A MONTE CARLO METHOD FOR THE STUDY OF FARM PLANNING PROBLEMS

Mårten Carlsson*, Bertil Hovmark†, and Ingvar Lindgren‡

A Monte Carlo method for studying farm planning problems is developed. The method allows the building of a flexible model including several objective functions, economies of scale, integer formulations and interactions between activities arising from, for example, rotational yield effects. The maximum number of activities in the solutions can be determined in advance. Minimum levels for the activities can be specified to eliminate levels of no practical interest. Many solutions are obtained between which the decision-maker can choose. A statistical analysis of the solutions for different levels of the objective functions gives an informative survey of the possible plans. Decision-making based on the Monte Carlo output may be of special interest with regard to "the behavioural theory of the firm".

The Monte Carlo method used here for studying farm planning problems is, broadly speaking, a process based on random number techniques for seeking and examining the maximum (maxima) and the region(s) near the maximum (maxima) of a function of several variables subject to given constraints. The method was first put forward by Lindgren and Carlsson [8]. The same idea has recently been adopted by different authors on similar problems [4], [5], [12]. A comparison of the method presented here with linear programming and programme planning methods has been undertaken by Stryg [10], [11] and by Dent and Thompson [4].

The present paper begins with a formal description of the Monte Carlo method (model and seeking process). Then the application of the method to a practical farm planning problem is discussed.

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* Department of Horticultural Economics, The Agricultural College of Sweden, Alnarp.

† Department of Economics and Statistics, The Agricultural College of Sweden, Uppsala.

‡ Professor of Physics, Chalmers University of Technology, Gothenburg, Sweden.

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1 DESCRIPTION OF THE MONTE CARLO METHOD

1.1 INTRODUCTION

A Monte Carlo or random number technique can in many cases be a useful tool for seeking maxima or minima of a function of several variables (see for example [1] and [2]). In its simplest form, this technique implies that each variable is randomly allotted a value within given limits, and the value of the function is calculated. This is repeated a great number of times, and the best result constitutes the solution. If constraints exist, it is of course necessary first to ensure that these are fulfilled. If they are not fulfilled, the solution is rejected. If the number of variables is large and if the functions are not too rapidly fluctuating, a great number of combinations of the variables exist which give the function values close to the maximum. In order to find one of these combinations, it may therefore be sufficient to test only a small part of all possible combinations, provided that the seeking is made randomly. However, in practical farm planning problems, this simple method is found to be very uneconomic. For this reason a more complicated method has been developed.

1.2 MATHEMATICAL DESCRIPTION

The Monte Carlo method we have developed for farm planning problems is based on the following model.

One or more objective functions are given

$$(1) \qquad Z(x_1, x_2, \ldots, x_j, \ldots, x_n)$$

where x_j is the level of the j-th activity. These functions can be of arbitrary form. The variables, which may be integers or non-integers, can either be in the region

$$x_j\ min \leqslant x_j \leqslant x_j\ max$$

or can be zero. Of the n variables, k are assumed to be *independent* activities, which means that their levels can be chosen independently of each other. The remaining n-k variables are *dependent* activities. Their levels are determined by the levels of the independent activities via the equation:

$$(2) \qquad x_{k+h} = \sum_{j=1}^{k} D_{h,j} x_j \qquad (h = 1, 2, \ldots, n-k)$$

$D_{h,j}$ can have almost arbitrary form and can also be a function of the levels of one or more activities.

The general form of the constraints is

$$(3) \qquad \sum_{j=1}^{k} R_{ij} \leqslant b_i \text{ where } \begin{cases} R_{ij} = q_{ij} + a_{ij}x_j & \text{if } x_j \neq 0 \\ R_{ij} = 0 & \text{if } x_j = 0 \quad (i = 1, \ldots, m) \end{cases}$$

where a_{ij} and b_i are non-negative constants; and

$q_{ij} + a_{ij}x_j\ min \geqslant 0$ but q_{ij} is otherwise arbitrary.

Note that the constraints in (3) above contain only the independent activities. However, if in the original formulation of the problem dependent activities appear in the constraints, they have to be eliminated by equation (2) so that the constraints will be of the form (3).

In principle it is possible to handle more complicated constraints than those given in (3) with the Monte Carlo method. However, with our our special seeking process, a more complex form of the constraints would involve considerable complications for reasons to be discussed later. Because the form of the constraints is already sufficiently flexible for our purposes, we have not yet studied more complicated forms empirically.

In table 1 a generic Monte Carlo matrix is given. Our Monte Carlo method now proceeds as shown in figure 1. The operation involves four phases:

Phase I. One of the k independent activities is chosen randomly and is randomly ascribed a level within the given limits. Then a check is made to ensure that all constraints (3) are satisfied. If not, the level of the activity is decreased sufficiently to fulfil the constraints. If the level is then smaller than the minimum, x_j *min*, it is set to zero. This procedure is repeated until a given number of activities have entered the solution (at a non-zero level) or until all independent activities have been tried once.

Phase II. In this phase the activities chosen in phase I are examined once more in the same order. The level of each activity is increased as much as the constraints allow.

Note that phase II causes all solutions to fall on the surface of the "sphere" formed by the constraints. In some cases solutions inside this "sphere" are also of interest. Phase II is then eliminated (see section 4.3).

Phase III. The levels of the dependent activities are calculated from equation (2).

Phase IV. The values of the objective functions (1) are calculated.

Phases I–IV can be illustrated by figure 2 which presents a traditional product-mix problem with two products, x_1 and x_2. The levels of these activities may lie in the regions x_1 *min* to x_1 *max* and x_2 *min* to x_2 *max* respectively, or they may be zero. The available resources allow x_1 to take any level below x_1 *max* and x_2 to take any level below x_2 *max* if the level of the other activity is zero. The constraints are of the form shown in equation (3). Since both x_1 and x_2 are assumed to have positive q_{ij} terms, no combination of the two activities above and to the right of the lines *AB*, *CD*, and *EF* in the diagram is feasible. All integer solutions within the constraints are encircled in the diagram. The only integer solutions which can be obtained if the adjustment procedure in phase II is used, are marked by heavy circles.

The seeking process in this simple case may be illustrated by the following examples. Of the two products x_2 might be chosen first, and randomly given the level x_2'. Then x_1 might be given the level x_1'. This gives point *P* in the diagram. This point is within the constraints. In phase II x_2 is first increased as much as possible with respect to the constraints.

TABLE 1

Generic Monte Carlo Matrix

		Independent activities					Dependent activities				
		P_1	…	P_j	…	P_k	P_{k+1}	…	P_{k+h}	…	P_n
Objective functions	Z_l	z		z		z	z		z		z
	Z_g	z	The objective functions can practically be of an arbitrary form.						z		z
	Z_n	z		z		z	z		z		z
Constraints	b_1	R_{11}		R_{1j}		R_{1k}					
	b_2	R_{21}	$R_{ij} = q_{ij} + a_{ij} \cdot x_j$ if $x_j > 0$ $R_{ij} = 0$ if $x_j = 0$			R_{2k}	Any parameters that may occur in this square have to be eliminated before the analysis by equation (2).				
	b_i	R_{i1}		R_{ij}		R_{ik}					
	b_m	R_{m1}		R_{mj}		R_{mk}					
Equation for calculation of dependent activities (eq. 2).		$D_{1,1}$		$D_{1,j}$		$D_{1,k}$	x				
		$D_{2,1}$	$D_{h,j}$ can be of almost an arbitrary form.			$D_{2,k}$					
		$D_{h,1}$		$D_{h,j}$		$D_{h,k}$			x		
		$D_{n-k,1}$		$D_{n-k,j}$		$D_{n-k,k}$					x
x_j *min* . .		x		x		x					
			These values are given according to economic considerations.								
x_j *max* . .		x		x		x					

This will give the level x_2'' (point Q). At this point Q it is still possible to increase the level of x_1 to x_1'' (point R). The point R will in this case be the solution. If instead x_2 had been given the value x_2^* and x_1

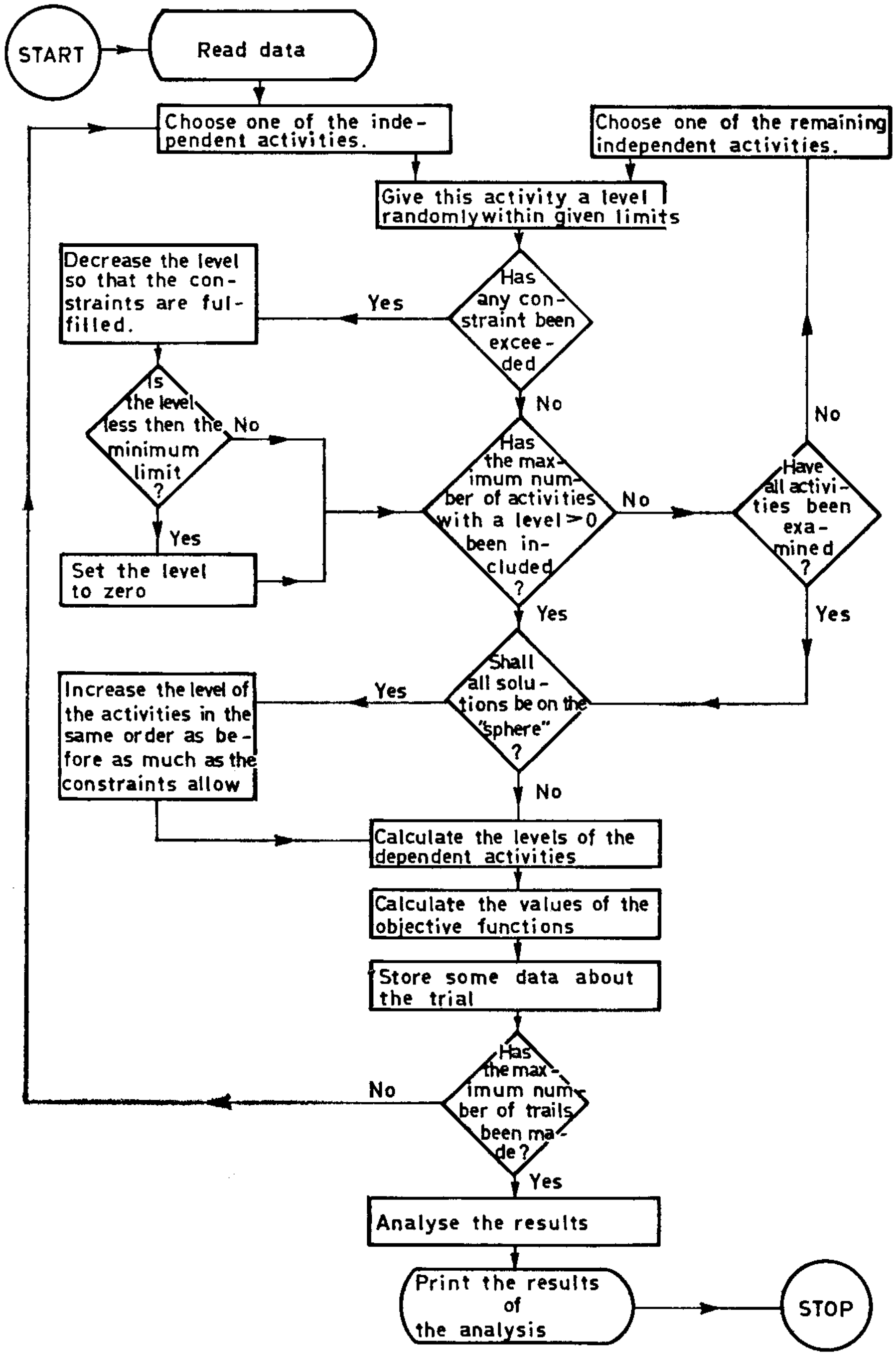

FIGURE 1

Flow Chart for the Monte Carlo Method

the value x_1* (point *S*), the constraints would have been exceeded. In this case the level of the last activity is decreased to x_1** (point *T*). The final solution in this case is point *U* (after applying phase II). If solutions inside the "sphere" are of interest, phase II above is eliminated. The two solutions would then be points *P* and *T*.

The main difference between this Monte Carlo method and the technique used in most other applications is that the levels of the activities are adjusted so that the constraints are automatically fulfilled. If the levels are chosen completely at random the constraints may be satisfied only in

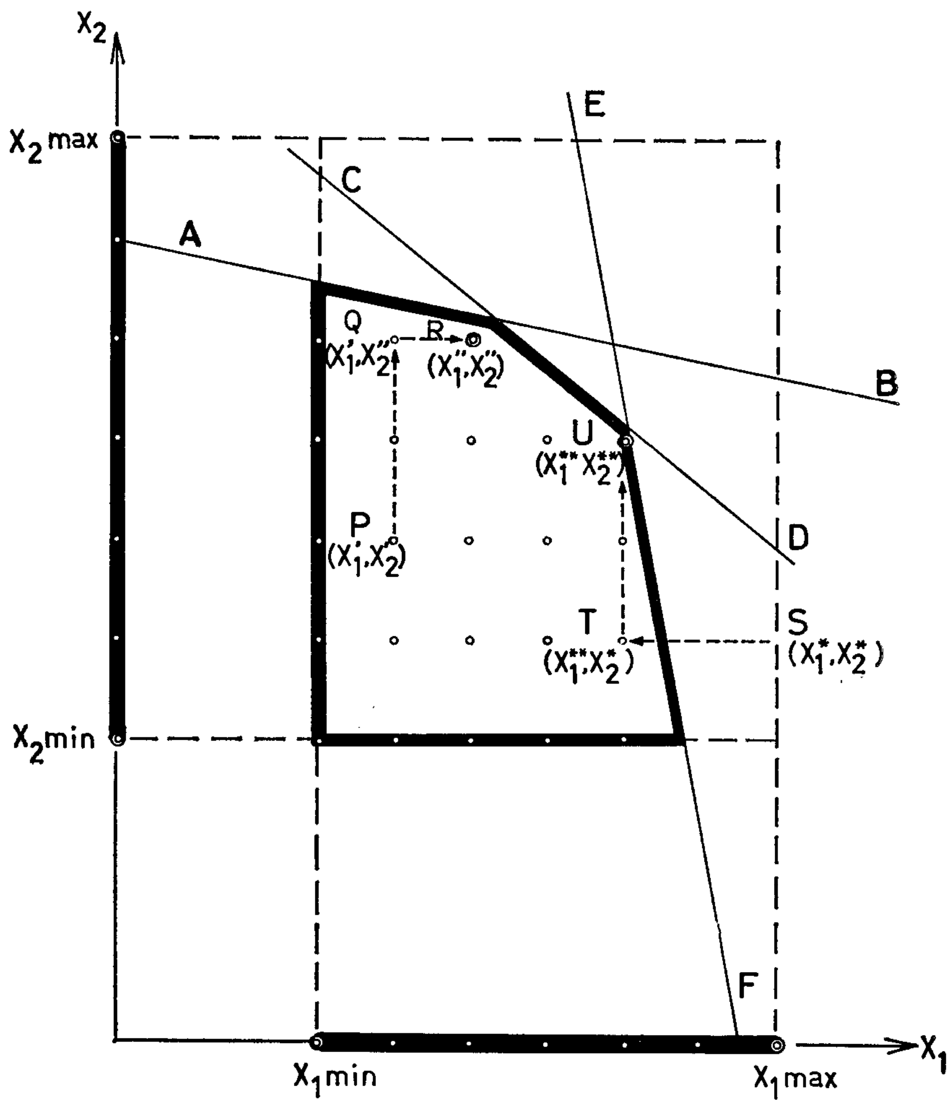

FIGURE 2

Diagram of a Simple Product Mix Problem Illustrating the Seeking Process in the Monte Carlo Method a very small fraction of the trials and most of the trials have to be rejected. Furthermore, if in our method the adjustment process in phase II is used, all the solutions will be on the surface of the "sphere" formed by the constraints. This means that no activity can be increased unless another activity is decreased. This adjustment procedure is particularly simple if the constraints are linear and only include the independent activities (see equation (3)).

2 A PRACTICAL EXAMPLE

2.1 DESCRIPTION OF THE PROBLEM

The planning example[1] chosen for demonstration of the Monte Carlo method is presented in table 2. The problem is to determine combinations of 9 vegetable ($P_1 - P_7$, P_{19}, P_{20}) and 11 animal ($P_8 - P_{18}$) activities. The feasible plans are limited by the resources ($b_1 - b_9$) and rotation limits ($b_{10} - b_{12}$).

Activities

Internal production and consumption can only be handled if there is also a market (selling and buying possibilities) for the product in question. In this example, there are no such markets for grassland products (hay, pasture). Therefore, animal activities which need grassland ($P_8 - P_{14}$) must be constructed as "combined activities" including both the animal and its need for grassland. The other activities need no special explanation (cf. an ordinary linear programming matrix).

Restrictions

Rows *A*, *B*, *E:* These restrictions for arable land, building area and rotation limits are of ordinary LP-form and thus need no special explanation. (Rotation limits are also included in the x_j max values in row *I*).

Row *C:* Restrictions of this kind are called "combination restrictions". They are used here to regulate the maximum number of animal activities that can share the same building. If $b = k$ and $q_j = 1$ for certain j, only k such activities can enter the solution. Restrictions of this kind can of course also be used to regulate the maximum number of activities from other subgroups of activities. The total number of activities in a solution is regulated by special input data. Notice that restrictions of this kind do not influence the level of the chosen activities.

Row *D:* In this row the general form $(q + ax)$ for labour restrictions is given. This form takes into consideration the fact that labour use per unit decreases with increasing level of the activity $(q/x + a)$.

Equation for calculation of dependent activities

Row *F:* *D* here stands for the "production of grassland" from the combined activities $P_8 - P_{14}$, and is used for calculation of the total grassland area in the plan (P_{19}, P_{20}). Notice that the combined activities include use of resources for arable land and labour for grass production; therefore P_{19} and P_{20} have no resource use.

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[1] In [8] a linear programming approximation of this example (proportional relations, only one objective function, etc.) is analysed and compared with the Monte Carlo solution.

Row G: Here D stands for production and consumption of grain, piglets, pork and so on. The differences between production and consumption constitute the selling and buying activities $P_{21} - P_{31}$. The possibility of giving D an almost arbitrary form is used here to let the yield of grain per hectare (P_1, P_2) vary with the total area used for these activities. In this case three yield values are given for P_1 and P_2. Which of these yields is used depends on the sum of x_1 and x_2 in the actual plan. A continuous function could also have been used if its form were known.

Minimum and maximum limits for activities

Rows H, I: In these rows minimum and maximum limits for the activity levels are given. The minimum limits are used to eliminate levels of no practical interest. The maximum limits often depend on rotation restriction ($P_1 - P_7$) or building capacities ($P_8 - P_{18}$). Maximum and minimum limits can also be used to eliminate levels for which some activity parameters are not valid.

Objective functions

The objective function Z_1 gives the *gross margin* for each calculated plan. As mentioned earlier, the objective function can be of arbitrary form. For P_1 to P_{20} gross margin per unit is constant. For P_{21} to P_{26} different values are given for selling and buying. For some of these activities (piglets, grain), the buying prices are discontinuous functions of the quantity bought (owing to quantity discounts). Depending on the total amount of bacon pigs sold per year (P_{31}), different quantity premiums are added to the gross margin.

The objective function Z_2 describes the *standard deviation* in gross margin and is discussed later under multi-objective problems.

2.2 THE MONTE CARLO OUTPUT

The calculated solutions are the primary output from the computer. For these solutions the values of Z_g, the activity-levels (x_j) and the amount of unused resources $\left(b_i - \sum_{j=i}^{k} R_{ij}\right)$ are stored on magnetic tape.

These data may be used for various further calculations. Some examples are described below.

First we consider a reduced problem with only one objective function, (Z_1). For such problems the following information is received:[2]

(a) All solutions (plans) with Z_1 above a certain value, or a given number of the solutions with the highest Z_1-values. For these solutions the values of Z_1, the activity levels (x_j), and the amounts of unused resources, are given (see table 3).

[2] The CDC-3600 computer calculates 20 solutions per second for this problem.

TABLE 2

				Independent activities			Dependent activities	
					Animal			
				Vegetable (excluding grassland)	With need for grassland	Without need for grassland	Grassland	Buying and selling
				P_1–P_7	P_8–P_{14}	P_{15}–P_{18}	P_{19}–P_{20}	P_{21}–P_{31}
	Row No. in original matrix and text	b_i	Z_1 Z_2	z z	z z	z z	z z	z z
A ..	1 Arable land	b	$\geqslant$	a	a			
B ..	2-4 Building area	b	$\geqslant$		a	a		
C ..	3-5 Combination restrictions ..	b	$\geqslant$		q	q		
D ..	6–9 Labour	b	$\geqslant$	$q + a$	$q + a$	$q + a$		
E ..	10–12 Rotation limits	b	$\geqslant$	a	a	a		
F ..	13–14 Grassland				D		x	
G ..	15–24 Selling-buying			D	D	D		x
H ..	x_j *min*			x	x	x		
I ..	x_j *max*			x	x	x		

NOTES FOR TABLE 2:

	Independent activities	*Unit*
P_1	Winter rye	1 ha.
P_2	Barley	1 ha.
P_3	Winter rape	1 ha.
P_4	Early potatoes	1 ha.
P_5	Late potatoes	1 ha.
P_6	Sugarbeets	1 ha.
P_7	Swedes	1 ha.
P_8	Milking cows I	1 head + required grassland area.
P_9	Milking cows II	1 head + required grassland area.
P_{10}	Heifers I	1 head + required grassland area.
P_{11}	Heifers II	1 head + required grassland area.
P_{12}	Young bulls for slaughter I	1 head + required grassland area.
P_{13}	Young bulls for slaughter II	1 head + required grassland area.
P_{14}	Baby beef	1 head + required grassland area.
P_{15}	Sows I	1 head.
P_{17}	Sows II	1 head.
P_{18}	Bacon pigs II	1 head.

	Dependent activities	*Unit*
P_{19}	Grassland for hay	1 ha.
P_{20}	Grassland for pasture	1 ha.
P_{21}	Fodder grain (rye and barley)	100 kg*.
P_{22}	Piglets	1 piglet*.
P_{23}	Heifers	1 heifer*.
P_{24}	Suckling calves	1 calf*.
P_{25}	Sugarbeet pulp	1 ton*.
P_{26}	Sugarbeet tops	1 ton*.
P_{27}	Fertilizer	100 kr (value)†.
P_{28}	Meat	100 kr (value)†.
P_{29}	Pork	100 kr (value)†.
P_{30}	Concentrate	100 kr (value)†.
P_{31}	Selling bacon pigs	1 pig‡.

* Selling and buying activities.

† Selling and buying activities only used for calculation of Z_2.

‡ Involves calculation of quantitative premium.

TABLE 3

Some Solutions

Sol. No.	Gross margin (Z_1)	Standard deviation (Z_2)	Activity No.*																				Unused labour hours (Resource No. 9)
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Ten best solutions with respect to $Z1$: Run No. 2.																							
a	101,164	43,541		14		1	7	5	5								33			313			447
b	101,151	43,691	1	13		2	6	5	5								33		27				315
c	100,889	42,436		14			8	6	4								33		27				415
d	100,682	42,379	1	13			8	6	4								33		27				351
e	100,636	34,211	1	9		1	5	5	5		14					12	33				1·74	4·06	7
f	100,571	44,154	2	12		1	7	5	5									383	27				425
g	100,439	36,003		9		2	6	4	5		14						33				1·26	4·06	23
h	100,360	43,352	4	10		1	7	5	5								33			313			395
i	100,360	36,080	5	4		2	6	5	5		10					18	33				1·62	2·90	26
j	100,342	30,917		9			5	6	4		21						33				1·89	6·09	1
Some extreme solutions: Run No. 2.																							
k	98,422	43,703	1	13		3	5	5	5									382	27				571
l	98,026	49,331	2	12		1	7	5	5									383		313			568
m	98,268	33,687	1	8		2	4	5	5		12		5			9	33				1·84	5·03	
n	98,080	34,020		7			8	5	4		11			9			33				2·25	5·62	59
Solutions (points) on the "efficiency line" (in figure 8, line 3): Run No. 3.																							
o	100,539	34,010		9		2	4	5	5		18						33				1·62	5·22	17
p	100,342	30,917		9			5	6	4		21						33				1·89	6·09	1
q	96,506	27,818		8			6	7	3		21						27				1·89	6·09	149
r	86,341	21,919		9		1	5	8	1		21						22				1·89	6·09	691
s	85,774	21,617	2	8			4	8	2		19					5	23				1·91	5·51	721
t	79,383	18,532	2	9	1		3	8	1		21						22				1·89	6·09	1,029
Original limits (Rows H and I in the matrix).																							
Min. limit	..	..	1	1	1	1	1	1	1	10	10	3	3	5	5	5	5	50	5	50			
Max. limit	..	..	5	16	8	8	8	8	5	22	22	27	27	27	33	33	33	383	27	313			

* For explanation of activities and units, see table 2.

(b) The number of solutions in different intervals of Z_1 (see figure 5a).

(c) Statistics on the activities in different intervals of Z_1 (see figure 3):

(i) Number (absolute and relative) of solutions in the interval which contain the relevant activity.

FIGURE 3

Graphs of the Statistics for Some Activities

On the horizontal axis the Z_1-values (gross margin in 1,000 Sw. kr.) are given. The vertical axis gives the activity levels. The figures below the horizontal axis give the relative abundance of solutions containing the activity in the relevant Z_1-interval. The upper line gives the realized maximum level in the relevant Z_1-interval, and the lower the corresponding minimum level. The dashed line gives the average level in solutions with $x_j \neq 0$. The vertical line on the left of the diagram gives the original interval between the maximum and minimum limits in the matrix.

FIGURE 3A

"PLUS" activity; Relative abundance above 96,000 *Sw. kr.* = 100 *per cent* (*Activity No.* 7: *Swedes*)

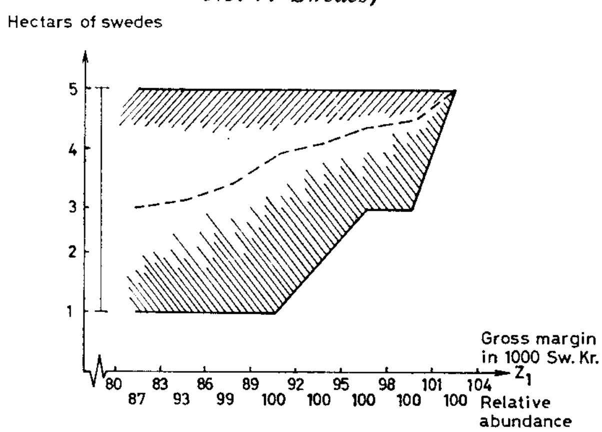


FIGURE 3B

"MINUS" activity; Relative abundance above 96,000 *Sw. kr.* = 0 *per cent.* (*Activity No.* 8: *Milking cows + required grassland area*)

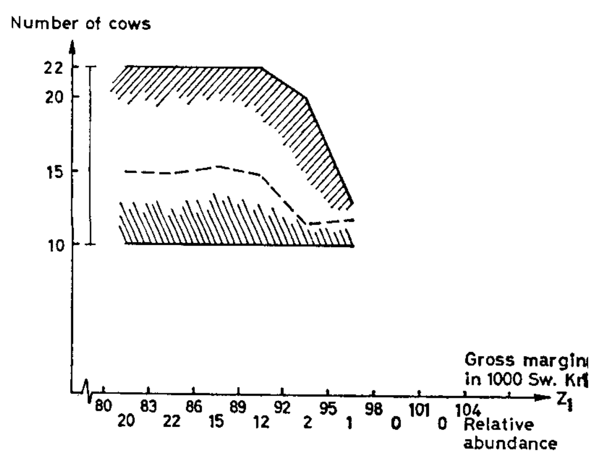
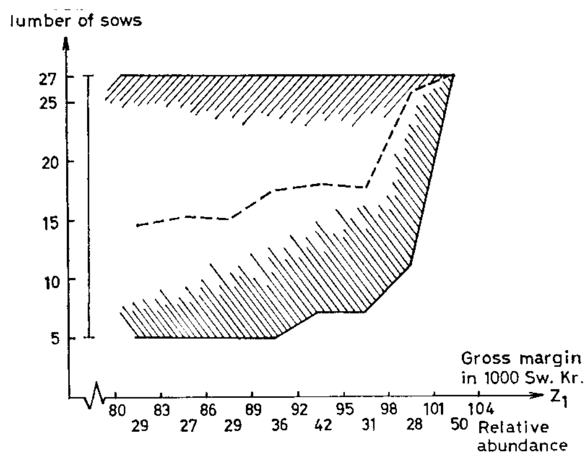


FIGURE 3C

"INDIFFERENT" activity; Relative abundance above 96,000 *Sw. kr. between* 0 *and* 100 *per cent.* (*Activity No.* 17: *Sows*)



(ii) Maximum and minimum and average levels of the activity in these solutions.

(d) Statistics on resources in different intervals of Z_1 (see figure 4):

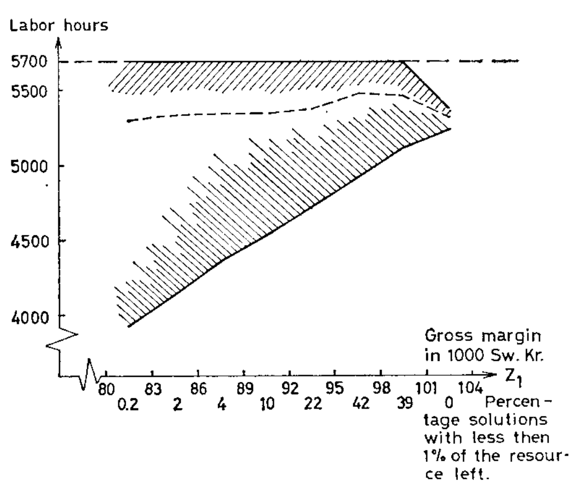
(i) Number of solutions in the interval (absolute and relative) with the relevant resource completely used (less than one per cent left).

(ii) Minimum, maximum, and average levels of unused activities in these solutions.

FIGURE 4

Labour Use

This graph shows total labour use during the year (resource No. 9) in solutions in different Z_1-intervals. 5,700 hours are available. The lower line gives the realized minimum labour use in different Z_1-intervals. The figures below the horizontal axis give the relative number of solutions in the relevant Z_1-interval in which the labour is completely used (i.e. less than one per cent left).



Among the solutions with high Z_1-values in our example above (e.g. 96,000 Sw.kr.)[3] three types of activities can be distinguished:

FIGURE 5A
Density of Solutions in the First Step (Run 1)

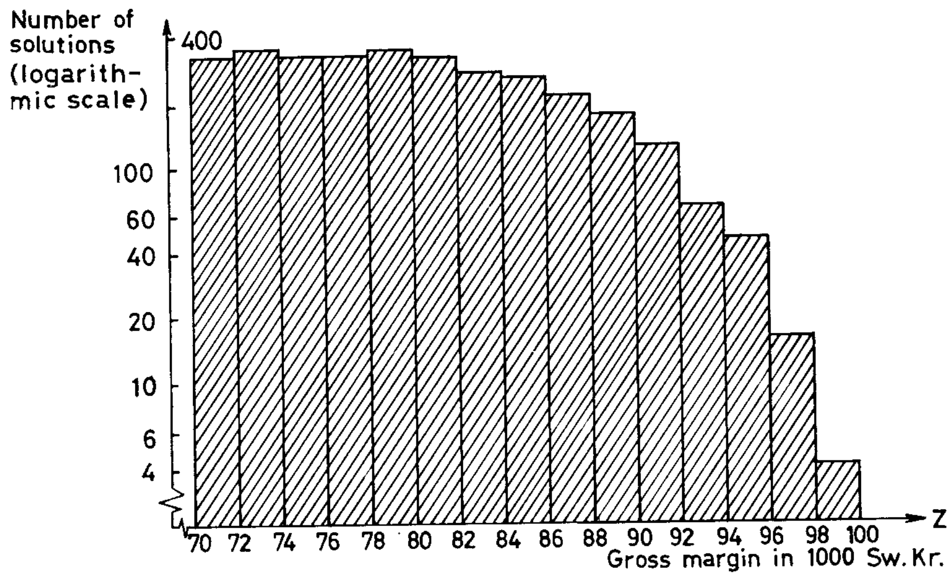
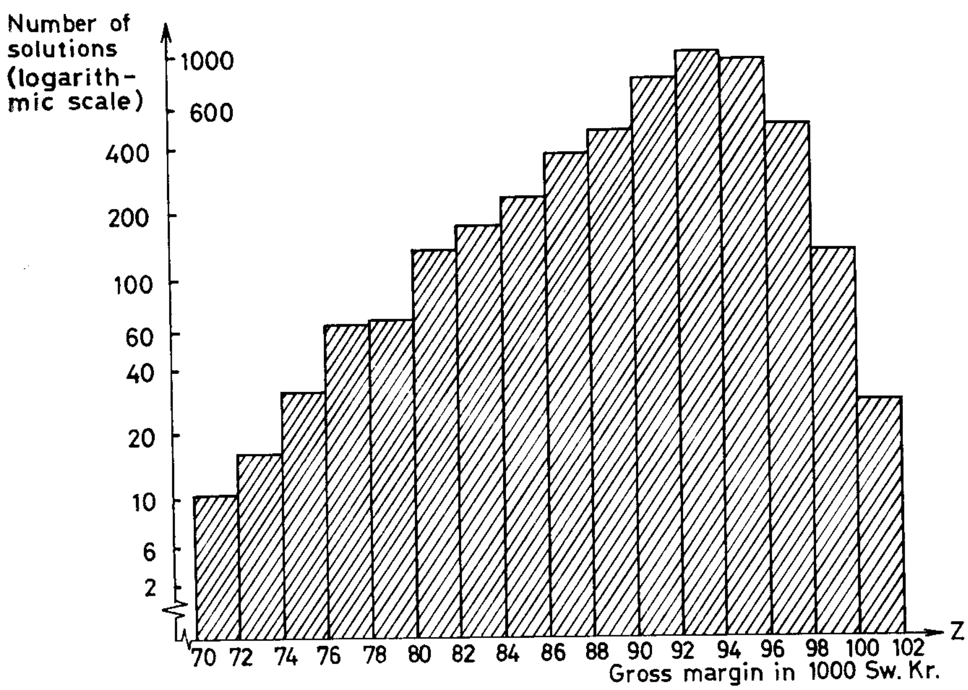


FIGURE 5B
Density of Solutions in the Second Step (Run 2)



[3] At current rates of exchange, \$A1 = 5·7 Swedish kroner approximately.

(a) "PLUS"—activities, which must enter the solution in order to yield a Z_1-value above the given limit (see figure 3a).
(b) "MINUS"—activities, which do not enter any solution above that limit (see figure 3b).
(c) "INDIFFERENT"—activities, which may or may not enter solutions above that limit (see figure 3c).

Of course this grouping of the activities cannot be definite, since it is based on sampled material. Nevertheless, it can be of great value for the decision maker and for the construction of a more effective seeking process in two or more steps (see next section) as can the information contained in the variation of the observed range for the activities at different levels of the objective function (see figure 3).

2.3 THE MONTE CARLO OUTPUT AND DECISION MAKING

Decision making based on the Monte Carlo output can of course be undertaken in a number of different ways. Some suggestions are given below.

(a) From the list of possible solutions with high Z_1-values (see table 3) the decision maker may choose one. This is very similar to decision making based on "programme-planning analysis".
(b) The decision maker may first analyse the grouping of activities in the "PLUS-, MINUS-, and INDIFFERENT-" categories. If the given Z_1-value for this grouping is satisfactory to the decision maker, he may proceed in one of two ways:
   (i) The decision maker may find the "PLUS and INDIFFERENT"-activities interesting. Then he can choose between the different combinations of them produced by the computer (e.g. from table 3).
   (ii) If the decision maker wants some MINUS-activity to be included in the plan, the statistics give information about the level of the objective function at which such plans may be found. If he is satisfied with this lower level he may then choose between the plans which contain the activity in question. If the decision maker wants more solutions containing a certain activity, this activity can be "forced" into the solution by artificial means. Corresponding information can also be obtained about the decrease in the level of the objective if the decision maker wants to exclude some PLUS-activity.
(c) Information concerning the resource use at different Z_1-levels can in some cases be of great value. This is illustrated for the resource "labour use during the year" in figure 4. As can be seen from this example a labour input of lower than 5,300 man-hours will limit the possible plans. Thus, this limiting resource has a non-zero marginal value product (cf. the shadow prices in linear programming). The steeper the slope of the lower line in the diagram, the higher is this value. In the example it can be estimated at about 15 Sw.kr./manhour and seems to be rather constant in the interval studied.

(d) As will be seen later, the value of other objective functions and their interdependence can also be of importance for the decision maker.

Finally, it should be emphasized that lack of empirical studies concerning decision making at the farm firm level makes it difficult to discuss the value of the Monte Carlo output for decision making. Intuitively our experiences from using the HUV-method [7]—the programme planning method most common in Sweden—suggest that most farmers would appreciate such a basis for their decision making. In particular, we believe that the presentation of a list of possible solutions with approximately the same value of the objective function could be more appealing to the decision maker than merely stating the very "best" solution. Also these results may be of interest with regard to the behavioural theory of the firm, with multi-goal and satisficing instead of maximizing as important components. This is particularly true when the decision maker is directly involved in the planning process by giving and changing satisfactory levels, steering the seeking process towards plans of interest to him, and interrupting the process when he finds further seeking not worthwhile.

3 A MULTI-STEP SEEKING PROCESS

3.1 DESCRIPTION OF THE PROCESS

If the decision maker is interested only in solutions with a Z_1-value above a given level (satisfaction level), a new and more efficient analysis can be performed. First, all MINUS-activities can be eliminated, and relatively uninteresting activities can be given only a small chance of entering the solution by the use of "weighting factors". Furthermore, it is possible to reduce the interval for the activities so that only levels appearing in solutions with a satisfactory Z_1-value are allowed. As will be shown later this reduction of the possible solutions strongly increases the relative abundance of solutions in satisfactory Z_1-intervals. This technique is called a *multi-step seeking process.* Let us consider this process now in more detail.

The meaning of *interval reduction* can be demonstrated with the help of figure 3. In satisfactory solutions (e.g. with $Z_1 > 95{,}000$) the activity illustrated in figure 3a always appears with levels within a reduced interval (3–5 ha) compared with the original interval (1–5 ha). In the next run the activity can be confined to this smaller interval. This interval reduction reduces the number of possible solutions and therefore increases the efficiency of the seeking process.

As the programme operates, the resources are gradually exhausted as the number of activities in the solution increases. Therefore, activities chosen at an early stage have a greater probability of entering the solution at high levels than activities chosen at a later stage. The order in which the activities are chosen can be influenced by giving the activities different weights. A large *weighting factor* implies that the activity has a high probability of being chosen at an early stage and hence a high probability of appearing at a high level. If the weighting factor is set equal to zero, the activity cannot enter the solution. By giving less economic activities small weights (but greater than zero), these activities will not prevent more economic activities from entering the solution. Nevertheless, at the end of phase I in the seeking process they will enter the solution if the constraints permit.

In the multi-step process we have used so far, the weight for an activity is set equal to its relative frequency in satisfactory solutions (p-weights) or equal to the square of these numbers (p^2-weights). The effect of using weights is demonstrated below.

Normally interval reduction and weights are used simultaneously. The effect of this on the planning problem can be seen from figure 5. After a first step (figure 5a), the thirty-one solutions with Z_1-value of greater than 95,000 Sw. kr. are analysed so that p^2-weights and reduced interval can be calculated for the second step. As can be seen from the diagrams, this second step has 25–30 times as many solutions in the interval $Z_1 > 95,000$. In the interval $Z_1 > 98,000$ the number of solutions is increased by a factor of 75 by the second step.

The same effects have been described by Lindgren and Carlsson [8] and Stryg [10], [11]. Other weights (p-, $p^{\frac{1}{2}}$-weights) and other interval reduction methods have also been tested. Trials have also been made to let the decision maker intuitively give the maximum and minimum values, or (for the weights) to use a ranking list based on gross margin of the activities per unit of different limiting resources or some other economic criterion. Experiences from these other methods are also partly discussed in these articles.

3.2 DIFFICULTIES WITH THE MULTI-STEP PROCESS

The method developed by us has to some extent an aim beyond that of ordinary maximizing procedures, namely to give a good picture of interesting solutions near the maximum. With this aim, it is obvious that multi-step processes have to be used carefully. Undesirable effects of interval reductions can occur, for instance, if some interesting activity levels do not happen to come into the solutions in the first run. Then these levels will be eliminated in the next step. Also the use of weights involves a risk of uncontrolled steering of the solutions. The risk of undesirable effects seems to be greatest if the equations for the resource use are not of the simple form ax, or if the objective functions have a complicated form, in which case local optima may occur. One way of reducing these difficulties is to decrease the level of satisfaction so that a greater number of solutions will be utilized as a basis for the second step. However this will also reduce the efficiency of this step. Other ways of reducing these unwanted effects have been tested. Before any general conclusions can be drawn, the effects—wanted and unwanted—of the multi-step process have to be investigated in more detail. As with other sampling techniques a compromise between security and costs must be effected.

4 MULTI-OBJECTIVE PROBLEMS

4.1 INTRODUCTION

As mentioned above, farmers often want to take more than one objective into consideration in their decision processes. In the Monte Carlo method each objective, or each main objective, is formulated in a separate objective function. Since the objective functions are calculated after the activity levels have been determined, the objective functions can be of almost arbitrary form and number. The intention is then to make a survey of the solutions that might be of interest to the decision maker. From this survey of feasible solutions the decision maker chooses the plan he subjectively finds best. It should be noted that sometimes an objective is to have special activity combinations or resource uses in the chosen plan. Such an objective need not, of course, necessarily be formulated as a special objective function, but can be taken into account when the decision maker chooses among different plans, or forces special activities into the plans by means of high weights, and so on. The description below refers to problems with two objective functions. Of these the first (Z_1) is to be maximized and the second (Z_2) is to be minimized. Of course, the discussion can easily be extended to more general multi-objective problems.

When handling more than one objective function, it is assumed that the decision maker cannot combine the various objective functions into a single function prior to receiving some information about possible plans, [3, p. 180]. For this reason it may be of interest to find the lines which connect plans with the best values of one of the objective functions for different values of the other function (see figure 6). Such a line we call an *efficiency line* [9, pp. 19–26], [6, pp. 557–8]. If the objectives

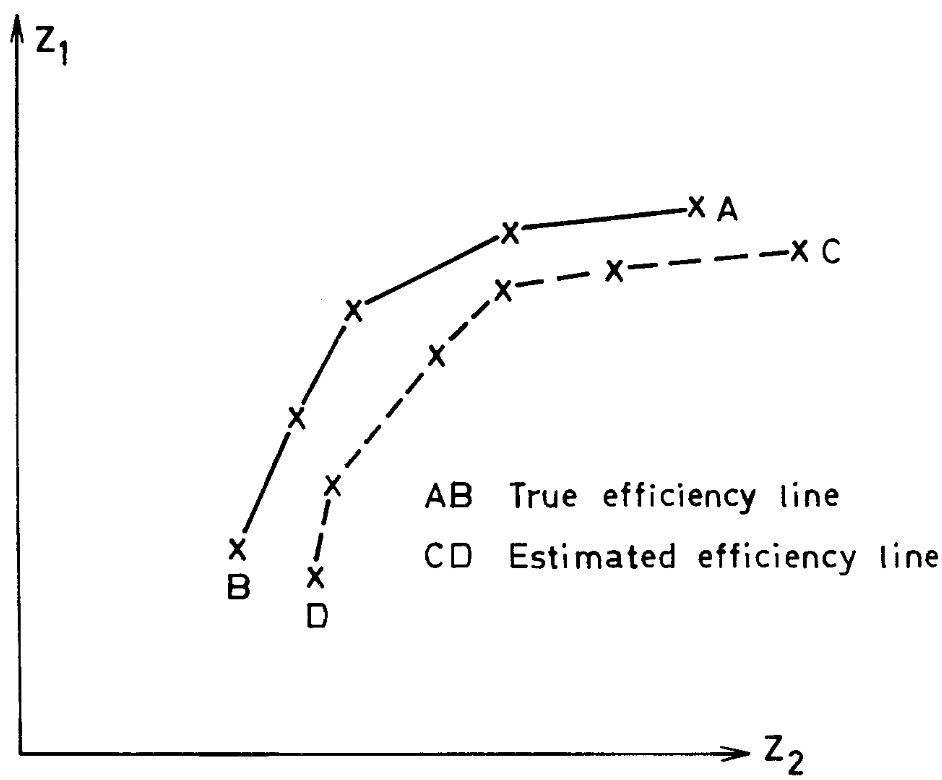


FIGURE 6

formulated in the objective functions are the only ones, the decision maker will always choose a plan on the efficiency line. Of course, if the decision maker has other "hidden" objectives he might equally choose a plan below this line.

If we knew all possible solutions, we could draw the true efficiency line (*AB* in figure 6). If we only have a sample of all possible solutions (as after a series of trials with the Monte Carlo method), we can only make an estimate of this line (*CD* in figure 6). A practical way of making this estimate is to determine the highest value of the function $(Z_1 - nZ_2)$ for different values of n.

4.2 A PRACTICAL EXAMPLE

In this example the first function is the gross margin (Z_1) as before. We introduce a second objective function (Z_2) which is a measure of the "risk" connected with different activity combinations. Risk is chosen here only to demonstrate the use of two objectives and no position is taken up about the value to a practical decision maker of such a risk calculation. The risk is calculated as the standard deviation of the gross margin. In this calculation consideration is taken of price variation in products and factors of production, yield variation, and correlation between different variation sources[4]. The existence of both yield variation and price variation for the same product is also considered.[5]
Five runs were undertaken, as shown in table 4. Runs 1 and 2 are the two steps in the multi-step process with a single objective function discussed above.

TABLE 4

Run No.	Interval reduction	Weighting factors	Phase II included	Number of solutions
1	No	"1-weights"	Yes	5,000
2	Yes	"p^2-weights"	Yes	5,000
3	Yes	"p^2-weights"	Yes	5,000
4	No	"1-weights"	No	5,000
5	Yes	"p^2-weights"	No	5,000

The distribution of the solutions in run 1 with respect to both objective functions is shown in figure 7a. It is seen that most of the solutions have

[4] $\sigma_{A+B} = \sqrt{\sigma_A^2 + \sigma_B^2 + 2\rho\sigma_A\sigma_B}$

[5] $\sigma_{p.q} = \sqrt{\mu_p^2\sigma_q^2 + \mu_q^2\sigma_p^2 + \sigma_p^2\sigma_q^2}$ (if p and q uncorrelated).

gross margin below 83,000 Sw.kr. and that the standard deviation is distributed between 17,000 and 47,000 Sw.kr. In general, greater gross margin gives greater risk.

Figure 7b shows the same distribution for run 2. For this run weights and interval reduction are based on the 31 solutions with the highest gross margin value in run 1. Most solutions in this run have higher gross margins but also greater risk than in run 1.

In figure 7c the distribution of the 5,000 solutions from run 3 is shown. In run 3 weights and interval reduction are based on 45 solutions in run 1 with a value of $Z_1 - Z_2 \geqslant 62{,}000$ Sw.kr. It is obvious that the solutions from run 3 have as a rule both higher gross margins and lower standard deviations than the solutions from run 1. Compared with run 2, run 3 yields fewer solutions with a high gross margin but more solutions with low risk. These results are natural consequences of the way in which the weights and new intervals for the activities are determined. By a proper use of the multi-step process it would seem possible to concentrate the solutions in almost any part of the feasible area that the decision maker finds especially interesting after studying the results of the basic run.

It is also found that the contour of the figures from the three runs is not very different. This implies that even the first step gives a good survey of the levels of the objective function that can be reached. The main effect of the second step is to increase the *density* of the solutions in interesting parts of the diagram.

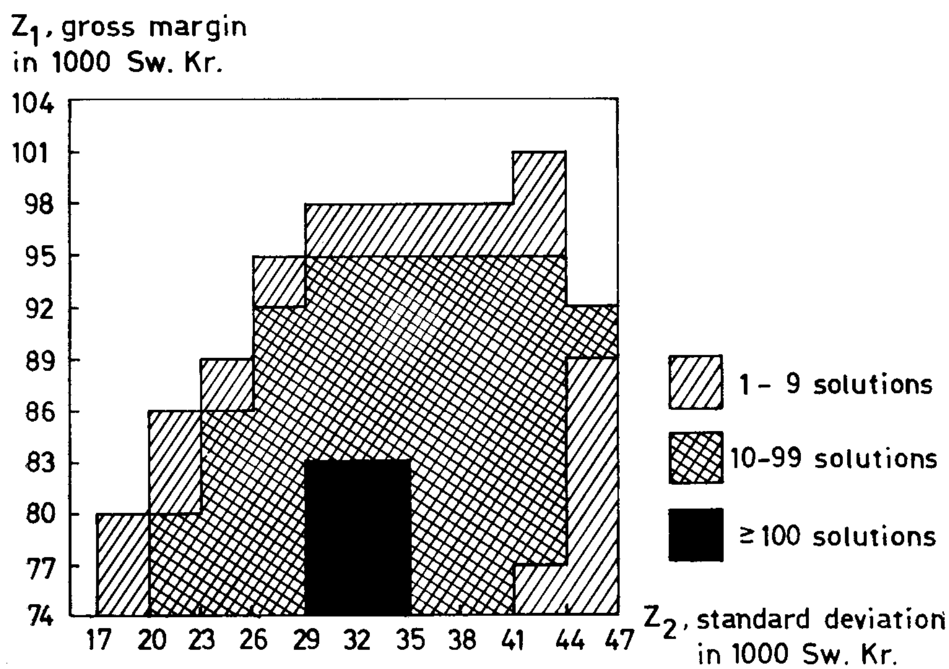


FIGURE 7A
Distribution of the Solutions in Run 1

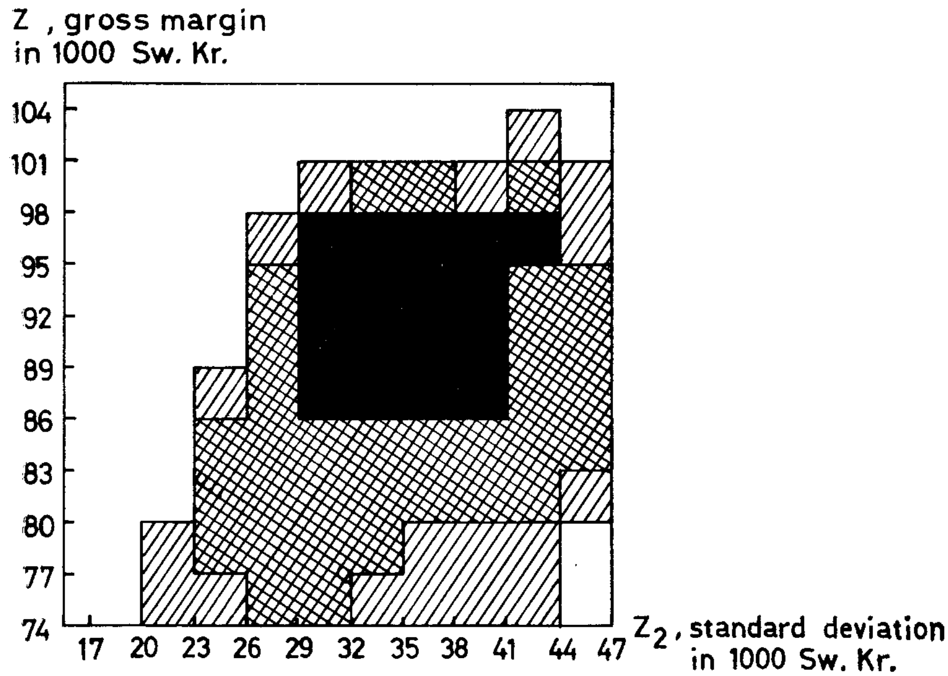


FIGURE 7B
Distribution of the Solutions in Run 2

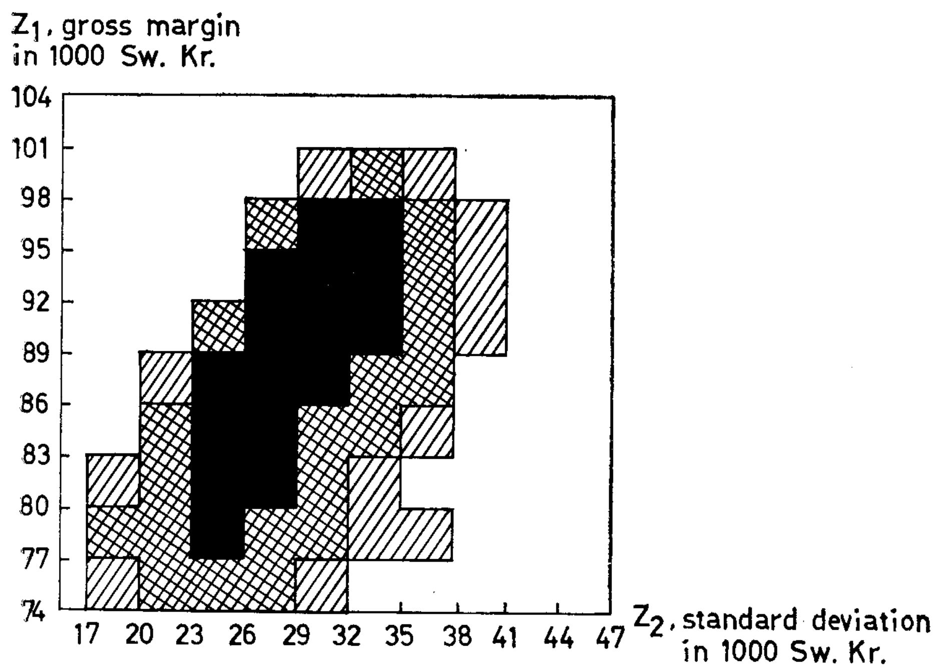


FIGURE 7C
Distribution of the Solutions in Run 3

4.3 A COMPLICATION IN MULTI-OBJECTIVE PROBLEMS

It can be shown that if in a two-objective problem both objective functions are linear, all activity combinations on the efficiency line will also fall on the surface of the "sphere" formed by the constraints. Under such circumstances some kind of adjustment procedure in the seeking process (phase II) should always be used in order to move points inside the "sphere" out to the surface. If one objective function is non-linear, this not necessarily the case.

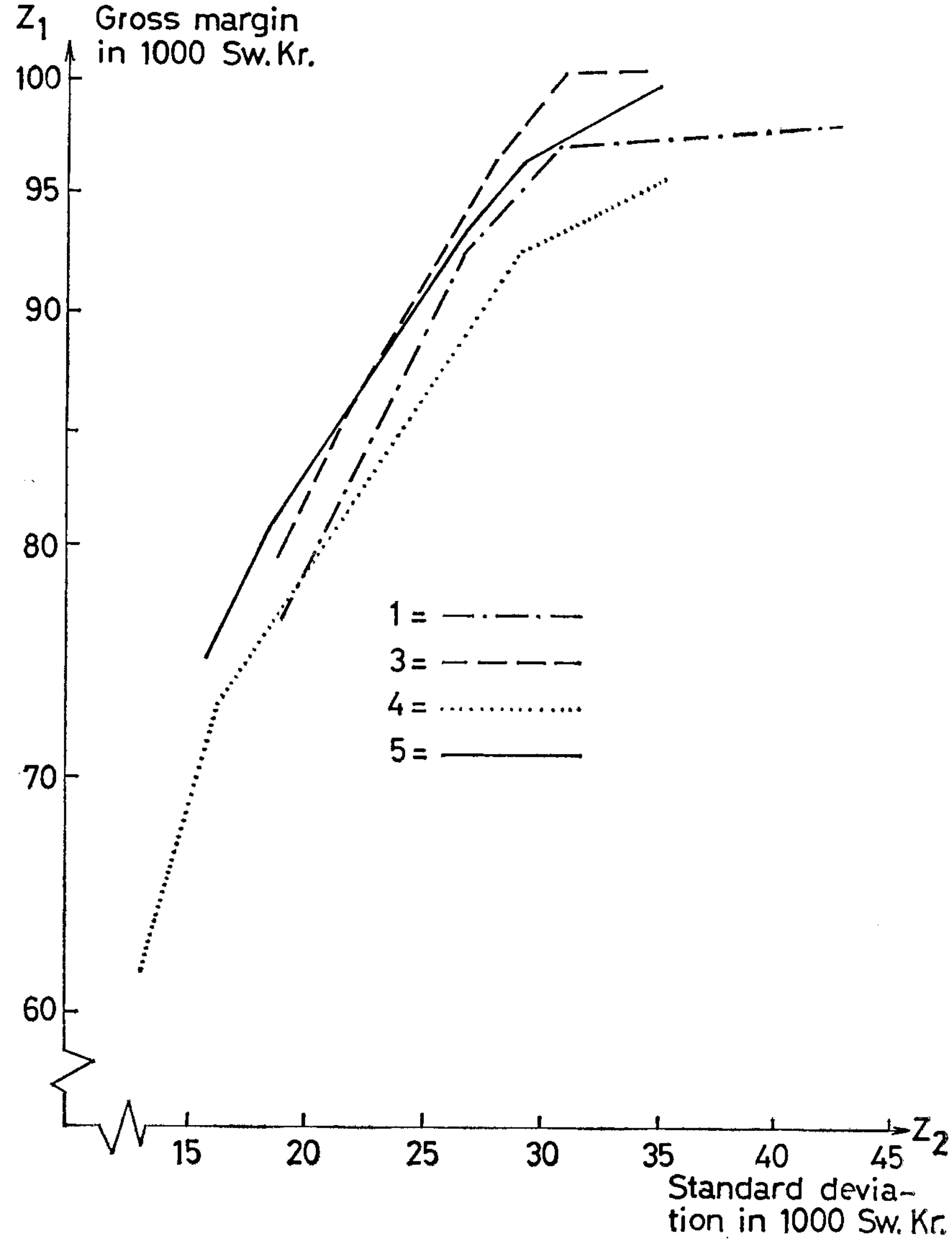


FIGURE 8
Estimated Efficiency Lines for Runs 1, 3, 4, *and* 5

In our example risk minimization is a non-linear objective function. Therefore, runs 4 and 5 are made without use of phase II in the seeking process. Run 4 is identical with run 1. In run 5 p^2-weights and interval reduction are based on 42 solutions with $Z_1 - Z_2 \geqslant 59{,}000$ Sw.kr. from run 4 and therefore corresponds to run 3 above. The estimated efficiency lines from these runs are shown in figure 8.

Figure 8 shows that the use of phase II will give a better estimate of the line close to the maximum value of the gross margin. In other parts of the diagram a better estimate is obtained without use of phase II.

Another effect which is not seen in the diagrams is that runs 1 and 3 have more solutions close to the lines than runs 4 and 5. These effects seem quite reasonable. When phase II is used, the better estimate of the efficiency line for solutions with high gross margin values and the increased number of solutions close to the line is an effect of the sample of solutions being taken from a smaller population than if phase II is not used. The poorer estimate in other parts of the efficiency line is a result of the bias this reduction of the population yields. As mentioned above, it is possible to emphasize different parts of the efficiency line by use of various combinations of the two objective functions as the basis for the second step.

The illustration above shows that the handling of several objectives might be quite complicated. Even if some general observations are possible, the best method for each particular planning problem can so far only be determined by trial and error.

5 FUTURE WORK

The Monte Carlo method presented in this article throws some interesting light on farm planning problems. However, new ways of looking at these problems have appeared during the development. Therefore more work must be done before it will be possible to give a final judgment concerning the advantages and disadvantages of the method. For example, it will be necessary to study in more detail the effects of different multi-step procedures. The model building is very flexible, allowing for development in different directions. One promising development involves the specification of activities producing output only for on-farm use. In combination with the integer formulation, such a possibility might give a method for evaluating the effects of different investments on a complete production plan.

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