WEATHER BASED INSTRUMENTS IN THE CONTEXT OF WHOLE FARM RISK MANAGEMENT

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Abstract

Recent and presumable future developments tend to increase the risks associated with farming activities. These include climate risks which have always played an important role in farming. Weather based instruments can be valuable tools to reduce the risk associate with unfavourable climatic events. However, a number of factors can limit the hedging effectiveness of these tools. These factors include basis risk, the impacts of remaining price uncertainty and diversification effects. The paper addresses the influence of each oft these factors. In its final part an integrated approach for a comprehensive assessment of weather derivatives and other hedging instruments is proposed that is based on the concept of portfolio optimisation.

Keywords

Downside risk, portfolio optimisation, risk management, risk-value models, weather derivatives.

1 Introduction

When the first weather derivatives appeared, i.e. the temperature based heating and cooling degree days contracts in the U.S., agriculture was soon detected as a promising field of application, since production quantities as well as input requirements are heavily dependent on weather patterns. Since then, in a number of theoretical studies the fundamentals have been laid out and several empirical analyses have indicated the potential of these new hedging instruments. However, despite these promising results, applications are still rare. One reason for this is certainly that it always takes some time for new instruments to enter the market. However, there may be more impediments for a wider adoption, as for example the fact that these tools provide financial compensation only for shortfalls of produced output or excessive input needs respectively, while other risks like unpredictable price changes remain unchanged. Furthermore, besides weather derivatives, farmers have other opportunities to influence the risk exposure of their firms. Among them are the choice of the production programme as well as marketing activities including forward pricing and hedging with futures and options. In total, all these opportunities comprise a portfolio of activities which eventually determines the extent of risk a farm operation is exposed to.

Because of a multitude of interdependencies, assessing the relative value of each instrument requires the consideration of the whole set of possible actions. Treating an instrument separately is likely to lead to an overestimation of its risk reducing potential. In our paper we address this issue. On the outset we systematize the risk management instruments available to

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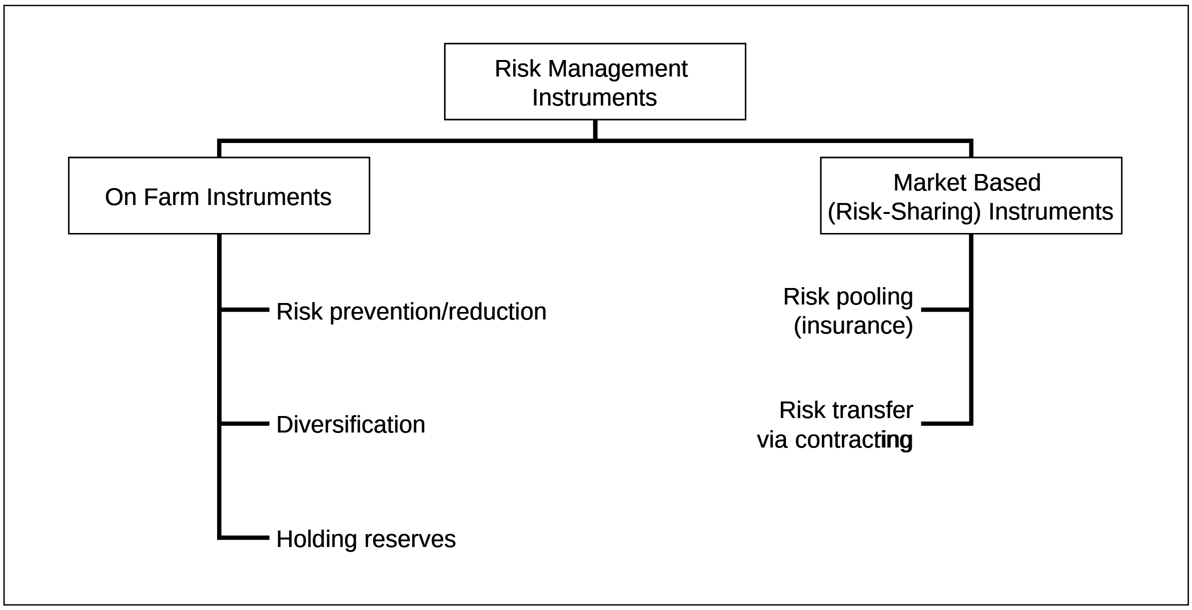
the farmer. Next, the major factors that influence the effectiveness of weather derivatives as hedging instruments will be addressed. These include basis risk, the impacts of remaining price uncertainty and diversification effects. In the last part of the paper we shall outline an approach that aims at assessing the value of weather derivatives and other hedging instruments comprehensively in the context of portfolio optimisation.

2 Risk management instruments

Farmers have a wide variety of possibilities to influence the risk associated with their operations. Following HARDAKER et al. (2004: 268ff) and BERG (2005), these can be broadly classified into on farm risk management instruments on one hand and market based or risk sharing instruments on the other hand (Figure 1). The former include all measures that aim at avoiding or reducing the exposure to risks, such as precautionary actions to prevent accidents, fire outbreaks or burglaries. Furthermore strategies to control pests and diseases in plant and animal production belong to this category. Spreading the risk through the diversification of farming activities is based on the fact the dispersion of the overall return can be reduced by selecting a portfolio of activities that have outcomes with low or negative correlations. Finally, building financial reserves aims at creating a risk bearing potential that allows compensating the effects of unfavourable events if necessary.

Figure 1: Risk management instruments



Risk sharing instruments presuppose the existence of market partners. If risk pooling is possible insurance contracts that certainly belong to the most popular risk management instruments may be the appropriate risk-sharing devices. In addition, risks can be shared with market partners by entering a contractual agreement. Popular examples include forward contracting of inputs and outputs as well as hedging with futures and options. Weather derivatives also belong to this group.

All these instruments are interdependent in the sense that the effect of a certain measure on the overall risk exposure depends on the constellation of all other instruments. For instance, a broadly diversified production programme limits the benefit of additional risk management instruments. In principal, this requires an integrated approach to risk management which considers the full set of risk management instruments simultaneously to ultimately arrive at an optimal mix of instruments.

3 Weather derivatives versus insurance contracts

While standard insurance contracts confirm indemnity payments in case of the occurrence of a damage, weather derivatives base their payoffs on the value that an underlying index takes on. Thus, they are equivalent with regular insurance contracts only in the absence of any basis risk. A simplistic example shall demonstrate the impacts of this circumstance.

As increasing values of weather indices often improve yields only up to a certain limit, we restrict our discussion to option contracts. In case of a long put option the payoff is given by

$$A = V \cdot Max\left[\, 0, (K - x) \,\right] \qquad (1)$$

where V denotes the tick size and K is the strike level. This payoff structure corresponds to a Leontief type production function (cf. BERG, 1997) that grows linearly with increasing x, until $x = K$ where the yield achieves its maximum.

The fair premium P_f of the option equals the discounted expected value of the payoff, $E(A)$, i.e.

$$P_f = e^{-r \cdot h}\, E(A) = e^{-r \cdot h}\, V\, E\left(Max\left[\, 0, (K - x)\right]\right) \qquad (2)$$

where the factor e^{-rh} discounts the payment over the duration h using the interest rate r. The expected value of the Max function, $E(Max[.])$, represents the weighted average of the payments that occur if the index falls above or below the strike level K, respectively. Since no payment occurs at index values above K we can write:

$$E\left(Max\left[\, 0, (K - x)\right]\right) = H(K) \cdot (K - E(x \mid x \leq K)) \qquad (3)$$

In equation (3) H marks the probability that x exceeds K. If $h(x)$ represents the density function of the weather index $H(K)$ is given by

$$H(K) = \int_{-\infty}^{K} h(x)\, dx \qquad (4)$$

If the index is normally distributed $H(K)$ becomes

$$H(K) = \Phi(z), \quad \text{with} \quad z = \frac{K - E(x)}{\sigma} \qquad (5)$$

where $\Phi(z)$ represents the standard normal distribution. We still have to determine the expected value of x, given that x falls below K as represented by the term $E(x \mid x \leq K)$. This is essentially the expected value of the distribution of x truncated above K. The expected value of the truncated normal distribution is (HARTUNG, 1998, p. 149)

$$E(x \mid x < K) = E(x) + \sigma \frac{-\phi(z)}{\Phi(z)} \qquad (6)$$

where $\Phi(.)$ is the standard normal distribution and $\phi(.)$ the respective density function.

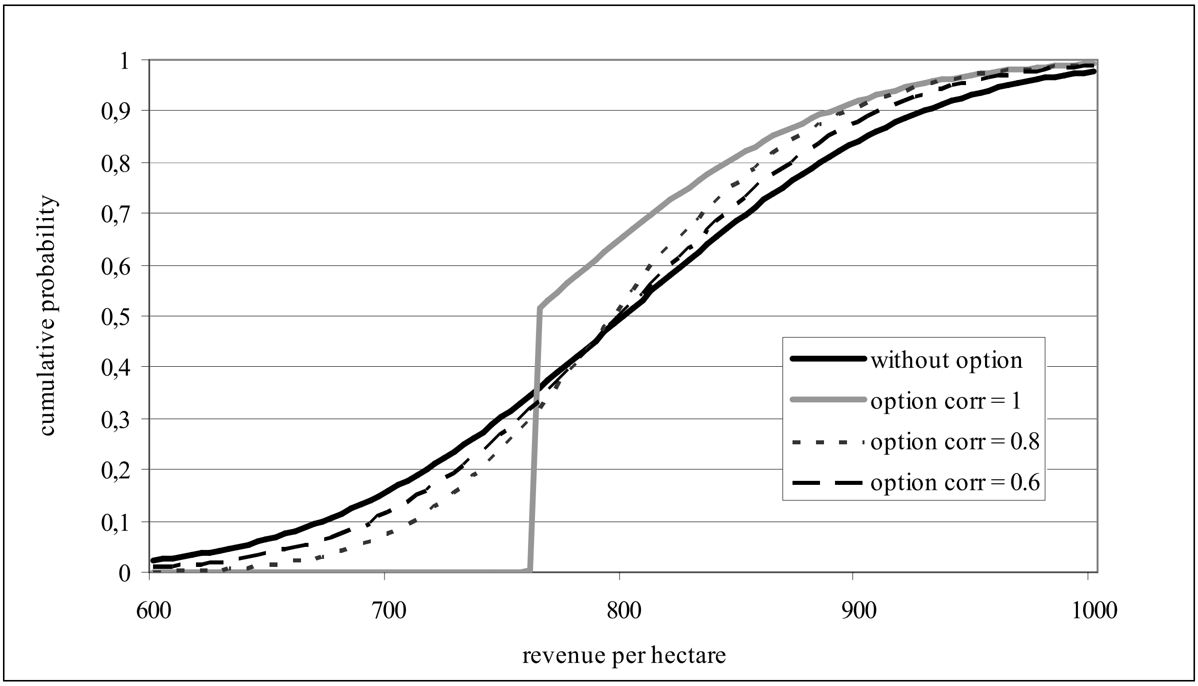
Following, we shall examine the effect that the option has on the total net return per ha W_p which comprises the market revenue plus the option payoff minus the fair premium P_f. It is given by

$$W_P = y\, p_y + V \cdot Max\left[0, (E(x) - x)\right] - P_f \qquad (7)$$

In the above formula y is the yield and p_y represents the product price. Now let y represent the yield of wheat which we assume to be normally distributed with a mean of 80 dt/ha and a standard deviation of 10 dt/ha. If the product price p_y is contractually fixed at 10 €/dt, with these assumptions, the distribution of the revenue corresponds to the solid black line in Figure 2 with an expected value of 800 €/ha and a standard deviation of 100 €/dt.

The weather index x may represent the amount of rainfall during a certain period and shall likewise be normally distributed with a mean of $E(x) = 100$ mm and the standard deviation $s = 125$ mm. Setting the strike level at the expected value, i.e. $K = 100$, we derive the probability $H(100) = 0.5$ and the conditional expectation $E(x \mid x \leq K) = 90$ mm. Thus, the average negative deviation of the index from K according to equation (3) is 5 mm. Multiplying by a tick size $V = 8$ €/mm yields a fair premium of 40 €/ha [1].

Figure 2: Impact of basis risk on the effectiveness of weather derivatives



Assuming that y and x are positively correlated random variables with the above characteristics, we can simulate the model of equation (7) stochastically. The simulation results are depicted in Figure 2. As can be seen from the graph, buying an option completely eliminates the downside risk if and only if we assume a perfect correlation between the yield and the weather index. In this case the weather derivative is equivalent to an insurance contract based on the individual yield. In turn, at correlations less than +1 – even though they may be close to one – very low revenues cannot be excluded anymore. Although the weather derivative always reduces the probability of low returns, it cannot secure a certain revenue because of the basis risk that is always present. This means that financial disasters caused by a local event, e.g. a hailstorm, flood or even pest damage, are still possible, although fairly unlikely. Weather derivatives can therefore reduce *profitability* risks but they cannot ensure *liquidity*. Likewise they cannot replace other types of disaster assistance. Naming them as index insurances may therefore be somewhat misleading.

4 Limits to the effectiveness of weather derivatives

The major factors that influence the effectiveness of weather derivatives as hedging instruments include basis risk, the impacts of remaining price uncertainty and diversification effects. These will be discussed in the following sections.

[1] Since all payments are evaluated at harvesting time discounting is not necessary.

4.1 Basis risk

Basis risk, in general, refers to the phenomenon that the payoffs of a derivative do not perfectly correspond to the shortfalls of the underlying exposure. In case of weather derivatives they may be either caused by an imperfect relationship between the weather index and the biological production process or by the fact that the index is monitored some distance away from site where the crop is grown. Latter is normally referred to as geographical basis risk while the former describes the local basis risk that remains even if monitoring takes place in close neighbourhood to the production site.

In the following, we deal with *local* basis risk, using the results of field trials where the weather data are recorded next to the experimental field. The example refers to starch potatoes that exhibit a remarkable dependency on weather variables, particularly rainfall, as can be seen from the correlation coefficients given in Table 1. While the accumulated rainfall between May and September is yield increasing, high temperatures during summer obviously have a negative impact. Trying different accumulation periods we found the highest correlation between the yield and the cumulative rainfall from May to September.

Table 1: Correlation coefficients

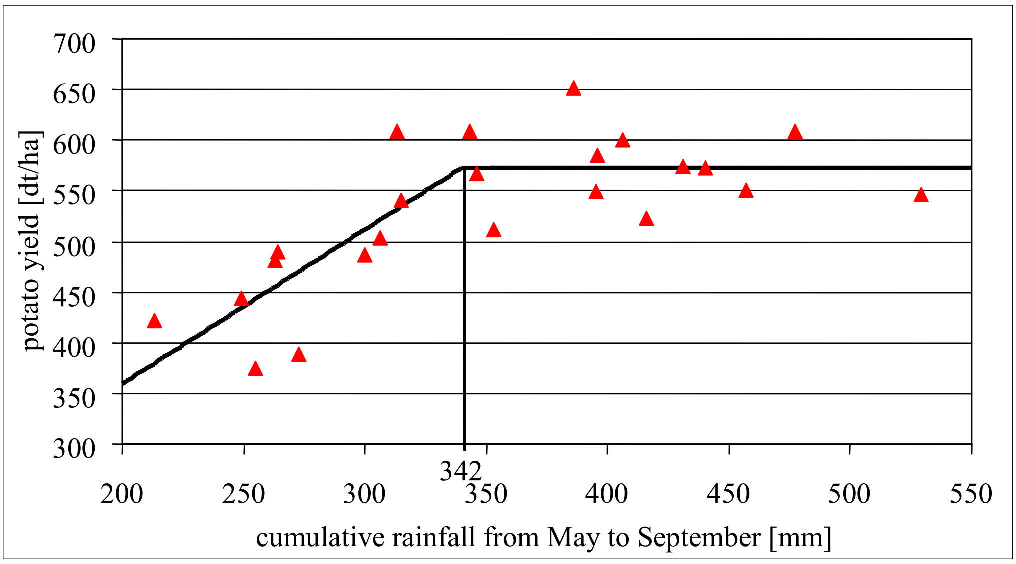
	Cumulative precipitation mm	Average temperature °C
April	-0.30	-0.01
May	-0.17	0.11
June	0.57	-0.20
July	0.47	-0.57
August	0.35	-0.24
September	0.27	0.07
May-September	0.67	0.02
Source: own calculation; data: chamber of Agriculture, Hanover		

Figure 3 depicts the relation between yield and precipitation using de-trended yield data of the years 1980 to 2002. According to the diagram, yield depression can be expected in years where the cumulative rainfall falls below around 340 mm. Above this amount yields remain largely constant. This relationship can be represented by a linear limited function:

$$y = Min\left[(a + b\,x), \hat{y}_{\max}\right] + e_B \qquad (8)$$

In the above equation y marks the estimated yield and x the cumulated rainfall from May to September. The parameters a and b are the constant and the slope of the linearly increasing function, respectively, $\hat{y}_{\max}$ is the maximum yield caused by increasing amounts of rainfall, and e_B represents an error term that accounts for the estimation error. Using least squares estimation leads to the parameters depicted in Table 2.

Figure 3: Yield response to rainfall



The graph in Figure 3 clearly indicates that a put option with the payoff structure given in equation (1) is an appropriate tool to hedge against the risk of low rainfall. The strike level K corresponds to the amount of rainfall that just leads to the maximum expected yield ($\hat{y}_{\max}$), i.e.:

$$K = \frac{\hat{y}_{\max} - a}{b} \tag{9}$$

The optimal tick size V can be expressed by the slope b and product price p_y:

$$V = b\ p_y \tag{10}$$

Since starch potatoes are subject to market regulations and because of the fairly low quality requirements the product price can be considered as almost deterministic. With these assumptions the revenue without derivative (W_0) is given by

$$W_0 = p_y\ y = p_y \left(Min\ [a + b\ x, \hat{y}_{max}\ ] + e_B \right) \tag{11}$$

In this equation the rainfall index x is a random variable. Thus, the variability of yield is determined by the variability of rainfall and the unexplained remaining variability e_B which then represents the basis risk. A χ^2 test of the residuals led to the conclusion that the normality hypothesis cannot be rejected at a 5 % error level. Thus, we assume e_B as normally distributed with mean 0 and a standard deviation of 43 dt/ha, as derived from the data.

Table 2: Parameters of yield response function and rainfall distribution

Yield response function					Rainfall May-Septemeber [mm]	
Parameters			Estimation error [dt/ha]			
$\hat{y}_{\max}$	[dt/ha]	573	Mean	0	Mean	353
a	[dt/ha]	55.3	Standard deviation	43	Standard deviation	82
b	[dt/ha/mm]	1.52	χ^2 *)	1.28	χ^2 *)	7.9
K	[mm]	342	Deg. of freedom	4	Deg. of freedom	8

*) Normality hypothesis accepted at 5 % error level

The total revenue with the put option (W_p) is composed of the market return as given in (11) plus the option payoff:

$$W_p = p_y \left(Min\,[\,a + b\,x, \hat{y}_{max}\,] + e_B \right) + V \cdot Max[0, (K - x)] - P_f$$

Using the relations given by (9) and (10) and rearranging the terms finally yields equation (12):

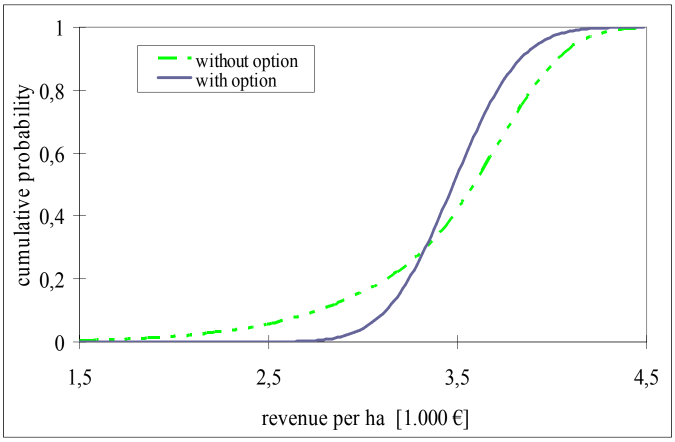
$$\begin{aligned} W_p &= p_y \left(Min[\,a + b\,x, \hat{y}_{max}\,] \; + e_B \right) + p_y\, b \cdot Max[0, (\,K - x\,)] - P_f \\ &= p_y \left(\hat{y}_{max} + Min[\,a + b\,x - (\,a + b\,K\,), 0\,] + e_B + Max[0, b(\,K - x\,)] \right) - P_f \\ &= p_y \left(\hat{y}_{max} + Min[\,b(\,x - K\,), 0\,] + e_B - Min[0, b(\,x - K\,)] \right) - P_f \\ &= p_y \left(\hat{y}_{max} + e_B \right) - P_f \end{aligned} \qquad (12)$$

As can be seen from equation (12), the total revenue with the put option is not dependent anymore on the rainfall index itself but only on the basis risk.

To compute the pair premium we need to analyse the historical rainfall data. The comparison of the empirical frequencies of the rainfall index from 1980 to 2002 with a normal distribution led to the conclusion that the normality hypothesis cannot be rejected at 5 % error level. Thus, the approach of equations (3) to (6) can be used to derive the fair premium. With the distributional parameters given in Table 2, an interest rate of 5 % p.a. and a duration of 5 months, the resulting fair premium amounts to 273 €/ha.

Figure 4 depicts the simulation results with and without the weather derivative[2]. The graph indicates that buying the option significantly reduces the risk of experiencing low returns. The standard deviation is almost cut by half and the 5 % and 10 % percentiles are notably shifted upward. The derivative turns the originally negatively skewed distribution into a symmetrical one, indicating that primarily downside risk is effectively reduced.

Figure 4: Simulation results for the potato example



	without option	with option
mean	3.483 €	3.480 €
Standard deviation	519 €	278 €
5% Percentile	2.457 €	3.023 €
10% Percentile	2.766 €	3.124 €
90% Percentile	4.026 €	3.838 €
95% Percentile	4.139 €	3.935 €
skewness	-1,14	0,00

The effect of the option would be less evident if the correlation was lower. The relatively high correlation is certainly influenced by the fact that the weather station is located next to the experimental field. While for temperature the geographical basis risk is less important (cf BERG et al. 2006a), in the case of rainfall it is certainly significant. MUßHOFF et al. (2005) have investigated the impact of geographical basis risk of rainfall for the state of Brandenburg using an empirically estimated de-correlation function. Their simulation results indicate that the risk reduction of a rainfall option – defined by the upward shift of the 5 % percentile –

[2] The results are based on 10000 random simulation runs.

drops by roughly 45 % if the index is measured in a distance of 25 km and by almost 70 % if the distance increases to 100 km.

While the geographical basis risk is something one has to live with since weather stations will rarely be located close to the field, the local basis risk may be reduced by using a portfolio of hedging instruments composed of set a of options based on different indexes. In our case the correlations given in Table 1 would call for the construction of additional put options based the monthly precipitation and call options based on the monthly temperature averages for June to August. Since the tick size *V* corresponds to the number of contracts with normalised payoffs, finding the best mixture then becomes a problem of portfolio selection.

4.2 Remaining price risk

While weather derivatives aim at reducing the risk associated with the uncertainty of yields, the price risk still remains with the farmer. Even if certain quantities are forward contracted at a fixed price a remaining risk can be caused by the fact that in case of low yields the producer is urged to purchase the shortfall quantity at uncertain market prices.

To analyse the impacts of price uncertainty we start looking at the difference in the variance of returns. From equation (12) the variance of revenues with the weather derivative can be derived as

$$Var\left(W_p\right) = p_y^2 \, Var\left(e_B\right) \tag{13}$$

where $Var(.)$ denotes the variance operator. Now let us assume that that the product price p_y is a normally distributed random variable. In this case we notice that for a product of random variables, i.e. $z = x \cdot y$, the variance of z can be computed by the following formula (BOHRN-STEDT and GOLDBERGER, 1969: 1439):

$$Var(z) = E(x)^2 \, Var(y) + E(y)^2 \, Var(x) + 2\, E(x)\, E(y)\, Cov(x,y) + Var(x)\, Var(y) + Cov(x,y)^2$$

The above formula, in which $Cov(x,y)$ represents the covariance between x and y, yields an exact measure of the variance if the density functions of the two random variables are symmetric. Otherwise the result is an approximation. Applying the above formula along with (12) and observing that the $E(e_B) = 0$ yields the variance of returns as:

$$Var\left(W_p\right) = E\left(p_y\right)^2 \, Var\left(e_B\right) + Var\left(p_y\right)\left[\, \hat{y}_{max}^2 + Var\left(e_B\right) \,\right] + Cov\left(p_y, e_B\right) \tag{14}$$

If the expected price $E(p_y)$ in (14) equals the deterministic price p_y in (13) the comparison of the two formulas shows that price uncertainty adds to the variance through the second and the third term of (13), where a negative correlation between price and yield reduces the variance as it constitutes a natural hedge[3]. Furthermore, the product of the second term indicates the interdependence between price uncertainty and the effectiveness of the weather derivative since $\hat{y}_{\max}$ is related to the contract parameters through (9).

To investigate the orders of magnitude of this interdependence, Monte Carlo simulation experiments were conducted using the former model, however, assuming a stochastic price p_y that is normally distributed with a mean of 6.55 €/dt and a standard deviation of 1 €/dt, representing a coefficient of variation of roughly 15 %. The simulation results are depicted in Table 3. Hedging effectiveness is measured by the reduction of the standard deviation through the derivative and by the upward shift of the 5 % percentile, the latter especially referring to reduction of downside risk. As can be seen from the figures, even a moderate volatility of prices cuts the risk reduction due to the weather derivative by more than half.

[3] Note that the expected value of a product of random variables too is increased by a positive and decreased by a negative covariance.

Table 3: Influence of price uncertainty on the effectiveness of weather derivatives

	Reduction of standard deviation		Shift of 5 % percentile	
	€/ha	%	€/ha	%
Deterministic price (6.55 €/dt)	241	46.4	566	23.0
Stochastic price *)	102	13.7	222	9.9

*) Normally distributed with mean 6.55 €/dt and standard deviation 1 €/dt

4.3 Diversification effects

Farmers have a variety of opportunities to influence the risk exposure of their operations. Among them the diversification of the production program plays an important role. This is particularly true for Europe where farms are typically set up as multi-commodity operations. If a diversified production program already exists additional hedging instruments are less valuable than in case of a high degree of specialisation.

This effect shall be illustrated in the following section, using an expected value-variance (EV) framework, i.e. we define the certainty equivalent (CE) as expected income minus a risk premium, where the latter is expressed using the PRATT approximation (cf. ROBISON and BARRY, 1987: 34). Assuming constant absolute risk aversion the certainty equivalent CE is given by

$$CE = E(y) - \frac{\lambda}{2} Var(y)$$

where $E(y)$ denotes expected income, $Var(y)$ is the variance of income and λ represents the coefficient of absolute risk aversion. For simplicity let us assume that the expected returns of all activities are the same, so we can limit the analysis to the variance. Considering n production activities realised in quantities q_i, the variance becomes

$$Var(y) = \sum_{i=1}^{n} \sigma_i^2 q_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i q_j \, cov_{ij} \quad (15)$$

where σ_i^2 represents the variance of the return of the i-th activity, and cov_{ij} denotes the covariance of returns between the activities i and j. If we assume a portfolio of activities in which all quantities are equal, i.e. $q_i = 1/n$ the above equation becomes

$$Var(y) = \frac{1}{n^2} \sum_{i=1}^{n} \sigma_i^2 + \frac{2}{n^2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} cov_{ij} \quad (16)$$

We now observe that a portfolio of n elements is comprised of $n(n\text{-}1)/2$ covariances. Thus we can define an average covariance as

$$\overline{COV} = \frac{2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} cov_{ij}}{n\,(n-1)} \quad (17)$$

On substituting the second term in equation (16) by this relation the variance of the portfolio becomes

$$Var(y) = \frac{1}{n^2} \sum_{i=1}^{n} \sigma_i^2 + \frac{n-1}{n} \overline{COV} \quad (18)$$

On introducing the average variance $\overline{\sigma}^2$ this equation further reduces to

$$Var(y) = \frac{1}{n}\overline{\sigma}^2 + \frac{n-1}{n}\overline{COV} \tag{19}$$

Let us assume identically distributed returns for all activities. From $cov_{ij}=\sigma_i \cdot \sigma_j \cdot \rho_{ij}$ where ρ_{ij} marks the correlation coefficient we can rewrite the average covariance as

$$\overline{COV} = \overline{\sigma}^2 \overline{\rho}$$

where $\overline{\rho}$ marks the average correlation coefficient. Equation (19) then becomes

$$Var(y) = \frac{1}{n}\overline{\sigma}^2 + \frac{n-1}{n}\overline{\sigma}^2\,\overline{\rho} = \frac{\overline{\sigma}^2}{n}\left(1 + (n-1)\overline{\rho}\right) \tag{20}$$

The above equations indicate that the portfolio risk decreases as n increases, however at diminishing rates. As the term $(n\text{-}1)/n$ approaches 1 for large n the portfolio variance reduces to the average covariance which is not diversifiable. If the returns are stochastically independent, i.e. the correlation coefficients and covariances are zero, the risk is completely diversifiable. If the correlation coefficients amount to +1 no diversification effect occurs as can be seen from equation (20). In turn, at correlation coefficients of -1 the portfolio variance completely vanishes already at n=2.

Now assume that in a production program composed of n commodities a derivative is introduced to hedge against weather risk for one commodity. This can be represented by replacing the i-th element in (18) by one that exhibits a reduced variance, i.e. σ_i^2 is replaced by $\tilde{\sigma}_i^2$. For simplicity, assume that the average covariance remains unchanged. Then the difference of the portfolio variance caused by the derivative is given by $Var(y) - Var'(y)$, where in $Var'(y)$ the reduced variance is considered. Expanding the summation in (19) we can write:

$$Var(y) = \frac{1}{n^2}\left(\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_i^2 + \ldots + \sigma_n^2\right) + \frac{n-1}{n}\overline{COV}$$

and

$$Var'(y) = \frac{1}{n^2}\left(\sigma_1^2 + \sigma_2^2 + \ldots + \tilde{\sigma}_i^2 + \ldots + \sigma_n^2\right) + \frac{n-1}{n}\overline{COV}$$

From this the difference $Var(y) - Var'(y)$ can be derived as:

$$Var(y) - Var'(y) = \frac{\sigma_i^2 - \tilde{\sigma}_i^2}{n^2} \tag{21}$$

From equation (21) we can see that within a portfolio of n activities the variance reducing power of a single derivative is downscaled by n^2. For a farm with a broadly diversified production program weather derivatives are therefore of much less value than for a highly specialized operation. Model calculations by SCHMITZ for a farm in Germany that grows five different crops clearly demonstrate this effect: a rainfall based weather derivative that is introduced for onions reduces the variance of the total profit only by 7.5 % (SCHMITZ, 2007: 123ff).

While this effect as such is fairly general, the extent to which it becomes palpable depends on the nature of the derivative. The relation of formula (21) applies only if the derivative is highly specific in the sense that it only effects the variance of returns of a single commodity. In most cases, though, the weather index will be correlated with the yields of several commodities which, in turn, will enhance the reduction of the portfolio variance. However, if such cross effects exist they must be considered in the construction and in the valuation of the derivatives, what further adds to the complexity of the problem.

5 Assessing the value of weather derivatives in the context of portfolio optimisation

The discussion so far has shown that an integrated approach to risk management is necessary that can best be characterised as portfolio optimisation. Portfolio selection is often associated with MARKOWITZ's approach of determining an expected value-variance-efficient frontier. In this setting the expected value serves as a measure of worth while the variance is used to assess the risk that must be assumed in order to achieve a certain level of expected income. Thus, the MARKOWITZ model can be viewed as a particular member of a more general class of models which are often referred to as *risk-value-models*. In general, the preference function of a risk-value model is defined as

$$\Phi(F_i(x)) = H\big(W[F_i(x)], R[F_i(x)] \big) \tag{22}$$

where $W[F_i(x)]$ is the measure of worth and $R[F_i(x)]$ represents the risk measure. $F_i(x)$ marks the cumulative distribution function of the risky prospect i and $H(.)$ determines the trade-off between risk and worth according to the decision maker's preferences. The usual assumption is that $H(.)$ grows with increasing worth and falls with increasing risk. Neither the value measure nor the risk measure depends on wealth. Only the trade-off function is wealth dependent. If the decision maker is able to specify the trade-off function, comparing pairs of distributions leads to an optimal choice. If $H(.)$ remains unspecified, it is still possible to determine the efficient set consisting of the distributions which are not dominated. A distribution $F_i(x)$ dominates the distribution $F_j(x)$ if the condition

$$W[F_i(x)] \geq W[F_j(x)] \quad \text{and} \quad R[F_i(x)] \leq R[F_j(x)]$$

holds with at least one strict inequality (FISHBURN 1977: 118). All non dominated alternatives lie on the efficient frontier which can be determined by solving the optimisation problem

$$\begin{aligned} & W[F(x)] \rightarrow Max! \\ & \text{subject to} \\ & R[F(x)] \leq c \end{aligned} \tag{23}$$

where c must be varied across all possible numerals of $R[F(x)]$.

While the appropriateness of risk measures is still controversially discussed in the relevant literature it is widely agreed that the expected value is the best measure of worth in risk-value models, i.e. $W[F(x)] = E[x]$. In the MARKOWITZ approach the risk measure is given by the variance, i.e. $R[F(x)] = E[(x-\mu)^2]$, where μ denotes the mean and $E[.]$ represents the expectation operator. The disadvantage of this approach is two-fold: First, the EV-approach yields similar results as the more general expected utility (EU) approach only if the distribution of outcomes is not very skewed. Secondly, when moments of the distribution are used as risk measures, the mean is (implicitly) considered as the relevant target and risk is quantified using the magnitude of deviations from this target. Since the target is determined endogenously these measures do not change if a certain amount d is added to an uncertain outcome X, i.e. $R[X]=R[X+d]$. Contrary, if the target is determined exogenously, adding a certain quantity to an uncertain prospect reduces the risk associated with it, i.e. $R[X]>R[X+d]$. Generally, one would consider a situation to be less risky if a certain income is earned in addition to the uncertain prospect.

This leads to a further class of risk measures which explicitly refer to downside-risk in the sense that only those outcomes are considered that are worse than some specific target. This class of measures dates back to the work of FISHBURN (1977) and was revisited by SARIN and WEBER (1993). Considering only the lower part of the distribution, these measures account for the downside-risk and are called *lower partial moments* (*LPM*). They are defined as

$$LPM_n(z) = \int_{-\infty}^{z} (z-x)^k f(x)\,dx \quad (k \geq 0) \tag{24}$$

Setting the target *z* and the order *k* of the LPM yields a specific measure. Basic cases that play an important role in applications, are obtained for $k = 0$, 1 and 2. Setting k=0 yields the *shortfall probability* $LPM_0(z)$ that is closely related with the *value-at-risk*:[4]

$$LPM_0(z) = \int_{-\infty}^{z} (z-x)^0 f(x)\,dx = F(z) \tag{25}$$

For k=1 the resulting measure is the *shortfall expectation*:

$$LPM_1(z) = \int_{-\infty}^{z} (z-x)^1 f(x)\,dx = E[z-x \mid x<z]\,F(z) \tag{26}$$

$LPM_1(z)$ denotes the (conditional) expected value of shortfalls multiplied by the probability of the occurrence of below target returns. Thus, it accounts for the probability as well as for the magnitude of shortfalls. Finally k=2 leads to the *shortfall variance*

$$LPM_2(z) = \int_{-\infty}^{z} (z-x)^2 f(x)\,dx = E[(z-x)^2 \mid x<z]\,F(z) \tag{27}$$

the square root of which denotes the *shortfall standard deviation*. Here the squared downside deviations from the target are considered in the risk measure.

The preference function of the risk-value model using the expected value $E[x]$ as value measure and a lower partial moment $LPM_k(z)$ as risk measure can be stated as

$$\Phi(F(x)) = E[x] - c\,LPM_k(z) \tag{28}$$

where c>0 denotes die weighting factor and *k* is the order of the LPM. Increasing *c* therefore means increasing risk aversion. SCHNEEWEIß (1967: 89ff) has shown that the corresponding utility function has the following form:

$$u(x) = \begin{cases} x & \text{if } x > z \\ x - c\,(z-x)^k & \text{if } x \leq z \end{cases} \tag{29}$$

As can be seen from (29) above the target level all three cases result in the same utility function which is given by $u(x) = x$. The differences between them occur in the range where *x* falls below the target. For k=0 the utility function is linearly increasing at a constant slope but has a discontinuity at the target *z*. This does neither allow a general statement about the decision maker's attitude towards risk nor is it consistent with decision theory. Therefore this measure receives no further attention in our paper.

The *shortfall expectation*, i.e. k=1, considers not only the shortfall probability but also its extent. The corresponding utility function is piecewise linear with the steeper slope in the lower part. Only if all possible outcomes fall either below or above the target level, respectively, the utility function implies risk neutral behaviour. Otherwise the shape of the utility function is approximately concave and therefore implies risk aversion.

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[4] For details on the value-at-risk concept see e.g. JORION (1997), MANFREDO and LEUTHOLD (1999)

The use of higher order LPMs, i.e. higher values of *k*, implies stronger local risk aversion in the lower part of the domain while above the target local risk neutrality remains. Using $LPM_2(z)$, i.e. the *shortfall variance*, the shortfalls are squared, thus giving particular weight to the higher losses. The corresponding utility function is quadratic in the range below the target level and therefore also implies risk aversion. Different from the former case, the utility function is strictly concave in the lower part.

From the above framework the shortfall expectation and the shortfall variance appear as suitable risk measures for risk averse decision makers. Since a desirable feature of any measure is that it has an obvious meaning for the decision maker the shortfall expectation is particularly appealing. Model experiments conducted by STARP have shown that at moderate degrees of risk aversion the results obtained by the expected value-LPM_1 approach are very similar to those of the EU approach (STARP, 2006; BERG and STARP, 2006). At higher degrees of risk aversion, however, the approaches yield different results because the LPM_1 model cannot approximate the utility function close enough. Using higher order LPMs might lead to improvements, but only at the expense of losing much of the understandability of the risk measure.

Implementing the general approach given in (23) using LPM_1 as risk measure implies that the expected profit enters the objective function while the risk measure is considered as a constraint. Thus, the objective function is to select the portfolio of activities **x** that maximizes the expected profit π

$$\max_{\mathbf{x}} \int_0^\infty \int_0^\infty \pi(\mathbf{p}, \mathbf{y}, \mathbf{x})\, g(\mathbf{p}, \mathbf{y} \mid \mathbf{\Omega})\, d\mathbf{p} d\mathbf{y} \tag{30}$$

subject to the resource constraints $\mathbf{A\,x} \le \mathbf{b}$ and the constraint on the risk measure $LPM_1(z) \le c$, where c is parameterised in order to compute the efficient frontier. In (30) the term $\pi(\cdot)$ denotes the profit function and $g(\cdot|\mathbf{\Omega})$ is the joint density function of prices and yields conditional on $\mathbf{\Omega}$, the set of information available when the portfolio is selected. The random price vector **p** consists of cash prices for all products and in addition futures and forward contract prices as far as they are available. The random yield vector **y** contains the individual crop yields. The resource constraints reflect the physical capacities of the farm as well as institutional constraints, e.g. rotational restrictions and agricultural policy regulations. Finally, the vector of activities **x,** besides the production processes also contains risk management measures including hedging price risks with futures and options and production risks with weather derivatives. While this approach is certainly complex it is the only way to assess the value of risk management tools comprehensively.

6 Conclusions

The discussion has shown that a comprehensive assessment of hedging instruments, including weather derivatives, requires an integrated approach as suggested in this paper. Since most research so far focuses on single instruments more research is needed to this end. Important aspects to be considered with regard to weather derivatives include structured (i.e. combined) contracts and cross effects resulting from the fact that most weather indexes are correlated with the yields of several crops.

Selecting an optimal portfolio of hedging instruments is a complex task. It can therefore be doubted that farmers – besides all other tasks they have to fulfil in their predominantly small to medium sized operations – will ever be able to successfully cope with this problem. Left to themselves they would certainly be overcharged. Instead one could imagine that other institutions, possibly formed under participation of agricultural commerce, the banking sector, insurance companies and the extension service, take over the task of creating and managing such portfolios that fit the needs of certain farm types. The farmers themselves would then only have to deal with one aggregate instrument aimed at reducing their downside-risk of income.

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