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Linking of Repeated Games. When Does It Lead to More Cooperation and Pareto Improvements?

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Linking of repeated games. When does it lead to more cooperation and Pareto improvements?

Summary

Linking of repeated games and exchange of concessions in fields of relative strength may lead to more cooperation and to Pareto improvements relative to the situation where each game is played separately. In this paper we formalize these statements, provide some general results concerning the conditions for more cooperation and Pareto improvements to materialize or not and analyze the relation between both. Special attention is paid to the role of asymmetries

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1 Introduction

There has developed an interest in the theory and applications of linking, also called 'interconnection'. The basic idea is the following. Consider a group of decision makers who are simultaneously involved in several different real world problems (issues). The standard approach is to consider the decision making process for each problem in isolation. In practice, however, the decision making process with respect to one problem is usually influenced by the decision making processes with respect to the other problems (spill-over effects or links). Discarding the links among the issues and analyzing the decision process on each issue separately rather than in a multi-issue decision making context is likely to lead to biased outcomes. Particularly, a single issue approach ignores the possibility that if the issues have compensating asymmetries of similar magnitudes, an exchange of concessions may allow and enhance cooperation which extends beyond cooperation in the single issue context. Some well-known real world examples of linking are the negotiations 'on land for peace' between Israel and Palestina and the deal on WTO membership and participation in the Kyoto agreement between the EU and Russia.

In the economics literature the notion of linking has been applied in the context of multimarket behavior in oligopolistic markets (see e.g. Bernheim and Whinston, 1990; Spagnolo, 1999) and of international environmental problems (see e.g. Folmer et al., 1993; Botteon and Carraro, 1998; Carraro and Siniscalco, 1999; Finus, 2001).

A game theoretical framework for the linking of repeated games was developed by Folmer et al. (1993) and by Folmer and von Mouche (1994). In Folmer and von Mouche (2000) the following themes for linking of repeated games were suggested: linking may sustain more cooperation,[1] may eliminate social welfare losses, may bring Pareto improvements and may facilitate cooperation. We observe that 'may' is used here to indicate that the characteristics of linking of repeated games mentioned do no hold unconditionally but depend on the particular nature of the problem at hand. However, to our best knowledge, the conditions under which these characteristics hold have not yet been thoroughly analyzed which is a major omission in the light of the practical and theoretical relevance of linking. Admittedly, some results about the conditions under which the characteristics of more cooperation and Pareto improvements hold can be found in Ragland (1995) and Just and Netanyahu (2000). However, these results are limited in scope because the settings in these publications concern the special case of linking of two repeated 2×2-bimatrix games.

The main purpose of this paper is to identify classes of isolated stages games for which the themes 'linking may sustain more cooperation' and 'linking may bring Pareto improvements' materialize or not. For that purpose we formalize the themes 'linking may sustain more cooperation'and 'linking may bring Pareto improvements'. Our results apply to the linking of an arbitrary number of repeated games with an arbitrary number of (the same) players. In section 2 we present preliminaries and introduce concepts. In section 3 we present figures that illustrate these concepts and that will be referred to in the next sections. In section 4 we discuss 'more cooperation' and in section 5 Pareto improvements. Section 6 concludes. Various proofs will be given in the appendix.

2 Preliminaries

Negotiation sets. Consider a game in strategic form among N players. That is, for each player $i \in \mathcal{N} := \{1, \dots, N\}$ we have a non-empty (action) set X^i and a real-valued (payoff) function f^i on the set of multi-actions $\mathbf{X} := X^1 \times \cdots \times X^N$. In order to avoid some technicalities we will restrict ourselves here often to what we call regular games in strategic form, which are games in strategic form that satisfy the following three assumptions. First, each payoff function is bounded. This assumption assures that the minimax payoff $\overline{v}^j$ of each player j is a well-defined real number. Second, without any loss of generality, we assume that $\overline{v}^j = 0$ for each player j. This assumption implies that a payoff vector (i.e. an element of $\mathbb{R}^N$) is individually rational if and only if it belongs to $\mathbb{R}^N_+$, i.e. the closed positive octant of $\mathbb{R}^N$. Third, denoting $\mathbf{f}(\mathbf{x}) := (f^1(\mathbf{x}), \dots, f^N(\mathbf{x}))$, the

[1] This is the counterpart of the theme 'repetition enables cooperation' for repeated games. 'More' is relative to the single issue case.

feasible set, i.e. the convex hull $\mathrm{co}(U)$ of the set $U := \{\mathbf{f}(\mathbf{x}) \mid \mathbf{x} \in \mathbf{X}\}$ of basic payoff vectors, is assumed to be closed. This condition is always satisfied in the case each action set is finite.[2]

For a regular game in strategic form Γ, the intersection of its set of individually rational payoff vectors and its feasible set is an important object. We call it here simply the negotiation set of Γ and denote it by H:[3]

$$H := \mathrm{co}(U) \cap \mathbb{R}^N_+.$$

The three assumptions presented above ensure that H is a compact set.[4]

Because each Nash equilibrium payoff vector of Γ is individually rational, H contains the set of Nash equilibrium payoff vectors. By $\mathrm{PB}(H)$ we denote the Pareto boundary of H and by $\mathrm{PB}_w(H)$ its weak Pareto boundary.[5] Because H is compact, $\mathrm{PB}(H) \neq \emptyset$ if H is non-empty. Also we have (see Appendix A.4)

$$\mathrm{PB}(H) = \mathrm{PB}(\mathrm{co}(U)) \cap \mathbb{R}^N_+. \tag{1}$$

Given a game in strategic form Γ we call a maximizer $\mathbf{x}$ of the total payoff function $\sum_{j=1}^N f^j$ a full-cooperative multi-action. The set of such multi-actions will be denoted by Y. It is easy to see that (see Appendix A.4) for a regular game in strategic form we have

$$Y \neq \emptyset. \tag{2}$$

Direct sum games and canonical mapping. Consider M games in strategic form ${}_1\Gamma, \dots, {}_M\Gamma$ among (the same) N players. We refer to them as isolated stage games and use pre-subscripts to refer to objects related to them. Let $\mathcal{M} := \{1, \dots, M\}$, the set of issues. Let ${}_kX^j$ be the action set of player j in ${}_k\Gamma$. Define for each $k \in \mathcal{M}$

$${}_k\mathbf{X} := {}_kX^1 \times \cdots \times {}_kX^N$$

and for each player j

$${}_*X^j := {}_1X^j \times \cdots \times {}_MX^j.$$

Moreover, define the mapping $\Psi : {}_1\mathbf{X} \times \cdots \times {}_M\mathbf{X} \to {}_*X^1 \times \cdots \times {}_*X^N$ by

$$\Psi(\begin{pmatrix} {}_1\mathbf{x} \\ \vdots \\ {}_M\mathbf{x} \end{pmatrix}) := ({}_*x^1, \dots, {}_*x^N).$$

Ψ is called the canonical mapping. Note that the canonical mapping is a bijection.

For M games in strategic form ${}_1\Gamma, \dots, {}_M\Gamma$ among N players, the trade-off direct sum game $(\oplus\Gamma)_\alpha$ is defined as the game in strategic form where player j has action set ${}_*X^j$ and his payoff function is given by[6]

$$f^j({}_*x^1, \dots, {}_*x^N) := \sum_{k=1}^M {}_kf^j({}_1x^1, \dots, {}_1x^N).$$

(In the case of two bimatrix games $(\oplus\Gamma)_\alpha$ is the tensor sum of the individual bimatrix games.) The set of possible payoffs vectors U_α of $(\oplus\Gamma)_\alpha$ equals $\sum_{k\in\mathcal{M}} {}_kU := {}_1U + \cdots {}_MU$.[7]

[2] Note that for a regular game in strategic form it is possible that its feasible set does not contain $\mathbf{0}$. Indeed, this for example holds for the regular bimatrix game $\begin{pmatrix} -2;\ 2 & 0;\ -4 \\ 1;\ -3 & -2;\ 0 \end{pmatrix}$.

[3] The negotiation set plays an important role in Folk theorems which relate to the geometric structure of the set of (average) subgame perfect Nash equilibrium payoff vectors for repeated games $< \Gamma >$ with Γ as stage game. In this context it is customary to assume that repeated games are with discounting and that each player has the same discount factor $\delta \in (0, 1)$. Finally, if we consider several repeated games below (with the same players) together, then it is assumed that in each of them the periods are the same and the discount factors are the same. For the purpose of this paper it is not necessary to go into the details of (technically complicated) Folk theorems. For this, we refer to, for example, Benoît and Krishna (1996).

[4] This set may be empty, as for example is the case for the bimatrix game in footnote 2.

[5] See appendix A.3 for Pareto boundaries.

[6] The α refers to the fact that in this formula the payoffs of the isolated games are added (with weights 1).

[7] For two subsets A, B of $\mathbb{R}^N$ its Minkowski sum $A + B$ is defined by $A + B := \{a + b \mid a \in A, b \in B\}$.

Let ${}_kE$ be the set of Nash equilibria of ${}_k\Gamma$, ${}_kY$ the set of full-cooperative multi-actions of ${}_k\Gamma$, E_α the set of Nash equilibria of $(\oplus\Gamma)_\alpha$ and Y_α the set of full-cooperative multi-actions of $(\oplus\Gamma)_\alpha$. It can be shown that (see Folmer et al., 1993; Folmer and von Mouche, 1994)

$$\Psi({}_1E \times \cdots \times {}_ME) = E_\alpha, \tag{3}$$

$$\Psi({}_1Y \times \cdots \times {}_MY) = Y_\alpha. \tag{4}$$

Suppose each ${}_k\Gamma$ is regular. Then $(\oplus\Gamma)_\alpha$ also is regular. The negotiation set of ${}_k\Gamma$ is

$${}_kH := \mathbb{R}_+^N \cap \mathrm{co}({}_kU).$$

Using the fact that a convex hull of a sum is the sum of the convex hulls, the negotiation set of $(\oplus\Gamma)_\alpha$ is

$$H_\alpha = \mathbb{R}_+^N \cap \sum_{k\in\mathcal{M}} \mathrm{co}({}_kU).$$

Linking. Again, let ${}_1\Gamma, \ldots, {}_k\Gamma$ be M regular games in strategic form and consider the repeated games $< {}_k\Gamma >$. Linking of the (isolated) repeated games $< {}_k\Gamma >$ is done by combining them into a repeated game $(\otimes\Gamma)_\alpha$, a so-called trade-off tensor game. This trade-off tensor game has as stage game the trade-off direct sum game $(\oplus\Gamma)_\alpha$.

In order to analyse the effects of linking, we define the aggregated negotiation set as

$$H_{\mathrm{ag}} := \sum_{k\in\mathcal{M}} {}_kH.$$

H_{ag} may be considered as the negotiation set when the M repeated games are not linked but merely aggregated. We remark that $H_{\mathrm{ag}} = \emptyset$ when some ${}_kH$ is empty. Because

$$\sum_{k\in\mathcal{M}} (\mathbb{R}_+^N \cap \mathrm{co}({}_kU)) \subseteq \sum_{k\in\mathcal{M}} \mathbb{R}_+^N \cap \sum_{k\in\mathcal{M}} \mathrm{co}({}_kU) = \mathbb{R}_+^N \cap \sum_{k\in\mathcal{M}} \mathrm{co}({}_kU) \tag{5}$$

it follows that

$$H_{\mathrm{ag}} \subseteq H_\alpha. \tag{6}$$

We observe that equality holds in (6) if and only if the $\subseteq$-symbol is a $=$-symbol in (5).

More cooperation and Pareto improvements. In Folmer et al. (1993) it is shown that Nash equilibria for each repeated game $< {}_k\Gamma >$ lead in a canonical way to a Nash equilibrium for the trade-off tensor game $(\otimes\Gamma)_\alpha$.[8] In general, the trade-off tensor game also has other (subgame perfect) Nash equilibria. Folk theorems are useful in order to investigate the question how many more subgame perfect Nash equilibria there are, particularly by focussing on the set $H_\alpha \setminus H_{\mathrm{ag}}$. This leads to the following definition:

Definition 1 There is an enrichment of the aggregated negotiation set if the strict inclusion $H_{\mathrm{ag}} \subset H_\alpha$ holds. $\diamond$

Hence, enrichment of the aggregated negotiation set can be interpreted as 'Linking sustains more cooperation'.

We call $\mathbf{u} \in \mathrm{PB}(H_{\mathrm{ag}})$ a (strong) expansion point of $\mathrm{PB}(H_{\mathrm{ag}})$ if there exists $\mathbf{w} \in H_\alpha$ such that[9] $\mathbf{w} \gg \mathbf{u}$ and a weak expansion point of $\mathrm{PB}(H_{\mathrm{ag}})$ if there exists $\mathbf{w} \in H_\alpha$ such that $\mathbf{w} > \mathbf{u}$. By EXP we denote the set of expansion points and by $\mathrm{EXP_w}$ the set of weak expansion points. Of course, $\mathrm{EXP} \subseteq \mathrm{EXP_w}$ and $\mathrm{EXP} \subseteq \mathrm{PB}(H_{\mathrm{ag}})$. Moreover, (see Appendix A.4)

$$\mathrm{EXP} = \mathrm{PB}(H_{\mathrm{ag}}) \setminus \mathrm{PB}_w(H_\alpha). \tag{7}$$

Below we shall only deal with strong expansion points.

[8]It is straightforward to show that this statement remains valid if one replaces 'Nash equilibrium' by 'subgame perfect Nash equilibrium'.

[9]For $\mathbf{a} = (a^1, \ldots, a^N), \mathbf{b} = (b^1, \ldots, b^N) \in \mathbb{R}^N$ we write $\mathbf{a} \geq \mathbf{b}$ if $a^i \geq b^i$ for all i. We write $\mathbf{a} > \mathbf{b}$ if $\mathbf{a} \geq \mathbf{b}$ and $\mathbf{a} \neq \mathbf{b}$. And we write $\mathbf{a} \gg \mathbf{b}$ if $a^i > b^i$ for all i.

Definition 2 We speak of partial expansion (of the Pareto boundary of the aggregated negotiation set) if $\emptyset \subset \mathrm{EXP} \subset \mathrm{PB}(H_{\mathrm{ag}})$. In the case $\mathrm{EXP} = \emptyset$ we say that there is expansion nowhere. Finally, in the case $\emptyset \subset \mathrm{EXP} = \mathrm{PB}(H_{\mathrm{ag}})$ there is expansion everywhere. $\diamond$

We observe that by virtue of Folk theorems the existence of an expansion point of $\mathrm{PB}(H_{\mathrm{ag}})$ is related to possible Pareto improvements. This may be interpreted as 'Linking brings Pareto improvements'.

Finally, we observe that if there is no enrichment of the aggregated negotiation set, i.e. if $H_{\mathrm{ag}} = H_\alpha$, then H_{ag} and H_α have the same Pareto boundaries and thus, by virtue of (7), $\mathrm{EXP} = \emptyset$.

3 Figures

In this section we present five figures that illustrate the concepts defined above. Moreover, we wil refer to these figures in sections 4 and 5. The figures present the linking of two repeated games, where the isolated stage games are (regular) 2×2- bimatrix games.

Figure 1 relates to the games

$$
{}_1\Gamma := \begin{pmatrix} 2;\ 1 & -3;\ 2 \\ 5;\ -1 & 0;\ 0 \end{pmatrix}, \qquad {}_2\Gamma := \begin{pmatrix} 1;\ 2 & -1;\ 5 \\ 2;\ -3 & 0;\ 0 \end{pmatrix}.
$$

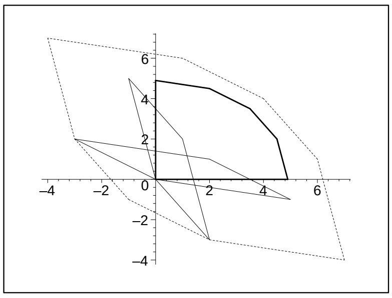
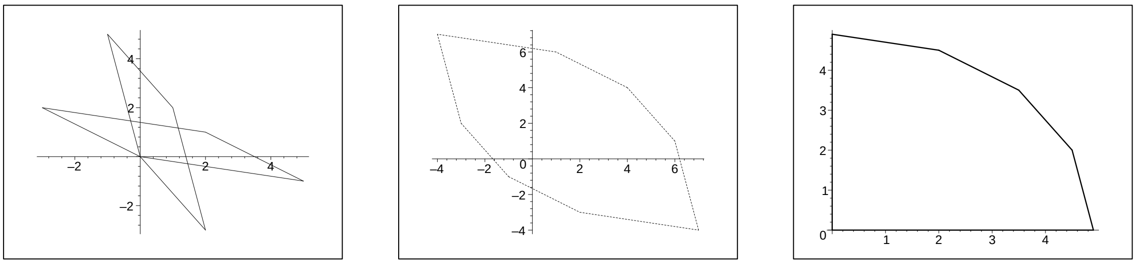


Figure 1: Expansion everywhere.

Figure 1, and also Figures 2 – 5, are to be interpreted as follows. Four polygons are drawn: the feasible sets $\mathrm{co}({}_1U), \mathrm{co}({}_2U)$, the sum of these two sets and the aggregated negotiation set $H_{\mathrm{ag}} = {}_1H + {}_2H$. Because the minimax payoff vectors for ${}_1\Gamma$ and ${}_2\Gamma$ are $\mathbf{0}$, the sets ${}_1H$ and ${}_2H$ can be distinguished. $H_{\mathrm{ag}} = {}_1H + {}_2H$ is the boldfaced polygon. Because the minimax payoff vector for $(\oplus\Gamma)_\alpha$ is $\mathbf{0}$, the set H_α can also be distinguished. For reasons of convenience these four sets for Figure 1 are drawn below.



The sets in the above three figures respectively concern $\mathrm{co}({}_1U)$ and $\mathrm{co}({}_2U)$, $\mathrm{co}({}_1U) + \mathrm{co}({}_2U)$ and $H_{\mathrm{ag}} = {}_1H + {}_2H$.

We note that in the case of Figure 1

$$(\oplus\Gamma)_\alpha = \begin{pmatrix} 3;3 & 1;6 & -2;4 & -4;7 \\ 4;-2 & 2;1 & -1;-1 & -3;2 \\ 6;1 & 4;4 & 1;2 & -1;5 \\ 7;-4 & 5;-1 & 2;-3 & 0;0 \end{pmatrix}.$$

Figure 2 relates to the two games

$${}_1\Gamma := \begin{pmatrix} 0;\ 2 & 3;\ 1 \\ -3;\ 0 & 0;0 \end{pmatrix},\ {}_2\Gamma := \begin{pmatrix} 0;\ 1 & 1;\ 0.5 \\ -2;\ 0 & 0;\ 0 \end{pmatrix}.$$

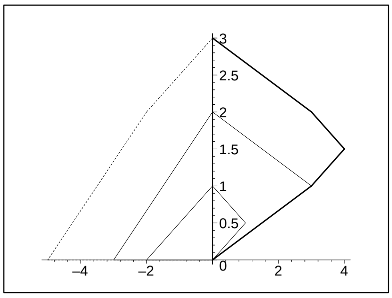


Figure 2: No enrichment of the aggregated negotiation set.

Figure 3 relates to the two games

$${}_1\Gamma := \begin{pmatrix} 7;\ 1 & -3;\ 3 \\ 10;\ -2 & 0;0 \end{pmatrix},\ {}_2\Gamma := \begin{pmatrix} 1;\ 7 & -2;\ 10 \\ 3;\ -3 & 0;\ 0 \end{pmatrix}.$$

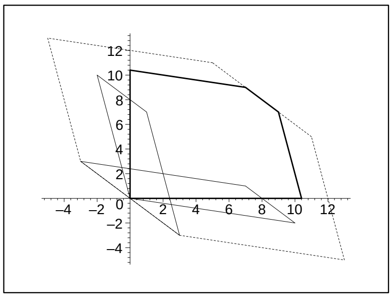


Figure 3: Partial expansion (non-symmetric isolated stage games).

Figure 4 relates to the two games

$${}_1\Gamma := \begin{pmatrix} 2;\ 2 & -2;\ 4 \\ 4;\ -2 & 0;0 \end{pmatrix},\ {}_2\Gamma := \begin{pmatrix} 2;\ 2 & -1;\ 1 \\ 1;\ -1 & 0;\ 0 \end{pmatrix}$$

Finally, Figure 5 relates to the two games

$${}_1\Gamma := \begin{pmatrix} 2;\ 2 & -2;\ 10 \\ 10;\ -2 & 0;0 \end{pmatrix},\ {}_2\Gamma := \begin{pmatrix} 3;\ 3 & -3;\ 4 \\ 4;\ -3 & 0;\ 0 \end{pmatrix}.$$

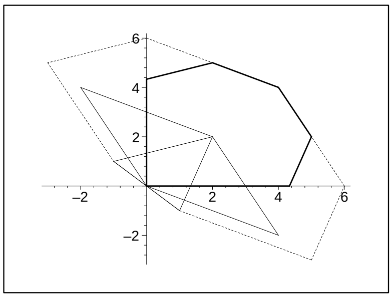


Figure 4: Enrichment of the aggregated negotiation set and expansion nowhere.

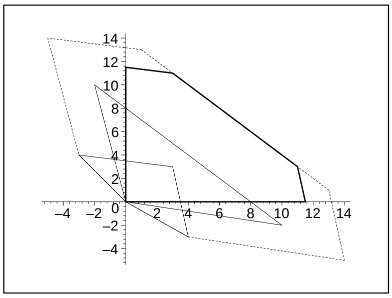


Figure 5: Partial expansion (symmetric isolated stage games).

4 Linking sustains more cooperation

The next theorem, proven in Appendix A.4, identifies three cases where linking does not lead to an enrichment of the aggregated negotiation set.

Theorem 1 *Each of the following conditions is sufficient for that there is no enrichment of the aggregated negotiation set.*

1. *For each k the payoff function of each player in ${}_k\Gamma$ is a positive multiple ${}_kr$ of its payoff function in ${}_1\Gamma$; this result holds in particular if all isolated stage games are identical.*
2. *In each isolated stage game each basic payoff vector is individually rational.*[10]
3. $H_\alpha = \emptyset$. $\diamond$

Theorem 1 is a negative result and clearly shows that the structure of the isolated stage game matters to achieve more cooperation. Figure 2 shows that there are situations of no enrichment of the aggregated negotiation set that are not covered by Theorem 1. In all other figures there is an enrichment.

Now we turn to the conditions under which a positive general result holds, i.e. linking leads to an enrichment of the aggregated negotiation set. For that purpose we present Theorem 2 as a first general result. This theorem deals with isolated stage games that have 'compensating asymmetries of exactly the same magnitude'. This notion is defined as follows. Given isolated stage games ${}_1\Gamma, \ldots, {}_N\Gamma$ (so $M = N$) we say that they have 'compensating asymmetries of exactly the same magnitude' if there are N permutations $\pi_1, \ldots, \pi_N$ of $\mathcal{N}$ with $\pi_1 := \mathrm{Id}$ (i.e. the identical permutation) such that for each $j \in \mathcal{N}$ one has $\{\pi_1(j), \ldots, \pi_N(j)\} = \mathcal{N}$ and such that ${}_k\Gamma := \pi_k({}_1\Gamma)$ $(k \in \mathcal{M})$. So each ${}_k\Gamma$ is a permutation of ${}_1\Gamma$ (see Appendix A.1 for permuted games), but not all $N!$ permuted games of ${}_1\Gamma$ are allowed.[11]

[10] Note that this is equivalent with 'in each isolated stage game each point of its feasible set is individually rational'.

[11] It should be noted that regularity of ${}_1\Gamma$ implies regularity of each ${}_k\Gamma$ and that if one of then is symmetric, all are such.

Another condition in Theorem 2 is that Γ has a defect (Folmer and von Mouche, 2000): a game in strategic form with bounded payoff functions has a j-defect (where $j \in \mathcal{N}$) if for player j no full-cooperative payoff vector is individually rational. The game has a defect if it has a j-defect for some j. Of course, a defect excludes the possibility that a Nash equilibrium is full-cooperative.[12] It also excludes the possibility that the game is symmetric and regular.[13]

Theorem 2 *Consider isolated regular stage games that have compensating asymmetries of exactly the same magnitude. If $\Gamma := {}_1\Gamma$ has a Nash equilibrium and a defect, then there is an enrichment of the aggregated negotiation set. Moreover, the game $(\oplus\Gamma)_\alpha$ has a Nash equilibrium for which there exists a full-cooperative unanimous Pareto improvement.* $\diamond$

The proof of Theorem 2 is given in Appendix A.4. Note that in Theorem 2 all the isolated stage games have a defect, but $(\oplus\Gamma)_\alpha$ does not have. Theorem 2 explains the enrichment of the aggregated negotiation set in Figure 1 (where Γ has a 2-defect). Figures 3–5 show that there are situations of enrichment of the aggregated negotiation set that are not covered by Theorem 2. We observe that Theorem 2 does not exclude the possibility that in the case the isolated stage games are symmetric (without having compensating asymmetries of exactly the same magnitude), there could be an enrichment of the aggregated negotiation set (Figures 4 and 5).

We note that in Figures 1, 3 and 5 the isolated stage games are prisoners' dilemma games,[14] but that this is not the case for Figure 4. Concerning this aspect:

Corollary 1 *Consider isolated regular stage games that are 2×2-bimatrix prisoners' dilemma games, with a unique full-cooperative multi-action that have compensating asymmetries of exactly the same magnitude, Then there is an enrichment of the aggregated negotiation set. Moreover, $(\oplus\Gamma)_\alpha$ has a Nash equilibrium for which there exists a full-cooperative unanimous Pareto improvement.* $\diamond$

Indeed, for this situation ${}_1\Gamma$ automatically has a Nash equilibrium and a j-defect for some j.[15]

5 Linking brings Pareto improvements

We have already seen that if there is no enrichment of the aggregated negotiation set, then there is expansion nowhere. A natural question now is whether enrichment of the aggregated negotiation set implies that there is an expansion point. The answer is 'no' as Figure 4 shows. Note that in this figure the Pareto boundary $\mathrm{PB}({}_2H)$ is the singleton $\{(2,2)\}$.

Theorem 1(2) implies that if in each isolated stage game each point of its feasible set is individually rational, then there is expansion nowhere. Also in Figure 2 there is expansion nowhere, but this can not be explained in this way. Individual rationality of each point of the feasible sets is a strong condition. In Theorem 4 there is a weaker condition that also guarantees expansion nowhere and explains expansion nowhere in Figure 2. The proof of Theorem 4 uses the technique of normal cones[16] and is a little bit complicated. Therefore, before we turn to this theorem, we state a special case of it, Theorem 3, for which we can provide a simple proof.

[12] In this sense one may say that a defect implies that each Nash equilibrium has a welfare loss. For such a game the welfare loss remains when we repeat the game. See Folmer and von Mouche (1994, Proposition 4.2.) for a precise statement.

[13] Here is a proof of this statement, by contradiction. Suppose Γ is symmetric, regular and has a j-defect. Then for each permutation π of $\mathcal{N}$ the game $\pi(\Gamma)$ has a $\pi^{-1}(j)$-defect. But $\pi(\Gamma) = \Gamma$, so Γ has an i-defect for each $i \in \mathcal{N}$. By (2) there exists a full-cooperative multi-action $\mathbf{y}$. Let $\mathbf{n}$ be a Nash equilibrium. Then one has (using the fact that each Nash equilibrium payoff vector is individually rational) $\sum_{j=1}^N f^j(\mathbf{n}) \geq \sum_{j=1}^N 0 > \sum_{j=1}^N f^j(\mathbf{y})$, a contradiction.

[14] We call a game in strategic form a prisoners' dilemma game if each player has a strictly dominant action and the strictly dominant equilibrium is not Pareto-efficient in the weak sense.

[15] The last statement is a direct consequence of the fact that for every 2×2-bimatrix prisoners' dilemma game the Nash equilibrium payoff for each player equals his minimax payoff.

[16] A more direct proof of Theorem 4 would be welcome.

Theorem 3 *If, in case $M = 2$, for each of the isolated stage games each point of the Pareto boundary of its feasible set is individually rational and at least one of these Pareto boundaries is a singleton, then* $\mathrm{PB}(H_\alpha) = \mathrm{PB}(H_{\mathrm{ag}})$ *and therefore there is expansion nowhere.* $\diamond$

For the proof of this theorem see Appendix A.4. The conclusion of expansion nowhere in Theorem 3 even holds for general M without the singleton assumption:

Theorem 4 *If for each of the isolated stage games each point of the Pareto boundary of its feasible set is individually rational, then there is expansion nowhere.* $\diamond$

Also for the proof of this theorem see Appendix A.4.

Figure 2 illustrates Theorem 4 and Figure 4 shows that there are situations of expansion nowhere that are not covered by Theorem 4. Note that in Figure 2 there even is no enrichment of the aggregated negotiation set (and that for player 2 the first isolated stage game 'is half the second one'). An important issue for further research is whether for the cases specified in Theorem 4 there always is no enrichment of the aggregated negotiation set.

Figures 3 and 5 show cases where there is partial expansion. Note that in Figure 1 there is expansion everywhere. Another interesting question for further research is whether expansion everywhere always holds in Theorem 2. An even more basic question is whether or not an expansion point always exists in Theorem 2.

Finally we note that even in case each isolated stage game is symmetric, there may be partial expansion as Figure 5 shows.

6 Conclusion

In this paper we have presented some general results on more cooperation and Pareto improvements which can be achieved by linking of repeated games. We have defined 'more cooperation' by the notion of enrichment of the aggregated negotiation set and 'Pareto improvement' by the notion of expansion point of the Pareto boundary of the aggregated negotiation set. Using these notions we have formalized for tensor games the theme 'linking may sustain more cooperation' and 'linking may bring Pareto improvements'.

We have shown that in the case linking brings Pareto improvements, it also sustains more cooperation but that the reverse does not hold in general. We have identified a class of isolated stage games for which linking does not sustain more cooperation and a class for which it does. In order to identify this last class we formalized the basic idea that an exchange of concessions may enhance cooperation if the issues have compensating asymmetries of similar magnitude. For this class all isolated stage games are asymmetric and permutations of each other and all have the property that each full-cooperative payoff vector is not individually rational. Concerning Pareto improvements, we derived (in the appendix) a characterization of expansion points in terms of positive normal cones and used this in order to identify a class where linking does not bring Pareto improvements. We showed that also in the case all isolated stage game are symmetric (but not identical), more cooperation and even partial expansion is possible.

The figures that we used for illustrating our results lead to interesting questions for further research:

A. How far can one deviate in Theorem 2 from the situation of (exact) permuted games? This would model the notion of 'similar magnitude' in the expression 'an exchange of concessions in issues that have compensating asymmetries of similar magnitude'.

B. Derive (interesting) sufficient conditions (like the conjecture in C) for the existence of expansion points.

C. If the isolated stage games have compensating asymmetries of exactly the same magnitude and one of them has a Nash equilibrium and a defect, is there then always expansion everywhere? More basically, we conjecture that there then always is at least one expansion point.

D. If for each of the isolated stage games each point of the Pareto boundary of its feasible set is individually rational, is there then no enrichment of the aggregated negotiation set?

Finally, we observe that although this paper is about game theory, the problems we deal with are in fact geometric problems related to Minkowski sums and intersections of convex sets. Therefore, basic research on linking should (also) relate to these topics.

A Appendices

Before turning to the proofs in Appendix A.4 we present some definitions and useful results. For those for which it is difficult to trace them in the literature we also give a proof.

A.1 Permuted games

Given a Cartesian product of sets $A_1 \times \ldots \times A_N$, we define for a permutation κ of $\{1, \ldots, N\}$ the mapping $T_\kappa : A_1 \times \cdots \times A_N \to A_{\kappa(1)} \times \cdots \times A_{\kappa(N)}$ by $T_\kappa(a_1, \ldots, a_N) := (a_{\kappa(1)}, \ldots, a_{\kappa(N)})$.

Let Γ be a game in strategic form and π a permutation of $\mathcal{N}$. We define the game in strategic form $\pi(\Gamma)$ (called a permuted game of Γ) as the game in strategic form where the action set Z^i of player i is $X^{\pi(i)}$ and his payoff function h^i is $f^{\pi(i)} \circ T_{\pi^{-1}}$. So,

$$h^i(z^1, \ldots, z^N) = f^{\pi(i)}(z^{\pi^{-1}(1)}, \ldots, z^{\pi^{-1}(N)}).$$

Finally, a game in strategic form Γ where each player has the same action set X is called symmetric if for each permutation π of $\mathcal{N}$ one has $\Gamma = \pi(\Gamma)$.

A.2 Normal cones

Let A be a non-empty subset of $\mathbb{R}^N$ and $\mathbf{x} \in \overline{A}$, i.e. $\mathbf{x}$ is an element of the topological closure of A. Then

$$N_A(\mathbf{x}) := \{\mathbf{d} \in \mathbb{R}^N \mid (\mathbf{y} - \mathbf{x}) \cdot \mathbf{d} \leq 0 \text{ for all } \mathbf{y} \in A\}.$$

$N_A(\mathbf{x})$ is a convex cone and is called the normal cone of A in $\mathbf{x}$. Moreover, we define for $\mathbf{x} \in \overline{A}$ the positive normal cone of A in $\mathbf{x}$ as

$$N_A^+(\mathbf{x}) := \{\mathbf{d} \in N_A(\mathbf{x}) \mid \mathbf{d} > \mathbf{0}\}.$$

Note that $\mathbf{0} \in N_A(\mathbf{x})$, but that $N_A^+(\mathbf{x})$ may be empty.

Let ${}_kA$ $(1 \leq k \leq M)$ be subsets of $\mathbb{R}^N$. It is straightforward to prove hat for ${}_k\mathbf{a} \in {}_kA$ $(1 \leq k \leq M)$, with $\mathbf{a} := \sum_{k=1}^N {}_k\mathbf{a}$, one has

$$N_{\sum_{k=1}^M {}_kA}(\mathbf{a}) = \cap_{k=1}^M N_{{}_kA}({}_k\mathbf{a}). \tag{8}$$

A.3 Pareto boundaries

Define the function $\mathcal{C} : \mathbb{R}^N \to \mathbb{R}$ by $\mathcal{C}(\mathbf{x}) := \sum_{l=1}^N x^l$. For a subset A of $\mathbb{R}^N$ we define $\tilde{A}$ as the set of maximizers of the restricted function $\mathcal{C} \restriction A$, i.e. of the function $\mathcal{C} : A \to \mathbb{R}$. Moreover, define $s(A) \in \mathbb{R} \cup \{-\infty, +\infty\}$ as the supremum of the function $\mathcal{C} \restriction A$. Closedness (boundedness) of A implies closedness (boundedness) of $\tilde{A}$ and if A is a non-empty compact subset of $\mathbb{R}^N$, then $\tilde{A}$ is non-empty and compact as well.

It is also straightforward to prove the following properties for all subsets A, B of $\mathbb{R}^N$:

$$\widetilde{\mathrm{co}(A)} = \mathrm{co}(\tilde{A}); \tag{9}$$

$$s(\mathrm{co}(A)) = s(A); \tag{10}$$

$$s(A + B) = s(A) + s(B). \tag{11}$$

For a subset A of $\mathbb{R}^N$ its (strong) Pareto boundary $\mathrm{PB}(A)$ is defined as the set of elements $\mathbf{a}$ of A for which there does not exist $\mathbf{c} \in A$ with $\mathbf{c} > \mathbf{a}$ whereas its weak Pareto boundary $\mathrm{PB_w}(A)$ is defined as the set of elements $\mathbf{a}$ of A for which there does not exist $\mathbf{c} \in A$ with $\mathbf{c} \gg \mathbf{a}$. Of course, $\mathrm{PB}(A) \subseteq \mathrm{PB_w}(A)$. For ∂A, the topological boundary of A, we have

$$\tilde{A} \subseteq \mathrm{PB}(A) \subseteq \mathrm{PB_w}(A) \subseteq \partial A.$$

So $\mathrm{PB}(A) \neq \emptyset$ if A is compact and non-empty.

Let A_k $(1 \leq k \leq M)$ be subsets of $\mathbb{R}^N$. It is easy to show that for $\mathbf{a}_k \in A_k$ $(1 \leq k \leq M)$, with $\mathbf{a} := \sum_{k=1}^{N} \mathbf{a}_k$, one has

$$\mathbf{a} \in \mathrm{PB}(\sum_{k=1}^{M} A_k) \;\Rightarrow\; \mathbf{a}_k \in \mathrm{PB}(A_k) \text{ for all } k.$$

Thus in particular

$$\mathrm{PB}(\sum_{k=1}^{M} A_k) \subseteq \sum_{k=1}^{M} \mathrm{PB}(A_k). \tag{12}$$

Lemma 1 *Let A be a compact subset A of $\mathbb{R}^N$. For each $\mathbf{a} \in A$ there exists $\mathbf{b} \in \mathrm{PB}(A)$ with $\mathbf{b} \geq \mathbf{a}$.* $\diamond$

Proof.— $Z := \{\mathbf{z} \in \mathbb{R}^N \mid \mathbf{z} \geq \mathbf{x}\}$ is closed. This implies that $Z \cap A$ is compact. Because $\mathbf{x} \in Z \cap A$, $Z \cap A \neq \emptyset$ and therefore also $\mathrm{PB}(Z \cap A) \neq \emptyset$. Take $\mathbf{y} \in \mathrm{PB}(Z \cap A)$. Then $\mathbf{y} \in Z$, so $\mathbf{y} \geq \mathbf{x}$. Also $\mathbf{y} \in \mathrm{PB}(A)$, because otherwise there would exist $\mathbf{b} \in A$ with $\mathbf{b} > \mathbf{y}$. Then we had $\mathbf{b} > \mathbf{y} \geq \mathbf{x}$, so $\mathbf{b} \in Z \cap A$ and $\mathbf{b} > \mathbf{y}$, which is a contradiction with $\mathbf{y} \in \mathrm{PB}(Z \cap A)$. Q.E.D.

Lemma 1 now will be used to derive further properties.

Lemma 2 *For two non-empty subsets A and B of $\mathbb{R}^N$ with $A \subseteq B$ and $\mathbf{a} \in \overline{A}$ one has:*
B compact and $\mathrm{PB}(B) \subseteq A \;\Rightarrow\; N_B^+(\mathbf{a}) = N_A^+(\mathbf{a})$. $\diamond$

Proof.— Because $A \subseteq B$ one has $N_B^+(\mathbf{a}) \subseteq N_A^+(\mathbf{a})$. By contradiction we prove that $N_B^+(\mathbf{a}) \supseteq N_A^+(\mathbf{a})$. So suppose $\boldsymbol{\gamma} \in N_A^+(\mathbf{a}) \setminus N_B^+(\mathbf{a})$. Now $(\mathbf{w} - \mathbf{a}) \cdot \boldsymbol{\gamma} \leq 0$ for all $\mathbf{w} \in A$, but not for all $\mathbf{z} \in B$. This implies that there is a $\mathbf{w} \in B \setminus A$ such that $\boldsymbol{\gamma} \cdot (\mathbf{w} - \mathbf{a}) > 0$. Because B is compact, there is, by Lemma 1, $\mathbf{b} \in \mathrm{PB}(B)$ such that $\mathbf{b} \geq \mathbf{w}$. Because $\boldsymbol{\gamma} > \mathbf{0}$, also $\boldsymbol{\gamma} \cdot (\mathbf{b} - \mathbf{a}) > 0$. So $\mathbf{b} \notin A$. But $\mathbf{b} \in \mathrm{PB}(B) \subseteq A$, which is a contradiction. Q.E.D.

In general the inclusion in (12) is not an equality. Here is a special case where equality holds:

Lemma 3 *$[A, B \subseteq \mathbb{R}^N,\ B$ compact and $\#\mathrm{PB}(B) = 1] \;\Rightarrow\; \mathrm{PB}(A + B) = \mathrm{PB}(A) + \mathrm{PB}(B)$.* $\diamond$

Proof.— Only $\supseteq$' remains to be proved. This we do by contradiction. So suppose $\mathbf{x} \in \mathrm{PB}(A) + \mathrm{PB}(B)$, but $\mathbf{x} \notin \mathrm{PB}(A + B)$. Write $\mathrm{PB}(B) = \{\mathbf{b}\}$. Let $\mathbf{a} \in \mathrm{PB}(A)$ such that $\mathbf{x} = \mathbf{a} + \mathbf{b}$. Because B is compact, there is for each $\mathbf{y} \in B$ an element of $\mathrm{PB}(B)$, i.e. $\mathbf{b}$, such that $\mathbf{y} \leq \mathbf{b}$. So $\mathbf{b} - \mathbf{y} \geq \mathbf{0}$ $(\mathbf{y} \in B$. Because $\mathbf{x} \in A + B$ and $\mathbf{x} \notin \mathrm{PB}(A + B)$, there is $\mathbf{d} \in A + B$ with $\mathbf{d} > \mathbf{x}$. Let $\mathbf{a}' \in A$ and $\mathbf{b}' \in B$ such that $\mathbf{d} = \mathbf{a}' + \mathbf{b}'$, Then $\mathbf{a}' > \mathbf{a} + (\mathbf{b} - \mathbf{b}') \geq \mathbf{a}$, so $\mathbf{a}' > \mathbf{a}$. But $\mathbf{a} \in \mathrm{PB}(A)$, a contradiction. Q.E.D.

Lemma 4 *Let $B, C \subseteq \mathbb{R}^N$ such that for no $\mathbf{c} \in C$ there exists $\mathbf{d} \in C^c$ with $\mathbf{d} > \mathbf{c}$. Then $\mathrm{PB}(B \cap C) = \mathrm{PB}(B) \cap C$.* $\diamond$

Proof.— "$\subseteq$": by contradiction. So suppose $\mathbf{a} \in \mathrm{PB}(B \cap C)$ and $\mathbf{a} \notin \mathrm{PB}(B) \cap C$. Because $\mathbf{a} \in B \cap C \subseteq C$, it follows that $\mathbf{a} \notin \mathrm{PB}(B)$. Now there is $\mathbf{b} \in B$ with $\mathbf{b} > \mathbf{a}$. Because $\mathbf{a} \in \mathrm{PB}(B \cap C)$, it follows that $\mathbf{b} \notin B \cap C$. Thus $\mathbf{b} \in C^c$, $\mathbf{a} \in C$ and $\mathbf{b} > \mathbf{a}$, which is a contradiction.

"$\supseteq$". Suppose $\mathbf{d} \in \mathrm{PB}(B) \cap C$. One has $\mathbf{d} \in B \cap C$. If we would have $\mathbf{a} \in B \cap C$ such that $\mathbf{a} > \mathbf{c}$, then, noting that $\mathbf{a} \in B$ and $\mathbf{d} \in B$, we would have a contradiction. Q.E.D.

Lemma 5 *Let A be a non-empty convex subset of $\mathbb{R}^N$. Then* $\mathbf{a} \in \mathrm{PB_w}(A) \Rightarrow N_A^+(\mathbf{a}) \neq \emptyset$. $\diamond$

Proof.— Define $B := \{\mathbf{x} \in \mathbb{R}^N \mid \mathbf{x} \geq \mathbf{a}\}$. One has $B^\circ = \{\mathbf{x} \in \mathbb{R}^N \mid \mathbf{x} \gg \mathbf{a}\}$ and thus $B^\circ \cap A = \emptyset$. B° and A are convex, non-empty and disjoint. Using a separation theorem, there exists an affine hyperplane that A and B° separates. Therefore there exists $\boldsymbol{\gamma} \in \mathbb{R}^n \setminus \{\mathbf{0}\}$ such that $\boldsymbol{\gamma} \cdot \mathbf{z} \leq \boldsymbol{\gamma} \cdot \mathbf{b}$ ($\mathbf{z} \in A,\ \mathbf{b} \in B^\circ$). Even now

$$\boldsymbol{\gamma} \cdot \mathbf{z} \leq \boldsymbol{\gamma} \cdot \mathbf{b} \ (\mathbf{z} \in A, \mathbf{b} \in B). \tag{13}$$

With $\mathbf{b} = \mathbf{a}$ it follows that $\boldsymbol{\gamma} \cdot \mathbf{z} \leq \boldsymbol{\gamma} \cdot \mathbf{a}$ ($\mathbf{z} \in A$). Now we prove by contradiction that $\boldsymbol{\gamma} > 0$. So (remembering that $\gamma \neq \mathbf{0}$) suppose $\gamma_i < 0$ for some i. For $\mathbf{b} \in B$ defined by $b_j := a_j$ ($j \neq i$) and $b_i := x$ aar $x \geq a_i$, we have

$$\boldsymbol{\gamma} \cdot \mathbf{b} = \sum_{j=1, j\neq i}^{n} \gamma_j a_j + \gamma_i x.$$

For x large enough this number is less than $\boldsymbol{\gamma} \cdot \mathbf{a}$, which is a contradiction with (13). Q.E.D.

A.4 Remaining proofs

Proof of (2). Because the game is regular, co(U) is closed, and bounded. So it is compact.[17] Because it is also non-empty, $\widetilde{\mathrm{co}(U)}$ also is non-empty and therefore, by (9), also $\tilde{U} \neq \emptyset$. Because of the general identity

$$\tilde{U} = \mathbf{f}(Y), \tag{14}$$

also $Y \neq \emptyset$. Q.E.D.

Proof of (1). '$\subseteq$': by contradiction. So suppose $\mathbf{u} \in \mathrm{PB}(H)$ and $\mathbf{u} \notin \mathrm{PB}(\mathrm{co}(U)) \cap \mathbb{R}^N_+$. Because $\mathbf{u} \in \mathbb{R}^N_+$, it follows that $\mathbf{u} \notin \mathrm{PB}(\mathrm{co}(U))$, Noting that $\mathbf{u} \in \mathrm{co}(U)$, there exists $\mathbf{w} \in \mathrm{co}(U)$ with $\mathbf{w} > \mathbf{u}$. Therefore $\mathbf{w} \in \mathbb{R}^N_+$ and thus $\mathbf{w} \in H$, which is a contradiction with $\mathbf{w} \in \mathrm{PB}(H)$.

'$\supseteq$': suppose $\mathbf{u} \in \mathrm{PB}(\mathrm{co}(U)) \cap \mathbb{R}^N_+$. Then $\mathbf{u} \in H$ and there does not exist $\mathbf{w} \in \mathrm{co}(U)$ with $\mathbf{w} > \mathbf{u}$. Thus there also dos not exist $\mathbf{w} \in H$ with $\mathbf{w} > \mathbf{u}$. Q.E.D.

Proof of (7). '$\subseteq$': suppose $\mathbf{u} \in \mathrm{EXP}$. Then $\mathbf{u} \in \mathrm{PB}(H_{\mathrm{ag}})$ and there exists $\mathbf{w} \in H_\alpha$ such that $\mathbf{w} \gg \mathbf{u}$. By (6), $\mathbf{u} \in H_\alpha$. Therefore $\mathbf{w} \notin \mathrm{PB}_w(H_\alpha)$.

'$\supseteq$': suppose $\mathbf{u} \in \mathrm{PB}(H_{\mathrm{ag}}) \setminus \mathrm{PB}_w(H_\alpha)$. By (6), $\mathbf{u} \in H_\alpha$. Because $\mathbf{u} \notin \mathrm{PB}_w(H_\alpha)$, there is an $\mathbf{w} \in H_\alpha$ with $\mathbf{w} \gg \mathbf{u}$. Thus $\mathbf{u} \in \mathrm{EXP}$. Q.E.D.

Proof of Theorem 1. 1. We check that equality in (5) holds. For $r := \sum_k {}_k r$ one has (with sums on $k \in \mathcal{M}$)

$$\sum(\mathbb{R}^N_+ \cap \mathrm{co}({}_kU)) = \sum(\mathbb{R}^N_+ \cap {}_kr\ \mathrm{co}({}_1U)) = \sum({}_kr\mathbb{R}^N_+ \cap {}_kr\ \mathrm{co}({}_1U)) = \sum {}_kr(\mathbb{R}^N_+ \cap \mathrm{co}({}_1U)) =$$

$$r(\mathbb{R}^N_+ \cap \mathrm{co}({}_1U)) = r\mathbb{R}^N_+ \cap r\mathrm{co}({}_1U)) = \mathbb{R}^N_+ \cap r\mathrm{co}({}_1U) = \mathbb{R}^N_+ \cap \sum({}_kr\ \mathrm{co}({}_1U)) = \mathbb{R}^N_+ \cap \sum \mathrm{co}({}_kU).$$

We observe that the fourth equality holds because $\mathbb{R}^N_+ \cap \mathrm{co}({}_1U)$ is convex and the seventh holds because $\mathrm{co}({}_1U)$ is convex.

2. Using ${}_kU \subseteq \mathbb{R}^N_+$ and $\sum_k \mathrm{co}({}_kU) \subseteq \mathbb{R}^N_+$ we obtain $\sum_k(\mathbb{R}^N_+ \cap \mathrm{co}({}_kU)) = \sum_k \mathrm{co}({}_kU) = \mathrm{co}(\sum_k {}_kU) = \mathbb{R}^N_+ \cap \mathrm{co}(\sum_k {}_kU) = \mathbb{R}^N_+ \cap \sum_k \mathrm{co}({}_kU)$.

3. Because of (6). Q.E.D.

Proof of Theorem 2. First a lemma:

Lemma 6 *Suppose the following two conditions hold:*

[17] Note that U need not be compact.

A. There exists an l such that no element of the convex hull of the full-cooperative payoff vectors of ${}_l\Gamma$ is individually rational,

B. The trade-off direct sum game $(\oplus\Gamma)_\alpha$ has an individually rational full-cooperative payoff vector. Let $\mathbf{b}$ be such a payoff vector.

Then $\mathbf{b} \in H_\alpha \setminus H_{\mathrm{ag}}$ and thus there is an enrichment of the aggregated negotiation set. $\diamond$

Proof.— Condition A comes down to $\mathrm{co}(\widetilde{{}_lU}) \cap \mathbb{R}^N_+ = \emptyset$ and Condition B to $\mathbf{b} \in \widetilde{U_\alpha} \cap \mathbb{R}^N_+$. Using (11) and the s-notation of Appendix A.3, we obtain

$$s(U_\alpha) = \sum_k s({}_kU).$$

Of course, $\mathbf{b} \in H_\alpha$.

Next we prove by contradiction that $\mathbf{b} \notin \sum_k {}_kH$. Suppose that $\mathbf{b} = \sum_k {}_k\mathbf{h}$ with the ${}_k\mathbf{h} \in {}_kH$. Using (10) we have for each $k \in \mathcal{M}$

$$\sum_j {}_kh^j \leq s(\mathrm{co}({}_kU)) = s({}_kU). \tag{15}$$

Because ${}_l\mathbf{h} \in \mathbb{R}^N_+$ it follows that ${}_l\mathbf{h} \notin \mathrm{co}(\widetilde{{}_lU})$ and so ${}_l\mathbf{h} \in \mathrm{co}({}_lU) \setminus \mathrm{co}(\widetilde{{}_lU})$. By virtue of (9) we have $\mathrm{co}(\widetilde{{}_lU}) = \widetilde{\mathrm{co}({}_lU)}$ and so ${}_l\mathbf{h} \in \mathrm{co}({}_lU) \setminus \widetilde{\mathrm{co}({}_lU)}$. Therefore, in (15) we have a strict inequality for $k = l$. Because $\mathbf{b} \in \widetilde{U_\alpha}$, one has $\sum_j b^j = s(U_\alpha)$. It follows that $s(U_\alpha) = \sum_k s({}_kU) > \sum_k \sum_j {}_kh^j = \sum_j \sum_k {}_kh^j = \sum_j b^j = s(U_\alpha)$, which is a contradiction. Q.E.D.

Now we will prove Theorem 2. We start by observing that if a regular game in strategic form has a j-defect, then no element of the convex hull of the full-cooperative payoff vectors is individually rational. Indeed, let I^j be the set of individually rational payoff vectors for player j. Having a j-defect means that $\tilde{U} \cap I^j = \emptyset$. Note that this is equivalent to $\mathrm{co}(\tilde{U}) \cap I^j = \emptyset$.[18] Finally, using (14) it follows that $\mathrm{co}(\mathbf{f}(Y)) \cap \mathbb{R}^N_+ = \emptyset$.

Because of the above observation and ${}_1\Gamma = \Gamma$, condition A of Lemma 6 holds for $l = 1$. The proof is complete if we show that $(\oplus\Gamma)_\alpha$ has a full-cooperative multi-action $\mathbf{Y}$ and a Nash equilibrium $\mathbf{N}$ such that $\mathbf{Y}$ is a Pareto improvement of $\mathbf{N}$. Indeed, denoting the payoff functions of $(\oplus\Gamma)_\alpha$ with $g^1, \ldots, g^N$, $\mathbf{g}(\mathbf{N})$ is individually rational and therefore $\mathbf{g}(\mathbf{Y})$ too. Let $\mathbf{n}$ be a Nash equilibrium of ${}_1\Gamma$. By virtue of (2), ${}_1\Gamma$ has a full-cooperative multi-action $\mathbf{y}$. Because ${}_k\Gamma = \pi_k(\Gamma)$, $T_{\pi_k}(\mathbf{n})$ is a Nash equilibrium of ${}_k\Gamma$ and $T_{\pi_k}(\mathbf{y})$ is a full-cooperative multi-action of ${}_k\Gamma$. Let

$$\mathbf{N} := \Psi(\begin{pmatrix} T_{\pi_1}(\mathbf{n}) \\ \vdots \\ T_{\pi_N}(\mathbf{n}) \end{pmatrix}), \quad \mathbf{Y} := \Psi(\begin{pmatrix} T_{\pi_1}(\mathbf{y}) \\ \vdots \\ T_{\pi_N}(\mathbf{y}) \end{pmatrix}).$$

By (3) and (4) we have that $\mathbf{N}$ is a Nash equilibrium of $(\oplus\Gamma)_\alpha$ and $\mathbf{Y}$ is a full-cooperative multi-action of $(\oplus\Gamma)_\alpha$. Because ${}_1\Gamma$ has a j-defect, $\mathbf{n}$ is not full-cooperative; (4) implies that $\mathbf{N}$ is not full-cooperative either. The payoffs in $\mathbf{N}$ are

$$g^i(\mathbf{N}) = \sum_{k=1}^N (f^{\pi_k(i)} \circ T_{\pi_k^{-1}})(T_{\pi_k}(\mathbf{n}) = \sum_{k=1}^N f^{\pi_k(i)}(\mathbf{n}) = \sum_{l=1}^N f^l(\mathbf{n}).$$

So each player has the same payoff, say a, in $\mathbf{N}$. In the same way one shows that each player has the same payoff, say b, in $\mathbf{Y}$. The total payoff in $\mathbf{N}$ is Na and that in $\mathbf{Y}$ is Nb. Because $\mathbf{N}$ is not full-cooperative it follows that $Na < Nb$, i.e. $a < b$ which implies that $\mathbf{Y}$ is a unanimous Pareto improvement of $\mathbf{N}$. Q.E.D.

[18] Here we use that for two subsets A and B of $\mathbb{R}^N$ with B^c convex: $A \cap B = \emptyset \Leftrightarrow \mathrm{co}(A) \cap B = \emptyset$.

Proof of Theorem 3. We may assume that $\#\,\mathrm{PB}(\mathrm{co}({}_2U)) = 1$. Next note that by (1)

$$\mathrm{PB}(\mathrm{co}({}_kU)) = \mathrm{PB}({}_kH)\ (k = 1, 2).$$

So also $\#\,\mathrm{PB}({}_2H) = 1$. And because, using (1 and (12), $\mathrm{PB}(\mathrm{co}(U_\alpha)) = \mathrm{PB}(\mathrm{co}({}_1U) + \mathrm{co}({}_2U)) \subseteq \mathrm{PB}(\mathrm{co}({}_1U)) + \mathrm{PB}(\mathrm{co}({}_2U)) \subseteq \mathbb{R}^N_+$, also

$$\mathrm{PB}(\mathrm{co}(U_\alpha)) = \mathrm{PB}(H_\alpha).$$

Now we obtain, noting that feasible sets and negotiation sets are compact, using Lemma 3,

$$\mathrm{PB}(H_\alpha) = \mathrm{PB}(\mathrm{co}(U_\alpha)) = \mathrm{PB}(\mathrm{co}({}_1U) + \mathrm{co}({}_2U)) =$$

$$\mathrm{PB}(\mathrm{co}({}_1U)) + \mathrm{PB}(\mathrm{co}({}_2U)) = \mathrm{PB}({}_1H) + \mathrm{PB}({}_2H) = \mathrm{PB}({}_1H + {}_2H) = \mathrm{PB}(H_{\mathrm{ag}}).\quad \text{Q.E.D.}$$

Proof of Theorem 4. First a lemma:

Lemma 7 *Suppose* $\mathbf{a} \in \mathrm{PB}(H_{\mathrm{ag}})$. *Then*

$$\mathbf{a} \in \mathrm{EXP}\ \Leftrightarrow N^+_{\mathrm{co}(U_\alpha)}(\mathbf{a}) = \emptyset.\ \diamond$$

Proof.— $\Rightarrow$. Let $\mathbf{c} \in \mathrm{PB}(H_\alpha)$ such that $\mathbf{c} \gg \mathbf{a}$. For all $\boldsymbol{\gamma} > \mathbf{0}$ one has $\boldsymbol{\gamma} \cdot (\mathbf{c} - \mathbf{a}) > 0$. Because $\mathbf{c} \in \mathrm{co}(U_\alpha)$, it follows that $\boldsymbol{\gamma} \not\in N^+_{\mathrm{co}(U_\alpha)}(\mathbf{a})$.

$\Leftarrow$. By Lemma 5 one has $\mathbf{a} \not\in \mathrm{PB_w}(\mathrm{co}(U_\alpha))$. Let $\mathbf{c} \in \mathrm{co}(\mathrm{U}_\alpha)$ with $\mathbf{c} \gg \mathbf{a}$. Since $\mathbf{a} \in \mathbb{R}^N_+$, also $\mathbf{c} \in \mathbb{R}^N_+$. This implies $\mathbf{c} \in H_\alpha$. Thus $\mathbf{a} \in \mathrm{EXP}$. Q.E.D.

Now we prove Theorem 4. According to Lemma 7 the proof is complete if we can prove that $N^+_{\mathrm{co}(U_\alpha)}(\mathbf{a}) \neq \emptyset$ for all $\mathbf{a} \in \mathrm{PB}(H_{\mathrm{ag}})$.

So suppose $\mathbf{a} \in \mathrm{PB}(H_{\mathrm{ag}}) = \mathrm{PB}(\sum_k {}_kH)$. By Lemma 5 one has $N^+_{\sum_k {}_kH}(\mathbf{a}) \neq \emptyset$. Because $\mathbf{a} \in \sum_k {}_kH$, there exists ${}_k\mathbf{a} \in {}_kH (k \in \mathcal{M})$ such that $\mathbf{a} = \sum_k {}_k\mathbf{a}$. With (8) one obtains

$$\cap_k N^+_{{}_kH}(\mathbf{a}) \neq \emptyset.$$

By assumption $\mathrm{PB}(\mathrm{co}({}_kU)) \subseteq \mathbb{R}^N_+$ for all k. Therefore $\mathrm{PB}(\mathrm{co}({}_kU)) \subseteq \mathbb{R}^N_+ \cap \mathrm{co}({}_kU) = {}_kH$. So we can apply Lemma 2 with $A = {}_kH$ and $B = \mathrm{co}({}_kU)$ and get

$$N^+_{\mathrm{co}({}_kU)}({}_k\mathbf{a}) = N^+_{{}_kH}({}_k\mathbf{a})\ (k \in \mathcal{M})$$

and therefore

$$\cap_k N^+_{\mathrm{co}({}_kU)}(\mathbf{a}) \neq \emptyset.$$

Applying again (8) one obtains $N^+_{\mathrm{co}(U_\alpha)}(\mathbf{a}) \neq \emptyset$. Q.E.D.

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