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Collusion and seasonality of market price - A case of fixed market shares

Sylwester Bejger

Faculty of Economics and Management
Nicholas Copernicus University of Torun, Poland
e-mail: sylw@uni.torun.pl

The paper develops a simple supergame model of collusion that focuses on the role of fixed (exogenous to game played) system of quantity market shares. Conclusions implied by the model could be used to motivate data - saving markers of collusion based on market price behavior. Following conclusions of the theoretical model we propose marker of collusion based on detecting changes in seasonal parameters of prices in periods of possible collusion. An empirical application of method has been done on well known data of Lysine cartel case.

JEL Classifications: L11, L13, L41

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Introduction

The collusive equilibrium of players in an industry may occur as a consequence of players' strategic interaction of the overt or tacit collusion nature. Although these two types of interaction are described by different models of game theory with various informative assumptions, their equilibriums are characterized by similar consequences for the industry, i.e.:

a. the occurrence of players' market power leading to the loss of social wealth (above-normal Price Cost Margin);

b. the limitation of competition and the impediment of the industry development.

Collusion constitutes a serious problem for the market economy. It not only generates welfare loss on a consumer side but seriously harms consumer's trust in a market system. Scale of collusion of various types is unknown but some researches, dealing hard core cartels mostly, show that it can be bigger than we thought so far (Connor and Helmers, 2006; Levenstein et al., 2004). Taking into account the fact how common and harmful collusion is, it seems natural that it should be quickly detected. Unfortunately, although theoretical models of overt or tacit collusion are described very well as research hypotheses concerning players' behavior, their empirical verification presents great difficulties. It happens mainly due to the fact that the players participating in collusion have an advantageous position over the observer in the form of private information. Moreover, the resources of public statistics are frequently (in Poland, for example) very humble on the disaggregation level of the industry or individual players. There are many methods by which we try to detect cartels (for good review see: Harrington, 2005; for review of econometric tools for various methods, see: Bejger, 2009). In author's opinion, good method should fulfill below mentioned postulates:

1. should be a part of coherent and systematic procedure[1] of detecting and estimating of market power in an industry, thus should have theoretical motivation implied by proper model of strategic interaction;
2. should use as small amount of publically presented statistical data as it's possible.

[1] For propositions of such procedures see for example (Oxenstierna, 1999; Bejger, 2009).

Group of methods which are especially useful for screening and verification tasks (following Harrington (2005, p.3) it means identification markets where collusion is suspected and providing evidences that observed behavior is in fact collusive) is so called markers of collusion (Harrington, 2005, p.25) i.e. specific patterns in economic processes that distinguish collusion from competition. These characteristic patterns concern:

- the relation between players' prices and market demand changes;
- price and market share stability;
- the relation between players' prices;
- investment in production capacity.

In the present paper such method is proposed and examined on empirical data. In the beginning section the supergame-based model of strategic interactions is developed. The model gives theoretical background for a collusion marker described in the following section ("Marker of collusion - econometric method"). Last sections examine the method empirically on well know Lysine conspiracy example and conclude the paper.

The model

It seems plausible that many price-fixing cartels do not use sophisticated methods of market sharing. Instead of these, very simple "rule of thumb" can be use. As de Roos (2004) noticed on a basis of Lysine1 cartel example, members of a cartel can agree to share market with market shares they enjoyed at the moment of initiating of a cartel and maintain these cartel quotas throughout the conspiracy period. There are evidence of such mechanism could work in Lysine cartel, vitamins C, E, A, B2 cartel but also in polish cement cartel. This last case could be the best confirmation of existence such market sharing mechanism because it was described in details in testimony of cartels members[2].

We would like to theoretically explore influence of such market sharing rule on strategy profile and possible market price movements. In earlier paper of similar topic[3] de Roos (2004) constructed the model of strategic interactions in a state space framework of Ericson and Pakes (1995) and then solve for the equilibrium using market specific values of parameters. Simulating optimal policy values he found among other that a firm entering a market characterized by collusion will tend to build up a market share comparable with its competitors before agreeing to collude (pp. 385). It means that price wars are possible in an equilibrium path and price dispersion in these periods is not affected by collusion.

We would like to develop a simple supergame model with above mentioned market sharing rule adopted and look for some analytical predictions on market price stability and movements.

Main assumptions

There are n players producing homogenous product. There is inverse demand function P(Q). Let us assume linear form of P(Q): $p = a - bQ$, where $Q = q_1 + q_2 + ... + q_n$. Cost functions of the players are defined as $C_i(q_i)$. Let us assume full symmetry of costs (the same production technology) and define linear cost function: $C = cq_i$ for $i = 1, 2, ..., n$ and $c = const$. We assume that $a > c > 0$.

Stage game g

It is assumed that stage game is finite, simultaneous-move game of complete information. Let us denote by $G = \{1,2, ...n\}$ set of players. Each player has convex and compact

[1] More about Lysine market and conspiracy see: Connor, J. (2000; 2001).

[2] See Decision of the President of the Office of Competition and Consumer Protection number Dok -7/2009, Warsaw, Republic of Poland.

[3] A little bit similar sharing rule has been used by Roller and Steen (2005) in purpose of explaining export market sharing.

action space Q_i with elements q_i, defined as supply (quantity) levels. For each player a stage game payoff function π_i is defined:

$$\pi_i: Q \to R, \tag{1}$$

where, $Q = \times_{i \in G} Q_i$.
Exact form of function (1) is defined as:

$$\pi_i = [a - bQ]q_i - cq_i \tag{2}$$

If we take into account cost symmetry we can define reaction function of the form:

$$R_i(Q_{-i}) = \frac{a - c - bQ_{-i}}{2b}, \quad i = 1,...,n. \tag{3}$$

where, $Q_{-i} = (n-1)q_i$.
For the system of n reaction functions (3) there exists1 unique pure actions profile:

$$q_i^{NC} = q^{NC} = \frac{a - c}{(n+1)b} \tag{4}$$

which form the Nash equilibrium of stage game g.
Payoffs in equilibrium (4) are defined as:

$$\pi_i^{NC} = \pi^{NC} = \frac{(a-c)^2}{(n+1)^2 b} \tag{5}$$

Superscript NC in (4) and (5) stands for Nash - Cournot equilibrium. In supergame framework (4) and (5) are called a punishment supply and payoff.
Let us assume now that players in an industry have historically determined (exogenously for the game played) quantity market shares:

$$s_i = \frac{q_i}{Q} \tag{6}$$

where, $\sum_{i=1}^{n} s_i = 1$. For aggregated system of functions (2):

$$\Pi = [a - bQ]Q - cQ \tag{7}$$

there exists unique quantity:

$$Q^m = \frac{a - c}{2b} \tag{8}$$

which maximizes value of (7). Let us name this quantity as monopoly supply and an aggregated payoff for Q^m , Π^m, as monopoly payoff. Quantity (8) is consistent with perfect collusion of players in an industry. For Q^m we have unique monopoly price p^m =

[1] π_i is strictly quasiconcave in q_i, which is a sufficient condition for uniqueness.

(a+c)/2. This is a text - book situation of ideal cartel agreement. In real world cartel episodes aggregate supply of cartel members is higher than that, mostly because of disturbances in information flow or players' cheating. Thus we can assume that average market price in collusive equilibrium is given as *r* part of monopoly price:

$$p = r\left[a - b\left(\frac{a-c}{2b}\right)\right] = r\frac{a+c}{2}, \tag{9}$$

where: $r \in (0;1]$.

Furthermore let us notice that for symmetric cartel equilibrium every player would have payoff of $\frac{\Pi^m}{n}$, but in a case of above defined exogenous market shares s_i and price (9) payoff of representative cartel member will be:

$$\pi_i^C = \frac{1}{4b} s_i (a^2 r^2 - 2a^2 r + 2acr^2 - 4acr + 4ac + c^2 r^2 - 2c^2 r) \tag{10}$$

with cartel quota:

$$q_i^C = s_i\left(\frac{a - (\frac{1}{2} r(a+c))}{b}\right) = -\frac{1}{b} s_i \left(\frac{1}{2} ar - a + \frac{1}{2} cr\right) \tag{11}$$

In a context of supergame models (10) and (11) we call as collusion payoff and quantity.

Cartel quotas (11) are not the best responses of players, i.e. are not actions in Nash equilibrium of a stage game. We can find the best responses using a best response correspondence:

$$N_i(Q_{-i}^C) = \left\{ q_i^D \in Q_i : q_i^D \in \arg\max_{q_i \in Q_i} \pi_i(q_i, Q_{-i}^C). \right\} \tag{11}$$

From our assumptions we can define:

$$Q_{-i}^C = (1 - s_i)\left(\frac{a - (\frac{1}{2} r(a+c))}{b}\right) \tag{12}$$

and formulate payoff function as:

$$\pi_i = \left[(a - b(q_i + (1 - s_i)\left(\frac{a - (\frac{1}{2} r(a+c))}{b}\right)) \right] * q_i - cq_i \tag{13}$$

Using FOC for (13) we can find best response of the player *i*:

$$q_i^D = -\frac{1}{4b}(2c - ar - 2as_i - cr + ars_i + crs_i) \tag{14}$$

It is quantity of player *i* with quota s_i assuming that she do not use that quota but every other players use their quotas. In a supergame context we name best response supply (14) as deviation supply of player *i*. We can define deviation payoff as:

$$\pi_i^D = \frac{1}{16b}(2c - ar - 2as_i - cr + ars_i + crs_i)^2 \tag{15}$$

For proper values of parameters we have $\pi^{NC} \leq \pi^C \leq \pi^D$.

Repeated game Γ(T)

Infinitely repeated game with observable actions (supergame) consists of repetition of a stage game *g* in T periods, where T = ∞. In a stage *t* players play a stage game *g*, choosing $q_i \in Q_i$. Actions of players are observable with some detection delay, Δ. Game Γ(T) is therefore a game with almost perfect information. Denote actions profile of stage *t* as $\boldsymbol{q}^t$. It implies that the set $H^t = (Q)^t$ histories h^t of a game for stage *t* is given. Additionally for *t* = 0, $h^0 = \varnothing$.

The set of terminal histories of the game H^∞ with elements h^∞ is also defined. A pure strategy S_i for player *i* is a sequence of maps $(S_i^t)_{t=0}^{\infty}$

$$S_i^t : H^t \rightarrow Q_i \tag{16}$$

(one for each stage *t*), that map possible histories in stage *t*, $h^t \in H^t$ to actions $q_i \in Q_i$.

For the games of oligopolistic competition common type of players' preferences are preferences with discounting, determined on terminal histories h^∞. Discount factor $\delta < 1$ is given as:

$$\delta = \frac{1}{(1+r)} = \mu e^{-r\Delta} \tag{17}$$

where: *r* - discount rate, μ - hazard rate - probability of o continuation of the game in a period *t+1*, Δ - length of a period (detection delay).

Discount factor δ is constant over time and is a common knowledge.

We can now define repeated game payoff function of player *i* (for pure strategies and preferences with discounting):

$$\Pi_i = \sum_{t=0}^{\infty} \delta^t \pi_i(q^t) \tag{18}$$

Equilibrium strategies of the repeated game

We assume that players use pure strategies (in a sense of (16)) which are trigger strategies with Nash punishment and are defined as:

$$S_i = \begin{cases} q_i^t = q_i^C(s_i) \quad gdy \quad q_j^\tau = q_{j\neq i}^C(1-s_i) \quad dla\ \tau = 0,...t-1 \\ q_i^t = q_i^{NC} \qquad\quad q_j^\tau = q_{j\neq i}^D(1-s_i) \quad dla\ \tau = 0,...t-1 \end{cases} \tag{19}$$

where: $s_i = s_i^0 = const$.

One should observe direct interdependence of actions and system of market shares defined exogenously. We can verbally explain strategy (19) for player *i* as: "choose as market supply the cartel quota quantity q^C (dependent on s_i) as long as other players do so, if deviation q^D of a single player is detected switch to punishment quantity q^{NC} for a rest of the game".

Profile of such strategies could form subgame perfect Nash equilibrium of the repeated game for sufficiently high discount factor[1]. For it to be the case the profile of strategies must satisfy Selten's subgame perfection condition. Common form of this condition for supergames is well known one - deviation principle in a form of inequality:

$$\frac{1}{1-\delta}\pi_i^Z \geq \pi_i^O + \frac{\delta}{1-\delta}\pi_i^{NC} \tag{20}$$

Thus collusive equilibrium with cartel quotas q^C could be sustained only if present value of stream of cartel payoffs is greater than present value of deviation and punishment for every player.

For the model we constructed there are two fundamental questions we want to examine:

- what is an influence of exogenously defined system of market shares si on stability of the potential cartel agreement,
- what is an influence of market size on average collusive market price.
- We use condition (20) to answer to above questions. We can rewrite (20) using (5), (10) and (15), thus we have:

$$\begin{aligned}&\frac{1}{4b}\frac{s_i}{\delta-1}(a^2r^2-2a^2r+2acr^2-4acr+4ac+c^2r^2-2c^2r)\geq\\&\geq\frac{1}{16b}(2c-ar-2as_i-cr+ars_i+crs_i)^2+(-\frac{1}{b}\delta\frac{(a-c)^2}{(\delta-1)(n+1)^2})\end{aligned} \tag{21}$$

what implies the limit value of s_i:

$$\begin{aligned}s_i \geq\ &\frac{1}{(\sqrt{\delta}-1)(\sqrt{\delta}+1)(n+1)(-2a+ar+cr)}(-2c-2\sqrt{\delta}(4a^2\delta+4c^2\delta+8c^2n-\\&-4c^2r+4c^2n+a^2r^2+c^2r^2+8ac-4a^2+a^2r^2n^2+c^2r^2n^2+2acr^2-8c^2nr+\\&+2a^2nr^2+2c^2nr^2-4c^2n^2r-8ac\delta-4acr+4acrn^2-4acn^2r+2acn^2r^2-\\&-8acnr)^{1/2}-2c\delta-2cn+ar+cr-2c\delta n+a\delta r+c\delta r+anr+cnr+a\delta nr+\\&+c\delta nr\end{aligned} \tag{22}$$

Limit value of s_i drives us to following conclusion.

Conclusion 1

Strategies (19) of repeated game Γ(Γ) are in subgame perfect Nash equilibrium if cartel quota of player *i* is greater than (22). Thus collusion in an industry could be started or sustained only if the smallest market share of some player *i* is greater than (22). As we see value of s_i is dependent on discount factor, number of players, cost, market price (given as some part *r* of the monopoly price) and, what is very interesting on market size *a*.

Conclusion 1 gives us an explanation of observed price wars in some industries. For example, if we assume that fixed system of market shares is prevailing cartel mechanism,

[1] It is a well known conclusion from „folk theorems" for infinitely repeated games. For a proof of such theorem with Nash punishment see for example: Fudenberg and Tirole (1996).

entrant player whose anticipated market share is smaller than (22) has no incentive to join the cartel and has to enlarge his quota at first, starting price war. This scheme of strategic behavior is consistent with history of Lysine cartel for instance, and confirms observations of de Roos (2004).

To answer to second question we have to estimate influence of market size changes on collusive market price. Let us assume deterministic changes in parameter a which can symbolize shifts in demand functions. We want to examine the influence of that shifts on market price r for fixed s_i, i.e. we want to determine what changes in price level should be to sustain collusion in an industry. To do this let us solve (21) as equality for r:

$$\begin{aligned} r = {} & \frac{1}{(n+1)(a+c)(-\delta+2s_i-\delta s_i^2+2\delta s_i+s_i^2+1)}(2c+2as_i^2-4a(-\delta-2\delta^2 s_i+ \\ & +\delta^2 s_i^2-2\delta s_i+\delta^2+2\delta n s_i^2+\delta n^2 s_i^2)^{1/2}+4c(-\delta-2\delta^2 s_i+\delta^2 s_i^2-2\delta s_i+\delta^2+ \\ & +2\delta n s_i^2+\delta n^2 s_i^2)^{1/2}-2c\delta+2cn+2as_i+2cs_i-2a\delta s_i^2+2ans_i^2-2c\delta n+2a\delta s_i+ \\ & +2c\delta s_i+2ans_i+2cns_i-2a\delta n s_i^2+2a\delta n s_i+2c\delta n s_i . \end{aligned} \tag{23}$$

To determine influence of enlarging or shrinking of the market, let us calculate limits of r given by (23) for a. First limit describes market shrinking as a drives to c:

$$\begin{aligned} \lim_{a\to c} r = {} & \frac{1}{2c(n+1)(-\delta+2s_i-\delta s_i^2+2\delta s_i+s_i^2+1)}2c+2cs_i^2-2c\delta+2cn+4cs_i-2c\delta s_i^2+ \\ & +2cns_i^2-2c\delta n+4c\delta s_i+4cns_i-2c\delta n s_i^2+4c\delta n s_i=1. \end{aligned} \tag{24}$$

The second limit describes enlarging the market to infinity:

$$\begin{aligned} \lim_{a\to\infty} r = {} & \frac{1}{(n+1)(-\delta+2s_i-\delta s_i^2+2\delta s_i+s_i^2+1)}(2s_i-4(-\delta-2\delta^2 s_i+\delta^2 s_i^2- \\ & -2\delta s_i+\delta^2+2\delta n s_i^2+\delta n^2 s_i^2)^{1/2}+2s_i^2-2c\delta+2ns_i-2\delta s_i+ \\ & +2ns_i^2-2\delta n s_i^2+2\delta n s_i . \end{aligned} \tag{25}$$

The value of limit (25) we found numerically, using parameterization of Lysine market from de Roos (2004). For that set o parameters and $n = 4$ players this value is 0,592. From (24) and (25) we have conclusion 2.

Conclusion 2

With fixed system of cartel quotas s_i if size of the market is growing the market price needed to sustain collusion is smaller, and if market size is shrinking the price needed to sustain collusion is greater than the starting collusive level of r. Thus to eliminate incentive to cheat, cartel price should depict some amount of rigidity in a case of market shrinking.

Conclusion 2 develops motivations of collusion markers relayed on structural changes in market price variance. If our model sufficiently well describes players' strategic behavior market price variability should be lower in cartel phase because of some rigidity of price in a periods of smaller market size.

To check conclusion 1 and 2 small numerical simulation have been done. We take into considerations de Roos' parameterization of the Lysine market and assume specific market

structure for n = 3. Table 1 contains these assumptions. Table 2 contains simulations for 20 periods (months).

TABLE 1. SIMULATION'S PARAMETERS

a	1.651
b	0.0857
c	0.527
δ	0.98
s_1, s_2, s_3	40%, 33%, 27%
r	0.9

TABLE 2. SIMULATION'S RESULTS

Period	A	B	C	D	E
1	1.651	1.583659	0.9801	0.9801	1.583659
2	1.646735	1.565427	0.978181	0.978181	1.565427
3	1.165027	0.02406	0.761412	0.761412	0.02406
4	0.905474	**-0.38096**	0.644613	0.716237	0.353834
5	1.227596	0.16627	0.789568	0.789568	0.16627
6	1.846239	2.504362	1.067957	1.067957	2.504362
7	1.504839	1.004751	0.914327	0.914327	1.004751
8	1.318246	0.403019	0.830361	0.830361	0.403019
9	1.356868	0.51493	0.847741	0.847741	0.51493
10	1.6119	1.419535	0.962505	0.962505	1.419535
11	1.273684	0.282095	0.810308	0.810308	0.282095
12	1.190405	0.079653	0.772832	0.772832	0.079653
13	1.193827	0.087366	0.774372	0.774372	0.087366
14	1.336498	0.455085	0.838574	0.838574	0.455085
15	1.539717	1.13431	0.930023	0.930023	1.13431
16	1.561634	1.218478	0.939885	0.939885	1.218478
17	1.905889	2.819283	1.0948	1.0948	2.819283
18	1.041524	**-0.20582**	0.705836	0.784262	0.653941
19	1.357413	0.516557	0.847986	0.847986	0.516557
20	1.121399	**-0.06485**	0.74178	0.8242	0.872737

Source: Author's own calculations on simulated data.

Note: A - parameter of market size *a;* B - difference beetwen LHS and RHS of incentive compatibility constraint (20), + collusion, (-) no incentive to collude for player with min s_i; C - market price for parameter r = 0.9; D - market price rigid to monopoly level r =1 in a periods of small market size; E - difference beetwen LHS and RHS of incentive compatibility constraint (20) for rigid market prices D for player with min s_i

We started from parameters from Table 1 (period 1) and then we changed value of parameter *a* (market size) to simulate periods of larger and smaller market. As it can be observed in Table 2, player with minimum market share (27%) has no incentive to collude in periods 4, 18 and 20 when collusive price is on 0.9 of monopoly price level. To maintain cartel agreement the price should be higher (for example on $r = 1$ level, it means monopoly level).

To reach this level cartel as a whole should limit the supply more than it is implied by maximization of payoff principle. We thus can observe some rigidity of prices in low demand periods. It also means lower variance of market price in collusive phase.

Marker of collusion - econometric method

We want to utilize theoretical predictions of previous section to motivate marker of collusive behavior. From Conclusion 2 we see that in cartel agreement phase variance of market price should be lower and price movement should not exactly follow market decreasing.

The works conducted so far that are connected with collusion detection on the basis of the detection of structural changes in variance included the application of the descriptive statistics methods for the comparison of variance level in collusion and competition phases (Abrantes-Metz, Froeb, Geweke, Taylor, 2006), application of ARCH / GARCH specification for the market price process together with additional 0-1 variable describing collusion and competition phases (Bolotova, Connor, Miller, 2008), application of Markov Switching Model of MS(M)(AR(p))GARCH(p,q) type (Bejger, 2010) and application of wavelet analysis (Bejger, Bruzda, 2010). All of the above mentioned papers focus on detection of structural changes in variance of market price.

In following paper we want to propose a different method based on observed market price movement[1]. As we see from theoretical model market price is rigid when market is getting smaller. We can describe such a shift in demand as a seasonal fluctuation (we can consider it deterministic). If some industry exhibits seasonal fluctuations of demand (which is an exogenous fact, know from economic theory) we can use seasonal price movement to detect or confirm cartel behavior of the players. Construction of the method is as follows:

- in the industry suspected of collusion detect possible phases of competition / cartel behavior
- split the sample of market prices into subsamples consistent with detected phases and check for significance of seasonal fluctuations. If we observe insignificant seasonal parameters in subsample of collusion it may be consistent with the strategic behavior described by the model suggested in previous section and could confirm possible cartel agreement.

As a method of detecting seasonal component in time series we use simple method of dummy variables. This method is often good enough to seasonally adjust the data and is easy to implement and interpret.

Empirical verification - Lysine cartel

As a test data set we want to utilize data on lysine prices from well - known conspiracy case. The data set includes monthly average lysine prices on the USA market in the period between 01/90 - 06/96[2]. Figure 1 depicts the prices with calendar dates and observations numbers.

In previous work (Bejger, Bruzda, 2010) we have been able to detect two switching points in variance of the process. The first one is localized about observation number 28 (increase of variance) and the second is about observation number 45 (decrease of variance). We concluded in cited paper that real cartel phase could start about September 1993 (45 - th. obs.). We know from market analysis (Connor, 2000, pp. 24) that demand for lysine is seasonal with the lowest level in first five months of the year and the lowest prices in the summer months. Thus, we proceed as follow:

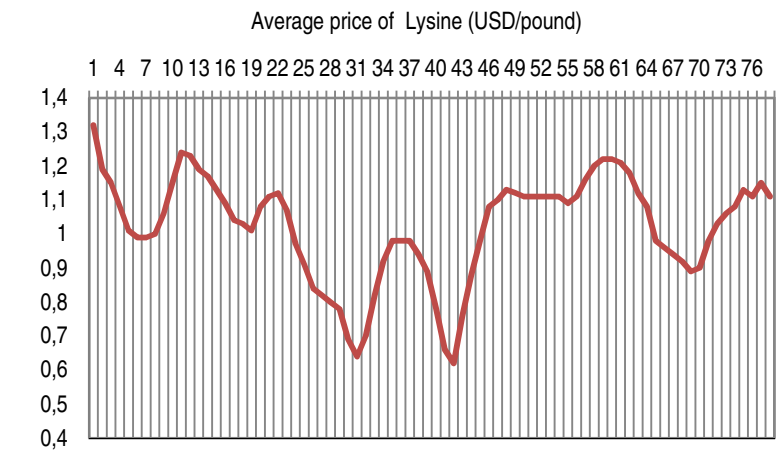
- check for seasonality in a whole sample,

[1] There are some other papers connected with demand movements and market price movements in dynamic game context (Green, and Porter, 1984; Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991; Knittel and Lepore, 2010) although no one of them consists a key assumption of fixed market shares system.

[2] The prices are from Connor, (2000), appendix A, Table A2.

- check for seasonality in subsamples: subsample number one dated from January 1990 to August 1993 (non-cartel period) and subsample number two dated from September 1993 to June 1996 (cartel period).

FIGURE 1. AVERAGE PRICE OF LYSINE (US MARKET)



Source: Data from CONNOR, (2000), APPENDIX A, TABLE A2.

For the steps listed above we use simple regression of price on constant and eleven seasonal (dummy) variables. We proceed as follows:

$$\hat{Y}_t = const. + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \alpha_4 D_{4t} + \alpha_5 D_{5t} + \alpha_6 D_{6t} + \alpha_7 D_{7t} + \alpha_8 D_{8t} + \\ + \alpha_9 D_{9t} + \alpha_{10} D_{10t} + \alpha_{11} D_{11t} + \alpha_{12} D_{12t} + u_t \qquad (26)$$

where: $\hat{Y}_t$ - average price of lysine; $D_{2t},...,D_{12t}$ - seasonal dummies.

As (26) implies, we treat January as the reference month so coefficients attached to the seasonal dummies are differential intercepts, showing by how much the average price in the month with dummy value of 1 differs from January.

Estimation results are shown in Tables 3, 4 and 5.

TABLE 3. REGRESSION RESULTS FOR A FULL SAMPLE

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CONST.	111.2857	5.596808	19.88378	0.0000
D2	-4.000000	7.915082	-0.505364	0.6150
D3	-6.285714	7.915082	-0.794144	0.4300
D4	-10.57143	7.915082	-1.335606	0.1863
D5	-15.14286	7.915082	-1.913165	0.0601
D6	-18.28571	7.915082	-2.310237	0.0240
D7	-20.78571	8.238279	-2.523065	0.0141
D8	-16.45238	8.238279	-1.997065	0.0499
D9	-10.95238	8.238279	-1.329450	0.1883
D10	-5.119048	8.238279	-0.621373	0.5365
D11	-1.452381	8.238279	-0.176297	0.8606
D12	-1.952381	8.238279	-0.236989	0.8134
R-SQUARED 0.198939				

Source: Author's own calculations

TABLE 4. REGRESSION RESULTS FOR A FIRST SUBSAMPLE
(NON - COLLUSIVE PERIOD)

Variable	Coefficient	Std. Error	T-Statistic	Prob.
CONST.	110.0000	8.543547	12.87522	0.0000
D2	-6.500000	12.08240	-0.537973	0.5943
D3	-10.25000	12.08240	-0.848341	0.4025
D4	-16.25000	12.08240	-1.344932	0.1881
D5	-22.75000	12.08240	-1.882904	0.0688
D6	-26.75000	12.08240	-2.213964	0.0341
D7	-25.00000	12.08240	-2.069125	0.0467
D8	-18.50000	12.08240	-1.531153	0.1356
D9	-10.33333	13.05048	-0.791797	0.4343
D10	-3.666667	13.05048	-0.280960	0.7805
D11	-0.333333	13.05048	-0.025542	0.9798
D12	-4.000000	13.05048	-0.306502	0.7612
R-SQUARED 0.286639				

Source: Author's own calculations

TABLE 5. REGRESSION RESULTS FOR A SECOND SUBSAMPLE
(COLLUSIVE PERIOD)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CONST.	113.0000	5.611586	20.13691	0.0000
D2	-0.666667	7.935981	-0.084006	0.9338
D3	-1.000000	7.935981	-0.126008	0.9009
D4	-3.000000	7.935981	-0.378025	0.7090
D5	-5.000000	7.935981	-0.630042	0.5352
D6	-7.000000	7.935981	-0.882059	0.3873
D7	-11.50000	8.872697	-1.296111	0.2084
D8	-11.50000	8.872697	-1.296111	0.2084
D9	-12.00000	7.935981	-1.512100	0.1447
D10	-7.000000	7.935981	-0.882059	0.3873
D11	-3.000000	7.935981	-0.378025	0.7090
D12	-0.333333	7.935981	-0.042003	0.9669
R-SQUARED 0.226108				

Source: Author's own calculations

At first we can observe statistically significant seasonal dummies in the full sample (Table 3). These are for June, July and August (with p-values less than 5%) and indicates fall of average prices in that months. At second, this is confirmed from June and July in non-collusive subsample (Table 4). Interestingly, all seasonal factors occurred insignificant in subsample 2 (collusive period, Table 5) which means that seasonal fall of prices was eliminated, prices stayed rigid. This price movement (seasonality "smoothing") is consistent with our theoretical predictions. We are of course aware of possible biases in estimation (small samples, assumption of deterministic seasonality) but we thing that this preliminary results could be promising for further research.

Conclusion

This paper developed a simple supergame model of collusion that focuses on the role of fixed (exogenous to game) system of market shares. Conclusions implied by the model could be used to motivate markers of collusion based on market price movements and

variability. One of the theoretical predictions is that to eliminate incentive to cheat, cartel price should depict some amount of rigidity in a case of shrinking of market size. On a basis of these findings we proposed simple method of confirmation/detection of cartel agreements in an industry. The method could be used as a marker of collusion and is based on detecting changes in seasonal parameters of prices in periods of possible collusion. We test the method on well known data of Lysine cartel case. We found some evidence of consistency of empirical conclusions (seasonality "smoothing") with the theoretical predictions. Our work has strong connection with practice as various methods of detecting cartels are still needed by many authorities. There are of course many ways for further exploration of the topic. On the theoretical side we leave for further research problem of influence of different penal codes, market parameters and structure on sustainability of collusion. On the empirical side one could use more sophisticated method of seasonality analysis (including stochastic seasonality models).

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