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Estimation of Consumer Demand Functions When the Observed Prices Are the Same for All Sample Units

by

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Giannini Foundation of Agricultural Economics

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Abstract

With the advent of the almost ideal demand system (AIDS) of Deaton and Muellbauer, the estimation of consumer demand functions revolves around specifications that use flexible functional forms of the indirect utility function. This dual approach has put on the backburner the traditional primal approach because the direct utility function exists only in a latent state. The lack of explicit, analytical invertibility of either system, however, is an indication that focusing exclusively on the dual side of the consumer problem is equivalent to disregard potentially important and independent information residing with the primal side. This paper suggests that efficient estimates (in the sense of using all the available information) of the demand functions require the joint estimation of all the primal and dual relations. The specification of this objective assumes that risk-neutral households maximize their expected utility subject to their expected budget constraint. This theoretical framework leads to a two-step procedure that produces consistent and efficient estimates of the model's parameters. The generality of the approach proposed here can handle also the frequently encountered case when all the sample units face the same observed commodity prices. Finally, we present a general solution of the nonlinear errors-in-variables problem with a novel estimation procedure that avoids the pitfalls of the traditional approach.

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Estimation of Consumer Demand Functions

When the Observed Prices Are the Same for All Sample Units

Introduction

Statistical surveys of consumer behavior rarely gather price information that is specific for each individual sample unit. How are, then, demand functions estimated in the absence of any price variability? The answer varies from the use of time series and panel data to construction of "unit value" prices. Indeed, this statistical reality is the consequence of the following conundrum of consumer theory. Theory suggests that observed commodity prices should be identical for households that make their consumption decisions in the same market environment, but econometric estimation of consumer demand functions requires a significant variability of prices without which the duality approach for their estimation is infeasible.

Another unsatisfactory aspect of the traditional approach to demand functions' estimation consists in the absence of any use of the primal information represented by the first-order-necessary conditions for utility maximization. With the advent of flexible functional forms such as the almost ideal demand system (AIDS) of Deaton and Muellbauer, the primal relations contribute independent information which exists only in a latent state. Hence, efficient estimators of the demand functions require the utilization of the complete system of primal and dual relations.

This paper contributes three main results. First, we propose the joint and novel estimation of primal and dual relations in order to obtain efficient estimates of systems of demand functions. Second, we present a theoretical and empirical framework that admits the estimation of demand functions also when the observed commodity prices are the same for all the sample units. Third, we offer a general solution of the nonlinear errors-in-variables problem that results from the specification of the empirical framework outlined above.

These results are obtained on the basis of plausible and general assumptions: The typical household is risk neutral and its objective is the maximization of the expected utility functions subject to an expected budget constraint. In other words, the household unit makes its consumption decisions through a planning process that uses expected information. The expectation process is specific to each household.

When the econometrician desires to measure commodity quantities and prices for the purpose of inferring the demand functions that were used by the sample households, measurement errors occur. We assume an additive error structure for each piece of sample information, that is, commodity quantities, prices and the household's disposable money income.

The resulting nonlinear errors-in-variables model will be estimated using a novel approach. Traditionally, errors-in-variables specifications include unobservable quantities that are replaced by their observable counterparts. This replacement step, however, induces undesired properties on the estimator with inconsistent estimates and other difficulties. In order to avoid these undesirable properties, we will not replace the latent expected quantities and prices. A two-phase procedure will first generate consistent estimates of those expected quantities and prices which, then, will be used as instrumental variables to produce efficient estimates of the model's parameters.

The Theoretical Framework

We assume that risk-neutral households maximize their expected utility subject to an expected budget constraint. Let $\mathbf{p}^e$ be the $(J \times 1)$ vector of expected prices and y^e the expected disposable income available to the risk-neutral household that solves the following problem

$$(1) \qquad U^*(\mathbf{p}^e, y^e) \overset{\text{def}}{=} \max_{\mathbf{x}}\{U^e(\mathbf{x}) \text{ s.t. } y^e = \mathbf{p}^{e\prime}\mathbf{x}\}$$

where $\mathbf{x}$ is a $(J \times 1)$ vector of commodity quantities. The first-order-necessary conditions are stated as

$$(2) \qquad \begin{aligned} \frac{\partial L}{\partial \mathbf{x}} &= U^e_{\mathbf{x}}(\mathbf{x}) - \lambda \mathbf{p}^e = \mathbf{0} \\ \frac{\partial L}{\partial \lambda} &= y^e - \mathbf{p}^{e\prime}\mathbf{x} = 0 \end{aligned}$$

where L is the Lagrangean function and λ is the Lagrange multiplier. After solving equations (2), the vector of commodity quantities $\mathbf{x}$ will become the vector of expected quantities $\mathbf{x}^e$. Similarly, the Lagrange multiplier λ will become the expected Lagrange multiplier λ^e which is equivalent to the marginal utility of money income. We assume an interior solution of equations (2) that will generate commodity demand functions with values

$$(3) \qquad \mathbf{x}^e = \mathbf{d}^e(\mathbf{p}^e, y^e).$$

In case the first-order-necessary conditions (2) do not admit an explicit, analytical solution, the demand functions (3) exist via the duality principle. The indirect utility function is obtained by inserting the optimal quantities (3) into the direct utility

$$(4) \qquad U^*(\mathbf{p}^e, y^e) = U^e(\mathbf{x}^e)$$

with the marginal utility of money income defined as $U^*_{y^e}(\mathbf{p}^e, y^e) = \lambda^e$.

The above development corresponds to the textbook discussion of consumer theory. The econometric specification of the model requires the specification of the error structure that accompanies any statistical measurement.

A Measurable Model of Consumer Demand

We assume that all the commodity quantities and prices and the disposable income are measured with errors. That is, the econometrician observes $\mathbf{x}$, $\mathbf{p}$ and y that bear an additive relation with their expected counterparts, that is $\mathbf{x} = \mathbf{x}^e + \square$, $\mathbf{p} = \mathbf{p}^e + \square$ and $y = y^e + \square_0$. The vector of errors $\mathbf{e}\square = (\square\square\square\square_0)$ is distributed as $\mathbf{e} \sim N(\mathbf{0}, \square)$. We further assume that the errors are independently distributed across sample units.

The measurable econometric model of consumer behavior can thus be assembled using all the primal and dual relations and the postulated error structure as follows:

Primal relations

(5) $$\mathbf{p} = U^e_{\mathbf{x}^e}(\mathbf{x}^e) / U^*_{y^e}(\mathbf{p}^e, y^e) + \square$$

(6) $$y = \mathbf{p}\square\mathbf{x}^e + \square_0$$

Dual relations

(7) $$\mathbf{x} = \mathbf{d}^e(\mathbf{p}^e, y^e) + \square$$

Error structure

(8) $$\mathbf{p} = \mathbf{p}^e + \square$$

(9) $$y = y^e + \square_0$$

(10) $$\mathbf{x} = \mathbf{x}^e + \square.$$

Equations (5) represent the latent first-order-necessary conditions involving commodity prices and the marginal utilities of goods. These relations are traditionally disregarded during the estimation of demand functions (7). However, in the case of model specifications involving flexible functional forms, relations (5) convey information independent from the demand functions (equations (7)). The pursuit of efficient estimates, therefore, requires the joint estimation of all the primal and dual relations of the consumer model.

Econometric Model

We assume a sample of N household with index $n = 1,...,N$. The estimation procedure proposed here is articulated in two-phases. The Phase I objective consists in obtaining consistent estimates of all the individual households' expected quantities and prices. These estimates, then, will be used as instrumental variables during Phase II to obtain efficient estimates of the model's parameters.

The Phase I specification assumes the form of a nonlinear errors-in-variables model to be estimated by nonlinear least-squares. Specifically, let $\square = (\square_{U^e}, \square_{U^e}, \square_{\mathbf{x}})$ be the vector of parameters that constitute the final goal of the estimation process. Then, the Phase I estimation problem is stated as follows:

(11) $$\min_{\square, p^e_{jn}, x^e_{jn}, y^e_n, \mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}_n' \mathbf{e}_n$$

subject to

(12) $$\mathbf{p} = U^e_{\mathbf{x}^e,n}(\mathbf{x}^e_n, \square_{U^e}) / U^*_{y^e,n}(\mathbf{p}^e_n, y^e_n, \square_{U^*}) + \square_n$$

(13) $$y_n = \mathbf{p}_n' \mathbf{x}^e_n + \square_{0n}$$

(14) $$\mathbf{x}_n = \mathbf{d}^e_n(\mathbf{p}^e_n, y^e_n, \square_{\mathbf{x}}) + \square_n$$

(15) $$\mathbf{p} = \mathbf{p}^e_n + \square_{\square}$$

(16) $$y_n = y_n^e + \square_{0n}$$

(17) $$\mathbf{x}_n = \mathbf{x}_n^e + \square_n$$

(18) $$\sum_{n=1}^{N} p_{jn}^e \square_{jn} = 0, \quad j = 1,...N$$

(19) $$\sum_{n=1}^{N} y_n^e \square_n = 0$$

(20) $$\sum_{n=1}^{N} x_{jn}^e \square_{jn} = 0, \quad j = 1,...N$$

The sample index was removed from the observed vector of prices in order to indicate that all the sample units face the same commodity prices. The solution of the Phase I model is not a trivial pursuit. For example, with *N=100* and *J=5,* the number of constraints is *(2J+1)(2N+1)+1=2211*. In the case of first-order-conditions that admit an explicit solution the number of constraints is reduced considerably because either the primal or the dual constraints are redundant. With a utility specification corresponding to the flexible AIDS model, all the primal and dual constraints are required for obtaining efficient estimates. The number of expected quantities and prices, expected money income, parameters and errors is somewhat larger than *2211*. The Phase I problem can be solved with commercially available computer applications for nonlinear programming such as GAMS (Brooke *et al.*)

Constraints (19)-(20) are the orthogonality conditions for the expected unobservables. The estimates of these quantities and prices $\hat{\mathbf{p}}_n^e, \hat{y}_n^e, \hat{\mathbf{x}}_n^e$ will be used as instrumental variables in the Phase II estimation problem. Let $\hat{\square}^{\square 1}$ be the estimated covariance matrix computed from the estimated residuals of Phase I. Then, the Phase II estimation problem is the following nonlinear seemingly unrelated (NSUR) equation model:

(21) $$\min_{\square,\mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}_n' \hat{\square}^{-1} \mathbf{e}_n$$

subject to

(22) $$\mathbf{p} = U^{e}_{\mathbf{x}^e,n}(\hat{\mathbf{x}}^{e}_{n}, \square_{U^e}) / U^{*}_{y^e,n}(\hat{\mathbf{p}}^{e}_{n}, \hat{y}^{e}_{n}, \square_{U^*}) + \square_n$$

(23) $$y_n = \hat{\mathbf{p}}_n^{e\prime} \hat{\mathbf{x}}^{e}_{n} + \square_{0n}$$

(24) $$\mathbf{x}_n = \mathbf{d}^{e}_{n}(\hat{\mathbf{p}}^{e}_{n}, \hat{y}^{e}_{n}, \square_{\mathbf{x}}) + \square_n .$$

The Phase II problem can be solved with conventional econometric packages such as SHAZAM (Whistler *et al.*). The consistency of the above nonlinear least-squares estimator has been discussed by Davidson and MacKinnon (1993) in their Theorem 5.1.

Empirical applications

We present two empirical applications of the methodology developed in the previous sections. The first example deals with a Cobb-Douglas utility function that admits an explicit solution of the first-order-necessary conditions. In this case, either the primal or the dual set of relations is redundant for the Phase I problem. It is not redundant, however, for the Phase II estimation problem, as each component of the error structure in the NSUR specification conveys independent information.

The second example deals with a flexible functional form of the indirect utility function such as the AIDS specification. In this case, the direct utility function exists only in a latent form and its econometric specification will be given as a second-order Taylor series expansion. Thus, this primal specification of the direct utility function conveys information that is independent from the dual set of relations and all the primal and dual relations, therefore, are required in the Phase I problem.

Cobb-Douglas Utility Function

The direct utility takes on the follow specification:

$$U^e(\mathbf{x}) = \sum_{j=1}^{J} \beta_j \log(x_j - \gamma_j) \tag{25}$$

where $x_j > \gamma_j \geq 0$ and the parameter γ_j represents a subsistence level of the corresponding good.

The Phase II econometric model takes on the following specification:

$$\min_{\theta, \mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}_n' \hat{\Sigma}^{-1} \mathbf{e}_n$$

subject to

Primal relations

$$\begin{aligned} p_j &= U^e_{\mathbf{x}^e_j, n}(\hat{\mathbf{x}}^e_{jn}, \theta_{U^e}) / U^*_{y^e, n}(\hat{\mathbf{p}}^e_{jn}, \hat{y}^e_n, \theta_{U^*}) + \varepsilon_{jn} \\ &= \frac{\beta_j}{\sum_{k=1}^{J} \beta_k} \frac{(\hat{y}^e_n - \sum_{k=1}^{J} \hat{p}^e_{kn} \gamma_k)}{(\hat{x}^e_{jn} - \gamma_j)} + \varepsilon_{jn} \end{aligned} \tag{26}$$

$$y_n = \sum_{j=1}^{J} \hat{p}^e_{jn} \hat{x}^e_{jn} + \varepsilon_{0n} \tag{27}$$

Dual relations

$$\begin{aligned} x_{jn} &= d^e_{jn}(\hat{\mathbf{p}}^e_{jn}, \hat{y}^e_n, \theta_{x_j}) + \nu_{jn} \\ &= \gamma_j + \frac{\beta_j}{\sum_k \beta_k} \frac{(\hat{y}^e_n - \sum_{k=1}^{J} \hat{p}^e_{kn} \gamma_k)}{\hat{p}^e_{jn}} + \nu_{jn} \end{aligned} \tag{28}$$

The Cobb-Douglas model of consumer behavior admits an explicit solution of the first-order-necessary conditions. We re-emphasize, however, that, during the Phase II estimation relations (26) and (28) convey independent information through different errors and their distributions. They are both required, therefore, if one wishes to obtain efficient estimates of the model's parameters.

AIDS Indirect Utility Function

The AIDS model is traditionally specified as the following expenditure function:

(29) $$\log E^e(\mathbf{p}^e, U^e) = (1 - U^e)\log[a(\mathbf{p}^e)] + U^e \log[b(\mathbf{p}^e)] = \log y^e$$

where $a(\mathbf{p}^e)$ and $b(\mathbf{p}^e)$ are price indexes defined (by Deaton and Muellbauer, 1980) as

(30) $$\log[a(\mathbf{p}^e)] = \alpha_0 + \sum_{k=1}^{J} \alpha_k \log p_k^e + \sum_{k}^{J} \sum_{j}^{J} \gamma_{kj} \log p_k^e \log p_j^e / 2$$

(31) $$\log[b(\mathbf{p}^e)] = \log[b(\mathbf{p}^e)] + \beta_0 \prod_{k=1}^{J} (p_k^e)^{\beta_k}.$$

The AIDS indirect utility function, therefore, can be written as

(32) $$U^*(\mathbf{p}^e, y^e) = \frac{\log y^e - \log[a(\mathbf{p}^e)]}{\beta_0 \prod_k (p_k^e)^{\beta_k}}$$

with the marginal utility of money income given by

(33) $$U^*_{y^e}(\mathbf{p}^e, y^e) = 1 / y^e \beta_0 \prod_k (p_k^e)^{\beta_k} = \lambda^e.$$

The Phase II estimation problem of the AIDS model can then be stated as follows:

$$\min_{\beta, \mathbf{e}_n} \sum_{n=1}^{N} \mathbf{e}_n' \hat{\Sigma}^{-1} \mathbf{e}_n$$

subject to

Primal relations

(34)
$$p_j = U^e_{\mathbf{x}^e_j, n}(\hat{\mathbf{x}}^e_{jn}, \beta_{U^e}) / U^*_{y^e, n}(\hat{\mathbf{p}}^e_{jn}, \hat{y}^e_n, \beta_{U^*}) + \varepsilon_{jn}$$
$$= \begin{bmatrix} \text{First derivative of 2nd order} \\ \text{Taylor series of } U^e_n(\hat{\mathbf{x}}^e) \end{bmatrix} \left[\hat{y}^e_n \beta_0 \prod_{k=1}^{J} (\hat{p}^e_{kn})^{\beta_k} \right] + \varepsilon_{jn}$$

(35) $$y_n = \sum_{j=1}^{J} \hat{p}^e_{jn} \hat{x}^e_{jn} + \varepsilon_{0n}$$

Dual relations

(36) $$x_{jn} = \frac{\alpha_j \hat{y}_n^e}{\hat{p}_{jn}^e} + \frac{\hat{y}_n^e}{\hat{p}_{jn}^e} \sum_{k=1}^{J} \gamma_{kj} \log \hat{p}_{kn}^e + \frac{\beta_j \hat{y}_n^e}{\hat{p}_{jn}^e} \log[\hat{y}_n^e / a(\hat{\mathbf{p}}_n^e)] + \varepsilon_{jn}.$$

The Taylor series expansion of the latent direct utility function provides the complementary primal information for obtaining efficient estimates of the AIDS parameters.

Conclusion

We have tackled threes neglected aspects of the econometric estimation of consumer demand functions. First, we have proposed a theoretical framework based upon the assumption that households are risk-neutral and maximize their expected utility subject to an expected budget constraint. We then assumed that all the sample information is measured with an additive error.

Secondly, we suggested that efficient estimates of the consumer demand functions require the joint estimation of primal and dual relations. As a byproduct of this general framework we obtain the possibility of estimating demand functions also when all the sample households face the same observed commodity prices. This statistical reality corresponds to a rather common event associated with many national surveys and censuses of consumer behavior.

Third, the resulting nonlinear errors-in-variables problem was solved using a two-phase approach that produces consistent and efficient estimates of the model's parameters. This novel approach avoids the pitfalls of the traditional method for handling errors-in-variables models where the latent, unobservable information is replaced by its observable counterpart. This replacement step induces undesirable properties on the estimator such as inconsistent estimates. The methodology presented in this paper is rather general and can be applied to several other micro-econometric contexts.

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