Efficiency and Redistribution in the French Comt´e Cheese Market

Pierre Merel
prmerel@ucdavis.edu

Paper prepared for presentation at the I Mediterranean Conference of Agro-Food Social Scientists. 103rd EAAE Seminar '*Adding Value to the Agro-Food Supply Chain in the Future Euromediterranean Space*'. Barcelona, Spain, April 23rd - 25th, 2007

Efficiency and Redistribution in the French Comté Cheese Market

Pierre R. Merel

University of California, Davis

École nationale du génie rural, des eaux et des forêts, Paris

Abstract

The purpose of this paper is to investigate the economic rationale underlying collective management and supply control mechanisms in European protected designations of origin (PDOs), with a focus on the French Comté cheese market. The argument is based on the presence of imperfect competition at the processing stage of the industry. Assuming that processors are able to exercise both oligopsony power in the procurement of the agricultural output and oligopoly power in the sale of the final product, I show that vertical integration between upstream producers and downstream processors is mutually beneficial as long as the integrated industry is able to exercise some degree of seller market power. I characterize instances where consumers also win from the integration process, so that the vertically integrated monopolist is less detrimental to social welfare than the separated industry. Even when consumers lose, the deadweight loss due to market power by the integrated monopolist may not be significantly higher than that arising from the joint exertion of oligopsony and oligopoly power by a separated processing sector. This argument provides a potential rationale to the government-sanctioned production control scheme currently in place in the French Comté cheese market.

Efficiency and Redistribution in the French Comté Cheese Market

1 Introduction

The purpose of this paper is to discuss the economic rationale underlying collective management and supply control mechanisms in European protected designations of origin (PDOs), with a focus on the French Comté cheese market. I begin with a brief presentation of the Comté cheese industry. I expose the supply control scheme currently in place in the Comté cheese market and the contractual relationships between producers of unripe cheese (upstream producers) and curing facilities (downstream processors). In section 3, I develop a theoretical model that explicitly accounts for the imperfectly competitive behavior of processors, and show that downstream integration of curing facilities by cheese producers, coupled with the exertion of market power by the resulting entity, may actually benefit all agents in the economy. When, however, consumers are hurt by the market power of the integrated entity, the resulting economy-wide deadweight loss ought to be compared to the deadweight loss that would arise if separated processors were free to exercise oligopsony and oligopoly power, a more realistic counterfactual than perfect competition in the case of Comté cheese. A brief simulation shows that failure to accurately consider the market conditions that would arise in the absence of the vertically integrated structure can lead to significant overestimation of the deadweight loss of the integration policy. Section 4 concludes.

2 The French Comté cheese market

Comté is one of the most popular cheeses in France, with annual production of about 50,000 metric tons, most of which is still consumed nationally. Comté is a pressed, cooked cheese made out of raw cow's milk, aged for at least 4 months, that comes in large wheels weighing between 66 and 106 lbs (République française, 1999). The specificity of Comté cheese was recognized by a Court decision in 1952, and its production was first codified in a 1958 regulation. Comté was introduced in the European PDO register in 1996, the date of creation of the register. The production process unfolds in three stages: milk production, cheese fabrication and cheese curing. All stages must take place within a delimited geographical region covering several districts ("cantons"), most of them located in the French Jura mountain. In January of 2005, there were about 3,300 milk producers, 190 cheese factories (called "fruitières") and 20 curing facilities involved in Comté cheese production (Comité interprofessionnel du Gruyère de Comté, 2005). Notably, 85% of cheese factories were owned by milk producers through cooperatives. The processing sector is characterized by heterogeneous curing facilities, with a few industrial entities covering a large share of the market (see table 1).

2.1 Production control

The Comté cheese industry is currently represented by a producer association, the Comité interprofessionnel du Gruyère de Comté (CIGC), whose stated missions are to guarantee the specificity of Comté cheese to consumers and to help Comté cheese producers maintain a sustainable activity in an economically sensitive region (Comité interprofessionnel du Gruyère de Comté, 2005). CIGC fulfills its first mission by controlling producers at various stages of the production process, filing lawsuits against imitators, and participating in the elaboration of regulations pertaining to Comté cheese production.

The second mission is fulfilled through technical assistance to producers, generic advertising and the promotion of Comté cheese in export markets. It is also met through the elaboration of a yearly production plan, the purpose of which is to restrict marketed quantities of Comté cheese. The production plan is subject to yearly approval by the government, and this approval is necessary for the supply control to be lawful. Indeed, the Conseil de la concurrence, the French authority in charge of implementing antitrust law, ruled against CIGC when they failed to obtain government support for their production scheme for the year 1995 (Conseil de la Concurrence, 1998).

2.2 Wealth redistribution through supply chain contracts

The rents generated through the supply control mechanism are redistributed among participants in the integrated industry through model contracts designed by CIGC and to be adopted between curing facilities and the cheese factories supplying them. The main purpose of these contracts is to set the price of unripe cheese.[1] The contract also compels the curing facility to purchase all cheese wheels produced by the cheese factory. Since the total supply of unripe cheese is fixed exogenously, curing facilities cannot act strategically, purchasing all the unripe cheese produced by their suppliers at the contractual price, bearing the aging costs and selling cured cheese at its market price.

3 Welfare analysis of the role of CIGC

In this section, I analyze the role of CIGC by focusing on the two functions described in section 2: supply control and rent redistribution between producers

[1] Since the vast majority of cheese factories are owned by milk producers, the price of unripe cheese determines the distribution of rents between milk producers and curing facilities.

and processors. To do so, I compare two alternative market situations: (i) the free-market situation, where producers of unripe cheese and curing facilities are separate entities, production of unripe cheese is unconstrained, unripe cheese is purchased by imperfectly competitive processors and sold to buyers after the aging process and (ii) the actual situation, where CIGC sets the total volume of production (under government supervision) and the price of unripe cheese, specified in the CIGC contract, determines the distribution of profits between processors and cheese producers.[2]

3.1 The theoretical model

I assume that unripe cheese is transformed by processors into cured cheese according to a Leontief technology. The quantity of unripe cheese produced is denoted by q and that of cured cheese by Q. Without loss of generality, I choose units so that one unit of unripe cheese is transformed into exactly one unit of cured cheese, so that $q = Q$. I denote the (constant) per unit cost of other inputs used to produce cured cheese from unripe cheese by c_0. Therefore, the marginal cost of producing one unit of cured cheese is equal to $c_0 + MC(Q)$, where MC is the marginal cost of producing one additional unit of unripe cheese. The inverse demand for cured cheese is denoted by $p(Q)$, and I assume that $p' < 0$. Following a common practice in the market power literature, I also assume that p and MC are linear.[3] I denote by η the absolute value of the elasticity of demand for cured cheeses, and by ϵ the elasticity of marginal cost (including the cost of inputs other than unripe cheese), both evaluated at the competitive equilibrium characterized by $p(Q) = c_0 + MC(Q)$. The price and quantity that would prevail under perfect competition are denoted by p^c and Q^c.

[2]The term "free market" refers to the no-intervention scenario and should not be confused with "perfect competition". That processors should behave in an imperfectly competitive fashion in the no-intervention scenario seems reasonable given the structure of the industry.

[3]See, for instance, Alston et al. (1997).

In the no-intervention scenario, the processing sector behaves as an oligopsony in the procurement of unripe cheese and an oligopoly in the sale of cured cheese. I adopt the usual parameterization that permits modeling of intermediate degrees of market power without specification of the oligopsony/oligopoly game. More precisely, I assume that the equilibrium condition at the processing stage is of the form:[4]

$$(1-\gamma)p(Q) + \gamma MR(Q) = (1-\xi)(c_0 + MC(Q)) + \xi ME(Q), \tag{1}$$

where γ and ξ are between 0 and 1 and represent the degree of seller and buyer market power being exercised by the processing industry, respectively. The function $MR(Q)$ represents the revenue to processors from the sale of an additional unit of cured cheese beyond Q units, and $ME(Q)$ represents the expenditure needed to produce an additional unit of cured cheese beyond Q units. For linear specifications of p and MC, those two functions are linear with slopes twice as steep as the slopes of p and MC, respectively.

In the real-world scenario, I assume that CIGC acts as an imperfect monopolist with market power parameter θ, that is, the pricing equation is of the form

$$(1-\theta)p(Q) + \theta MR(Q) = c_0 + MC(Q), \tag{2}$$

and I allow the monopoly parameter θ to take on values smaller than 1.[5]

[4]This condition is similar, for instance, to the equilibrium condition used by Huang and Sexton (1996), Alston et al. (1997) and Zhang and Sexton (2002) to represent the oligopsonistic and oligopolistic behavior of processing firms.

[5]The extent of market power being exerted by CIGC is likely to be limited by the government's role in approving the production plan.

3.2 Profitability of the vertically integrated monopoly for the production sector

For downstream processors and upstream producers to jointly agree to participate in the integrated monopoly scheme, CIGC has to leave upstream producers and downstream processors at least as well-off as under the oligopsony/oligopoly scenario. A necessary condition for this participation constraint to hold is that the joint profits under the monopoly scenario are at least as large as those under the oligopsony/oligopoly scenario. It can be shown that this condition is equivalent to[6]

$$(\theta - \theta_1)(\theta - \theta_2) \leq 0, \tag{3}$$

where $\theta_1 = \gamma + \frac{\eta}{\epsilon}\xi$ and $\theta_2 = \frac{1+\frac{\eta}{\epsilon}-\frac{\eta}{2\epsilon}(\gamma+\frac{\eta}{\epsilon}\xi)}{\gamma+\frac{\eta}{\epsilon}\xi+\frac{\eta}{2\epsilon}}$. Therefore, in order for the monopoly situation to benefit the industry in aggregate, the value of the monopoly parameter θ must lie between two bounds, θ_1 and θ_2, that depend on the values of γ and ξ as well as the ratio of elasticities $\frac{\eta}{\epsilon}$. In addition, θ must lie between 0 and 1. I denote the set of admissible values for θ by $\Theta(\gamma, \xi)$, where the dependence on the elasticity ratio has been omitted for notational convenience. Note that θ_1 is the sum of the oligopoly parameter and the oligopsony parameter normalized by the elasticity ratio, and thus represents the overall degree of market power of processors, expressed in terms of oligopoly power. That is, *mathematically*, the equilibrium quantity offered by an oligopsonist/oligopolist with market power intensities γ and ξ is equal to the quantity that would be offered by a sole oligopolist with market power intensity $\gamma + \frac{\eta}{\epsilon}\xi$. I will refer to θ_1 as the "oligopoly-equivalent degree of overall market power" or simply the "overall degree of collusion" of processors, not to be confused with γ, the oligopoly power of processors. Note that contrary to γ and ξ, θ_1 need not be smaller than 1.

[6] All derivations are available from the author upon request.

For values of γ and ξ such that $\gamma + \frac{\eta}{\epsilon}\xi \leq 1$, the minimal market power intensity that makes the monopoly scenario profitable is equal to θ_1. Since $\gamma \leq \theta_1 \leq 1$ in this range, this means that a vertically integrated structure will benefit the industry only if it is capable of exerting a degree of seller market power that is strictly larger than the degree of oligopoly power of the separated processing sector. Therefore, in this range, removal of the social inefficiency due to underemployment of the agricultural input by oligopsonistic processors does not benefit producers in aggregate, because the overall degree of collusion of the industry in the oligopsony/oligopoly scenario, measured by θ_1, is less than 1. Thus, underemployment of the unripe cheese input by processors, which directly translates into a higher price for cured cheese, benefits the industry as a whole. As a result, the industry can only benefit from vertical integration if it is allowed to compensate for the loss of this beneficial effect by exerting a higher degree of oligopoly power than that of the separated processing sector. This finding somewhat departs from previous results in the literature on vertical integration, which traditionally links suppression of inefficiencies within the production sector through vertical integration to both higher producer profits and lower consumer prices, the "double marginalization" effect (Greenhut and Ohta, 1976; McGee and Bassett, 1976; Perry, 1978a,b). The intuition behind our result is that oligopsony power by processors can have a positive effect on industry profits by bringing output price closer to the joint profits-maximizing level. This effect is absent when output price is fixed exogenously, or if processors behave as a perfect cartel in the separated scenario (i.e., if their perceived marginal revenue curve coincides with their true marginal revenue curve), as is usually assumed in the literature.

For values of γ and ξ such that $1 \leq \gamma + \frac{\eta}{\epsilon}\xi \leq \frac{1+\frac{\eta}{\epsilon}}{\frac{\eta}{2\epsilon}}$, the minimal market power intensity that makes the monopoly scenario profitable is equal to θ_2. In addition, if $\xi \leq \frac{1+\frac{\eta}{\epsilon}-\gamma(\gamma+\frac{\eta}{\epsilon})}{\frac{\eta}{\epsilon}(\gamma+\frac{\eta}{2\epsilon})}$, $\theta_2 \geq \gamma$, which means that the minimal degree of market power to be exercised by the integrated monopoly must be strictly greater than the oligopoly power exercised by the separated processing sector in the free-market scenario. In this case, even though the oligopoly-equivalent degree of overall market power is greater than 1, the industry benefits from the price-enhancing effect of the oligopsony power in the free-market scenario. Therefore, it needs to be compensated by exerting a degree of seller market power greater than γ in the integrated monopoly scenario. By contrast, if $\xi \geq \frac{1+\frac{\eta}{\epsilon}-\gamma(\gamma+\frac{\eta}{\epsilon})}{\frac{\eta}{\epsilon}(\gamma+\frac{\eta}{2\epsilon})}$, $\theta_2 \leq \gamma$, meaning that the industry benefits from the elimination of the oligopsony inefficiency and can thus profitably exercise a degree of seller market power lower than γ in the integrated monopoly scenario.

For values of the demand and supply elasticities such that $\frac{\eta}{\epsilon} \geq 1$, there exists a set of values of γ and ξ such that even the weakest degree of monopoly power, that is, $\theta = 0$, would enable producers to be better off than under the oligopsony/oligopoly scenario. This situation arises for values of γ and ξ satisfying $\theta_2 \leq 0$, that is, $\gamma + \frac{\eta}{\epsilon}\xi \geq \frac{1+\frac{\eta}{\epsilon}}{\frac{\eta}{2\epsilon}}$.

The above results are summarized in figure 1.

3.3 Impact of the vertically integrated monopoly on consumers and welfare

Results from section 3.2 show that as long as the vertically integrated monopolist is able to exercise a sufficiently high degree of market power over buyers of the final product, upstream producers and downstream processors can, starting from the oligopsony/oligopoly scenario, benefit from vertical integration. In this section, we examine the effect of this integration on consumer surplus and social welfare.

Since $\theta_1 = \gamma + \frac{\eta}{\epsilon}\xi$ represents the overall degree of collusion in the oligopsony/oligopoly scenario, measured in terms of seller market power, consumers benefit from the vertical integration process as long as the resulting monopoly power θ is lower than θ_1. This condition is compatible will increased aggregate producer welfare only if $\gamma + \frac{\eta}{\epsilon}\xi \geq 1$, that is, the oligopoly-equivalent degree of overall market power in the industry is larger than 1, so that aggregate profits can be brought closer to monopoly profits by a decrease in output price. In this case, vertical integration in the production sector has the potential to benefit all classes of agents in the economy: upstream and downstream producers through the decrease in an inefficiently large oligopoly-equivalent degree of overall market power, and consumers through lower price and thus greater quantity. Consequently, when $\gamma + \frac{\eta}{\epsilon}\xi \geq 1$ the integrated monopoly constitutes a Pareto improvement compared to the free-market situation. Note that as long as $1 \leq \gamma + \frac{\eta}{\epsilon}\xi \leq \frac{1+\frac{\eta}{\epsilon}}{\frac{\eta}{2\epsilon}}$, the integrated industry needs to exert some positive degree of market power for the participation constraint to be satisfied.

In contrast, when $\gamma + \frac{\eta}{\epsilon}\xi < 1$, vertical integration cannot benefit producers and consumers at the same time, since producer profitability in this case requires a higher overall degree of seller market power and thus a higher output price than in the oligopsony/oligopoly scenario. The effect of vertical integration on social welfare exactly mirrors that on consumer welfare, because the social inefficiency in each scenario is measured by the oligopoly-equivalent degree of overall market power being exerted at the industry level. Therefore, when $\gamma + \frac{\eta}{\epsilon}\xi < 1$ and the industry as a whole benefits from vertical integration, consumer surplus necessarily decreases, and so does social welfare.

In all instances, the first-best solution in terms of social welfare would be to vertically integrate the industry and impose a market power intensity of zero. However, Pareto improvements in this case may require transfers from consumers (who would always win) to producers (who would lose as soon as

$\gamma+\frac{\eta}{\epsilon}\xi < \frac{1+\frac{\eta}{\epsilon}}{\frac{\eta}{2\epsilon}}$). Unlike within-industry transfers through the contractual price of unripe cheese, transfers between consumers and producers would be difficult to implement in this case, which may explain why the government allows exertion of a certain degree of market power by the integrated industry.

3.4 Social inefficiency of the action of CIGC

In this section, I focus on the case where the integrated monopoly acts in the interest of producers and processors, but harms consumers by reducing quantity beyond the oligopsony/oligopoly level, that is, I assume that $\gamma + \frac{\eta}{\epsilon}\xi < 1$. Since I do not have empirical estimates of the demand or supply elasticities and can only make inferences about the severity of oligopsony and oligopoly power that would emerge in the absence of the vertically integrated industry structure, there is no particular reason to favor this case relative to the alternative $\gamma + \frac{\eta}{\epsilon}\xi \geq 1$. However, it is useful to evaluate the social cost of the current integration policy for a set of plausible values of the parameters satisfying $\gamma + \frac{\eta}{\epsilon}\xi < 1$. To accomplish this objective, I compare the deadweight loss arising from the exertion of market power by the integrated industry (denoted DWL^i) to that arising from the oligopsony and oligopoly power of unconstrained processors (denoted DWL^o). This comparison illustrates the error that an analyst would make if he mistakenly assumed that downstream processors behave competitively in the no-intervention scenario.

Table 2 compares the deadweight loss measures for a set of plausible values of parameters η, ϵ, θ, γ and ξ. I could not find empirical estimates of the Marshallian demand and supply elasticities for Comté cheese, and thus allow the ratio $\frac{\eta}{\epsilon}$ to take on the values 0.5, 1 and 2.[7] I do not have access to an

[7] There exist close substitutes to Comté cheese, such as Emmental cheese, both in consumption and production, so that I would expect both the demand and supply of Comté cheese to be relatively elastic, thus justifying the choice to use values of $\frac{\eta}{\epsilon}$ that do not depart too far from 1.

estimate of the market power being exercised by the industry in the real-world situation. Therefore, I allow the market power parameter θ to take on a wide range of values, but impose the condition that the industry must be at least as well-off as in the free-market scenario. Regarding the value of the oligopsony and oligopoly parameters, I impose a very weak degree of oligopoly power ($\gamma = 0.01$) and a moderate degree of oligopsony power ($\xi = 0.2$). The concentration ratio of the 4 largest firms in the curing industry is quite large (CR4 = 0.61 in 2006), as is the market share of the largest firm (30% in 2006).[8] Imposing $\xi = 0.2$ is equivalent to assuming a symmetric Cournot game with only 5 players. Even if the 5 largest firms in the industry do not account for the totality of Comté cheese production (they accounted for approximately 70% of total production in 2006), setting $\xi = 0.2$ does not seem unreasonable given the large heterogeneity among those firms. Notably, the largest firm in the industry produces three times the output of the second largest firm.

The difference $DWL^i - DWL^o$, indicated in table 2, measures the real social cost of moving from the no-intervention scenario to the current industry structure. The difference between DWL^i and $DWL^i - DWL^o$ measures the error made by an analyst who would erroneously assume competitive behavior for processing firms in the no-intervention scenario. The ratio $\frac{DWL^i - DWL^o}{DWL^i}$, reported in the last column of table 2, can be interpreted as the share of the deadweight loss falsely measured relative to the competitive case that reflects the actual cost of moving from the oligopsony/oligopoly situation to the integrated industry. Depending on the parameter settings, the error may be quite large. For instance, when $\frac{\eta}{\epsilon} = 1$, $\gamma = 0.01$ and $\xi = 0.2$, the real deadweight loss from a vertically integrated entity with market power $\theta = 0.3$ is less than 50% of that calculated relative to the perfectly competitive case. If we increase $\frac{\eta}{\epsilon}$ up

[8] Those numbers are based on the CIGC classification. According to this classification, only firms handling more than 400 metric tons are considered to be curing facilities.

to 2 and consider a market power intensity of 0.5, this figure falls to 30%. The relative magnitude of the error increases with the value of $\gamma + \frac{\eta}{\epsilon}\xi$, which reflects the severity of the oligopoly-equivalent degree of overall market power exerted by the processing sector in the free-market scenario.

4 Conclusion

In this paper, I developed an argument in favor of the current supply control mechanism in the Comté cheese sector, based on the presence of imperfect competition at the processing stage of the industry, a feature that may be shared by other PDOs or agricultural commodity markets. I supported the view that the current industry organization should not only be considered as a regulated cartel, but also as a vertically integrated entity between cheese makers (upstream producers) and curing facilities (downstream processors), the latter of which would otherwise behave as oligopsonists in the procurement of the upstream input, and may also exert some oligopoly power in the sale of the output.

Confirmation of the hypothesis that the observed policy is justified from a social welfare perspective would require in particular the empirical measurement of the demand and marginal cost elasticities. This is the focus of ongoing research by the author.

References

Alston, J. M., Sexton, R. J., and Zhang, M. (1997). The Effects of Imperfect Competition on the Size and Distribution of Research Benefits. *American Journal of Agricultural Economics*, 79(4):1252–1265.

Comité interprofessionnel du Gruyère de Comté (2005). Le C.I.G.C. Presentation leaflet, France.

Conseil de la Concurrence (1998). Décision No 98-D-54 du 8 juillet 1998 relative à des pratiques relevées dans le secteur de la production du gruyère de Comté. *Bulletin Officiel de la Concurrence, de la Consommation et de la Répression des Fraudes*, 2(Janvier 1999).

Greenhut, M. L. and Ohta, H. (1976). Related Market Conditions and Interindustrial Mergers. *The American Economic Review*, 66(3):267–277.

Huang, S.-Y. and Sexton, R. J. (1996). Returns to an Innovation in an Imperfectly Competitive Market: Application to Mechanical Harvesting of Processing Tomatoes in Taiwan. *American Journal of Agricultural Economics*, 78(3):558–571.

McGee, J. S. and Bassett, L. R. (1976). Vertical Integration Revisited. *Journal of Law and Economics*, 19(1):17–38.

Perry, M. K. (1978a). Related Market Conditions and Interindustrial Mergers: Comment. *The American Economic Review*, 68(1):221–224.

Perry, M. K. (1978b). Vertical Integration: The Monopsony Case. *The American Economic Review*, 68(4):561–570.

République française (1999). Décret du 30 décembre 1998 relatif à l'appellation d'origine contrôlée "Comté". *Journal Officiel de la République Française*, 3(Janvier 1999):201–203.

Zhang, M. and Sexton, R. J. (2002). Optimal Commodity Promotion when Downstream Markets Are Imperfectly Competitive. *American Journal of Agricultural Economics*, 84(2):352–365.

Tables

Size	Number of firms	Production (tons)	% of total production
fruitières	47	194	0.4
5-99 tons	55	1389	2.8
100-999 tons	21	8312	16.6
1000-1999 tons	1	1492	3
2000-2999 tons	4	10438	20.8
3000-3999 tons	1	3096	6.2
$>$ 4000 tons	4	25212	50.3
total	133	50133	100

Table 1: Distribution of curing facilities according to size (2005). A small number of cheese factories also age part of their production for their own retail activity. According to the CIGC classification, only facilities handling more than 400 tons of cheese are considered curing facilitites. Source: CIGC.

$\frac{\eta}{\epsilon}$	γ	ξ	θ_1	θ	DWL^i	DWL^o	$DWL^i - DWL^o$	$\frac{DWL^i - DWL^o}{DWL^i}$
1	0.01	0.2	0.21	0.7	0.067	0.009	0.058	0.866
1	0.01	0.2	0.21	0.5	0.040	0.009	0.031	0.774
1	0.01	0.2	0.21	0.3	0.017	0.009	0.008	0.469
2	0.01	0.2	0.41	0.7	0.054	0.022	0.032	0.596
2	0.01	0.2	0.41	0.5	0.031	0.022	0.009	0.292
0.5	0.01	0.2	0.11	0.7	0.076	0.004	0.072	0.954
0.5	0.01	0.2	0.11	0.5	0.047	0.004	0.043	0.925
0.5	0.01	0.2	0.11	0.3	0.021	0.004	0.017	0.832
0.5	0.01	0.2	0.11	0.2	0.010	0.004	0.007	0.663

Table 2: Measures of deadweight loss from monopoly power. The demand elasticity η is set to 1 in all calculations. All measures are expressed relative to the value of output p^cQ^c.

Figures

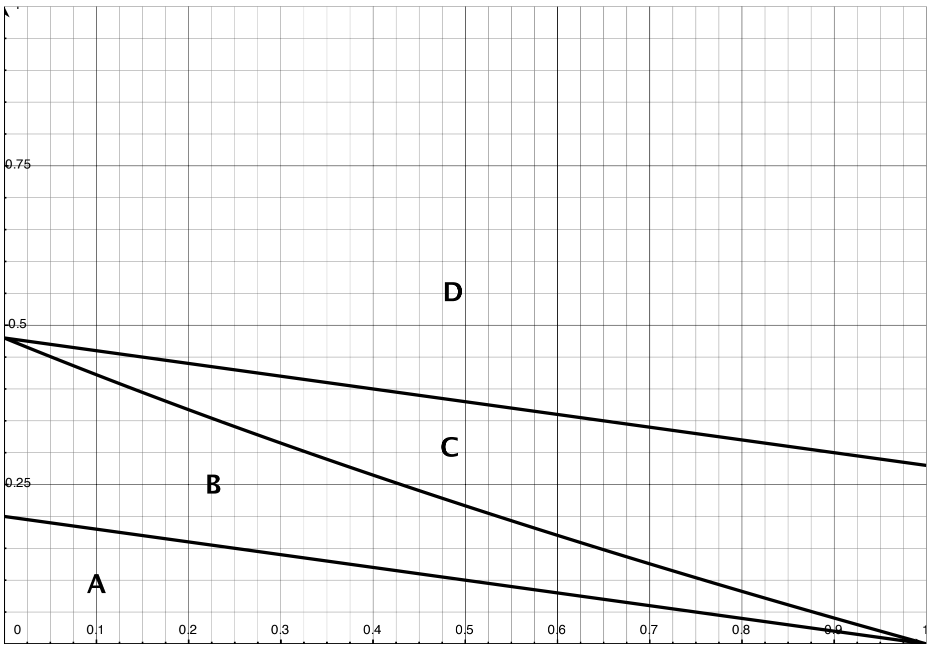

Figure 1: Admissible values of θ for $\frac{\eta}{\epsilon} = 5$. The x-axis represents $\gamma \in [0, 1]$ and the y-axis $\xi \in [0, 1]$. In region A, $\gamma + \frac{\eta}{\epsilon}\xi \leq 1$ and $\Theta(\gamma, \xi) = [\theta_1, 1]$. In region B and C, $1 \leq \gamma + \frac{\eta}{\epsilon}\xi \leq \frac{1+\frac{\eta}{\epsilon}}{\frac{\eta}{2\epsilon}}$ and $\Theta(\gamma, \xi) = [\theta_2, 1]$, with $\theta_2 \geq \gamma$ in region B and $\theta_2 \leq \gamma$ in region C. In region D, $\gamma + \frac{\eta}{\epsilon}\xi \geq \frac{1+\frac{\eta}{\epsilon}}{\frac{\eta}{2\epsilon}}$ and $\Theta(\gamma, \xi) = [0, 1]$.