A SIMULATION APPROACH TO RISK ASSESSMENT IN INVESTMENT ANALYSIS

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A simulation technique is advanced as a means of determining the probability of achieving various possible financial outcomes when assessing alternative investments. To this end a model is constructed and applied to a proposed investment in pasture improvement. Results are contrasted with a deterministic budget approach. The model uses triangular distributions to derive probabilistic estimates for the stochastic events considered. Possible fields of application of interest to agricultural economists are discussed.

1 STATEMENT OF THE PROBLEM

Applications of operations research techniques to investment and planning situations in agriculture are growing in number. Recent contributions to this Review have included a number of planning procedures for deriving alternative farming systems [4], [7], and [25]. The decision maker is able to choose among these alternatives according to his subjective preferences. While these techniques are undoubtedly promising, they need further adaptation to meet the plea of Harle [11] to provide an explicit statement of variability among investment plans along with their profitability[1]. The objective of this paper is to attempt to assess a particular plan in the light of this further requirement.

Byrne *et al* [3] have outlined two broad groups of models which include risk assessments in planning and investment situations. Distinction between these groups is on the basis of whether each alternative is reduced to a single figure-of-merit or presented as a probability distribution of outcomes. The classical single figure-of-merit approach only deals with risk implicitly by such methods as discount rate or length of project life adjustments and related measures[2]. However it

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† Department of Agriculture, University of Queensland. This study which was partially supported by a grant from the Rural Credits Development Fund of the Reserve Bank is part of a larger project.

This larger project is wholly supported from the above Fund. The authors are indebted to C. Higham and I. Horton for computer programming assistance.

[1] Carlsson *et al* [4] include a limited approach to this problem by incorporating an analytical non-simulation approach to risk assessment in the second part of their paper.

[2] Solomon has highlighted the inability of classical D.C.F. methods to deal with the extreme sensitivity common in dynamic situations. "When uncertainty concerning costs, revenues, or project life exists, classical measures of capital investment return are limited in value. Sensitivity analysis indicates that relatively small over estimates and under estimates create relatively large errors in discounted rate of return for different types of return schedules." [30, p. B334.]

is possible to deal with probabilistic occurrences explicitly within the models available in the first group. Certain optimizing models based on mathematical programming methods are relevant here[3]. In the second group are the models that develop probability distributions concerning the range of outcomes for the investment concerned. Unlike mathematical programming models these usually employ simulation techniques[4] and do not optimize[5].

This paper applies a model belonging to the latter group to a farm planning and investment situation. It is closely allied to studies by Hertz [14], Meredith [19], and Murphy [21]. Nevertheless it differs in at least one significant respect from previous analyses. This is in the method it employs to derive probabilistic estimates for the values of critical parameters. The methodological approach is first outlined, and data and results from a published study are used to illustrate and contrast the results from the simulation model[6]. Other possible applications of interest to agricultural economists are suggested.

2 THE METHOD OF APPROACH

2.1 GENERAL

For investment analysis in a practical setting the authors do not favour mathematical programming models that result in a single figure-of-merit. Such models are generally based on maximizing some measure of utility, for example, expected return subject to a variability of return constraint, or are formulated to minimize risk subject to a satisfactory return constraint. They are somewhat inflexible in that an explicit statement of risk preference is generally needed *a priori*. Further, the authors believe these models are more difficult to use than the method suggested here.

Presenting the outcome of projects as probability distributions allows added flexibility to the decision-maker. Only when faced with the set of possible outcomes can a decision on trade-offs legitimately be made. As Hertz states:

> For example, would it (management) prefer a virtual certainty of no loss coupled with a virtual ceiling on gains over 20 per cent after taxes—or would it accept a one-in-ten chance of significant loss for the sake of a one-in-ten chance of very high gain? [13, p. 102.]

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[3] These include linear programming under uncertainty, stochastic programming, and chance-constrained programming.

[4] A discussion of the simulation technique in relation to other mathematical approaches available for economic analysis is provided in [22] and [18].

[5] Included here are Hertz [14] "Risk Analysis", Hess and Quigley [16] "Venture Analysis", and Hespos and Strassman [15] "Stochastic Decision Trees".

[6] Haug and Hirst [12] used D.C.F. methods to examine the outcome of investment in pasture improvement in the Queensland Spear Grass Zone. The authors acknowledge the co-operation of Messrs Haug and Hirst in providing further data.

Taking the analysis a step further, such trade-offs suggest the possibility of ranking procedures. Studies incorporating this step have suggested measures such as "efficiency frontiers" and "indifference systems" [6], [13], and [24].

2.2 SUBJECTIVE PROBABILITIES

The investment proposal investigated in this study may be viewed as a system which is influenced by stochastic events. To analyse such a system it is necessary to estimate probability distributions. Because of the nature of the planning situation such distributions must be based on subjective probabilities. Although there is still some controversy concerning this approach to decision making, it is becoming more widely used in operations research studies and emphasis has now shifted to finding reliable estimators of these probabilities[7].

Smith has reviewed the literature and noted the extensive use of subjective probabilities both in an explicit and implicit sense.

One observation stands out:

> Despite the obvious potential utility of subjectively derived probability distributions, a review of associated literature reveals that there exists a conspicuous scarcity of treatments devoted to the obtaining of these required distributions. It is important to note, however, that the scarcity refers to lack of discussion of an adequate method for developing subjective probabilities in *general* situations. [28, p. B238.]

Smith ranks Hertz's approach of obtaining estimates of the likelihood of various outcomes by questioning the experts involved as one of the best methods available, but considered it was not sufficiently definitive. He proposed a more precise method based on statistical ranking procedures. Subsequently, Green suggested a less circuitous method that could lead to estimates consistent with those of Smith, but states:

> . . . I believe the problem of choosing a best method for eliciting subjective probability distributions is still wide open. [9, p. B251.]

However it is agreed that any method should be logical, definitive and consistent. Simon emphasised the necessity for restricting requirements to within the capabilities of forecasters. He noted:

> . . . we may reasonably expect managers to be able to estimate expected values of variables, but it is dubious whether they can often estimate other characteristics of probability distributions (variances or higher moments); [27, p. 52.]

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[7] "If subjective probabilities are implied in real behaviour, then it is far better to make these explicit in a systematic manner—even if they may not be readily extractable." [24, p. 13.] For additional support see [13], [14], and [23]. Psychologists investigating the existence and performance of subjective probabilities have shown many deviations in experimental situations from rational usage of subjective weightings. However the unaided information processing capabilities of humans are likely limitations in many of these situations [8].

Sprow [31] in his pioneering contribution outlined further criteria in selecting a method or distribution leading to acceptable subjective weightings[8]. Dealing specifically with cash flow forecasts in an investment situation, he argues that there is an absence of empirical evidence to support the adoption *a priori* of any particular distribution to describe financial outcomes. In keeping with Simon's viewpoint, Sprow noted that any distribution adopted should be defined (uniquely if possible) by parameters that can be unambiguously specified and understood by those required to estimate them. Additionally it should be capable of taking on a skewed form, depending on the information available. Finally, it would be advantageous if the distribution were amenable to mathematical manipulation and computer techniques.

Given the criteria listed above, the triangular probability function comes closest to satisfying such requirements. It is suggested that the PERT beta [17] is the next most satisfactory, but while a simple range of forecasts is all that is necessary to completely specify the triangular distribution, additional assumptions with respect to the standard deviation are necessary for the PERT beta [10] and [17]. Sprow has utilized both the triangular and PERT beta in investment analyses and recommends the triangular[9]. Other distributions, for example the normal distribution [16], have been used in investment analysis situations. However the authors believe the normal distribution is severely limited for such use. Firstly, it cannot be skewed. Secondly, the need to predict higher moments and not simple parameters may exceed forecasters' capabilities.

Weibull gives further weight to the authors' acceptance of the triangular distribution. He emphasizes the pragmatic assumptions taken for granted when statisticians apply any distribution. In most cases he doubts the sense of speaking of the "correct" distribution function. He argues that there are very few populations that conform to the theoretical basis of any distribution. He states:

> Furthermore, it is utterly hopeless to expect a theoretical basis for distribution functions of random variables such as strength properties of materials or of machine parts or particle sizes, the "particles" being fly ash, Cyrtoideae, or even adult males, born in the British Isles.
>
> It is believed that in such cases the only practicable way of progressing is to choose a simple function, test it empirically, and stick to it as long as none better has been found. [34, p. 283.]

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[8] Two approaches can be employed in deriving subjective weightings. Either the method utilized builds up a distribution from particular subjective opinions regarding outcomes [28] and [20], or a specific probability function is considered operational [31] and [16].

[9] Other papers which use the triangular distribution include [29], [32], and [33]. An interesting example of incorporating use of both subjective and objective probabilities viz., PERT beta and Poisson distributions is given in [1].

2.3 THE TRIANGULAR DISTRIBUTION

The triangular distribution is unimodal[10] and is uniquely defined by estimates of the following parameters of the variable[11]:

(i) The minimum value

(ii) The "most likely" value

(iii) The maximum value.

The distribution is illustrated in figure 1 where

x—the value of the particular variable;

a—the minimum value of x;

b—the "most likely" value of x;

c—the maximum value of x.

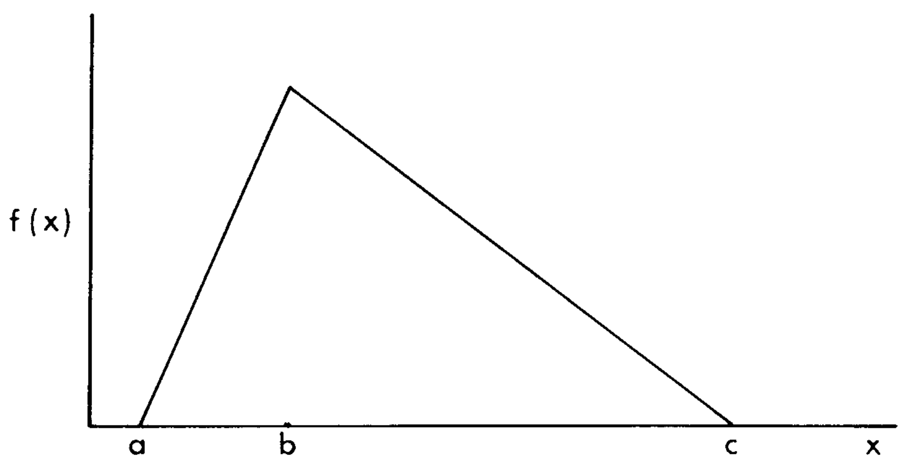

FIGURE 1: *Triangular Density Function*

Thus the triangular distribution is suitable when the information available suggests a central tendency and when there is sufficient data to enable estimation of the "modal" value of the variable and the

[10] The techniques of Lee M. Smith [28] and Morrison [20] are not restricted to formulating unimodal distributions. While they claim this is an advantage of their respective methods, the authors believe this point to be of marginal significance in the situations where we are advocating application of the triangular distribution.

[11] An alternative method for defining a triangular density function from subjective estimates is given in [32].

upper and lower limits[12]. In view of the fact that single-value estimates are in accepted use, the range limits should be relatively easy to determine. As Hertz states:

". . . it is easier to guess with some accuracy a range rather than a single value." [14, p. 100.]

The probability density function of the triangular distribution is given by:

$$f(x) = \frac{2(x - a)}{(c - a)(b - a)} , \quad a \leqslant x \leqslant b$$

$$f(x) = \frac{2(x - c)}{(c - a)(b - c)} , \quad b \leqslant x \leqslant c$$

where x, a, b, and c are as defined above.

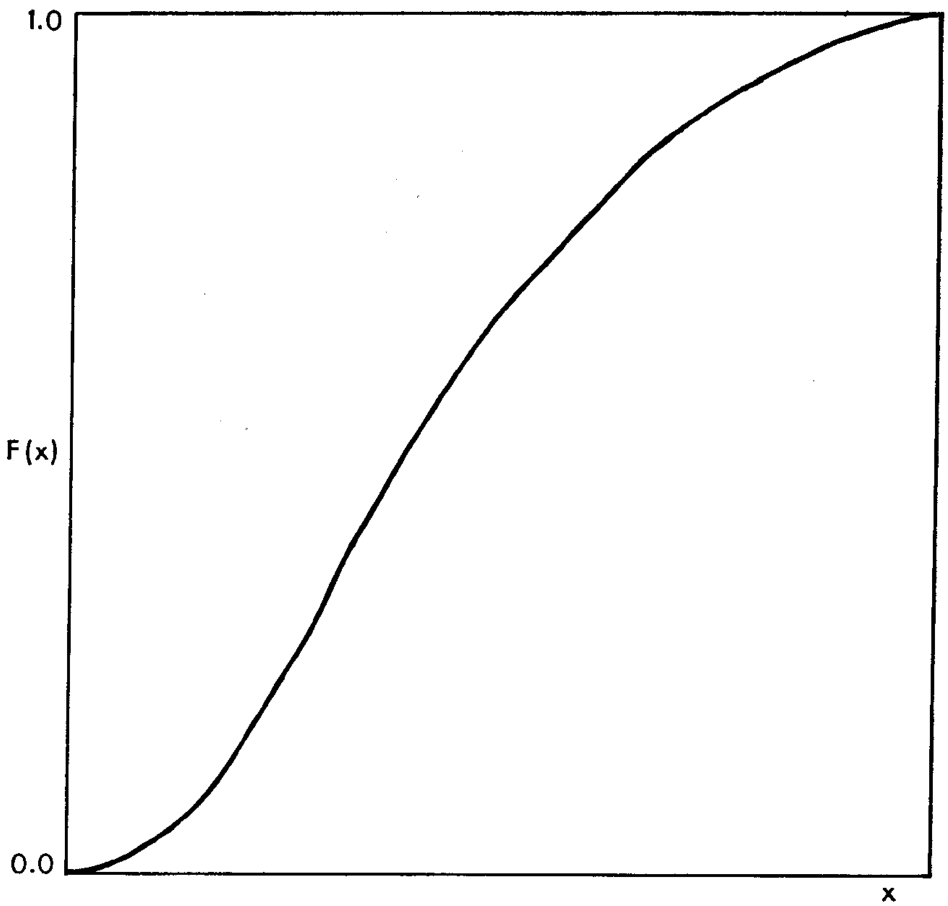

FIGURE 2: *Cumulative Probability Function for a Triangular Distribution*

[12] In cases where the range for the variable can be specified but not the "most likely" value, Walstrom *et al* [33] recommend the use of the uniform probability distribution.

The cumulative distribution function $F(x)$ is given by the integration:

$$F(x) = \int f(x)\,dx$$

such that $F(a) = 0$ and $F(c) = 1$.

Therefore:

$$F(x) = \frac{(x-a)^2}{(c-a)(b-a)}, \quad a \leqslant x \leqslant b$$

$$F(x) = 1 - \frac{(x-c)^2}{(c-a)(c-b)}, \quad b \leqslant x \leqslant c.$$

Solving for x in terms of the ordinate $F(x)$, and the known parameters a, b, and c, we get:

$$x = a + [F(x)\,(c-a)\,(b-a)]^{\frac{1}{2}}, \quad a \leqslant x \leqslant b$$

$$x = c - [(1 - F(x))\,(c-a)\,(c-b)]^{\frac{1}{2}}, \quad b \leqslant x \leqslant c.$$

In this form, a value of the stochastic variable can be determined by random selection of the ordinate.

2.4 THE SIMULATION APPROACH

The second stage in the approach used here involves incorporating subjective weightings on stochastic events into a model of the system under investigation. This system involves investment over time so that the model should be dynamic in nature. The system is subject to non-controllable stochastic influences e.g. market prices, fluctuations in pasture productivity. The overall problem requires a model classified by Samuelson as "stochastic and historical" [26, p. 316] and [22, p. 18]. It is extremely difficult in most realistic cases to solve this class of problem by mathematical methods [11] and [22]. Monte-Carlo simulation offers a practicable solution[13].

Briefly, the model comprises a number of stochastic processes. Parameters entering into the simulation are chosen by Monte-Carlo selection and combined according to the functional relationships of the model. The combination of these values for a particular simulation run determines an outcome. A cumulative distribution of a number of such outcomes is then constructed. Thus the output describes the range of possible results with the probability of their occurrence.

[13] In the simplest projects where only one path containing a large number of events defines the system under consideration, recourse to the Central Limit Theorem can be made. For examples see [21] and [35], but the technique has obvious limitations. In the example outlined in this paper the authors have allowed the possibility of "poor strikes" in the case of pasture establishment, thus retarding project output. This constitutes divergence of paths and negates use of the Central Limit Theorem [31].

3 AN EMPIRICAL APPLICATION

3.1 THE INVESTMENT PROPOSAL

To facilitate comparison of this approach with the presently utilized D.C.F. capital budgeting methods the authors chose Haug and Hirst's example [12]. This assessment of the profitability of long-term pasture improvement in the Spear Grass Zone was selected for the following reasons:

(i) It contains most of the basic data and assumptions involved, and is readily available to the interested reader to contrast with the present article;

(ii) The presentation of results is based on "triple-price" calculations i.e. "optimistic", "pessimistic", and "most likely". This method of making some allowance for uncertainty is one of the common approaches undertaken to risk assessment and serves as a useful comparison;

(iii) The authors believe it is a realistic assessment of planning possibilities in the area.

The results reported from the simple simulation model utilized in this study should not be taken as a last word. The authors' application is concerned with using only one of the alternative investment proposals in [12] in order to demonstrate the technique involved.

The only modification introduced into the basic assumptions, and incorporated in the simulation model, was an allowance for a setback of animal turn-off due to unsuccessful Townsville Lucerne establishment. Basic to Haug and Hirst's proposal was the strip-sowing of 250 acres of native pasture with Townsville Lucerne each year for ten years. This assumed:

> . . . that the strips amount to one-third of the area to be developed and that it would take five years to cover the intervening land by natural spread, . . . resulting in the improvement after 15 years of that half of the property which carried one beast to 12 acres or better before improvement. [12, p. 118.]

This assumption was not discarded in the simulation, but was regarded as the most "optimistic" outcome—the "most likely" being one poor strike and the "most pessimistic" being five poor strikes. Each poor strike has the effect of setting back the cattle breeding and store fattening program by one year. Thus allowance for unsuccessful establishment makes it possible that the fully developed situation may not be reached before year 20.

A related innovation regarding the uncertainties of pasture establishment was also incorporated. It has been postulated that poor strikes are more likely to occur in the early years of a pasture improvement program[14]. The overall impact of such assumptions affected the budgeted increase in turn-off numbers. These are dependent on the success of the pasture development phase. This then leads to its concomitant effect on the proposal's cash flows. Turn-off weights were treated independently. The modifications regarding the problems of pasture establishment are considered to have moved the model a step nearer reality.

Certain cost parameters were assumed to be known with certainty[15]. They are subject to little uncertainty in real world situations and enter

TABLE 1

Parameter Ranges for Critical Variables

	Minimum Possible	"Most Likely"	Maximum Possible
(1) No. of "Poor Strikes"	0	1	5
(2) Year of "Poor Strike"	Year 1	Year 1	Year 10
(3) Beef Prices (weighted average)	$16.50	$18.00	$19.50
(4) Store Purchases and Sales—			
(a) Purchase weight	285 lb	300 lb	315 lb
(b) Sale as forward stores (average weight gain)	152 lb	160 lb	168 lb
(c) Sale as fats (average weight gain)	304 lb	320 lb	336 lb
(5) Property Bred Cattle—Average dressed weight at sale—			
(a) Spays	494 lb	520 lb	546 lb
(b) Cows	428 lb	450 lb	472 lb
(c) Fats	589 lb	620 lb	651 lb
(d) Bulls	665 lb	700 lb	735 lb
(6) Life of Investment	20 years	30 years	60 years

[14] The authors have incorporated these possibilities after discussion with tropical pasture experts of the Agriculture Department, University of Queensland. In their experience a critical factor is lack of management expertise in the early years leading to ineffective broad-scale establishment. Selection of such outcomes was achieved as follows:

$N = [G(x)]$ where N = the value of the outcome;

$G(x)$ = the cumulative distribution function based on an appropriate triangular density function.

[ ] indicates that the largest integer less than $G(x)$ is selected as input into the model, as $G(x)$ is a continuous function.

[15] These are denoted in [12] as:

(a) All fixed costs.

(b) Variable costs:

(i) Freight rate per animal;

(ii) Selling charges per animal;

(iii) Spaying costs for animals treated;

(iv) Other costs per head of animals.

(c) Pasture improvement costs.

the revenue equation of the model at the levels detailed in Haug and Hirst. The other parameters were deemed critical and treated stochastically to find their influence on overall profitability. In the simulation, triangular distributions were set to such parameter ranges Table 1 lists critical parameters and indicates the ranges assumed.

3.2 THE SIMULATION—PROGRAMMING AND OPERATION

The resulting cash flows from five hundred computer simulation runs were stored on magnetic tape for subsequent input into a second program to calculate internal rates of return. Other parameters deemed relevant to the investment proposal under consideration (e.g. simulated peak deficits) were obtained as printout from the basic program[16]. A truncated flow chart of the overall simulation is presented as diagram 1.

3.3 RESULTS

An advantage of the simulation approach is that numerous project evaluation indices can be extracted at little additional cost. For the purpose of this paper only six sets of outcomes have been presented. They were chosen on the basis of comparability with the results reported in [12] and are:

(1) Internal Rate of Return

(2) Peak Deficit

(3) Cumulative Surplus at the end of Year 15

(4) Cumulative Surplus at the end of Year 20

(5) Year of Peak Deficit

(6) Payback Period.

To aid comparison with the deterministic study, the results arising from the three discrete price levels assumed by Haug and Hirst are detailed in table 2, along with the expected values, range and standard deviations of the outcome distributions from the simulation.

The simulation results for the first four indices have been plotted as cumulative distributions in figures 3, 4, 5, and 6. In addition, all simulation results are presented in tabular form in tables 3 to 8. In the interest of brevity, only internal rate of return results are presented in the text as figure 3 and table 3, while other results are in the appendix.

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[16] Tax considerations would alter these results. To conform to the presentation in [12] no account was taken of this item. The authors have incorporated taxation factors in a study presently underway.

TABLE 2

Comparison of Results

	Simulation Results			B.A.E.	Results
	Expected Value	Standard Deviation	Range	Average Beef Price	Outcome
(1) Internal Rate of Return ..	17·24%	1·37%	13·08 to 19·52 per cent	$ 16.50 18.00 19.50	17 per cent 19 per cent 20 per cent
(2) Peak Deficit	$13,468	$3,646	$5,989 to $27,316	16.50 18.00 19.50	$24,016 $12,078 $5,553
(3) Cumulative Surplus* at end of Year 15	$66,591	$10,747	$28,407 to $91,673	16.50 18.00 19.50	$33,532 $73,417 $105,295
(4) Cumulative Surplus* at end of Year 20	$129,733	$11,877	$90,398 to $158,640	16.50 18.00 19.50	$82,287† $136,907† $181,225†
(5) Year of Peak Deficit ..	6th year	..	3rd to 10th year	16.50 18.00 19.50	6th 5th 3rd
(6) Pay Back Period‡.. ..	10 years	..	7 to 13 years	16.50 18.00 19.50	12 9 7

* As defined in [12].

† Not stated explicitly in [12]. Estimated on the basis of financial performance recorded for first year after development.

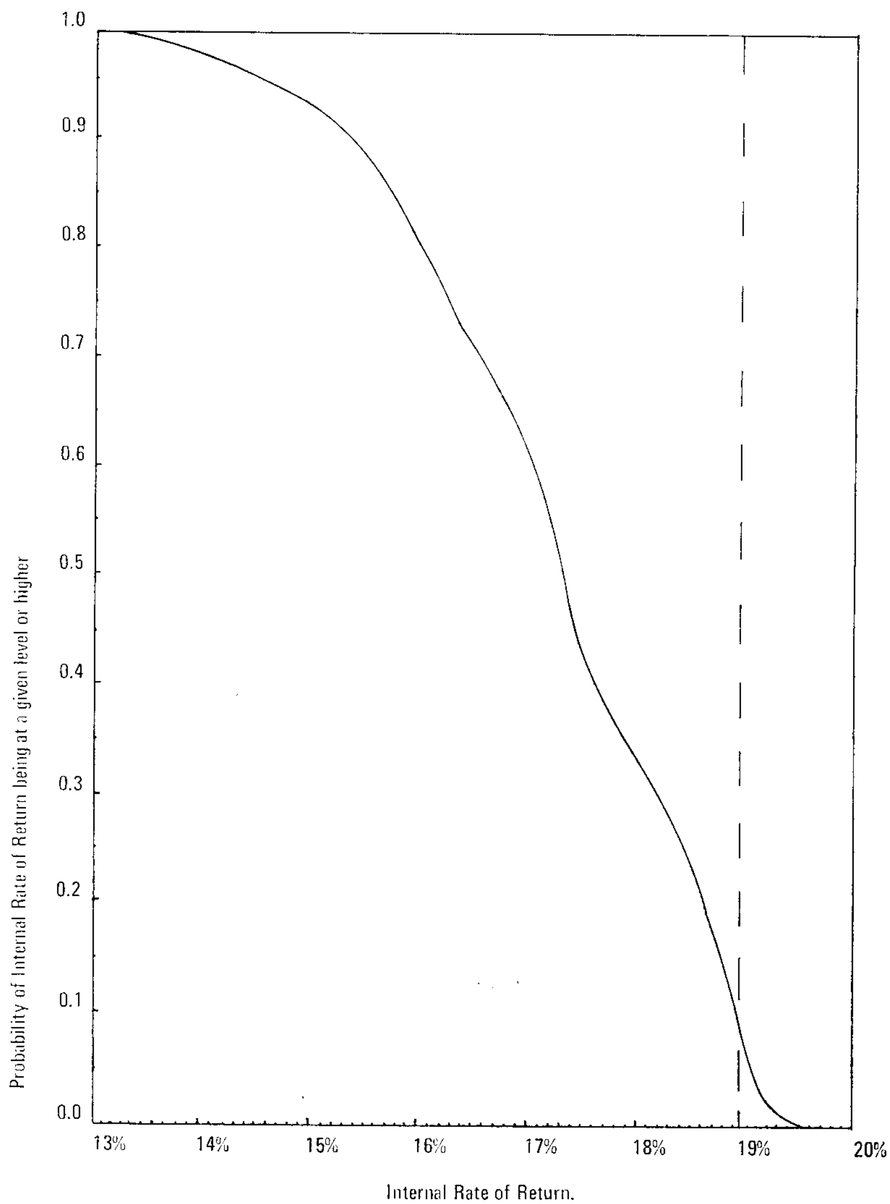
‡ Defined as the year during which the "cumulative surplus" changes from negative to positive.

TABLE 3

Selected Readings from the Internal Rate of Return—"More Than" Ogive

Probability of achieving ⩾	Per cent
90 per cent	15·39
75 per cent	16·30
50 per cent	17·35
25 per cent	18·48
10 per cent	18·93

FIGURE 3: *Internal Rate of Return—"More Than" Ogive*



(Note: Vertical broken line indicates result in [12] at average price of $18.00 per 100 lb)

3.4 DISCUSSION

With Haug and Hirst's assessment, the decision-maker is unable to forecast the likelihood of achieving an outcome within the ranges indicated by their triple-price calculations for the different project measures[17]. They report a possible range from 17 per cent to 20 per cent in the internal rate of return. Simulation results allowing stochastic events illustrate a wider range (13·08 per cent to 19·52 per cent). More importantly, the probability of obtaining an internal rate of return larger than 17·35 per cent is only 50 per cent. Chances of obtaining greater than 18·93 per cent are only one in ten.

The peak deficit is often critical in investment proposals. Haug and Hirst estimate a range for this parameter of between $5,553 to $24,016. In practical terms such a range may prove unsatisfactory for negotiating finance. Results from the simulation are helpful here. They show a 90 per cent probability of requiring greater than $9,320 (which is approximately 1·7 times the minimum requirements detailed in Haug and Hirst), while there is a 50 per cent probability of requiring more than $12,820 but only a 10 per cent chance of requiring $18,370 or greater.

When consideration is given to the payback period (Table 8), the simulation results indicate a 6 per cent probability of being in deficit after eleven years. On the other hand there is a 6·2 per cent probability that repayment will be achieved by the end of year 7. These results contrast with the seven and twelve year payback periods for the range of beef prices given in [12].

Such comparisons indicate the extra information yielded by the simulation. This could be decisive in assessing investment merits. As well, models of the type used here are capable of extending further information relevant to decision-making. While not reported here, it is a relatively simple matter to provide, for example, expected values and standard deviations of annual net cash flows and net present values and to investigate possible repayment strategies. Simulation models are ideal tools to test results generated by posing questions of the "what-if" type. "What if better management pushes up calving percentages?" "What if more reliable estimates could be obtained about particular stochastic variables e.g. sale prices for different classes of stock?" This could lead to a narrowing of the range of the distributions governing important variables and lead to isolating which elements in the system contribute most to reducing risk i.e. those elements which reduce the spread of the final outcome. Such elements should have priority for further research.

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[17] The range of possible results for all indices reported by Haug and Hirst is attributable to variations in their price parameter alone. Results from the simulation (based in the main on using data with only a slight range either side of the above authors' deterministic parameters) are due to interaction between stochastic outcomes of all such parameters. While no study has been undertaken of which component contributes most to variation in the simulation results, the allowance for setback due to poor strikes is regarded as significant.

Finally, as demonstrated in previous studies [14], [19], [21], and [35], probability approaches are superior to the usually employed single figure-of-merit methods when examining investment possibilities. Arranging the cumulative outcome distributions of selected criteria for alternative investments on the same axes enables a contrast and thus more informed decision making.

4 CONCLUDING REMARKS

In the first part of this paper it was shown that the triangular distribution is an acceptable and useful method for deriving subjective probabilities of critical parameters for investment analysis. It has been proposed as an improvement on alternative methods postulated to date. The second part of the article was concerned with incorporating the triangular distribution into a model describing a particular investment system, using Monte Carlo methods to quantify the inherent uncertainty of outcomes.

It is the belief of the authors that, compared with advanced mathematical programming models, specifically those dealing with uncertainty, the present formulation is more suitable as a "front-line" method. Computer programs with generality to a wide range of investment decisions can be written that could bring this method within the reach of the farm management adviser. Farm management service centres are the logical places for such development. With widespread availability, cost should not be beyond the medium-sized farm business.

The method suggests many areas of application. For example, it could provide a more complete assessment of drought strategies than that now given by the parametric budgeting approach. There is no reason why public investment e.g. in a Benefit-Cost framework should not be amenable to analysis by this technique, for example [2]. A study in this field is now underway, [5]. The method could go another step and link with both programming frameworks and utility analysis. Further, for a single firm, or for that matter in a macro situation, it is possible to forecast the potential impact of a particular investment on the existing operations of the firm or sector, [32] and [35].

REFERENCES

[1] ASHER, D. T., "A Linear Programming Model for the Allocation of R and D Efforts", *IRE Transactions on Engineering Management*, Volume EM 9, No. 4 (December, 1962), pp. 154–157.

[2] BUREAU OF AGRICULTURAL ECONOMICS, *The Economics of Land Development in the Belyando-Suttor Rivers Region*, Queensland (B.A.E., 1968).

[3] BYRNE, R., A. CHARNES, W. W. COOPER AND K. KORTANEK, "Some New Approaches to Risk", *The Accounting Review*, Volume 43, No. 1 (January, 1968), pp. 18–37.

[4] CARLSSON, M., B. HOVMARK AND I. LINDGREN, "A Monte Carlo Method for the Study of Farm Planning Problems", this *Review*, Volume 37, No. 2 (June, 1969), pp. 80–103.

[5] CASSIDY, P. A. AND R. C. GATES, *A Simulation/Benefit Cost Assessment of Land Use Alternatives—Wide Bay Area, Queensland*, (University of Queensland, 1969, in preparation).

[6] DEAN, G. W., "Decision Theory Models in Range Livestock Research", in *Conference Proceedings of the Committee on Economics of Range Use and Development, Western Agricultural Economics Research Council*, Report No. 8 (August, 1966), pp. 111–138.

[7] DENT, J. B. AND P. F. BYRNE, "Investment Planning by Monte Carlo Simulation", this *Review*, Volume 37, No. 2 (June, 1969), pp. 104–120.

[8] EDWARDS, W. AND A. TVERSKY, eds., *Decision Making*, Penguin Modern Psychology Series (Middlesex: Penguin Books, 1967).

[9] GREEN, P. E., "Critique of Ranking Procedures and Subjective Probability Distributions", *Management Science*, Volume 14, No. 4 (December, 1967), pp. B250–B252.

[10] GRUBB, F. E., "Attempts to Validate PERT Statistics or 'Picking on PERT' ", *Operations Research*, Volume 10, No. 6 (Nov.-Dec., 1962), pp. 912–915.

[11] HARLE, J. T., "Towards a More Dynamic Approach to Farm Planning", *Journal of Agricultural Economics*, Volume 19, No. 3 (September, 1968), pp. 339–346.

[12] HAUG, N. AND G. HIRST, "Pasture Improvement Budgeting in the Queensland Spear Grass Zone", *Quarterly Review of Agricultural Economics*, Volume 20, No. 3 (July, 1967), pp. 113–126.

[13] HERTZ, D. B., "Investment Policies that Pay Off", *Harvard Business Review*, Volume 46, No. 1 (Jan.-Feb., 1968), pp. 96–108.

[14] ——— "Risk Analysis in Capital Investment", *Harvard Business Review*, Volume 42, No. 1 (Jan.-Feb., 1964), pp. 95–106.

[15] HESPOS, R. F. AND P. A. STRASSMAN, "Stochastic Decision Trees for the Analysis of Investment Decisions", *Management Science*, Volume 11, No. 10 (August, 1965), pp. B224–B259.

[16] HESS, S. W. AND H. A. QUIGLEY, "Analysis of Risk in Investments Using Monte-Carlo Techniques", *Chemical Engineering Symposium Series: Statistics and Numerical Methods in Chemical Engineering*, Volume 59, No. 42 (1963), pp. 55–63.

[17] MACCRIMMON, K. R. AND C. A. RYAVEC, *An Analytical Study of the PERT Assumptions* (RAND Corporation: Memorandum RM-3408-PR, December, 1962).

[18] MCMILLAN, C. AND R. F. GONZALEZ, *Systems Analysis: A Computer Approach to Decision Models*, (Homewood: Richard D. Irwin, 1968), pp. 1–35.

[19] MEREDITH, G. G., "Project Evaluation Under Risk and Uncertainty", *The Australian Accountant*, Volume 36, No. 6 (June, 1966), pp. 305–313.

[20] MORRISON, D. G., "Critique of Ranking Procedures and Subjective Probability Distributions", *Management Science*, Volume 14, No. 4 (December, 1967), pp. B253–B254.

[21] MURPHY, M. C., "A Stochastic Approach to Investment Appraisal", *The Farm Economist*, Volume 10, No. 7 (1968), pp. 304–318.

[22] NAYLOR, T. H., J. L. BALINTFY, D. S. BURDICK AND K. CHU, *Computer Simulation Techniques* (New York: Wiley, 1968), pp. 16–20.

[23] NORTON, J. H., "The Role of Subjective Probability in Evaluating New Product Ventures", *Chemical Engineering Symposium Series: Statistics and Numerical Methods in Chemical Engineering*, Volume 59, No. 42 (1963), pp. 49–54.

[24] OFFICER, R. AND J. ANDERSON, "Risk, Uncertainty and Farm Management Decisions", this *Review*, Volume 36, No. 1 (March, 1968), pp. 3–19.

[25] POWELL, R. A. AND J. B. HARDAKER, "Sub-Optimal Programming Methods for Practical Farm Planning", this *Review*, Volume 37, No. 2 (June, 1969), pp. 121–129.

[26] SAMUELSON, P. A., *Foundations of Economic Analysis*, (Cambridge: Harvard University Press, 1948), Ch. 11.

[27] SIMON, H. A., "The Role of Expectations in an Adaptive or Behavioristic Model", in M. S. Bowman ed., *Expectations Uncertainty and Business Behavior* (New York: Social Science Research Council, 1968), pp. 49–58.

[28] SMITH, L. H., "Ranking Procedures and Subjective Probability Distributions", *Management Science*, Volume 14, No. 4 (December, 1967), pp. B236–B249.

[29] SMITH, M. B., "Estimate Reserves by Using Computer Simulation Analysis", *The Oil and Gas Journal*, Volume 66, No. 11 (March 11, 1968), pp. 81–84.

[30] SOLOMON, M. B. JNR., "Uncertainty and Its Effects on Capital Investment Analysis", *Management Science*, Volume 12, No. 8 (April, 1966), pp. B334–B339.

[31] SPROW, F. B., "Evaluation of Research Expenditures Using Triangular Distribution Functions and Monte Carlo Methods", *Journal of Industrial and Engineering Chemistry*, Volume 59, No. 7 (July, 1967), pp. 35–38.

[32] SWIRLES, J. AND L. A. LUSZTIG, "Capital Expenditure Decisions Under Uncertainty", *Cost and Management*, Volume 42, No. 8 (September, 1968), pp. 13–19.

[33] WALSTROM, J. E., T. D. MUELLER AND R. C. MCFARLANE, "Evaluating Uncertainty in Engineering Calculations", *Journal of Petroleum Technology*, Volume 19, No. 12 (December, 1967), pp. 1595–1603.

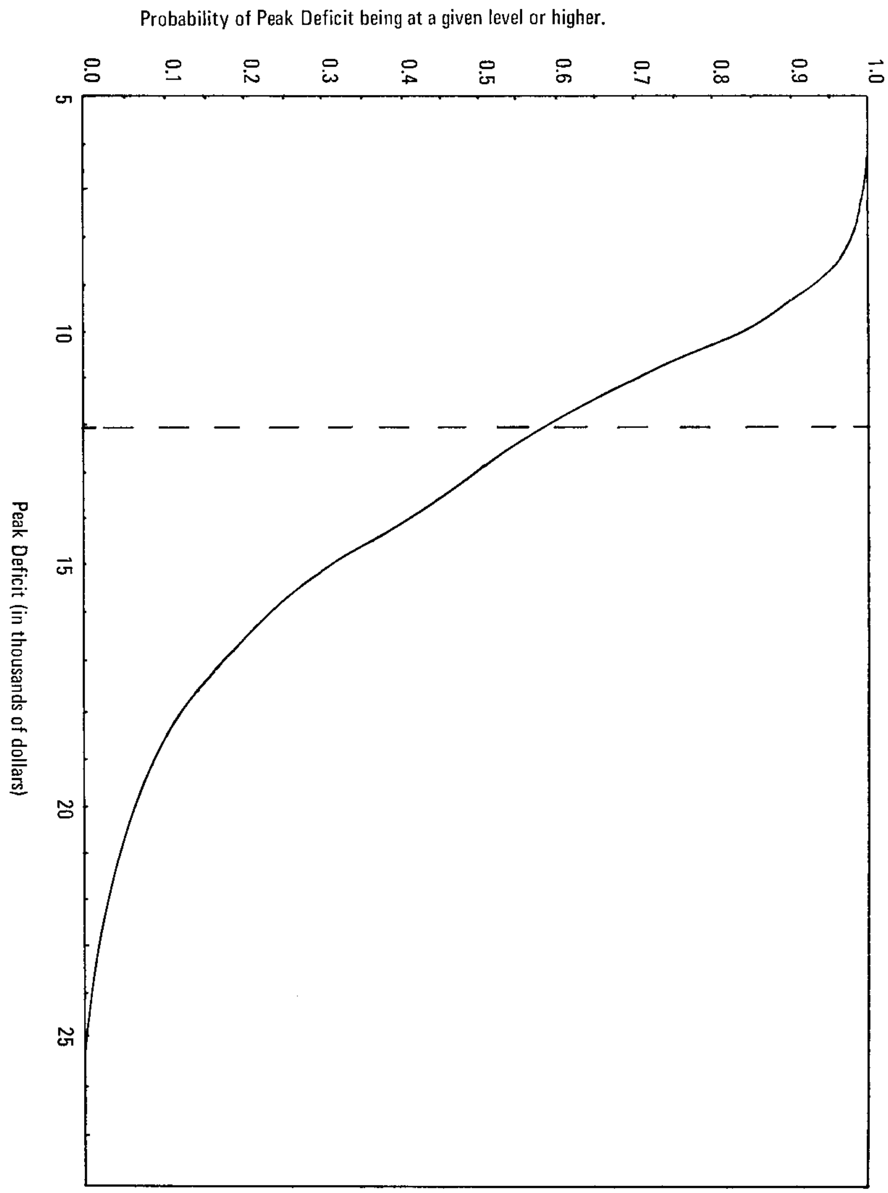
[34] WEIBULL, W., "A Statistical Function of Wide Applicability", *Journal of Applied Mechanics*, Volume 18, No. 3 (September, 1951), pp. 293–297.

[35] WHITLAM, G. B., "Capital Budgeting", (Paper presented at the Queensland Department of Primary Industries Regional Agricultural Economists Conference, Brisbane, July, 1969).

[36] WOODS, D. A., "Improving Estimates That Involve Uncertainty", *Harvard Business Review*, Volume 44, No. 4 (July-August, 1966), pp. 91–98.

APPENDIX

FIGURE 4—*Peak Deficit—"More Than" Ogive*



(Note: Vertical broken line indicates result in [12] at average price of $18.00 per 100 lb)

TABLE 4

Selected Readings from the Peak Deficit "More Than" Ogive

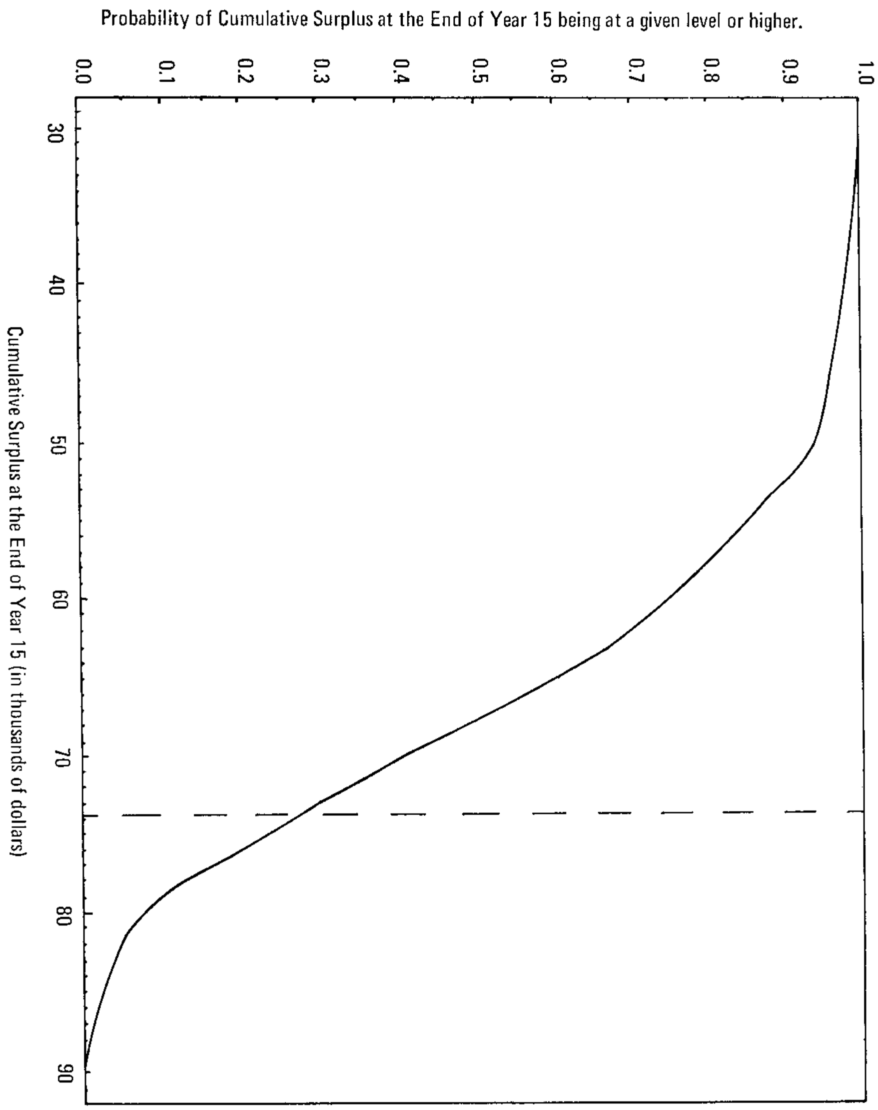
Per cent Probability of achieving ≥	$
90 per cent	9,320
75 per cent	10,570
50 per cent	12,820
25 per cent	15,680
10 per cent	18,370

TABLE 5

Selected Readings from the Cumulative Surplus at End of Year 15—*"More Than" Ogive*

Per cent Probability of achieving ≥	$
90 per cent	52,370
75 per cent	60,230
50 per cent	67,620
25 per cent	74,700
10 per cent	78,980

FIGURE 5—*Cumulative Surplus at End of Year 15—"More Than" Ogive*



(Note: Vertical broken line indicates results in [12] at average price of $18.00 per 100 lb)

TABLE 6

Selected Readings from the Cumulative Surplus at the End of Year 20—"More Than" Ogive

Per cent Probability of achieving ⩾	$
90 per cent	113,780
75 per cent	122,300
50 per cent	130,860
25 per cent	138,350
10 per cent	143,160

TABLE 7

Year of Peak Deficit—Simulation Results

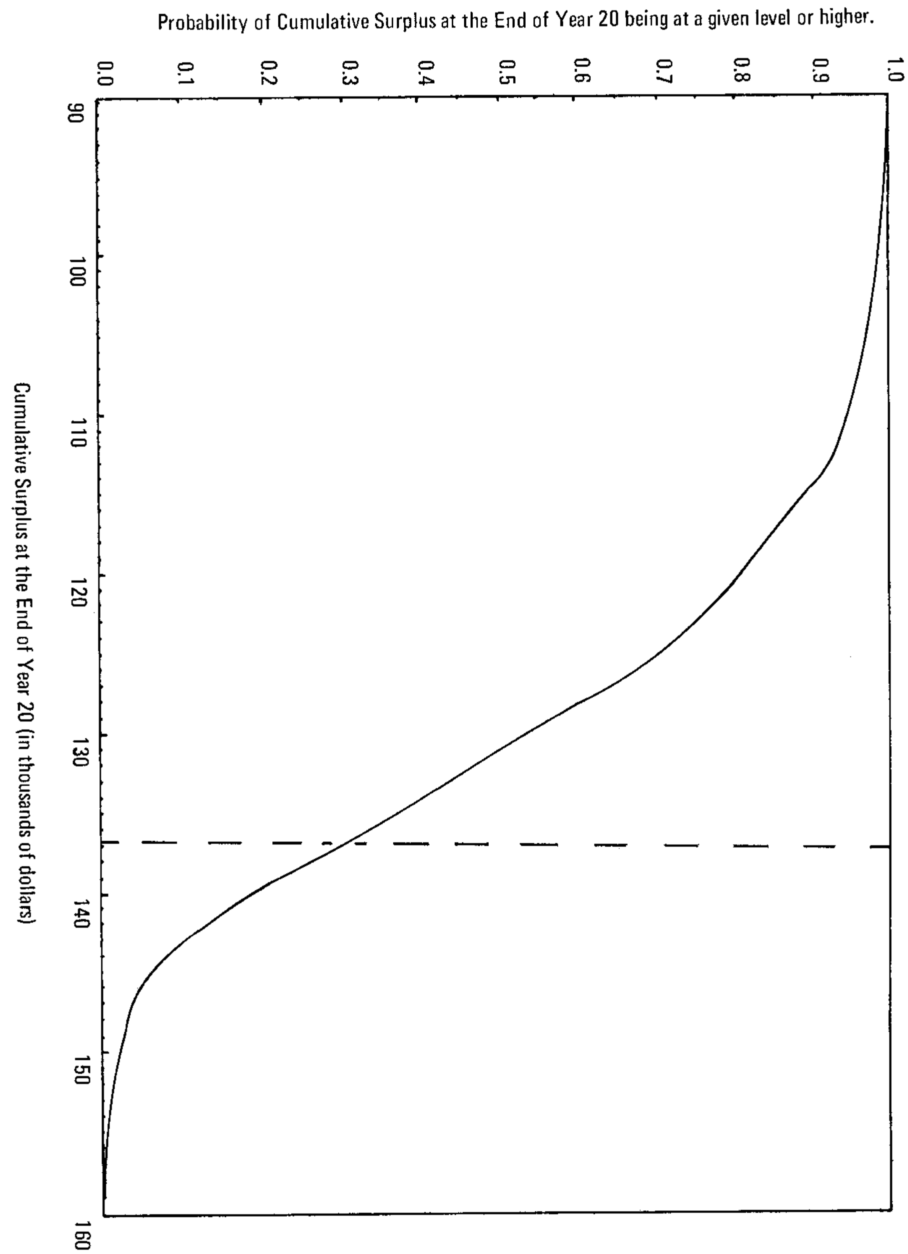
"More Than" Probability	Year
100·0 per cent	2
92·0 per cent	3
80·0 per cent	4
65·4 per cent	5
38·4 per cent	6
18·0 per cent	7
4·2 per cent	8
0·8 per cent	9
0·0 per cent	10

TABLE 8

Pay Back Period—Simulation Results

"More Than" Probability	Year
100·0 per cent	6
93·8 per cent	7
73·8 per cent	8
48·0 per cent	9
24·0 per cent	10
6·0 per cent	11
1·6 per cent	12
0·0 per cent	13

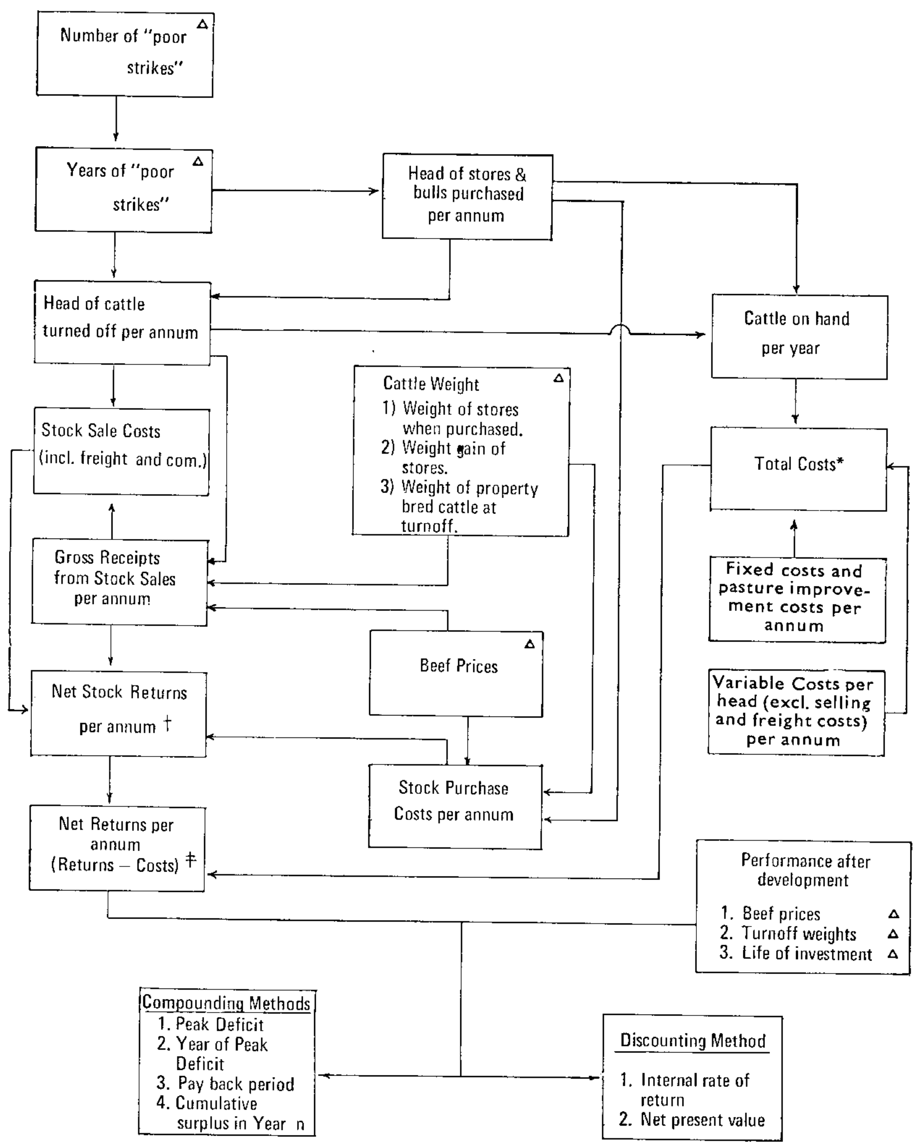
FIGURE 6—*Cumulative Surplus at the End of Year 20—"More Than" Ogive*



(Note: Vertical broken line indicates results in [12] at average price of $18.00 per 100 lb)

DIAGRAM 1

Simulation Model Flow Chart



△ Indicates the Monte-Carlo selection of the variable from a triangular function.

* Corresponds to item A in Table 4 of [12]

† Corresponds to item B in Table 4 of [12]

‡ Corresponds to item (B-A) in Table 4 of [12]