A Note Comparing Single-Index Models and Quadratic Programming Models for Farm Planning Under Risk

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Single-index models from portfolio theory have previously been adapted for risk efficient farm planning in North America. The potential for using single-index models in farm planning is considered in this paper both theoretically and in the light of two illustrative Australian case studies. It is concluded that single-index models have no significant computational or other advantages over full quadratic programming portfolio selection models for farm planning and may produce relatively poor plans and poor assessments of the risks associated with those plans.

1. Introduction

A common planning problem in finance, farming and other contexts is that of choosing amongst various combinations of risky and often interrelated activities, that is, the problem of identifying an optimal portfolio. Although most economists would view the optimal portfolio as the one which maximises the decision maker's utility function, procedures used for portfolio selection have been somewhat pragmatic. In this paper, the role of 'single-index' portfolio models in farm planning is considered.

Modern portfolio theory dates back to Markowitz (1952) who showed how the standard deviation of a portfolio of stocks could be reduced by choosing stocks which do not 'move together'. He then presented the basic principles of portfolio construction based on the assumption that decision makers preferred greater expected return and less risk of return, developing the notion of mean-variance (EV) 'efficient' portfolios: portfolios such that no other portfolio exists having less (/as little) risk (variance) and as much (/more) expected return. Markowitz showed that quadratic programming (QP) can be used to obtain the set or frontier of EV efficient portfolios.

At about the same time, Heady (1952) identified the role that diversification on farms could play in reducing risk to achieve stability in farm incomes. Whilst not developed under the title of portfolio selection, Heady's concepts are quite similar to the independent work of Markowitz.

It is now known that, from a utility theory perspective, the set of EV efficient portfolios contains the optimal portfolios for all risk averse decision makers seeking to maximize utility of income if the activity returns are normally distributed. For other types of distributions of returns however, the EV efficient set generally holds the optimal portfolios only for decision makers with quadratic utility functions. Thus, the set of EV efficient portfolios is generally not identical to the set of utility efficient portfolios.

While the notion of utility efficiency is well developed in the stochastic dominance litera-

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Review coordinated by Bob Farquharson.

ture (for example Drynan 1986), and procedures for the identification of the efficient sets for some special classes of utility functions and particular forms of uncertainty are being devised (see for example Hardaker *et al* 1991), procedures for identifying the full set of utility efficient portfolios remain elusive. In practice, portfolio theory continues to be applied largely within an EV framework.

Even in this context, considerable difficulty was experienced with the use of QP codes to solve the portfolio selection problem until the early 1970s when the Rand Corporation code became available to researchers (see Takayama and Batterham 1972). These early difficulties led researchers interested in EV portfolio selection to seek alternative methods for identifying the set of efficient portfolios. This work was initiated by Sharpe (1963, 1970) who showed that a significant amount of the variation in the returns to activities could be captured via the relationship that the return from each activity has with some common factor, thereby allowing the use of 'single-index' models to approximate the variance measure of risk in portfolio selection problems.

Single-index models have the advantage that activity (and portfolio) risk can be specified with many fewer parameters than are necessary in specifying the variance-covariance matrix required in the general QP model for portfolio selection. A further advantage of some versions of the single-index model is that the set of efficient portfolios can be determined using linear programming (LP). In the past, LP codes were more widely available, generally more familiar to researchers, and more reliable than QP codes. Index models became relatively widely used for portfolio selection, at least in the scholarly finance literature (see Harrington 1987).

Agricultural economists concerned with farm planning under risk have also sought simple alternatives to QP and a number of North American studies based on single-index models have been reported (examples are Collins and Barry 1986; and Turvey, Driver and Baker 1988). Australian agricultural economists have been active in the search, but have chosen means other than single-index models for simplification, most notably MOTAD models, focus-loss models, and linear-segmented objective functions (Hardaker *et al* 1991). The application of single-index models in a New Zealand farm setting has been reported in the Australian literature (Johnson 1992), but there seems to be no published paper exploring their application in Australian farm planning.

Several hypotheses could be put forward to explain the relative lack of interest in single-index models, ranging from lack of awareness, inapplicability of the assumptions to farm planning, difficulties in application, to a simple lack of any incentive to adopt such models. It is not the intention in this paper to search for an explanation of the past behaviour of our profession. Instead, the main aim is to examine what these models have to offer farm planning to-day by presenting the basics of single-index models and some results from illustrative case studies. Two versions of the Sharpe single-index model are outlined. The models are used to derive EV efficient portfolios for two farm case studies in central and southern New South Wales and these sets of portfolios are compared to those produced by QP.

2. The Quadratic Programming Model

Quadratic programming can identify the set of EV efficient farm plans under the usual farm planning assumption of a known linear constraint set. The QP model can be expressed as follows:

Maximise $Z = c'x - \lambda x'Qx$

subject to $Ax \leq b$

and $x \geq 0$

where Z is the objective function,

c is the vector of expected activity returns,

Q is the variance-covariance matrix of activity returns,

x is the vector of activity levels,

λ is a parameter which is varied to alter the relative weights on expected return and variance of returns to produce the EV frontier,

A is a matrix of input-output coefficients,

b is the vector of resource constraints or requirements.

The QP model requires knowledge of the mean and variance of the gross margin for each activity considered and of the covariances between gross margins. As these parameters are unknown, sample means and variance-covariances based on historical time series of gross margins are commonly used as estimates. This introduces a further element of uncertainty that strictly should be taken into account, but which is typically ignored in deriving EV efficient plans.

3. Single-Index Models

Sharpe's single-index model (1963,1970) represents a special case of the QP model for deriving the EV frontier. The model is based on the assumption that each activity's return (R_i) is related to some common factor (R_m) and dependent on a random element (ε_i):

(1) $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$

In farm planning situations R_i in equation (1) represents the gross margin for the ith activity; α_i a constant; β_i the expected change in R_i in response to a change in R_m; R_m the index; and ε_i an error term.

According to Sharpe, the common factor R_m should be the single most important factor influencing returns. In the work on index models in finance, this factor is the 'market portfolio return'. One or other of the available financial indices is used as a proxy for the market portfolio. Relationships among securities are derived from common relationships with the index. In farm planning, Collins and Barry (1986) and Turvey, Driver and Baker (1988) used a reference farm portfolio for R_m. However any factor believed to explain a significant amount of the joint variation in activity returns could be used. If, for example, the set of feasible activities consisted only of dryland winter cropping alternatives, rainfall throughout the growing season or subsoil moisture may provide a reasonable explanation of the variations in gross margins.

The error term ε_i reflects factors unique to the individual activity itself and unrelated to the level of the index. Its expected value is assumed to be zero. The expected return to activity i is then:

(2) $E_i = \alpha_i + \beta_i E_m$

where E_m is the expected level of the index.

The risk associated with activity i, measured by the variance of returns ($\sigma_{Ri}2$), can be divided into two parts, systematic (or market) risk, and non-systematic (or unique) risk:

(3) $\sigma_{Ri}2 = V(\alpha_i + \beta_i R_m + \varepsilon_i)$

$$= V(\beta_i R_m) + V(\varepsilon_i)$$

$$= \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon i}^2$$

where σ_m^2 is the variance of the common factor or index,

$\sigma_{\varepsilon i}^2$ is the variance of the error term, and

V(.) also denotes variance.

The first term in (3) is the systematic risk. The second term is the risk unique to the particular activity. The systematic risk for a particular activity depends on its ß coefficient and the variation in the common factor. No covariance terms appear in (3) because the unique factors are assumed independent of the index.

The returns from any two different activities will be correlated because of their joint dependence on the common single index:

(4) $Cov_{ij} = \beta_i \beta_j \sigma_m^2$

The set of efficient farm plans can be calculated via parametric QP using the variance-covariance matrix formed with the variances and covariances defined above. The various parameters are unlikely to be known and will usually need to be estimated, for example by ordinary least squares regressions of activity gross margins on the index.

In summary, given an appropriate index, Sharpe's single-index model allows the mean and variance of a portfolio to be calculated with knowledge only of the variance of the common factor ($\sigma^2{}_m$) and of the following parameters for each activity: its expected return (E_i); the responsiveness of return to changes in the level of the index (β_i); and its unique or non-systematic risk ($\sigma_{\varepsilon i}^2$). The ready availability of activity betas, and hence reduced primary information requirements, have given this model considerable appeal in the finance area.

3.1 The Diagonal Model

A particular portfolio of n activities at levels X_i, i=1...n, has an expected return

(5) $E_p = \sum_{i=1}^{n} X_i E_i$

and variance

(6) $\sigma_p^2 = (\sum_{i=1}^{n} X_i \beta_i)^2 \sigma^2{}_m + \sum_{i=1}^{n} X_i^2 \sigma_{\varepsilon i}^2$

The form of the expression for the portfolio variance suggests that the QP problem in n activities with a variance-covariance matrix based on a single-index can be rewritten as one in n+1 activities with a diagonal variance-covariance matrix. That is, if an n+1th activity is defined (along with an additional constraint) as the beta-weighted sum of the first n activities ($X_{n+1} = \Sigma X_i \beta_i$), portfolio expected return and variance are correctly calculated when this new activity is given a zero expected gross margin and the variance-covariance matrix is defined as diagonal with the unique risks in the first n positions and the variance of the common factor in the n+1th position. This diagonal representation of the single-index model is usually used in preference to the non-diagonal equivalent since it avoids pre-QP calculation of the systematic risks and covariances.

3.2 The Minimise Portfolio Beta Model

Sharpe (1963) argued that when a portfolio of activities is well diversified, non-systematic risk becomes relatively small and, as an approximation, can be ignored. Returns to all activities are effectively assumed to be perfectly correlated. Equation (6) reduces to:

(7) $\sigma_p^2 = (\sum_{i=1}^{n} X_i \beta_i)^2 \sigma^2{}_m$

Portfolio variance will then be minimised when portfolio beta ($\Sigma X_i \beta_i$) is minimised. The problem can therefore be solved using LP.

Often there will be no a priori reason to believe that a well-diversified portfolio is optimal (Frankfurter and Booth 1985) and thus there will be a danger that the error in approximating the diagonal model may not be insignificant. The more relevant concern is whether the implied variance-covariance matrix can approximate well the general or full variance-covariance matrix and so identify portfolios which are close to being EV efficient with respect to the latter matrix.

A major advantage of the minimise portfolio beta model is the further reduction in parameters needed to specify the model. A measure of each activity's beta, the risk or variance of the index, and each activity's expected return are the only items needed to identify efficient portfolios. The ability to solve the problem using LP is an added bonus.

4. The Case Studies

The analysis is based on two sets of historical gross margins obtained for two mixed livestock and cropping areas within New South Wales. The raw data along with the mean and variance-covariance estimates for the two farms are outlined in Appendix 1.

In the first case study, the gross margins were based on price, yield and variable cost records for the property 'Bull Plain' near West Wyalong in the South West Slopes and Plains. Additional data for activities considered in the farm planning exercise, but not previously pursued on the property, were obtained from neighbouring properties that have similar resources and productive capacity. Price figures were indexed to 1990 prices using an index of prices received by farmers published by the Australian Bureau of Agricultural and Resource Economics (ABARE) and detrended in order to provide a measure of stochastic variation about the expected prices at any time. Variances were assumed to be unchanging over time and were estimated as if the observations represented the entire population of possible gross margins. Costs were assumed to be constant at 1990 levels. Information that could not be provided from the property records or from neighbouring properties was obtained from district records obtained from the local office of New South Wales Agriculture.

In the second case study, the gross margins were based on historical records of price, yield and variable costs associated with a number of farm activities undertaken in the Orange District in the Central Tablelands. The data relates to the period 1975 to 1989. For the cropping activities considered, average yield data from the Shire of Weddin as published by the Australian Bureau of Statistics, served as the basis of production variability over the period. The use of Shire data will obviously reduce estimates of the variability of yields, and hence gross margins, as compared to estimates derived from farm level data.

Grain prices were based on Sydney retail feed ingredient bi-monthly estimates provided by New South Wales Agriculture. Sheep enterprise returns reflected variability in wool production per head, wool prices, live weight sale and purchase prices. Livestock values were based upon information provided by New South Wales Agricultures' monthly Homebush saleyard reports and the New South Wales Meat Industry Authority state-wide monthly averages. Statistics provided by the Australian Wool Corporation were used to formulate production estimates as well as wool price variability over the period. Additional information was collected from New South Wales Agriculture staff in the district, and from a number of producers.

The choice of the factor to serve as the single index is potentially important since the observed performance of the single-index models will depend not just on the applicability of a single-index model but also on the appropriateness of the particular index used. Single-index models could be made to appear better than they really are by selecting an index based on some form of factor analysis of the gross margins data. Such an approach to defining an index would seem inappropriate, for apart from giving a biased view of performance, it would seem to be of little relevance when a major argument for considering single-index models is simplified analysis. An advantage of the approach in a research study, however, would be that it would give some indication of the maximum potential of single-index models The alternative approach, and the one which would be used in practice, is to identify the factor ex-ante. As already noted, this factor could be some climatic variable or a performance index for a reference portfolio. A naive and somewhat arbitrary specification of the index has been used in the case studies, the index simply being the average gross margin across all activities under consideration.

The beta parameters were estimated by ordinary least squares regressions of activity gross margins on the index and then taken as if known with certainty. Details of the simple regressions are provided in Appendix 2. The unique variance for an activity was calculated as the difference between the variance for the activity's gross margin and the activity's systematic risk.

5. Results

5.1 Case Farm 1

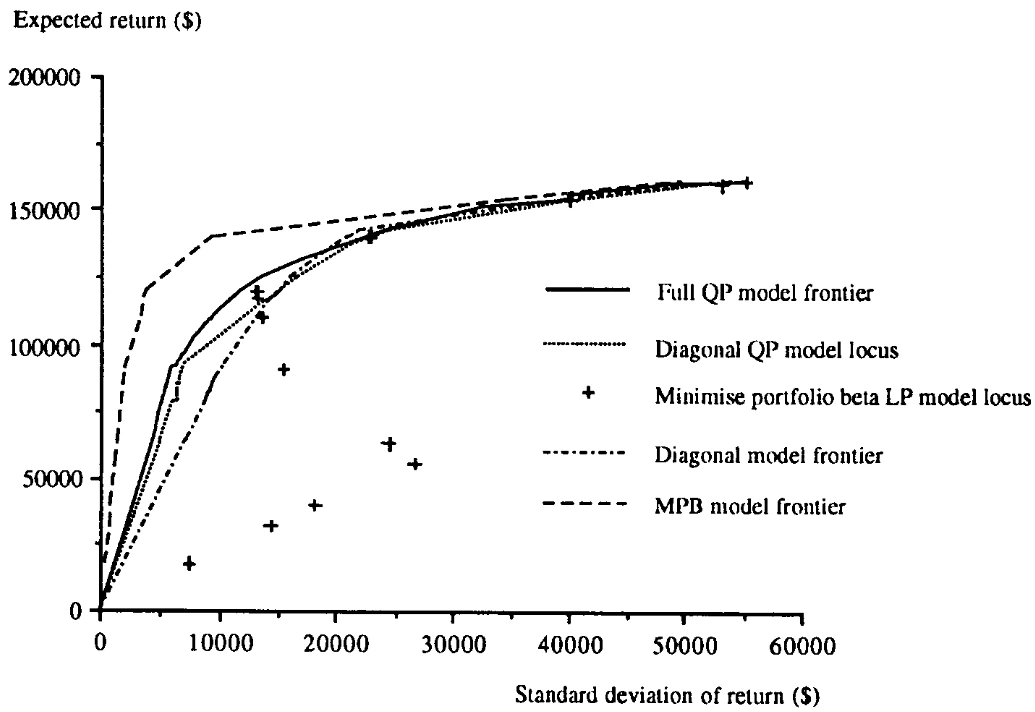
Frontiers were determined for each of the three models using parametric LP and QP. The mean-standard deviation frontiers from the three models are displayed in Figure 1 along with the mean-standard deviation loci calculated on the basis of the full variance-covariance matrix for the plans derived from each of the models. Full results are available from the authors.

Comparing the frontiers, the minimise portfolio beta model necessarily has a frontier lying to the left of that for the diagonal model since the former model has the same covariances as the latter but has smaller variances. Depending on how well the single-index models capture the full covariance structure, these models may have frontiers lying to the right or to the left of the QP model's frontier (the 'real' frontier). In the event, the minimise portfolio beta model understates the minimum risk levels for different levels of expected gross margin, whereas the diagonal model generally overstates the risk.

The two loci calculated using the full variance-covariance matrix and the frontier solutions from the single-index models necessarily lie to the right of the real frontier. Differences here indicate opportunity costs in using single-index models. The diagonal model produced plans which were close to being EV efficient over the whole range of expected gross margin. This is consistent with the results of the studies of Collins and Barry (1986) and Turvey, Driver and Baker (1988). In effect, the average gross margin index has been able to capture most of the joint variation in activity returns.

The mean-standard deviation locus associated with the plans derived from the minimise portfolio beta model lies close to the real frontier only over the 'upper' parts of the frontier where risk is relatively lightly weighted by the decision maker. The selection of portfolios that are actually quite EV inefficient when risk is given greater weight reflects the significance of the substantial non-systematic risk ignored by this model and indicates the opportunity to lower overall risk by accepting more

Figure 1: Mean-standard Deviation Frontiers and Loci for Case Farm 1



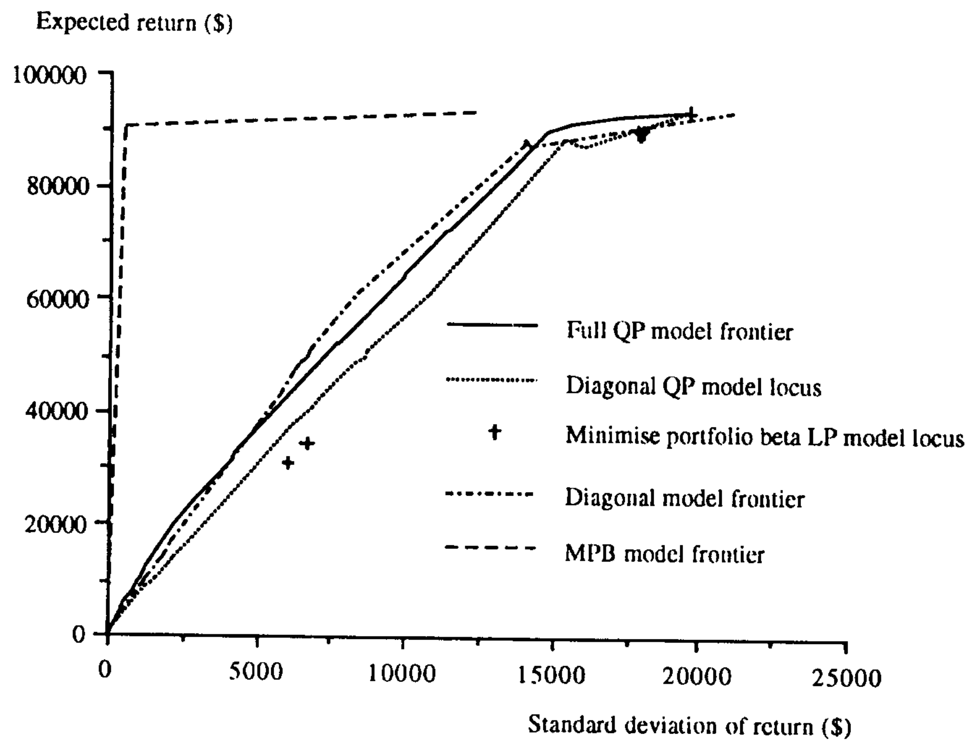
systematic risk in reducing non-systematic risk. In the lower expected income portfolios, oats, a low systematic risk crop, dominates the solutions. But the R^2 statistic for oats gross margin regressed on the index is zero to two decimal places (Appendix 2). The index therefore has little ability to predict the income for a portfolio containing only the activity oats, and the minimise portfolio beta model cannot estimate well the real variance of such a portfolio. It is not until the activities barley and yearlings (R^2 of 0.02 and 0.11 respectively) enter the solutions and oats is forced out, that the portfolios approach the frontier generated by the full QP model.

5.2 Case Farm 2

The results, shown in Figure 2, exhibit analogous patterns to those of the first case study. Both index models produce portfolios which are reasonably efficient, the diagonal model again producing the better plans. The minimise portfolio beta model performs relatively better than in the first case study, though it seriously understates the risk associated with its plans. Its solutions are generally dominated by one activity, merino wethers, which has a zero systematic risk. The wethers activity also happens to have a high expected return. It dominates the solutions up to an expected return of $90,000. It also has relatively low unique risk, making it an attractive activity in all three models. Except for the 'do nothing' portfolio, the quality of the approximations (in terms of relative efficiency of plans) is again better for the higher expected return portfolios.

The near-linear shape of substantial sections of the frontiers reflects the dominance of the wethers activity and relatively few changes in

Figure 2: Mean-standard Deviation Frontiers and Loci for Case Farm 2



wethers activity and relatively few changes in the optimal basis as expected income increases.

6. Discussion

In both case studies the plans derived from the diagonal single-index model generally lie closer to the real frontier produced by the full QP portfolio selection model than do the plans from the minimise portfolio beta LP model. This result is somewhat different from that of the Turvey, Driver and Baker (1988) study where the diagonal single-index model and the minimise portfolio beta model provided similar approximations of the real frontier.

Mean-standard deviation loci developed from single-index models might be expected to differ from the real frontier whenever the 'errors' in using the two approximations of the full variance-covariance are large. Thus, the diagonal model may 'fail' if the assumption of a single index is inappropriate; and the minimise portfolio beta model may produce a locus different from that of the diagonal model when its further assumption of zero-valued unique variances is inappropriate. One might expect that this locus would differ from the real frontier if either of the two assumptions is violated and be more different when both assumptions are violated. Potentially, though, errors in the two assumptions could offset. Case study 1, for example, does show one instance in which the minimise beta portfolio gets closer to the real frontier than does the diagonal model.

For any of these differences to be expressed, the constraint set must allow sufficient flexibility for the optimum solution to adjust to the altered variance-covariance matrices. Collins and Barry (1986) have noted this. The nature for our results differing from those of Turvey *et al* (1988). Our constraint sets were developed independently of the risk analysis and, while relatively simple, are of a size and style of construction commonly used in modelling for Australian farm planning research.

A more likely factor contributing to different results is our use of detrended data whereas Turvey *et al* (1988) analyse raw data series. Common trends in raw data series would effectively amount to a common factor, thus contributing to apparently better performance of the single-index models when using raw data. Systematic risk certainly represents a relatively smaller part of total risk in our study than was the case in the North American study.

Previous authors have claimed several advantages of the single-index models. These advantages can be classified as computational advantages and data collection/model specification advantages. The computational advantages include the need for a smaller capacity computer and less computer processing time (Collins and Barry 1986) and the relative simplicity of the models. However, over the last decade there has been enormous progress in personal computing technology. This technology has been accepted, in part at least, as a tool for farm planning purposes. Reliable and user-friendly software for QP and general nonlinear programming is now available[1], casting doubt as to the need for single-index models. Running a general QP portfolio selection model is now no more difficult than running the diagonal model. The minimise portfolio beta model is somewhat easier to specify and solve, requiring only LP, but this computational saving is relatively minor to-day. This study has thrown some doubt on the usefulness of the plans it generates, at least in the two case studies.

The potential advantages associated with the fewer parameters needed to construct the single-index models are largely irrelevant to farm planning situations. In contrast to the situation for financial markets, there are no published beta and residual error estimates available for agricultural activities. Without published betas and unique variances, the same raw data (or even additional data if the index is not based on the gross margins series) that is needed to construct a full QP portfolio selection model is still needed in estimating the beta coefficients (and the residual error variances for the diagonal model) in the single-index models.

The likelihood of widely applicable indexes being developed for Australian farms seems remote. Such an index could be based on rainfall or other general influence on gross margins, or it might be based on the gross margin performance of a standard or reference portfolio. But with gross margins varying as a result of both yield and price movements, it is unlikely a single factor will explain well the covariance structure of activity returns. Regional variations in weather and agronomic factors would certainly limit the spatial applicability of any particular climatic or reference portfolio index and its associated betas. Regular developments of new varieties and other technology and continually changing pest problems would also serve to alter any published betas and unique variances.

Ultimately, only further experience with the use of single-index models and different indexes will determine the utility of such models. Monte Carlo studies of the performance of different forms of index under a variety of data generating models for activity gross margins may be useful. The two small case studies reported here prove little, but do add weight to the view that there is more to be lost than gained in using single-index models for farm planning.

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[1] Most farm management researchers now use MINOS (see Murtagh and Saunders 1983), LINDO (see Schrage 1991) or the Rand code for QP problems. However there are other commercial QP packages available for microcomputers.

References

COLLINS, R.A. and P.J. BARRY (1986), 'Risk analysis with single-index portfolio models: an application to farm planning', *American Journal of Agricultural Economics* 68(1), 152-61.

DRYNAN, R.G. (1986), 'A note on optimal rules for stochastic efficiency analysis', *Australian Journal of Agricultural Economics* 30(1), 53-62.

FRANKFURTER, G.M. and G.G. BOOTH (1985), 'Further evidence of the role of nonsystematic risk in efficient portfolios', *Quarterly Review of Economics and Business* 25(2), 38-48.

HARDAKER, J.B., S. PANDEY and L.H. PATTEN (1991), 'Farm planning under uncertainty: a review of alternative programming models', *Review of Marketing and Agricultural Economics* 59(1), 9-22.

HARRINGTON, D.R. (1987), *Modern Portfolio Theory, The Capital Asset Pricing Model and Arbitrage Theory: A User's Guide*, Prentice-Hall, Englewood Cliffs.

HEADY, E.O. (1952), *Economics of Agricultural Production and Resource Use*, Prentice-Hall, Englewood Cliffs.

JOHNSON, R.W.M. (1992), 'Risk and the farm firm: a corporate finance view', *Review of Marketing and Agricultural Economics* 60(1), 9-21.

MARKOWITZ, H.M. (1952), 'Portfolio selection', *Journal of Finance* 7, 77-91.

MURTAGH, B.A. and M.A. SAUNDERS (1983), 'MINOS 5.0 User's Guide', Department of Operations Research, Stanford University.

SCHRAGE, L. (1991), *User's Manual for Linear, Integer, and Quadratic Programming with LINDO*, 5th Edition, Scientific Press, South San Francisco.

SHARPE, W.F. (1963), 'A simplified model for portfolio analysis', *Management Science* 9(2), 277-293.

SHARPE, W.F. (1970), *Portfolio Theory and Capital Markets*, McGraw-Hill, New York.

TAKAYAMA, T. and R.L. BATTERHAM (1972), *Portfolio Selection and Resource Allocation for Financial and Agricultural Firms with the Rand QP360 Quadratic Programming Code*, Agricultural Economics Research Report 117, Department of Agricultural Economics, University of Illinois.

TURVEY, C.G., H.C. DRIVER and T.G. BAKER (1988), 'Systematic and nonsystematic risk in farm portfolio selection', *American Journal of Agricultural Economics* 70(4), 831-36.

Appendix 1: Historical Gross Margins Data and Derived Risk Parameters

Table A1.1: Case One Farm

Activity Gross Margins ($/ha for crop activities, and $/hd for livestock)

Year	Wheat	Oats	Barley	Field peas	Wool	First cross steers	Yearling steers	Market return*
1982	282	181	201	92	23	44	270	156
1983	118	144	191	-65	26	42	198	93
1984	186	54	158	199	29	40	304	139
1985	129	56	132	132	25	35	322	119
1986	220	75	138	127	27	35	265	127
1987	362	93	166	219	41	57	277	174
1988	202	104.	169	153	52	67	204	136
1989	199	168	213	101	37	38	226	140
1990	124	129	171	88	18	28	239	114
Mean	202	112	171	116	31	43	256	133

* Simple average of per unit returns of each activity.

Table A1.2: Case One Farm

Variance-covariance Matrix

	Wheat	Oats	Barley	Field peas	Wool	First cross steers	Yearling steers	Market
Wheat	6385	338	282	3714	344	516	672	1750
Oats	338	2203	1174	-2146	-39	-12	-1211	44
Barley	282	1174	740	-956	28	35	-690	88
Field peas	3714	-2146	-956	6706	353	330	2065	1438
Wool	344	-39	28	353	109	112	-150	108
First cross steers	516	-12	35	330	112	150	-149	140
Yearling steers	672	-1211	-690	2065	-150	-149	1840	339
Market	1750	44	88	1438	108	140	339	558

Table A1.3: Case One Farm

Risk Parameters for Single-index Models

Activity	Beta	Systematic Risk Beta*Beta * Var (market)	Unique variance
Wheat	3.135	5487	898
Oats	0.079	3	2200
Barley	0.157	14	726
Field peas	2.576	3704	3002
Wool	0.194	21	88
First cross steers	0.251	35	115
Yearling steers	0.608	206	1634
Market	1.000	558	

Table A1.4: Case Two Farm

Activity Gross Margins ($/ha for crop activities and $/hd for livestock)

Year	Wheat	Oats	Barley	Triticale	Canola	Lupins	Wethers	Merino ewes	Second cross lambs	Yearling	Market
1975	272	194	73	172	331	86	45	22	35	30	126
1976	242	207	114	234	267	111	47	22	53	22	132
1977	57	18	-45	171	18	87	49	32	59	23	47
1978	544	69	26	198	342	152	46	39	65	44	152
1979	235	56	34	16	116	-63	40	30	52	50	57
1980	145	108	3	262	-48	-36	33	26	55	69	62
1981	388	164	97	248	-121	195	36	28	49	51	113
1982	-70	-63	-158	-67	-132	-121	40	19	22	37	-49
1983	512	148	100	357	166	236	34	26	47	69	169
1984	188	80	16	185	80	336	40	32	40	56	105
1985	253	104	40	346	139	274	31	30	37	55	131
1986	132	129	17	195	236	136	35	26	55	30	99
1987	146	99	6	164	150	123	61	34	55	23	86
1988	209	147	83	208	266	190	52	31	51	29	127
1989	131	92	32	176	193	131	40	31	46	31	90
Mean	226	103	29	191	134	122	42	29	48	41	96

Table A1.5: Case Two Farm

Variance-covariance Matrix

	Wheat	Oats	Barley	Triticale	Canola	Lupins	Wethers	Merino ewes	Second cross lambs	Yearling steers	Market
Wheat	25786	5684	7578	9893	10111	9553	-168	290	627	1061	7042
Oats	5684	4736	4265	4858	5250	3854	2	-51	174	-25	2875
Barley	7578	4265	4561	4914	5254	4574	-12	34	253	119	3154
Triticale	9893	4858	4914	11511	4131	9390	-226	103	345	620	4554
Canola	10111	5250	5254	4131	22713	5987	398	190	503	-881	5366
Lupins	9553	3854	4574	9390	5987	15280	-38	257	123	302	4928
Wethers	-168	2	-12	-226	398	-38	64	15	27	-93	-3
Merino ewes	290	-51	34	103	190	257	15	26	34	2	90
Second cross lambs	627	174	253	345	503	123	27	34	115	-21	218
Yearling steers	1061	-25	119	620	-881	302	-93	2	-21	261	135
Market	7042	2875	3154	4554	5366	4928	-3	90	218	135	2836

Table A1.6: Case Two Farm

Risk Parameters for Single-index Models

Activity	Beta	Systematic Risk Beta*Beta * Var (market)	Unique variance
Wheat	2.48	17485	8301
Oats	1.01	2914	1822
Barley	1.11	3508	1053
Triticale	1.61	7313	4198
Canola	1.89	10153	12560
Lupins	1.74	8565	6716
Wethers	0.00	0	64
Merino ewes	0.03	3	24
Second cross lambs	0.08	17	98
Yearling steers	0.05	6	255
Market	1.00	2836	

Appendix 2: Estimation of Beta Coefficients

Table A2.1: Case One Farm

Regression output:	Wheat	Oats	Barley	Field peas	Wool	First cross steers	Yearling steers
Constant	-214.44	100.99	150.24	-226.31	5.09	9.38	175.05
Standard error of estimate	32.04	50.14	28.81	58.57	10.03	11.45	43.21
R Squared	0.86	0.00	0.02	0.55	0.19	0.23	0.11
Number of observations	9	9	9	9	9	9	9
Degrees of freedom	7	7	7	7	7	7	7
β coefficient	3.14	0.08	0.16	2.58	0.19	0.25	0.61
Standard error of co-efficent	0.48	0.75	0.43	0.88	0.15	0.17	0.65

Table A2.2: Case Two Farm

Regression output:	Wheat	Oats	Barley	Triticale	Canola	Lupins	Wethers	Merino ewes	Second cross lambs	Yearling steers
Constant	-13.96	5.65	-78.15	36.04	-49.01	-45.22	41.99	25.45	40.56	36.67
Standard error of estimate	94.55	44.30	33.68	67.24	116.30	85.04	8.28	5.04	10.26	16.57
R squared	0.68	0.62	0.77	0.64	0.45	0.56	5.00E-05	0.11	0.15	0.024
Number of observations	15	15	15	15	15	15	15	15	15	15
Degrees of freedom	13	13	13	13	13	13	13	13	13	13
β co-efficient	2.48	1.01	1.11	1.61	1.89	1.74	-1.00E-3	0.03	0.08	0.05
Standard error of co-efficent	0.47	0.22	0.17	0.34	0.58	0.43	0.04	0.03	0.05	0.08