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A Rule of Thumb for Controlling Invasive Weeds: An Application to Hawkweed in Australia

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Abstract

We use a bang-bang optimal control model to derive a rule of thumb for an optimal management of invasive weeds, in terms of the marginal benefits and costs of various control actions. Instead of determining the size of infestation under an optimal surveillance measure, the rule specifies the types of land where an invasive weed should be first prevented from establishment, and under what conditions control should be initiated. The types of land are modeled via the heterogeneous vulnerability of land to the weed and likely infestation. This easy-to-use rule is applied to determine how hawkweed should be controlled in Australia, across three potential control strategies: containment, eradication and no action. We investigate this rule-of-thumb in both deterministic and stochastic settings. With uncertainty, when calculating the threshold of when and how to act, we take into account the fact that delaying a control action will incur not only larger damage and a potentially larger spread but also a higher cost from uncertainty in the spread of the weed itself. The land value threshold is thus given by the unit cost of keeping a weed off a parcel of land times the difference between the interest rate and the current weed spread rate plus the effect of uncertainty. An application to hawkweed in Australia is provided. The rule specifies that hawkweed should be immediately eradicated in all types of agricultural lands they currently occupy where the potential damage is larger than 15AUD/ha/year. This generates a full eradication strategy under broad parameter values. Though the cost of removing hawkweeds is significant, it is overwhelmed by the damage if Hawkweeds spread to higher value agricultural land.

JEL Classification:

Keywords: Stochastic optimal control, biosecurity, invasive weed management, hawkweed.

I. Introduction

Invasive weeds are significant economic and ecological threats in many countries. Their invasion is characterized often results in relatively fast spread and the elimination of habitats of other species. In addition, the threats of some invasive weeds might have been underestimated as many invasive weeds are starting with very low spread rate for a long time, even decades before invading aggressively. Many authors[1] have agreed that invasive weeds must be closely monitored for timely actions.

Actions in a weed management strategy are commonly classified into three broad approaches, namely eradication, containment and no action. A natural question regarding an optimal weed management strategy is what approach should be applied. A common solution of this question is to identify the size of invasive weed infestation where each type of control action is appropriate. For example, Woldendorp and Bomford (2004) observe that the feasible sizes of most successful eradication campaigns are less than 4 ha. Cacho et al. (2004) derive two critical size thresholds for an infestation above which eradication and containment should no longer be economically efficient.

Despite clear technical meanings of the three approaches[2], what they actually mean depends on the geographical scope attached to them. More specifically, a strategy to contain a weed in one village and to clear it off elsewhere, is containment to a nation planner, no action to the first village and eradication to the others. Similar problems arise with different geographical scales such as suburbs, towns, states and countries. For this reason, a practical strategy must specify not only how large a weed infestation should be before action, but also where it should be contained. In other words, spatial information should be incorporated.

A model with full spatial information is often too expensive regardless of the fact that we rely on reasonably simplified assumptions on the dynamics of weeds and their spread. Full spatial models are extremely hard, if not impossible, when the geographical area of study is as large as a state and or a country. In addition, such a model, if existent, is unlikely to provide a simple and specific guidance rule for weed management strategies. Hence we do not directly address the question "what is the optimal size of infestation" but, rather, "on what types of land should a weed be kept off". Under

[1] See, for example, Woldendorp and Bomford (2004), Simberloff (2003), Cunningham and Brown (2006), Brinkley and Bomford (2002).

[2] Bomford and O'Brien (1995) develop six criteria for assessing a successful eradication and Simberloff (1997) observes that a management plan is often specifically aimed at eradication of a species, but the methods are the same as those that would be used to reduce a population to an economically or ecologically acceptable level.

these circumstances, anyone interested in controlling a weed can specify what should be done by comparing the marginal cost and benefit of an immediate action.

We develop the concepts of marginal cost and marginal benefit of an immediate action via an optimal control problem with bang-bang solutions. Our model originates from two commonly observed characteristics of a weed invasion, as distinct from epidemic diseases: (i) a weed often spreads from a low-value area to higher value one, and (ii) an invasive weed usually has a sleeping period with a relatively low invasion rate before spreading aggressively. Therefore a general rule is that an invasive weed should be contained in low value lands with relatively 'low aggression' and eradicated elsewhere.

In our formulation, the value of land is modeled via the heterogeneity in the vulnerability of an area to a weed. The rule of thumb we derive here specifies a threshold in the land value above which weed should be eradicated. Hence, this rule can be applied in any actual infestation of any density and size as a guidance of invasive weed management strategies.

The remaining of the paper is organized as follows. Section II summarizes some technical models on optimal weed management. In section III, we present the bang-bang optimal control problem and derive the rule to determine the land value threshold for weed eradication. The problem is first formulated in a deterministic setting for simplicity and then extended to include uncertainty. Section 4 applies the rule to Hawkweed in Australia. Section 5 provides summary remarks.

II. A survey on technical models of optimal weed management

The first applications of dynamic optimization to disease control can be found in the early 1980s. Fisher and Lee (1981) use dynamic programming to solve an optimal crop rotation problem regarding the vulnerability to weed and disease infestation in wheat. Shoemaker (1982) uses a high dimension stochastic dynamic programming formulation for optimal control of a univoltine pest population. Both of these highlight the importance of spending money early to have a better state for the future, or to obtain eradication.

Following this trend, some papers applying dynamic programming to the technicality of weed control were first published in the 1990s. For example, Pandey and Medd (1991) use a two-state-one-control problem to show that herbicide treatment should be increased but at a diminishing rate as weeds become denser. Sells (1995) indicates that the dynamic programming model is more powerful than using simulation in that optimum weed control strategies are calculated from a wide range of possible decision options over the full range of weed seed banks, including the variable nature of herbicide sprays.

The added emergence of invasive weeds has induced important publications about optimal invasive weed controls in the 2000s. Jones and Cacho (2000) and Wu (2001) show that weed control in a dynamic model should be more extensive than in a static model and will result in higher returns. Taylor and Hastings (2004) use a 60 year time horizon optimal control problem to find that at low and medium budgets, it was necessary to remove the low-density plants first to achieve eradication, but if more money was available then the optimal strategy was to prioritize high-density areas. Jayasuriya and Jones (2008) use a logistic growth and constant dispersal rate dynamic model to show how control effort depends on the budget and infestation in 50 year time horizon problem.

Our model is, unlike all above mentioned, formulated in continuous time setting to facilitate the derivation of the marginal cost and marginal benefit of controlling a weed. In addition, we use infinite time horizon as many invasive weeds, including hawkweed in our case study, are initially ‘sleeping’ (Brinkley and Bomford 2002) taking several decades to spread aggressively.

III. Model Formulation

1. Weeds dynamics, controlling cost and the dynamic optimization problem

Consider an area of any geographical size, normalized to one, so a small patch of land can be represented by a number $s \in [0,1]$. The area is spatially heterogeneous with respect to the land's marginal value (productivity) such that $MP_{land}(s) = f(s) \geq 0$. As the geographical size can be chosen arbitrarily, we assume, without the loss of generality, that the land's productivity is monotonically increasing with respect to the land position:

$$f'(s) \geq 0 \tag{1}$$

If an area [0,s] is occupied by weeds, then the annual damage is[3]:

$$d(s) = \int_0^s f(x)dx \tag{2}$$

[3] By this, we assume that weeds start in the least productive area and spread to more productive land as commonly observed.

If we denote the weed growth function as $g(s) \geq 0$ with $g(0) = 0$ and the area cleared in a year should a controlling action occurs as $h \in [0, s]$, then the dynamics of weeds can be represented by the following equation:

$$\dot{s} = g(s) - h \tag{3}$$

If the unit cost of clearing the weed is *c*, then total instantaneous cost will be $d(s) + c \times h$ and the problem is to choose how to control the weed (*h*) so as to minimize the life-time present value of the total cost, discounted at an annual rate ρ, over an infinite time horizon:

$$min_{h(t)} \int_0^\infty e^{-\rho t}[d(s) + c \times h]dt \text{ subject to equation (3)} \tag{4}$$

The current-value Hamiltonian function for problem (4) is

$$H(s, h, \lambda) = d(s) + c \times h + \lambda[g(s) - h] \tag{5}$$

The first order conditions are

$$\frac{\partial H(s, h, \lambda)}{\partial h} = 0 \text{ or } c = \lambda \tag{6}$$

$$\frac{\partial H(s, h, \lambda)}{\partial s} = \rho\lambda - \dot{\lambda} \text{ or } f(s) + \lambda g'(s) = \rho\lambda \tag{7}$$

$$\frac{\partial H(s, h, \lambda)}{\partial \lambda} = \dot{s} \text{ or } \dot{s} = g(s) - h \tag{8}$$

In order to have the second order condition for a minimization satisfied, the Hamiltonian function in equation (5) must be convex. We assume this in equation (9) but will discuss the relaxation of this assumption later.

$$f'(s) + cg''(s) > 0 \tag{9}$$

The Hamiltonian function in equation (5) is linear with respect to the control variable so problem (4) has a bang-bang solution. To determine this bang-bang solution, we combine equations (6) and (7) to give:

$$f(s) + cr(s) = \rho c \tag{10}$$

where $r(s) = g'(s)$ measures a weed aggression, i.e. how a weed growth responses to its infestation. We will analyze this equation to determine the best strategy among eradication, no action or containment below.

2. Eradication, No Action or Containment

Equation (10) contains the marginal benefit and marginal cost of controlling a weed. The LHS of this equation represents the marginal benefit of an immediate action. It consists of the avoided productivity loss, $f(s)$, that would be caused by a marginal invasion and the increase in the intervention cost $cr(s)$ caused by the change in the weed aggression $r(s)$. The RHS of the equation is the marginal cost of starting to control the weed, which is the interest rate forgone ρc. As the marginal benefit is an increasing function with respect to the invaded area while the marginal cost is a constant, there are three possibilities as illustrated in Figure 1.

First, if the marginal benefit of controlling a weed is always larger than the marginal cost, then the optimal strategy is to eradicate the weed as early as possible. The net cost of a late intervention can be represented by the net present value of the shaded area in Figure 1.a. As shown in the Figure, the condition for this 'earliest eradication' strategy is:

$$f(0) + cr(0) \geq \rho c \tag{11}$$

where $r(0)$ is the intrinsic growth rate of the weed[4].

Second, if the marginal benefit of controlling a weed is always smaller than the marginal cost, as shown in Figure 1.b, the optimal strategy is to ignore the weed. In this situation, the productivity lost by the weed invasion and the weed aggregation are sufficiently small. The condition for this situation is:

$$f(1) + r(1) \leq \rho c \tag{12}$$

[4] As $g(s) \approx g'(0)s$ by the Mclaurin's expansion, the growth rate $r = \frac{ds/dt}{s} \approx g'(0)$.

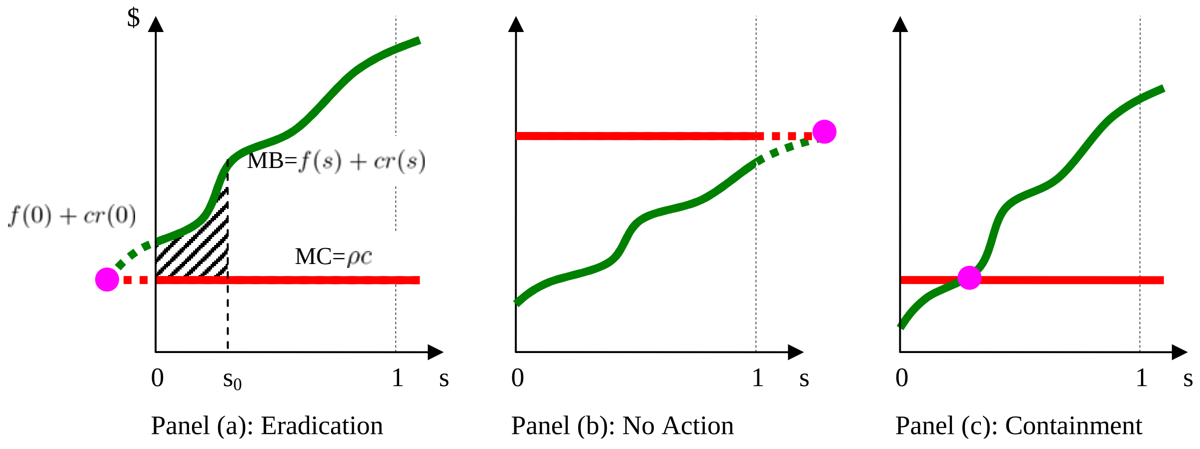

Figure 1. Three possibilities for the optimal strategy

The third possibility for optimality, an interior solution, occurs when the conditions in equations (11) and (12) are not satisfied. The optimal point for containment is where the marginal benefit intersects the marginal cost, as shown in Figure 1.c. If the current infestation is small, the best strategy is to wait until the optimal point is reached and contain the weed there. In this case, it is not efficient to spend money too early on a sleeping weed with a small infestation of low-value land. On the other hand, if the current infestation is already above the optimal point, the best strategy is to immediately destroy some infestation to that point. Late eradications may result in a net loss when the weed 'wakes up' and aggressively invades high-value land. We summarize the three possibilities in Table 1.

Condition	Optimal strategy
$f(0) \geq c[\rho - r(0)]$	As early eradication as possible
$f(1) \leq c[\rho - r(1)]$	Doing nothing
$f(s^*) = c[\rho - r(s^*)]$ with $s^* \in (0,1)$	Optimal containment at s^*. If $s_0<s^*$, wait. If $s_0>s^*$, immediately destroy infestation to s^*.

Table 1. Three possibilities for the optimal strategy

Table 1 highlights the rule of thumb to determine the threshold of land value where an invasive weed should be kept off. The threshold is the unit cost of eradicating a weed times the gap between the interest rate and the weed aggression. The rule also makes clear that the weed growth rate being smaller than the interest rate is not the only condition that supports early eradications. Even when the growth rate is smaller, early eradications can still be the optimal if the productivity lost due to initial invasion is sufficiently large.

It should be noted that the conditions in Table 1 provide a consistent guide on what should be done though analyses of a problem with different geographical scales may be classified into different cases. For example, a weed is reaching its carrying capacity (hence has a low aggression) in a small village with low value land and Table 1 suggests the village of an ignorance as per case 2. However, a national planner may see wed incursion of a village is only the initial stage of an aggressive process of the weed to invade higher value areas, evaluate the weed to case 3 and wait until the optimal containment point is reached. Hence, both the village and the national planner agree that nothing should be done in the village but the latter may contain the weed in the future. Similarly, a high-value-land village wants to eradicate a weed while a national planer wants containment. Though using different words, they both mean no infestation should be left in a high-value-land and the weed should be contained in low-value areas.

3. Multiple equilibrium

If the assumption for convexity in equation (9) is not satisfied over the whole domain [0,1], multiple equilibriums may arise as illustrated in Figure 2. In this situation, there are three intersections of the marginal benefit and marginal cost but only points A and C present minima. Point B presents a maximum.

The choice of equilibrium depends on the current infestation. At points to the left of A, the optimal decision is to wait until A is reached and at points to the right of C, the most valuable invaded land must be cleared immediately. If the initial infestation is between A and C, moving back to A implies spending money now for less damage in the future compared to waiting until C is reached. In this situation, the choice will depend on various factors such as the distance to A and C, invading speed, production damage difference between points A and C and so on.

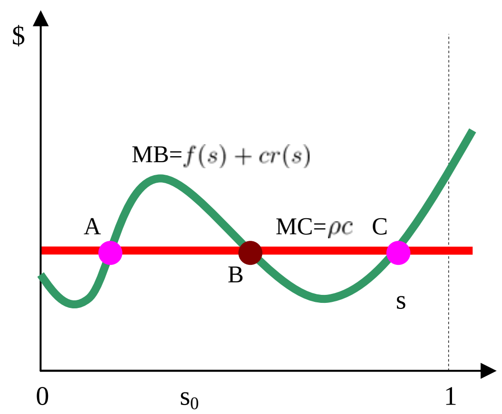

Figure 2. Multiple equilibria

4. Uncertainty

An important characteristic of weed invasion is that it depends on many uncontrolled factors such as wind, temperature, rainfall and unintentional human activities. These factors create uncertainties in the spread rate of a weed. In addition, they may enable the weed to escape from where it is contained with a non trivial probability (Baxter et al., 2007). To capture these uncertainties, we modify the dynamics of weeds in equation (3) to include uncertainties such that:

$$ds = [g(s) - h]dt + \mu(s)dw + \phi(s)dq \tag{13}$$

where dw is a standard Brownian motion, dq is a Poisson diffusion process with arrival rate β, $\mu(s)$ is the state-dependent standard error of the Brownian motion and $\phi(s)$ is the magnitude of the Poisson diffusion.

The FOCs derived from the Hamiltonian function, similar to section 1 give the condition for the marginal productivity of an invaded land where eradications should start with:

$$f(s^*) = c[\rho - r(s^*) - \beta\phi'(s^*)] \tag{14}$$

The difference between equation (14) and its deterministic version, equation (10), is the term $\beta\phi'(s^*)$ which represents the effect of uncertainty. This equation implies that when calculating the threshold, we need to take into account the fact that delaying a control action will incur not only larger damage and higher aggression but also a higher burden from uncertainties in the weed spread. Therefore, the land value threshold is the unit cost of keeping a weed off times the difference between the interest rate and the current weed aggression plus the uncertainty effect. We will calibrate equation (14) to Hawkweed in Australia.

IV. Application to Hawkweed in Australia

1. Hawkweed and its potential impact in Australia

Invasive hawkweeds are a group of invasive weeds originating from Europe and have become world-wide weeds, causing serious problems in New Zealand, the United States, Canada, and Japan. Biological characteristics of these weeds allow them to survive and grow in various types of habitats and more importantly create ecological threats such as biodiversity and productivity loss. Hawkweed infestation covers 500,000 ha in New Zealand's South Island (Hunter, 1991). In the United States, hawkweed infestation is estimated at 480,000 ha (Ducant et al., 2004), to be growing by 16% per year with $58 million in control costs (Baker and Giroday, 2006).

In Australia, four species of hawkweeds have been found in four states (Victoria, New South Wales, Act and Tasmania) in 2400ha with the area of occupancy of 28ha (Report, 2010). Though there has not been any publication on the economic damage actually incurred, there is consensus on the potential huge impact of hawkweeds, if not controlled. For example, Brinkley and Bomford (2002) estimate that 14.3 million ha of agricultural land are in highest area for orange hawkweed invasion with the production value of $1.25 billion. Cunningham et al. (2003) estimate the area at risk is 1.2 million ha with production value of 1.77 billion AUD and profit of 0.3 billion AUD. Beaumont et al. (2009) predict that climate change will contract climate range of Hawkweeds overall but much of the Australian Alps, which contain large contiguous tracts of reserves and many endemic species, will continue to retain climatically suitable areas for hawkweeds through to 2070.

2. Baseline parameter values

The most cost effective method of hawkweed control is with the use of herbicides applied by spot spraying or wick-wiping to reduce the risk of off-target damage (Report, 2010)[5]. Tordon 22K at 2.25 L/ha or Grazon at 3.8 L/ha provide very good control of orange hawkweed (MFF, undated). Grazon has been approved to be used in Australia (Report, 2010). Given the price of this herbicide around $350/5L and one-day labor cost to apply it to 1ha, the cost for one treatment is approximately $500/ha. As hawkweed seeds in soil lose viability fairly quickly (DARA 1999, as cited in Cunningham and Brown, 2006), applying the herbicide twice a year during the spring and summer may keep a place free from Hawkweed, so we approximate the treatment cost is $c = 1000 AUD/ha$.

Hawkweed is known to be a fast spreading. The spread of hawkweed in the US can be up to 16% per year (Baker and Giroday, 2006 and Wilson and Prather, undated). The area covered by mouse-ear hawkweed in New Zealand had increased by 50% during the period from 1982 to 1992 (Johnstones et al., 1999). Several authors have modeled the spread of hawkweed in Australia by spatial simulation techniques, for example Beaumont et al. (2009) and Williams et al. (2008) but a specific measurement of Hawkweed spread aggression in Australia of Hawkweeds is still not published.

[5] Hawkweed can be controlled by hand removal, however this needs to be undertaken carefully to ensure no stolons or rhizomes are left in the soil or spread to other areas.

However, some stylized facts regarding the spread of Hawkweeds in Australia have been recognized. Brinkley and Bomford (2002) classify Hawkweeds as a sleeping weed with relatively low spread rate compared to what it may have. DAWA (1999), as cited by Cunningham and Brown (2006), indicates that the wind dispersed seed has a normal annual dispersal distance less than one kilometer, though Morgan (2000) recognize some populations have established more than 1 km from the presumed source. In this case study, we assume that at the current infestation the weed will double after 25 years (if controlled) implying an annual spread rate 3%.

Given its wide distribution in Victoria and Tasmania and the fact that it has been sold in nurseries in these states as well as in New South Wales and Queensland (Cunningham and Brown, 2006), uncontrolled factors regarding the spread of Hawkweed are significant. Therefore, we assume that a the arrival rate and magnitude of Poisson certainty are 0.1 and 0.05s implying that uncontrolled factors increase the Hawkweed infestation approximately once every 10 years. The annual interest rate is assumed to be 5%. The parameters are reported in Table 2.

Variables	Notation	Values
Arrival rate of Poisson diffusion	λ	0.1
Magnitude of Poisson diffusion	$\phi(s)$	0.05s
Weed aggregation	$r(s)$	0.03
Annual interest rates	ρ	0.05
Treatment cost	c	$1000/ha

Table 2. Baseline parameters for Hawkweed invasion

3. Results and practical implications

Substituting the baseline parameters in Table 2 into equation (14) gives a result that Hawkweed should be eradicated in all areas where its damage is larger than 15AUD/ha/year. Comparing this threshold with the values of land use in Table 3 indicates that Hawkweed should be eradicated unless in deserted land. A very similar conclusion can be drawn with a range of parameter values as reported in a sensitivity analysis in Table 4.

Land use	Value (AUD/ha/year)	Information sources
Grazing	30 AUD	Cunningham et al. (2003)
Environment purpose	50 AUD	Stoneham et al. (2004)
Cropping (wheat)	170 AUD	Cacho et al. (2004)
Horticulture	500 AUD	Brinkley and Bomford (2002)

Table 3. Value of different land use

Treatment cost \ Aggression	35 years to double	24 years to double	18 years to double	<15 years to double
800AUD/ha/year	20.0	12.0	4.0	0
900AUD/ha/year	22.5	13.5	4.5	0
1000AUD/ha/year	25.0	15.0	5.0	0
1200AUD/ha/year	30.0	18.0	6.0	0

Table 4. Sensitivity analysis on the land value threshold (Unit: AUD/ha/year).

Two points should be noted on the practical implications of the threshold we calculate here. First, the damage caused by Hawkweeds in one hectare depends on its density so this should be taken into account. If Hawkweeds spread over one hectare of a grazing land but is not enough dense to cause a damage of 16AUD/ha/year, then it is not economically efficient to apply herbicide over one hectare to remove it immediately.

Second, the density-dependent damage should be calculated over the size of area that herbicide must be applied to remove the weed. Adding a weed-free area to reduce to the average weed density and hence the average damage per hectare will lead to misleading result. For example, when the weed is found in a village causing a damage of 30AUD/ha at grazing land, adding two adjacent weed-free villages into this calculation will reduce the average damage to 10AUD/ha, well below the threshold. The later calculation is clearly biased toward delaying treatment unless the herbicide must be applied in all villages to eradicate the weed in the first one.

V. Summary Remarks

Using an optimal control model with bang-bang solutions, we develop the concepts of the marginal benefit and marginal cost of controlling invasive weeds. The key feature of our model is the heterogeneity in the land value and the variable aggression of the weed spread. The optimal strategy is to contain an invasive weed in a low-value land with low invasive aggression.

By comparing the marginal cost and benefit, we derive a simple rule guiding the optimal weed management strategy. The rule determines a threshold in the damage per unit of land over which the weed should be eradicated. Therefore, it can be applied in any invaded areas without requiring complicated spatial information.

When applied to sleeping hawkweed in Australia, the rule specifies that hawkweeds should be immediately eradicated in all types of agricultural lands they fully occupy. Though the cost of removing hawkweeds is significant, it is overwhelmed by the damage if Hawkweeds spread to agricultural land. Uncertainties, including spread of hawkweed by human activities are also factor that induces early eradication before the weed invades more productive area.

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