SENSITIVITY OF PLANT LOCATION SOLUTIONS TO CHANGES IN RAW PRODUCT SUPPLIES

W. O. McCarthy, D. C. Ferguson, and P. A. Cassidy*

A method is outlined to test the stability of a plant location solution to changes in regional production. Supplies are generated within a probabilistic framework and used as input data for the usual otherwise static model. This approach also yields additional information on the probability of actual plant throughput exceeding designed throughput. Such types of information further assist policy decisions.

1 THE PROBLEM

Recent contributions to this Review by Weinschenck, Henrichsmeyer, and Aldinger [15] and Ferguson and McCarthy [6] have indicated the flexibility and range of models and solution techniques that are available for processing plant or warehouse location problems.

A shortcoming of all the empirical studies cited is that they fail to indicate the effect on the derived solution of changes in data inputs. Data for the usual processing plant location study normally include raw product supply and final product demand, transport charges from supply regions to processing centres and then to final demand and processing plant cost functions.

Recently, however, Toft, Cassidy, and McCarthy [14] have outlined a test relating to changes in processing costs, and Ladd and Halvorson [10] have dealt briefly with variations in some parameters of the simple Stollsteimer model. The purpose of this note is to outline one possible procedure for testing the sensitivity of plant location solutions to changes in raw product supplies from the point of view of (i) stability of locations and (ii) likely variations in throughput.

It is suggested that the assumption of completely fixed supply inherent in plant location models is particularly inappropriate in agricultural situations because of the dominating influence of environment on production. Thus supply is probably more subject to variation than transport or processing costs. Accordingly if policy decisions are to be made on the basis of plant location studies, then the consequences of likely changes in supply need to be carefully assessed.

Consider, for example, the current agitation for wool supply management[1] to solve the price problem of wool growers. Such restrictions could readily be incorporated into the model. Or, if supply management became a respectable policy measure, the technical question of optimum location of production (*or* the cost of non-optimum location) could be easily solved.

---

* Respectively, Lincoln College, Australian Wool Board, and London School of Economics. The authors appreciate helpful comments from colleague John Rodgers.

[1] *Australian Financial Review*, May 20, 1971, p. 6.

The empirical example used here to illustrate the sensitivity test is the study of Ferguson and McCarthy [6]. This was concerned with determining the size, number, and location of wool selling centres in Australia and used the transhipment formulation of the transportation model as the analytical framework.

2 A METHOD OF APPROACH

2.1 GENERAL

The basic model was static and retrospective (hereafter called retrospective) in the sense that the solution was derived using data for a past time period—namely the 1967–68 season. One possibility for introducing greater reality is subjectively to assume changes in supply levels for particular regions (for example based on the application of improved technology) and observe the effect on the optimum solution. This approach was discussed briefly by Dent [4], but may not be rewarding because it does not introduce time in a particularly meaningful way. It is suggested in this paper that a preferred alternative is a probabilistic approach.

The retrospective model is placed within this framework by establishing appropriate probability distributions for production in each of the thirty-two supply regions which together comprise total supply for the model. Variates are then generated at random from these distributions. Each set of one variate from each of the thirty two distributions constitutes the basic supply data. These are then incorporated with other input data (transport and processing costs) and the problem solved to establish the optimum location pattern for that particular supply. This process is repeated many times. Export demand (estimated as a residual) is varied in each case to meet the model restriction that total supply must equal total demand. The "optimum" plans so generated are then used to determine the sensitivity of the retrospective model locations to changes in supply as well as to estimate likely variations in throughput for plants at these locations.

2.2 APPROPRIATE PROBABILITY DISTRIBUTIONS

It is suggested that the approach outlined above is a realistic one. The major consideration is choice of appropriate probability distributions. More precisely, what distributions best describe the supply data? If there is a choice, then ease of estimation is another aspect to be considered.

Recent studies in investment and operations research utilize a range of distributions. Thus Sprow [12] and Cassidy, Rodgers, and McCarthy [2] use the triangular, Hess and Quigley [8] the normal, and Hertz [7] the generalized.[2] Kaplan [9] uses both the uniform (rectangular) and the Erlangian and Asher [1] the Poisson and the PERT beta.[3]

---

[2] Also referred to as the frequency distribution and when the nodes of each frequency class are joined is known as the frequency polygon.
[3] The PERT beta is of the triangular type but difficult to use in generating random variates.

Of these, the uniform is the simplest, assuming an equal probability of occurrence within the bounds of the distribution. However it is difficult to visualize a definite probability for a variable at the extreme upper or lower bound of the distribution but no probability for a variable slightly greater or less than these bounds. The normal distribution does not have this characteristic nor does it have a cut off point. The triangular is more flexible again. It is not necessarily symmetrical and is easily defined. It is also simple to use when generating random variables. The generalized has no pre-determined shape. All that is required is the specification of "n" node points.

However as Sprow [12] has noted in an investment analysis context and Day [3] for agricultural situations, choice of a function properly rests on the characteristics of the data under examination rather than *a priori* reasoning. Accordingly, in this study, supply data are plotted and choice of appropriate distributions then made.

2.3 DATA GENERATION

To gain information on the distribution of wool production over time for each of the 32 supply regions, histograms were prepared using data from 1950 onwards. Data prior to 1950 were not included because it was considered that technology before this time—particularly in the higher rainfall zones—would result in unrealistically low values being used for parameter estimation. Time trends were determined and extrapolated for 10 years to provide a guide for upper bounds for the histograms. Each histogram was assessed and classed into one of the frequency distributions discussed above. It was found that all fell either into the triangular or generalized groups. The distribution function together with the individual supply region histogram data then formed the basis for generating random supplies[4] for each of the 32 supply regions. This process was repeated 200 times, thus providing 200 sets of supply data for the location model.[5]

3 RESULTS AND DISCUSSION

3.1 LOCATION OF PROCESSING CENTRES

The retrospective model required four iterations to reach a stable solution. With (in effect) 200 models the total cost of running each to stability was estimated to be prohibitive. Accordingly the first iteration of each was run and inspected. Those with (subjectively) atypical solutions were noted and run to stability. The atypical solutions were those with country centres having large (low cost) throughputs compared with the retrospective model and those in which centres located at ports had small (high cost) throughputs. These situations were considered the most likely to result in a different final solution. Eight cases were

[4] A programme for the generation of random variates is available from the authors. The general procedure is discussed in texts such as Mize and Cox [11], Tocher [13]. Aspects are outlined more adequately in Dent and Anderson [5], chapter 3.

[5] This procedure implies that variations in supply between regions are independent. This is a complex question with plausible arguments for and against, and is at present being further investigated. Factors to take into account here include size, location, and development of regions.

considered to be of this type and were run to stability. Only one of the flows generated differed from the flows of the retrospective model. In this case, wool from Dubbo moved to Brisbane instead of Melbourne. The reason was that in this instance Brisbane had a large throughput compared with Melbourne, and consequently per bale costs in Brisbane were cheaper than those in Melbourne. Such a result was not unexpected. A previous analysis of the dual had indicated that flow from Dubbo was sensitive to transport rates. The reduced processing costs in Brisbane, relative to those in Melbourne, produced the same effect as reducing the transport rate from Dubbo to Brisbane. No other changes in the optimal flows occurred. It was therefore concluded that the original model plant location pattern was stable to likely changes in regional supply.

3.2 VARIATIONS IN THROUGHPUT

The procedure used to estimate likely variations in annual throughput of complexes established in the suggested locations was as follows.

Twenty[6] sequential sets of generated supply data, beginning at a randomly selected point in the 200 generated sets, were used as input for models which were run to stability. The resulting throughputs for each centre were tabulated and the mean of these throughputs calculated. These are given in column 1 of table I. It is considered that this mean is a more appropriate throughput on which to base individual plant size than the point estimate of the retrospective model.

TABLE 1

Suggested Mean Size and Probability of Actual Throughput Exceeding Designed Size (Bales per Annum)

Centre	1 Suggested mean size	2 Probability of actual > mean throughput	3	4	5
			Probability of exceeding the following throughput		
			·25	·10	·05
Brisbane	1,155,065	·44	1,300,000	1,340,000	1,348,000
Newcastle	173,586	·64	182,000	188,000	190,000
Sydney	310,902	·42	320,000	338,000	350,000
Melbourne	1,727,475	·52	1,850,000	1,877,000	1,887,000
Hobart	114,108	·68	125,000	130,000	134,000
Portland	174,332	·58	187,000	195,000	198,500
Adelaide	604,557	·61	620,000	630,000	650,000
Albany	266,331	·52	310,000	355,000	375,000
Fremantle	623,397	·56	670,000	700,000	720,000

Minimum total cost occurs when actual plant throughput is equal to the designed throughput. However, given production uncertainty, this will likely be a rare occurrence. If actual throughput falls below

[6] A greater number of runs may have been desirable but again cost was a critical factor.

designed throughput, per bale capital costs are high but per bale operating costs are reduced. Conversely, if throughputs exceed designed capacity per bale, capital costs fall but are more than offset by rising per bale operating costs. As operating costs tend to increase rapidly after throughput exceeds designed capacity, management may only wish to be faced with a situation in which expected throughput exceeds designed capacity only (say) 10 per cent of the time.

The probability approach allows the determination of complex sizes which fulfil such requirements. Histograms of throughput at each suggested selling centre can be prepared and transformed into distributions showing the probability of throughput being more than a specified size.

Figures 1 and 2 illustrate such ogives for Brisbane and Adelaide.

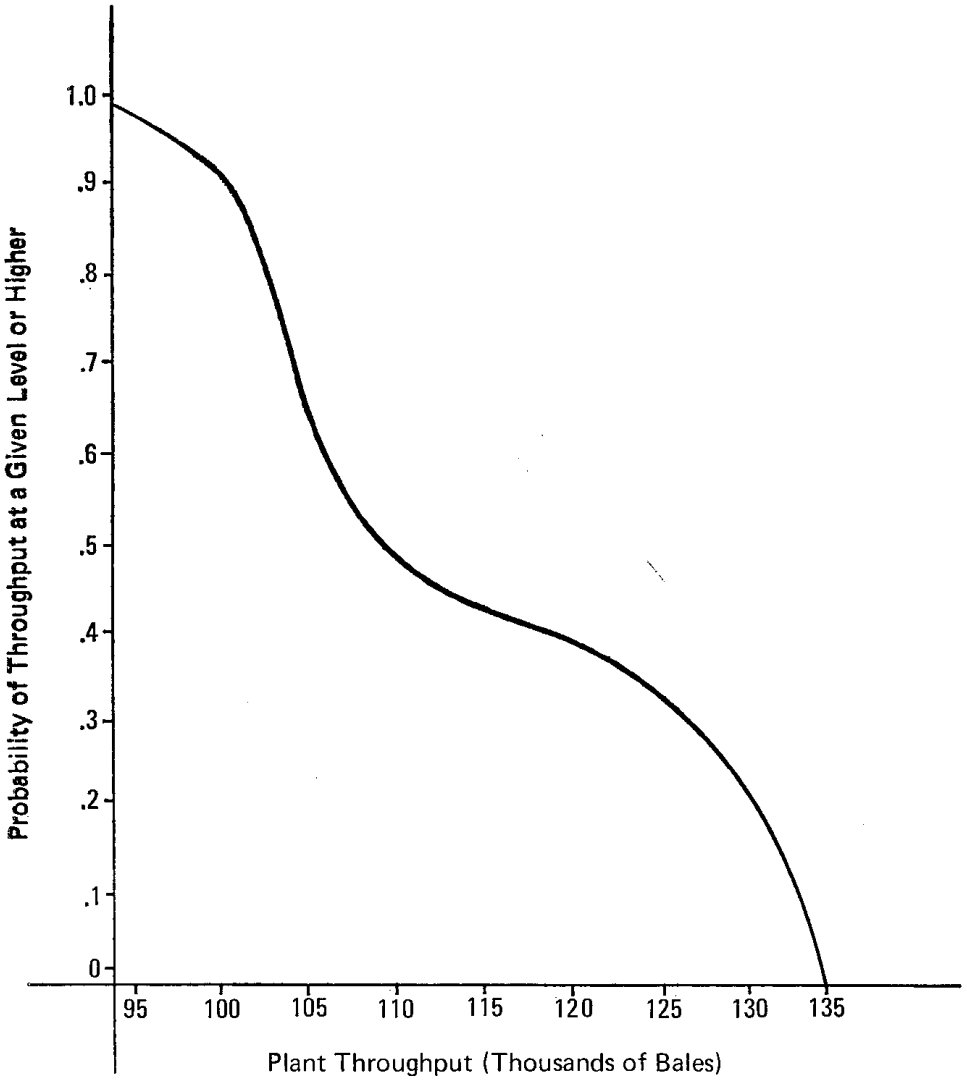

FIGURE 1: *Probability of throughputs at a given level or higher—Brisbane*

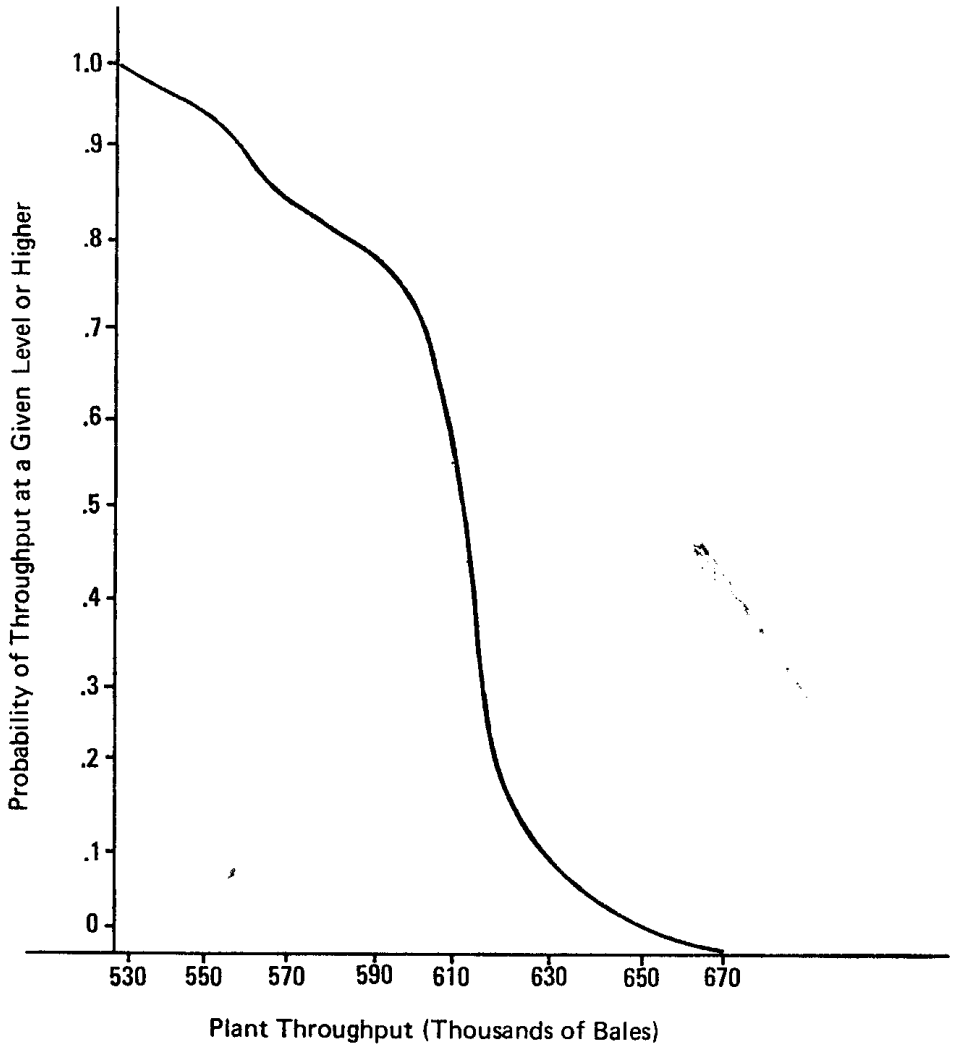

FIGURE II: *Probability of throughputs at a given level or higher—Adelaide*

Table 1 also includes data (columns 4, 5, 6) regarding plant capacity to meet the requirement that actual throughput exceeds designed throughput in 25 per cent, 10 per cent, and 5 per cent of cases only.

Consider Brisbane with a suggested (designed) mean size of 1,155,065 bales. There is a ·25 probability that actual throughput will exceed 1,300,000 bales and a ·05 probability of exceeding 1,348,000 bales.

When plant capital and operating costs are considered along with such information, management can decide on their best course of action.

3.3 CONCLUSION

The probability approach used here is proposed as a useful extension for policy makers concerned with plant location problems because it provides further information for decision making. While this note

deals with only one input (supply) the method is also applicable to transport and processing costs. Or, it could be used in conjunction with other tests such as the static one of Toft, Cassidy, and McCarthy [14] if it was not considered appropriate to assign probabilities to all inputs.

REFERENCES

[1] ASHER, D. T., "A Linear Programming Model for the Allocation of R and D Efforts", *IRE Transactions on Engineering Management*, Volume EM 9, No. 4 (December, 1962), pp. 154–157.

[2] CASSIDY, P. A., RODGERS, J. L., and MCCARTHY, W. O., "A Simulation Approach to Risk Assessment in Investment Analysis", *Review of Marketing and Agricultural Economics*, Volume 38, No. 1 (March, 1970), p. 24.

[3] DAY, R. H., "Probability Distributions of Field Crop Yields", *Journal of Farm Economics*, Volume 47, No. 3 (August, 1965), pp. 713–741.

[4] DENT, W., "Sensitivity Analysis of Wool Transportation", *Australian Economic Papers*, Volume 5, No. 2 (December, 1966), pp. 211–219.

[5] DENT, J. B. and ANDERSON, J. R., *Systems Analysis in Agricultural Management* (Sydney: John Wiley, 1971).

[6] FERGUSON, D. C. and MCCARTHY, W. O., "A Spatial Analysis Approach to Size and Location of Australian Wool Selling Centres", *Review of Marketing and Agricultural Economics*, Volume 38, No. 4 (December, 1970), pp. 153–156.

[7] HERTZ, D. B., "Risk Analysis in Capital Investment", *Harvard Business Review*, Volume 42, No. 1 (Jan.-Feb., 1964), pp. 95–106.

[8] HESS, S. W. and QUIGLEY, H. A., "Analysis of Risk in Investment Using Monte-Carlo Techniques", *Chemical Engineering Symposium Series: Statistics and Numerical Methods in Chemical Engineering*, Volume 59, No. 42 (1963), pp. 55–63.

[9] KAPLAN, S. and BARISH, N. N., "Decision-Making Allowing for Uncertainty of Future Investment Opportunities", *Management Science*, Volume 13, No. 10 (June, 1967), pp. 569–577.

[10] LADD, G. W. and HALVORSON, M. P., "Parametric Solutions to the Stollsteimer Model", *American Journal of Agricultural Economics*, Volume 52, No. 4 (November, 1970), pp. 578–586.

[11] MIZE, J. H. and COX, J. G., *Essentials of Simulation*, Prentice-Hall International Series in Industrial Engineering and Management Science, (W. G. Ireson, ed.), (Englewood Cliffs: Prentice-Hall, 1968).

[12] SPROW, F. B., "Evaluation of Research Expenditures Using Triangular Distribution Functions and Monte Carlo Methods", *Journal of Industrial and Engineering Chemistry*, Volume 59, No. 7 (July, 1967), pp. 35–38.

[13] TOCHER, K. D., *The Art of Simulation*, Electrical Engineering Series (English Universities Press Ltd, 1963), p. 184.

[14] TOFT, H. I., CASSIDY, P. A., and MCCARTHY, W. O., "Sensitivity Testing and the Plant Location Problem", *American Journal of Agricultural Economics*, Volume 52, No. 3 (August, 1970).

[15] WEINSCHENCK, G., HENRICHSMEYER, W., and ALDINGER, F., "The Theory of Spatial Equilibrium and Optimal Location in Agriculture: a Survey", *Review of Marketing and Agricultural Economics*, Volume 37, No. 1 (March, 1969), pp. 3–70.