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Primal-Dual Estimation
of a Linear Expenditure Demand System

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Abstract

Efficient estimates require the utilization of all the available theoretical and statistical information. This fact suggests that econometric models based on an explicit optimization theory might achieve more efficient estimates when all the primal and dual relations are used for a joint estimation of the model's parameters. We present a discussion of this idea using a Linear Expenditure System (LES) of consumer demand. We assume that the risk-neutral household chooses its consumption plan on the basis of expected information. Some time after that decision, the econometrician attempts to measure quantities and prices and in so doing commits measurement errors. Hence, the econometric model is an errors-in-variables nonlinear system of equations for which there is no known consistent estimator. We propose an easy-to-implement estimator and analyze its empirical properties by a Monte Carlo simulation that shows a relatively small bias.

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Executive Summary

This paper deals with efficient estimates of the parameters in consumer demand systems. It is well known that such estimates are obtained if and only if one makes use of all the available theoretical and statistical information.

We assume that consumer households are risk-neutral and maximize their expected utility subject to an expected budget constraint. The idea of dealing with expected information is not new and it allows the drawing of a clear distinction between the specification of the theoretical model and the specification of the statistical structure of the error terms in the intervening econometric model. The process of expectation formation belongs to the individual consumer and needs not be further specified, given the risk-neutrality assumption.

We view the econometric process of estimating systems of demand functions as a two-step approach. First, the consumer chooses his consumption plan according to the theoretical model specified above. Second, the econometrician intervenes some time later to collect information about the choices made by a sample of consumers and, in so doing, he introduces measurement errors on all the sample information involved, namely quantities and prices of real goods and disposable income.

The theoretical development, therefore, must be coupled with the specification of the error structure associated with the observed quantities of real goods, their observed prices and disposable income to form the econometric model of interest. The traditional approach to demand analysis is to assume that only the observed quantities of real goods are measured with error. In contrast, we allow for all the sample quantities, prices and disposable income to be measured with error. This is not an assumption but rather an empirical fact of data collection. This primal-dual approach was originally suggested by Paris (2003d).

The specification of the primal-dual approach requires the utilization of all the primal and dual relations of consumer theory in order to provide estimable equations for all the error terms involved on both the observed quantities and prices of the real goods. This process of data generation and collection constitutes a novel primal-dual specification of the econometric model for consumer demand. This proposition is the main contribution of the paper. It departs in a radical way from the traditional approach that uses only the dual side of consumer theory.

The second contribution of the paper is a suggestion for an efficient estimating procedure that implements the primal-dual specification articulated above. Presumably, there may exist alternative estimation approaches (Bayesian, for example) and the procedure presented here is only one of them.

The estimation approach is divided into two phases. The objective of Phase I is the estimation of all the expected quantities and prices for each sample unit. These estimates are then used in Phase II for obtaining efficient estimates of the model parameters. The Phase II specification of the consumer demand model takes on the form of a nonlinear seemingly unrelated (NSUR) equation system. Hence, it can be estimated by a maximum likelihood approach (alternatively, an iterated feasible generalized least-squares procedure).

The model selected for illustrating the primal-dual approach is the Linear Expenditure System (LES), with two sets of parameters: The first set of nonnegative

parameters, say the beta parameters, constitute the structural parameters of the Cobb-Douglas utility function that characterizes the LES model. Identification of these parameters requires that they add up to unity. The second set of parameters, say the gamma parameters, one for each good, are interpreted as the "subsistence" quantity of the corresponding good. The restriction on these parameters is that be nonnegative and have a value that is inferior to the minimum sample quantity of the corresponding good.

An interesting and novel by-product of the primal-dual approach is that the system of demand functions must be specified in terms of the quantities of real goods. In other words, an expenditure share specification is not admitted because, now that all the observed quantities and prices are measured with error, the complexity of the corresponding error term is unyielding and unnecessary. This specification of the primal-dual demand system implies that it is no longer necessary to drop an equation because the variance/covariance matrix of the residuals is, in general, not singular.

The sample data utilized for the empirical implementation of the LES model deal with 119 Italian households that made their consumption choice over four goods: bread and cereals, meat, beverages, and other food. The estimates of the primal-dual model indicate a considerable gain in efficiency when compared with the estimates of the traditional dual model.

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Introduction

A well-known proposition asserts that efficient estimates of an econometric model are obtained if and only if one makes use of all the available theoretical and statistical information. Keeping this proposition in mind, we investigate the task of estimating a Linear Expenditure System (LES) of consumer demand (Klein and Rubin, 1947-48, Stone, 1954) using the traditional approach and a novel approach based upon the joint estimation of all the primal and dual relations of the standard consumer theory. This primal-dual approach was originally suggested by Paris (2003a, 2003b, 2003c, 2003d), who also developed an analogous rationale and methodology for estimating factor demand and output supply functions. We will also present the extension of the primal-dual approach to an Almost Ideal Demand System (AIDS).

We assume that consumer households are risk-neutral and maximize their expected utility subject to an expected budget constraint. The idea of dealing with expected information is not new and it allows the drawing of a clear distinction between the specification of the theoretical model and the specification of the statistical structure of the error terms in the intervening econometric model. The process of expectation formation belongs to the individual consumer and needs not be further specified, given the risk-neutrality assumption.

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We view the econometric process of estimating systems of demand functions as a two-step approach. First, the consumer chooses his consumption plan according to the theoretical model specified above in terms of expected information. Second, the econometrician intervenes some time later to collect information about the choices made by a sample of consumers and, in so doing, he introduces measurement errors on all the sample information involved, namely quantities and prices of real goods and disposable income.

The theoretical development, therefore, must be coupled with the specification of the error structure associated with the observed quantities of real goods, their observed prices and disposable income to form the econometric model of interest. The specification of the primal-dual approach requires the utilization of all the primal and dual relations of consumer theory in order to provide estimable equations for all the error terms involved on both the observed quantities and prices of the real goods. This process of data generation and collection constitutes a primal-dual specification of the econometric model for consumer demand. This proposition is the main contribution of the paper. It departs in a radical way from the traditional approach that uses only the dual side of consumer theory.

The second contribution of the paper is a suggestion for an estimating procedure that implements the primal-dual specification articulated above. The resulting econometric specification is a nonlinear errors-in-variables system of equations. The nonlinear error-in-variables problem has received a great deal of attention in recent years (Amemiya,1985; Amemiya and Fuller, 1988; Hsiao, 1989; McFadden, 1989; Hsiao and Wang, 2000; Gorrieroux and Monfort, 1993; McFadden and Ruud, 1994; Carroll,

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Ruppert and Stefanski, 1995; Kukush and Zwanzig, 2002; Kukush, Markovsky and van Huffel, 2002). In spite of all these interesting efforts, a general and easy-to-implement consistent estimator for the general errors-in-variables problem has yet to be discovered. In particular, no discussion of a nonlinear system when all the variables are measured with error seems to have appeared in the literature.

Given this background, we propose the following approach to estimation of the primal-dual model of consumer theory under measurement error. We suggest an easy-to-estimate two-phase procedure that, in phase II, may be regarded as a (NSUR) nonlinear seemingly unrelated equation model. The statistical properties of this estimator are presently unknown. For this reason, we analyze the estimator by means of a Monte Carlo (MC) experiment that mimics very closely the empirical model at hand and shows that the bias is very small.

The estimation approach is divided into two phases. The objective of Phase I is the estimation of all the expected quantities and prices for each sample unit. These estimates are then used in Phase II for obtaining efficient estimates of the model parameters. The Phase II specification of the consumer demand model takes on the form of a nonlinear seemingly unrelated (NSUR) equation system. Hence, it can be estimated by an iterated feasible generalized least-squares procedure.

The model selected for illustrating the primal-dual approach is the Linear Expenditure System, with two sets of parameters: The first set of nonnegative parameters, say the beta parameters, constitute the structural parameters of the Cobb-Douglas utility function that characterizes the LES model. Identification of these parameters requires that they add up to unity. The second set of parameters, say the gamma parameters, one for

each good, are interpreted as the "subsistence" quantity of the corresponding good. The restriction on these parameters is that be nonnegative and have a value that is inferior to the minimum sample quantity of the corresponding good. The gamma parameters are important determinants of the price elasticities of demand.

An interesting by-product of the primal-dual approach is that the system of demand functions must be specified in terms of the quantities of real goods. In other words, an expenditure share specification is not admitted because, now that all the observed quantities and prices are measured with error, it takes the full primal-dual system of relations to estimates the moments of the error distribution. This specification of the primal-dual demand system implies that it is no longer necessary to drop an equation because the variance/covariance matrix of the residuals is, in general, not singular.

The sample data utilized for the empirical implementation of the LES model deal with 119 Italian households that made their consumption choice over four goods: bread and cereals, meat, beverages, and other food. The estimates of the primal-dual model indicate a considerable gain in efficiency when compared with the estimates of the traditional dual model.

The Theory

We postulate risk-neutral consumers who maximize their expected utility subject to an expected budget constraint. The idea of dealing with expected information is not new. The process of expectation formation belongs to the individual consumer and needs not be further specified, given the risk-neutrality assumption. Hence, let $\mathbf{p}^e$ be the $(J \times 1)$

vector of expected prices and y^e the expected disposable income available to the risk-neutral household. We assume that the consumer solves the following problem

(1)
$$U^*(\mathbf{p}^e, y^e) \overset{\text{def}}{=} \max_{\mathbf{x}}\{U^e(\mathbf{x}) \text{ s.t. } y^e = \mathbf{p}'^e\mathbf{x}\}$$

where $\mathbf{x}$ is a $(J \times 1)$ vector of commodity quantities. The first-order-necessary conditions corresponding to problem (1) are

(2)
$$\frac{\partial L}{\partial \mathbf{x}} = U^e_{\mathbf{x}}(\mathbf{x}) - \lambda\mathbf{p}^e = \mathbf{0}$$
$$\frac{\partial L}{\partial \lambda} = y^e - \mathbf{p}'^e\mathbf{x} = 0$$

where L is the Lagrangean function and λ is the Lagrange multiplier associated with the budget constraint. Equations (2) constitute the primal relations. Solution of equations (2) (if it exists) transforms the vector of commodity quantities $\mathbf{x}$ into a vector of expected quantities $\mathbf{x}^e$. Similarly, the Lagrange multiplier λ becomes the expected Lagrange multiplier λ^e taking on the meaning of marginal utility of money income, that is $U^*_{y^e}(\mathbf{p}^e, y^e) = \lambda^e$, where $U^*(\cdot)$ is the indirect utility function. We assume an interior solution of equations (2) that will generate commodity demand functions with values

(3)
$$\mathbf{x}^e = \mathbf{d}^e(\mathbf{p}^e, y^e).$$

Equations (3) constitute the dual relations. In the case of a flexible specification of the demand functions (3), such as in the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), the first-order necessary conditions (2) exist only in a latent form. This suggests that the first order necessary conditions convey independent information that is not fully utilized in the conventional estimation of the AIDS demand system.

The Error Structure

The theoretical development of the previous section must be coupled with the specification of the error structure associated with the observed quantities of real goods, their observed prices and disposable income to form the econometric model of interest. The traditional approach to demand analysis is to assume that only the observed quantities of real goods are measured with error. In contrast, we allow for all the sample quantities, prices and disposable income to be measured with error. This is not an assumption but rather an empirical fact of data collection. In other words, the econometrician observes $\mathbf{x}$, $\mathbf{p}$ and y that bear an additive relation with their expected counterparts, that is $\mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon}$, $\mathbf{p} = \mathbf{p}^e + \boldsymbol{\nu}$ and $y = y^e + v_0$. The vector of errors $\mathbf{e}' = (\boldsymbol{\varepsilon}', \boldsymbol{\nu}', v_0)$ is distributed as $\mathbf{e} \sim N(\mathbf{0}, \Sigma)$. We further assume that the errors are independently distributed across sample units.

The combination of the statistical model specified above and the primal and dual relations of the previous section can be summarized in the following econometric structure:

The statistical model

$$(4) \qquad \mathbf{p} = \mathbf{p}^e + \boldsymbol{\nu}$$

$$(5) \qquad y = y^e + v_0$$

$$(6) \qquad \mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon}$$

subject to the theoretical constraints

$$(7) \qquad \mathbf{p}^e = U_{\mathbf{x}^e}^e(\mathbf{x}^e, \boldsymbol{\beta}_{U^e}) / U_{y^e}^*(\mathbf{p}^e, y^e, \boldsymbol{\beta}_{U^*})$$

$$(8) \qquad y^e = \mathbf{p}'^e \mathbf{x}^e$$

$$(9) \qquad \mathbf{x}^e = \mathbf{d}^e(\mathbf{p}^e, y^e, \boldsymbol{\beta}_{\mathbf{x}^e}).$$

Suppose that both the observed and the expected quantities and prices were known. Then, the problem of estimating the parameters $\boldsymbol{\beta}_{U^e}, \boldsymbol{\beta}_{U^*}$ and $\boldsymbol{\beta}_{\mathbf{x}^e}$ would be a straightforward application of a nonlinear seemingly unrelated (NSUR) equation procedure to the following model

$$\mathbf{p} = U^e_{\mathbf{x}^e}(\mathbf{x}^e, \boldsymbol{\beta}_{U^e}) / U^*_{y^e}(\mathbf{p}^e, y^e, \boldsymbol{\beta}_{U^*}) + \boldsymbol{\nu}$$

$$y = \mathbf{p}'^e \mathbf{x}^e + \nu_0$$

$$\mathbf{x} = \mathbf{d}^e(\mathbf{p}^e, y^e, \boldsymbol{\beta}_{\mathbf{x}^e}) + \boldsymbol{\varepsilon}.$$

Unfortunately, expected quantities and prices are rarely, if ever, recorded and it is necessary to obtain an estimate of them before achieving the final objective. The estimation problem, then, consists in the estimation of all the expected quantities, expected prices and expected disposable income together with the parameters $\boldsymbol{\beta}_{U^e}, \boldsymbol{\beta}_{U^*}$ and $\boldsymbol{\beta}_{\mathbf{x}^e}$ that specify the utility and the demand functions.

The Econometric Model

We assume a sample of N household with index $n = 1, \ldots, N$. There are probably alternative approaches suitable for estimating the specification presented in the previous section. The estimation procedure proposed here is articulated in two-phases. The Phase I objective consists in obtaining estimates of all the individual households' expected quantities and prices. These estimates, then, will be used during Phase II to obtain the final estimates of the model's parameters.

The Phase I specification minimizes the least-squares objective function. Specifically, let $\boldsymbol{\beta} = (\boldsymbol{\beta}_{U^e}, \boldsymbol{\beta}_{U^*}, \boldsymbol{\beta}_{x^e})$ be the vector of parameters that constitute the final goal of the estimation process. Then, the Phase I estimation problem is stated as follows:

$$(10) \qquad \min_{\boldsymbol{\beta}, p^e_{jn}, x^e_{jn}, y^e_n, e_n} \sum_{n=1}^{N} \mathbf{e}'_n \mathbf{e}_n$$

subject to

$$(11) \qquad \mathbf{p}_n = \mathbf{p}^e_n + \mathbf{v}_n$$

$$(12) \qquad y_n = y^e_n + v_{0n}$$

$$(13) \qquad \mathbf{x}_n = \mathbf{x}^e_n + \boldsymbol{\varepsilon}_n$$

and

$$(14) \qquad \mathbf{p}^e_n = U^e_{\mathbf{x}^e,n}(\mathbf{x}^e_n, \boldsymbol{\beta}_{U^e}) / U^*_{y^e,n}(\mathbf{p}^e_n, y^e_n, \boldsymbol{\beta}_{U^*})$$

$$(15) \qquad y^e_n = \mathbf{p}'^e_n \mathbf{x}^e_n$$

$$(16) \qquad \mathbf{x}^e_n = \mathbf{d}^e_n(\mathbf{p}^e_n, y^e_n, \boldsymbol{\beta}_{x^e}).$$

The estimates of expected quantities and prices $\hat{\mathbf{p}}^e_n, \hat{y}^e_n, \hat{\mathbf{x}}^e_n$ will be used in the Phase II estimation problem. The objective function of Phase II is a weighted least-squares specification. Let $\hat{\Sigma}^{-1}_{)y(}$ be the estimated covariance matrix computed from the estimated residuals of the first order necessary conditions and of the demand functions of Phase I, where the error associated with the budget constraint is omitted. Thus, the error vector is $\mathbf{e} = (\mathbf{v}, \boldsymbol{\varepsilon})$. The error associated with the budget constraint has been omitted because in Phase II the expected expenditure is known and there is no error term to minimize. Then, the Phase II estimation problem consists in the application of the NSUR procedure to the following model:

(17)
$$\min_{\boldsymbol{\beta}, \mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}'_n \hat{\Sigma}_{)y(}^{-1} \mathbf{e}_n / NJ$$

subject to

(18)
$$\mathbf{p}_n = U^e_{\mathbf{x}^e,n}(\hat{\mathbf{x}}^e_n, \boldsymbol{\beta}_{U^e}) / U^*_{y^e,n}(\hat{\mathbf{p}}^e_n, \hat{y}^e_n, \boldsymbol{\beta}_{U^*}) + \mathbf{v}_n$$

(19)
$$\mathbf{x}_n = \mathbf{d}^e_n(\hat{\mathbf{p}}^e_n, \hat{y}^e_n, \boldsymbol{\beta}_{\mathbf{x}}) + \boldsymbol{\varepsilon}_n$$

where N is the number of observations and J is the number of equations. Iteration to convergence of the covariance matrix $\hat{\Sigma}_{)y(}^{-1}$ will produce the final estimates of the parameter vectors $\boldsymbol{\beta}_{U^e}, \boldsymbol{\beta}_{U^*}$ and $\boldsymbol{\beta}_{\mathbf{x}}$. The model's specification presented above assumes that the errors in different equations are equally weighted. A preliminary Monte Carlo analysis suggests that different weighting schemes have a negligible impact upon the parameter estimates.

We point out that an important stopping rule of the iteration process is provided by the convergence of the objective function to the number 1 since $tr\mathbf{U}'\Sigma^{-1}\mathbf{U} = tr\mathbf{U}\mathbf{U}'\Sigma^{-1} = NJ$, where the operator tr is the trace and $\mathbf{U} = [\mathbf{e}_1, ..., \mathbf{e}_J]$. That is, the optimal convergence point of the objective function is known a priori. We have found empirically that convergence to the number 1 is faster than the convergence to the number NJ, which is the reason for the form of the objective function specified in equation (17).

The Linear Expenditure System

In the Linear Expenditure System (LES) originally suggested by Klein and Rubin (1947-48) and estimated by Stone (1954), the direct utility takes on the following Cobb-Douglas specification:

$$(20) \qquad U^e(\mathbf{x}) = \sum_{j=1}^{J} \beta_j \log(x_j - \gamma_j)$$

where $x_j > \gamma_j \geq 0$ and the parameter γ_j represents a subsistence level of the corresponding good and where the nonnegative parameters β_j must sum up to unity, $\Sigma_j \beta_j = 1$, for identification purposes.

The Phase II econometric model takes on the following specification:

$$(21) \qquad \min_{\boldsymbol{\beta}, \mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}'_n \hat{\Sigma}^{-1}_{)y(} \mathbf{e}_n / NJ$$

subject to

Primal relations

$$
\begin{aligned}
(22) \qquad p_{jn} &= U^e_{\mathbf{x}^e_j,n}(\hat{\mathbf{x}}^e_{jn}, \boldsymbol{\beta}_{U^e}) / U^*_{y^e,n}(\hat{\mathbf{p}}^e_{jn}, \hat{y}^e_n, \boldsymbol{\beta}_{U^*}) + v_{jn} \\
&= \beta_j \frac{(\hat{y}^e_n - \Sigma_{k=1}^{J} \hat{p}^e_{kn} \gamma_k)}{(\hat{x}^e_{jn} - \gamma_j)} + v_{jn}
\end{aligned}
$$

Dual relations

$$
\begin{aligned}
(23) \qquad x_{jn} &= d^e_{jn}(\hat{\mathbf{p}}^e_{jn}, \hat{y}^e_n, \boldsymbol{\beta}_{x_j}) + \varepsilon_{jn} \\
&= \gamma_j + \beta_j \frac{(\hat{y}^e_n - \Sigma_{k=1}^{J} \hat{p}^e_{kn} \gamma_k)}{\hat{p}^e_{jn}} + \varepsilon_{jn}
\end{aligned}
$$

The LES demand functions are homogeneous of degree zero in prices and income and satisfy the Slutsky equation if all the estimated parameters $\tilde{\beta}_j$ and $\tilde{\gamma}_j$ are positive and $\Sigma_j \tilde{\beta}_j = 1$, as required by the above specification. Hence, no hypothesis testing is possible for these important properties of demand functions.

The LES model of consumer behavior admits an explicit solution of the first-order-necessary conditions. This means that, during the Phase I estimation, either the primal or the dual relations are redundant. We re-emphasize, however, that, during the

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Phase II, relations (22) and (23) convey independent information through different errors and their distributions. They are both required, therefore, for a complete specification of the econometric model, if one wishes to obtain more efficient estimates of the model's parameters. In the case of a flexible functional form specification such as the AIDS model, for example, neither the primal nor the dual relations are redundant even during Phase I.

The Traditional LES Model and Its Estimation

In contrast to the primal-dual specification discussed above, the traditional estimation of the LES model assumes that prices and disposable income are measured without error. Hence, the problem reduces to the estimation of the (dual) system of demand functions (23), with a considerable loss of efficiency, as will be illustrated in the empirical section.

Traditionally, consumer demand systems have been estimated in expenditure share form. The reason for selecting this specification is probably based upon the empirical fact that household surveys often record only the expenditure on the various categories of goods and not the corresponding quantities and prices, separately. The primal-dual approach presented above, however, does not admit such a specification of the demand system because the error terms of the demand equations would become unnecessarily complex and the individual error terms could not be estimated. Furthermore and contrary to the share specification, the primal-dual approach does not require dropping any equation because the corresponding variance/covariance of the disturbance terms is not singular. The justification of this proposition goes as follows. Demand equations (23) have the structure of a nonlinear seemingly unrelated (NSUR)

equation specification. Estimates of this model, obtained by an iterated Feasible Generalized Least Squares procedure are stated as $\tilde{\varepsilon}_{nj}, \tilde{\beta}_j, \tilde{\gamma}_j$ and $\Sigma_j \tilde{\beta}_j = 1$. Then, it must be that:

$$p_{nj}x_{ij} = p_{nj}\tilde{\gamma}_j + \tilde{\beta}_j(y_n - \Sigma_k p_{nk}\tilde{\gamma}_k) + p_{nj}\tilde{\varepsilon}_{nj}$$

$$\Sigma_j p_{nj}x_{nj} = \Sigma_j p_{nj}\tilde{\gamma}_j + \Sigma_j \tilde{\beta}_j(y_n - \Sigma_k p_{nk}\tilde{\gamma}_k) + \Sigma_j p_{nj}\tilde{\varepsilon}_{nj}$$

$$y_n = y_n + \Sigma_j p_{nj}\tilde{\varepsilon}_{nj}$$

$$0 = \Sigma_j p_{nj}\tilde{\varepsilon}_{nj}$$

QED. That is, the restriction $\Sigma_j p_{nj}\varepsilon_{nj} = 0$, which in a model stated in terms of good expenditures (the dependent variable is $p_{nj}x_{nj}$) corresponds to the "restriction" $\Sigma_j p_{nj}x_{nj} = y_n$ (met by all sample observations), is naturally satisfied in model (23) and the variance/covariance matrix $\tilde{\Sigma} = \left[\sum_n \tilde{\varepsilon}_{nj}\tilde{\varepsilon}_{nk} / N \right]$ is, in general, not singular.

In summary, when distinct quantities and prices of all commodities are available for every sample unit, it is more appropriate to estimate the demand system in quantity form than in either expenditure or share form. Because the LES model is nonlinear in the parameters, a change of scale may make a good deal of difference both upon the estimates and their standard errors.

Empirical Results

A cross-section sample of 119 consumers who made their choices over bundles of four real goods (bread and cereals, meat, beverages, other foods) was utilized for the estimation of the above LES model. Information on quantities and expenditures on the four goods was available for each sample unit. Hence, it was possible to compute the corresponding commodity prices (unit values) for each consumer. The estimation of the

Phase II was performed using the SHAZAM 9 econometric package and the iterated NSUR procedure.

Table 1 presents the results of the Phase II estimation of the primal-dual model along with the estimates of the LES model obtained using the traditional approach. We adopted the out-of-sample prediction procedure suggested by Fuller (1980) for nonlinear models and estimated the prediction of the last observation, number 119. The values of the estimated beta parameters in the two models are within a rather narrow neighborhood of each other. However, the values of the gamma parameters are quite different: all the values of the gamma parameters of the traditional LES model violate the theoretical restriction that $\min_i(x_{ij}) > \gamma_j \geq 0$, while all the values of the gamma parameters in the primal-dual model satisfy those conditions. The values of the t-ratios of the primal-dual model are consistently and considerably higher than the corresponding t-ratios of the LES model. Hence, the results of Table 1 support the proposition that the estimation of the LES consumer model produces more efficient estimates when using the primal-dual approach.

The efficiency gain documented by the results of Table 1 is measured by the increase in the value of t-ratios to which there corresponds a smaller value of the variance of the estimates.

In both cases the variance/covariance matrix is well conditioned as indicated by the condition number associated with each of the models. Belsley *et al.* have suggested that collinearities may begin to arise when the condition number is near or above 30.

Table 1. Iterated NSUR Estimates of the LES model

(t-ratios in parenthesis)

Parameters	Traditional LES model	Primal-Dual LES model
Bread & cereals β_1	0.04500 (4.495)	0.07516 (52.57)
Meat β_2	0.18879 (11.77)	0.15372 (414.44)
Beverages β_3	0.20320 (12.12)	0.20301 (346.52)
Other foods β_4	0.56300 (21.32)	0.56812 (368.69)
Bread & cereals γ_1	0.81685 (21.739)	0.17288 (19.435)
Meat γ_2	0.38904 (10.82)	0.15485 (256.29)
Beverages γ_3	1.3493 (7.351)	0.05220 (6.598)
Other foods γ_4	2.1027 (10.579)	0.03015 (2.728)
Log-Likelihood	-137.9925	1121.573
Condition Number of Σ	8.706	20.587

Table 2 presents the estimates of the out-of-sample prediction for all the dependent variables in the respective models. As stated above, we chose to predict the values of observation N. 119. Overall, the predictions of the primal-dual model appear to be closer to the actual values than those of the traditional model.

Table 2. Out-of-Sample Prediction of Observation 119 in the LES model

(t-ratios in parenthesis)

Dependent Variables	Observation 119 Actual Values	Traditional LES model	Primal-Dual LES model
Bread & cereals	1.15	0.91965 (2.7405)	0.9819 (3.87)
Meat	1.20	0.52380 (3.548)	0.5286 (3.10)
Beverages	2.50	2.0376 (2.585)	2.5123 (37.27)
Other foods	1.10	3.0158 (5.612)	1.6847 (13.54)
Price of Bread & cereals	0.6	------	0.3737 (1.656)
Price of Meat	1.9	-------	1.7783 (34.14)
Price of Beverages	0.4	-------	0.3564 (1.813)
Price of Other food	0.8	-------	1.5087 (7.695)

Table 3 presents the estimated variance/covariance matrix of the residuals. It appears that the variances of the residuals in any demand equation of the primal-dual specification are considerably smaller than the corresponding variances in the traditional model.

Table 3. Variance/covariance matrices of the residuals

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SIGMA MATRIX of the traditional LES model
        q₁              q₂              q₃              q₄

q₁    0.10928
q₂    0.11551E-01   0.20517E-01
q₃   -0.95759E-01  -0.14903E-01    .60383
q₄   -0.56190E-01  -0.43189E-01   -.13240         .28027


- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
SIGMA MATRIX of the primal-dual LES model

        p₁        p₂        p₃        p₄        q₁        q₂        q₃        q₄

p₁   0.59366E-01
p₂  -0.17408E-03  0.25097E-01
p₃  -0.58309E-02  0.25474E-03  0.40408E-02
p₄  -0.11953E-01 -0.66197E-02 -0.29823E-02  0.13974E-01
q₁   0.43796E-01  0.52107E-02 -0.46102E-02 -0.95242E-02  0.46482E-01
q₂  -0.86016E-03  0.69884E-02  0.81152E-04 -0.14204E-02  0.21219E-02  0.24322E-02
q₃  -0.17338E-01 -0.17054E-02  0.10305E-01 -0.92647E-02 -0.14462E-01 -0.13820E-03  0.33890E-01
q₄  -0.20982E-01 -0.14904E-01 -0.44839E-02  0.18958E-01 -0.19124E-01 -0.40583E-02 -0.14132E-01  0.34153E-01
```

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Finally, Table 4 presents the expenditure and price elasticities evaluated at the sample mean. It can be shown that the expenditure elasticity, η_j, in the LES model is equal to

$$\eta_j = \beta_j / w_j$$

where $w_j = p_j x_j / y$ is the expenditure share of the j-th good. Similarly, the Marshallian own price elasticity, e_{jj}^M, can be stated as

$$e_{jj}^M = -1 + (1 - \beta_j)(\gamma_j / x_j)$$

while the Marshallian cross-price elasticities are equal to

$$e_{jk}^M = -\beta_j (p_k \gamma_k / p_j x_j).$$

Hence, if γ_j is positive, as postulated by the original specification and as we would expect for food categories, the demand for the j-th commodity is inelastic. The corresponding Hicksian price elasticities can be stated as

$$e_{jk}^{H} = e_{jk}^{M} + \eta_j w_k.$$

The Hicksian price elasticities are relevant for a welfare analysis of the consumer problem.

The first two commodities involved in the empirical analysis (bread and cereals, meat) are food categories that could be regarded as "necessity" goods. Table 4 shows that the expenditure elasticities of both models reflect this expectation. From the magnitude of the expenditure elasticities beverages and "other food" are to be considered "luxury" commodities. Both the Marshallian and the Hicksian own price elasticities of the primal-dual model are more responsive than those of the traditional LES model. The Hicksian cross-price eslasticities of the primal-dual model have larger absolute values than the corresponding elasticities of the traditional model.

Notice that while the Marshallian cross-price elasticities indicate that all the commodities are gross complements (this characteristic is implied by the LES model when all the beta and gamma parameters are positive), the Hicksian cross-price elasticities indicate that all the commodities are gross substitutes (this result is not implied by the LES model).

Table 4. Expenditure and (Marshallian and Hicksian) Price Elasticities

(evaluated at the sample means)

Expenditure Elasticities	LES Traditional model	LES Primal-Dual model
Bread & cereals	0.4209	0.7749
Meat	0.9890	0.7690
Beverages	1.0518	1.0756
Other food	1.1060	1.1044

Marshallian price elasticities: traditional LES model

	q_1	q_2	q_3	q_4
q_1	−0.1472597	−0.0500327	−0.0426791	−0.1577379
q_2	−0.0919421	−0.4463100	−0.1059385	−0.3915389
q_3	−0.0996054	−0.1345430	−0.5678010	−0.4241732
q_4	−0.0993272	−0.1341671	−0.1144477	−0.6586734

Marshallian price elasticities: Primal-Dual model

	q_1	q_2	q_3	q_4
q_1	−0.8221749	−0.0389523	−0.0032695	−0.0044655
q_2	−0.0160878	−0.7784319	−0.0033353	−0.0045554
q_3	−0.0212071	−0.0523811	−0.9833059	−0.0060049
q_4	−0.0216506	−0.0534766	−0.0044886	−0.9951954

Hicksian price elasticities: traditional LES model

	q_1	q_2	q_3	q_4
q_1	−0.1022577	0.0303274	0.0386461	0.0565331
q_2	0.0137867	−0.2575100	0.0851289	0.1118748
q_3	0.0128371	0.0662456	−0.3646010	0.1112067
q_4	0.0189162	0.0769801	0.0992353	−0.0956734

Hicksian price elasticities: Primal-Dual model

	q_1	q_2	q_3	q_4
q_1	−0.7470549	0.1159914	0.1429720	0.3941417
q_2	0.0584390	−0.6247119	0.1417512	0.3909037
q_3	0.0830681	0.1626987	−0.7803059	0.5473078
q_4	0.0854132	0.1673550	0.2039403	−0.4270854

Monte Carlo Analysis of the Primal-Dual LES model

The two-phase primal-dual estimator discussed in previous sections is simple to implement but its statistical properties are currently unknown. Hence, given the complexity of the errors-in-variables system of nonlinear equations discussed above, it is of interest to make an empirical assessment of the extent by which the two-phase primal-dual estimator may perform within the context of the LES model.

To this end, we performed a Monte Carlo analysis of the primal-dual estimator using the approximate estimates of the LES model presented in Table 1 as the benchmark parameter values. In particular, the beta parameter values in the Monte Carlo experiment were selected as $(\beta_1 = 0.1, \beta_2 = 0.2, \beta_3 = 0.2, \beta_4 = 0.5)$ while the gamma parameter values were chosen as $(\gamma_1 = 0.17, \gamma_2 = 0.15, \gamma_3 = 0.20, \gamma_4 = 0.15)$. The latent expected prices and expected total expenditure were chosen as the sample information that was perturbed by measurement errors within a wide range of 10 to 30 percent of the base sample values. The sample size of the Monte Carlo experiment, then, was equal to the 119, the number of sample observations. With the latent prices and total expenditure so defined, equations (23) were used in their expected specification to generate the latent expected quantities of the LES model. Then, 300 draws of measurement errors from normal distributions $N(0, \sigma)$ as given in tables 5, 6 and 7 were used to define the observed price and quantity information of the Monte Carlo runs.

For comparison, the LES model was also estimated in its traditional form, that is, as a system of demand functions represented by equations (23), using the same observed quantity and price information generated for the primal-dual estimator. In other words, if we assume that both prices and quantities are measured with error and we estimate the

LES model by the two alternative estimators, which of the two estimators should be preferred?

The performance of the primal-dual and the traditional estimators was assessed by means of the mean squared error (MSE) loss criterion as defined in Judge et al. (1980, p. 26). They show that an overall MSE measure of a model can be decomposed in the familiar two components as the trace of the covariance matrix and the trace of the squared bias.

Tables 5, 6 and 7 present the results of the Monte Carlo analysis with three different structures of error distributions. The comparison between the primal-dual estimator and the traditional dual estimator shows two distinct patterns of parameter estimates. The β coefficients are measured rather precisely in all cases with the traditional dual estimator exhibiting a smaller MSE loss in Tables 5 and 6 but a larger MSE in Table 7. However, the squared bias of the β coefficients is smaller (by an order of magnitude) for the primal-dual estimator in all the three Tables. The γ coefficients seem more difficult to estimate correctly and it is important to recall that these coefficients are crucial determinants of the demand's price elasticities. The MSE loss and the squared bias of the γ coefficients are much larger for the traditional dual estimator. As the error size increases (Table 6), the γ MSE loss of the traditional estimator becomes very large in absolute terms and relative to the variance loss. This event occurs also in Table 7. Overall, the bias of the primal-dual estimator remains relatively small for both the beta and gamma parameters under the three scenarios. Table 8 expands the results of Table 7 reporting on the MSE, variance and squared bias of each individual parameter. The results show a very small bias for both the beta and the gamma parameters.

There remains to discuss the identification problem of the primal-dual estimator. The complexity of the errors-in-variables in a system of nonlinear equations makes a general treatment difficult to deal with in analytical terms. Therefore, we have approached the identification of the primal-dual estimator from an empirical point of view. That is, if the Phase-I and Phase-II estimation problems have unique optimal solutions, the model is certainly locally identified. One way to verify empirically this unique optimal solution property is to re-estimate the model using different starting values. Indeed, in all the explorations of the parameter space we obtained the same optimal solution. Hence, we are rather confident that the primal-dual estimator used in the LES model presented above is identified. A concomitant way to explore the identification problem is to examine the frequency diagrams of the beta and gamma parameters of the LES model in the Monte Carlo simulation experiment. If the frequency is peaked sufficiently around the mean value one can reasonably conclude that the model is identified, as lack of identification would correspond to a flat or even a multiple-peak distribution. In figure 1 we report the distributions of the beta and gamma parameters of the primal-dual model corresponding to the measurement error assumptions of Table 7. Also this empirical evidence suggests that the primal-dual model is identified.

Although Monte Carlo analyses have only a limited validity, the structure of the experiments reported above mimic very closely the structure of the original LES problem. Hence, the results of these Monte Carlo experiments throw some interesting light on a very complex statistical problem. The preliminary conclusion is that, for this sample of data applied to the LES model, the primal-dual estimator may be preferable to the traditional dual estimator under a rather wide range of measurement errors because of

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the more precise estimates of the γ parameters. As prices and quantities are almost surely measured with error, the Monte Carlo evidence presented here suggests that the primal-dual estimator, which attempts to deal explicitly with errors on both quantities and prices, may have a significantly smaller bias than the traditional LES estimator which assumes away fifty percent of the problem.

Table 5. Performance comparison between the primal-dual and the traditional estimators of the LES model (averages over 300 samples, $\sigma \equiv$ standard error)

Parameter	Distribution of Measurement Errors $N(0,\sigma)$	Parameter True Values	Primal-Dual Estimator	Traditional Dual Estimator
β_1	$N(0,.05)$	.10	.10153	.10132
β_2	$N(0,.15)$	.20	.19395	.20332
β_3	$N(0,.04)$	.20	.20231	.20767
β_4	$N(0,.10)$	.50	.50221	.48769
γ_1	$N(0,.10)$	.17	.14895	.24025
γ_2	$N(0,.05)$	.15	.15897	.19534
γ_3	$N(0,.15)$	.20	.14674	.30975
γ_4	$N(0,.15)$	.15	.11047	.46277
β MSE loss			.0016143	.0005204
β Variance loss			.0015651	.0002975
β Squared bias			.0000492	.0002229
γ MSE loss			.0219131	.1410271
γ Variance loss			.0169909	.0241679
γ Squared bias			.0049222	.1168592

Table 6. Performance comparison between the primal-dual and the traditional estimators of the LES model (averages over 300 samples, $\sigma \equiv$ standard error)

| Parameter	Distribution of Measurement Errors $N(0,\sigma)$	Parameter True Values	Primal-Dual Estimator	Traditional Dual Estimator
β_1	$N(0,.10)$	.10	.09816	.09470
β_2	$N(0,.20)$	.20	.20354	.22355
β_3	$N(0,.10)$	.20	.20218	.20035
β_4	$N(0,.10)$	.50	.49612	.48139
γ_1	$N(0,.10)$	.17	.15399	.46191
γ_2	$N(0,.10)$	.15	.12181	.28595
γ_3	$N(0,.20)$	.20	.10969	.64485
γ_4	$N(0,.20)$	.15	.11058	1.05442
β MSE loss			.0111978	.0062146
β Variance loss			.0111621	.0052855
β Squared bias			.0000357	.0009292
γ MSE loss			.0530731	1.3591495
γ Variance loss			.0423443	.2395930
γ Squared bias			.0107288	1.1195565

Table 7. Performance comparison between the primal-dual and the traditional estimators of the LES model (averages over 300 samples, $\sigma \equiv$ standard error)

Parameter	Distribution of Measurement Errors $N(0,\sigma)$	Parameter True Values	Primal-Dual Estimator	Traditional Dual Estimator
β_1	$N(0,.10)$	.10	.09956	.09545
β_2	$N(0,.20)$	.20	.20478	.22819
β_3	$N(0,.10)$	.20	.20282	.19901
β_4	$N(0,.10)$	.50	.49284	.47735
γ_1	$N(0,.10)$	.17	.16232	.45499
γ_2	$N(0,.10)$	.15	.12900	.27771
γ_3	$N(0,.05)$	.20	.20843	.64981
γ_4	$N(0,.05)$	.15	.19008	1.05253
β MSE loss			.0021637	.0070402
β Variance loss			.0020813	.0057108
β Squared bias			.0000823	.0013295
γ MSE loss			.0260812	1.3296031
γ Variance loss			.0239038	.2151904
γ Squared bias			.0021774	1.1144128

Table 8. MSE, Variance, and Squared Bias for each parameter of the Primal-Dual Estimator (From Table 7). 300 samples.

Parameter		MSE	Variance	Squared Bias
Bread & Cereals	β_1	0.0000732	0.0000730	0.0000002
Meat	β_2	0.0002524	0.0002295	0.0000229
Beverages	β_3	0.0010640	0.0010560	0.0000079
Other Food	β_4	0.0007741	0.0007228	0.0000513
Bread & Cereals	γ_1	0.0016289	0.0015699	0.0000590
Meat	γ_2	0.0027959	0.0023546	0.0004412
Beverages	γ_3	0.0034389	0.0033678	0.0000711
Other Food	γ_4	0.0182176	0.0166115	0.0016061

Figure 1. Distribution of beta and gamma parameters in the Monte Carlo analysis
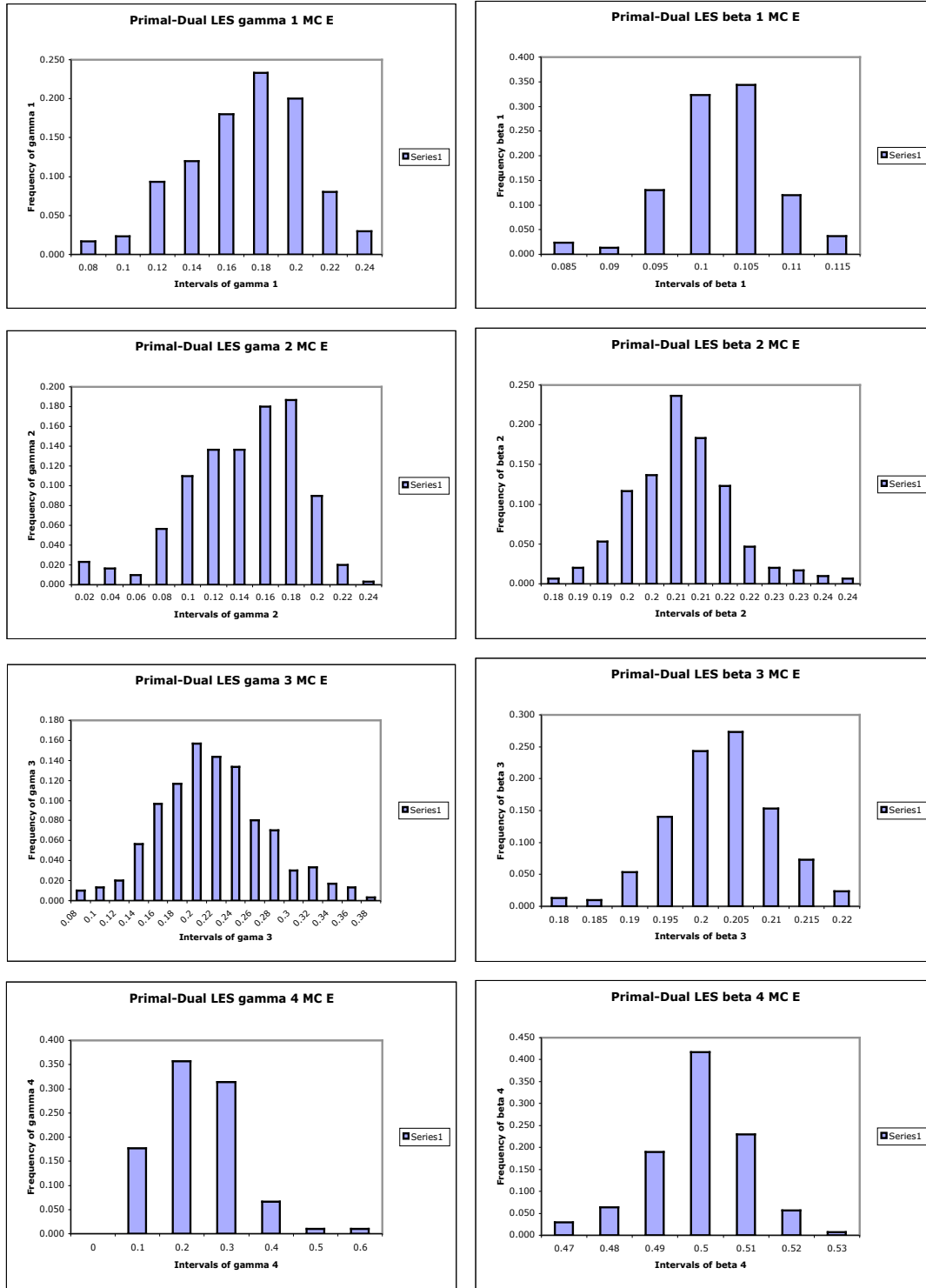
The Primal-Dual Estimator of the AIDS Model

The AIDS model is traditionally specified as the following expenditure function:

$$(24) \qquad \log E^e(\mathbf{p}^e, U^e) = (1 - U^e)\log[a(\mathbf{p}^e)] + U^e \log[b(\mathbf{p}^e)] = \log y^e$$

where $a(\mathbf{p}^e)$ and $b(\mathbf{p}^e)$ are price indexes defined (by Deaton and Muellbauer, 1980) as

$$(25) \qquad \log[a(\mathbf{p}^e)] = \alpha_0 + \sum_{k=1}^{J} \alpha_k \log p_k^e + \sum_{k}^{J}\sum_{j}^{J} \gamma_{kj} \log p_k^e \log p_j^e / 2$$

$$(26) \qquad \log[b(\mathbf{p}^e)] = \log[a(\mathbf{p}^e)] + \beta_0 \prod_{k=1}^{J} (p_k^e)^{\beta_k}.$$

The AIDS indirect utility function, therefore, can be written as

$$(27) \qquad U^*(\mathbf{p}^e, y^e) = \frac{\log y^e - \log[a(\mathbf{p}^e)]}{\beta_0 \prod_{k} (p_k^e)^{\beta_k}}$$

with the marginal utility of money income given by

$$(28) \qquad U_{y^e}^*(\mathbf{p}^e, y^e) = 1/y^e \beta_0 \prod_{k} (p_k^e)^{\beta_k} = \lambda^e.$$

The Phase II estimation problem of the AIDS model can then be stated as follows:

$$\min_{\boldsymbol{\beta}, \mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}_n' \hat{\Sigma}^{-1} \mathbf{e}_n$$

subject to

Primal relations

$$(29) \qquad \begin{aligned} p_{jn} &= U_{\mathbf{x}_j^e n}^e(\hat{\mathbf{x}}_{jn}^e, \boldsymbol{\beta}_{U^e}) / U_{y^e n}^*(\hat{\mathbf{p}}_{jn}^e, \hat{y}_n^e, \boldsymbol{\beta}_{U^*}) + v_{jn} \\ &= \left[\text{Taylor series of } U_{\mathbf{x} n}^e(\hat{\mathbf{x}}^e)\right]\left[\hat{y}_n^e \beta_0 \prod_{k=1}^{J}(\hat{p}_{kn}^e)^{\beta_k}\right] + v_{jn} \end{aligned}$$

Dual relations

$$(30) \qquad x_{jn} = \frac{\alpha_j \hat{y}_n^e}{\hat{p}_{jn}^e} + \frac{\hat{y}_n^e}{\hat{p}_{jn}^e} \sum_{k=1}^{J} \gamma_{kj} \log \hat{p}_{kn}^e + \frac{\beta_j \hat{y}_n^e}{\hat{p}_{jn}^e} \log[\hat{y}_n^e / a(\hat{\mathbf{p}}_n^e)] + \varepsilon_{jn}.$$

The Taylor series expansion of the latent direct utility function provides the complementary primal information for obtaining efficient estimates of the AIDS parameters. The results will be reported in another paper.

Errors-in-Variables with Replicated Measurements

The principal source of the statistical difficulties (unknown properties) associated with the estimator presented in previous sections lies with the number of latent variables which increases indefinitely, in parallel with the number of sample observations. If it were possible to obtain replicate measurements of these latent variables, a consistent estimator would be readily available. Thus, we reformulate the primal-dual LES model as a panel data specification. In this enterprise, we follow Collado (1997) who assumes that the population is divided in cohorts, $c = 1,...,C$, according to certain characteristics. Furthermore, the cohorts have fixed membership. The sample observations are thus given the index $r_c = 1,...,R_c$ and $c = 1,...,C$. Furthermore, and only for simplicity, let assume that we have sampled the same number of households from each cohort. In this case, the sample observations bear the double index rc and the model assumes the appearance of a special panel data specification where the latent variables are associated with replicated measurements. Therefore, the Phase I specification of the LES models becomes

(31)
$$\mathbf{p}_{rc} = \mathbf{p}_c^e + \mathbf{v}_{rc}$$

(32)
$$y_{rc} = y_c^e + v_{0rc}$$

(33)
$$\mathbf{x}_{rc} = \mathbf{x}_c^e + \boldsymbol{\varepsilon}_{rc}$$

(34)
$$x_{jrc} = d_{jrc}^e(\hat{\mathbf{p}}_{jrc}^e, \hat{y}_{rc}^e, \boldsymbol{\beta}_{x_j}) + \varepsilon_{jrc}$$
$$= \gamma_j + \beta_j \frac{(\hat{y}_{rc}^e - \Sigma_{k=1}^{J} \hat{p}_{krc}^e \gamma_k)}{\hat{p}_{jrc}^e} + \varepsilon_{jrc}$$

which can be re-written in terms of the averages of the sample information with respect to the number of sample replicates in each cohort.

The original sample of 119 households was divided into 40 cohorts on the basis of the total income expenditure. Admittedly, other more stable characteristics might be preferable. The empirical results are given in Table 8.

Table 8. Iterated NSUR Estimates of the LES model of Cohorts

(t-ratios in parenthesis)

Parameters	Traditional LES Cohort model	Primal-Dual LES Cohort model
Bread & cereals β_1	0.05267 (6.648)	0.09358 (72.509)
Meat β_2	0.19870 (9.661)	0.18365 (422.89)
Beverages β_3	0.14562 (7.676)	0.18920 (257.51)
Other foods β_4	0.60211 (18.881)	0.53356 (443.36)
Bread & cereals γ_1	0.39419 (5.2626)	0.17167 (14.463)
Meat γ_2	-0.0209 (0.471)	0.04478 (17.024)
Beverages γ_3	0.58899 (2.8667)	0.14727 (9.011)
Other foods γ_4	-0.17425 (0.431)	0.06818 (3.265)
Log-Likelihood	18.6052	584.8549
Condition Number of Σ	9.237	37.003

In comparison to the Table 1, the results of the cohort model presented in Table 8 indicate that the estimates of the traditional LES model are somewhat unstable. Furthermore, there is a considerable loss of significance. In contrast, the estimates of the primal-dual model have about the same high level of significance of the model reported in Table 1.

Since we know for certain that the estimator of the errors-in-variables model based upon replicated measurement of the latent variables is consistent and the empirical estimates are not very different between the two error specifications, we advance the conjecture that also the estimator of the errors-in-variables model without replicated measurement is consistent. The proof of this conjecture is the subject of another paper.

Conclusion

We presented a primal-dual estimator of a LES model of consumer behavior that, in principle, ought to be more efficient than any other estimator based upon the traditional dual approach. The model specification assumes the natural structure of a nonlinear errors-in-variables system of equations for which no easy-to-implement consistent estimator is available. We formulate the estimation problem as the minimization of the sum of squared errors in the direction of all available prices and quantities.

The estimator is articulated into two phases. In phase I, the un-weighted sum of squared errors is minimized subject to the theoretical nonlinear equations and the statistical error structure. The minimization is carried out with respect to the structural parameters of the LES model and the latent variables expressing the expected quantities and prices of the individual household.

In phase II, the estimates of the latent variables are used in an iterated nonlinear seemingly unrelated equation specification involving all the primal and dual relations.

It is conjectured that the primal-dual estimator is consistent. In order to explore the performance of the estimator we use a Monte Carlo analysis on a model that mimics very closely the nature of the sample information. The conclusion of this Monte Carlo analysis is that the primal-dual estimator has a very small bias in relatively small samples.

To gauge the performance of this estimator (whose statistical properties are still unknown) with a consistent estimator, we assume that the available sample information can be interpreted as a collection of replicate measurement of population cohorts' behavior. In this way, the model is akin to a panel data model whose estimator is known to be consistent. The primal-dual estimates of the LES model obtained with the cohort estimator are very close to those obtained with the original estimator. This evidence lends further support to the conjecture that the single-measurement primal-dual estimator is also consistent.

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