled variables away from values which are optimal in the absence of uncertainty (or when certainty equivalents are used), such as conservative stocking rates which McArthur and Dillon [9] explain on the basis of risk aversion, can also arise in risk-neutral situations if production functions are of a particular nature. In practice, both risk preferences *and* "real" factors are likely to cause systematic bias in decisions made under uncertainty. So also are the rules-of-thumb and surrogate procedures which are adopted by farmers. Theoretical and empirical work on decision-making must take account of these three factors.

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