INVESTMENT PLANNING BY MONTE CARLO SIMULATION

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The development of Monte Carlo programming as a farm planning method is reviewed. The possibility of applying this method and linear programming to investment planning is discussed in relation to two main types of problem. It is concluded that Monte Carlo programming is well suited to planning problems involving investment in fixed equipment and machinery partly because of the need for integer solutions and partly because of the largely exclusive nature of the investment opportunities. Neither of these points can be directed towards the investment problem in land improvement. Present Monte Carlo methods tend to be cumbersome for this type of problem though slightly adapted algorithms may prove to be of considerable value.

1 INTRODUCTION

Most operations research techniques used in farm planning involve the specification of a precise goal which must be maximized. Typically, this has involved the maximization of some measure of net income. It has become clear, however, that the objectives of farmers, particularly their long term objectives, cannot be reduced to a single criterion (Carrington [2]). The list of factors which influence the selection of a farm plan will include the stability of the plan in a changing economic and physical environment, the ease with which the business can be expanded (or in some cases contracted), the need for new capital investment, the position on the farmer's concept of social scale which a particular way of farming permits, as well as short term profitability and maintenance of family income. It would be difficult to supply quantitative data which would adequately describe these criteria even if a technique were available for incorporating them into the planning procedure. In any case many such criteria are probably best left to the subjective assessment of the farmer. Therefore, the alternative to an optimizing approach is to offer the farmer a number of feasible plans. These plans should be similar in terms of whichever criterion is considered to be most appropriate (this may be usefully termed the *primary criterion*). In addition, values of a number of other criteria may be calculated for each plan and presented together with the value of the primary criterion (these can be termed *secondary criteria*). The farmer may then make a choice from the range

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of plans offered with some knowledge about the financial and physical implications of each plan.

This approach has been made using linear programming methods by Powell and Hardaker [11], and by Donaldson and Webster [6] using Monte Carlo methods. In these instances, the two methodological approaches achieve very similar ends though each has its particular advantages and limitations. The relative methodological merits of linear programming and Monte Carlo programming have been adequately discussed by Dent and Thompson [5] and by Strygg [13] and need no further attention here.

2 BASIC MECHANISM OF MONTE CARLO PROGRAMMING

2.1 THE METHOD

Since the application of Monte Carlo methods to farm planning is not widely documented it is appropriate to discuss the basic mechanisms at this point. Monte Carlo programming, which is the title given to these applications, does not involve a specific algorithm in the same sense as linear programming. There are many variations on the basic theme. Essentially Monte Carlo programming involves random sampling from all feasible plans within the boundary of constraints relevant to the farm. For most planning situations there are a very large number of feasible alternative combinations of enterprises. To be realistic then, the method must at least review a fairly large sample of plans. The manner in which this is generally accomplished has been set out in some detail by Thompson [14].

In the Monte Carlo programming approach some or all of the selected activities can be restricted to integer levels. The integer step size may vary from one activity to another. In addition all relationships of the Monte Carlo programming matrix may take the form:

$$b \geqslant \sum_{j=1}^{m} q_j + a_{ij}x_j \tag{1}$$

and the primary criterion may take any functional form, i.e.[1]

$$z = f(x_1, x_2, x_3, \ldots x_m) \tag{2}$$

These characteristics of Monte Carlo programming offer some advantages not available when using a linear programming approach particularly in problems concerned with capital investment.

It is in the use of the sample plans thrown up by the random procedures that most variations on the theme occur. These variations have, of course, arisen because of the different areas of application of the method. The application by Donaldson and Webster [6] represents a conceptually

[1] Generally this function will be of the linear programming form, $c'x$, where c and x are column vectors. Another form that may be useful is that used in quadratic (risk) programming. This is $c'x - \theta x'Vx$ where V is the variance-covariance matrix of c and θ is some risk aversion constant.

simple case wherein the random combinations of activities generated are simply reviewed. A running store of the top group of plans (i.e. in terms of the primary criterion) is kept and continually upgraded. From this "top" group of plans the farmer can select one, bearing in mind the calculated secondary criteria as well as further subjective opinions about the presented group.

The work of Lindgren and Carlsson [10] represents a more sophisticated use of the randomly generated output from Monte Carlo programming. Here the whole sample of randomly generated plans is stored in the computer and is subsequently classified in various ways. The two basic forms of classification, "activity analysis" and "criterion analysis", are indicated in a general sense by Dent [4] and in relation to a specific example by Carlsson, Hovmark, and Lindgren [1]. An "activity analysis" examines the importance of specific enterprises in generating varying levels of profit from the farm as a whole and gives some indication of the way in which various enterprises combine and partially substitute for one another as the profit level of the farm is increased. The "criterion analysis" permits the primary criterion to be plotted as computer output against any single or any pair of secondary criteria. Carlsson, Hovmark, and Lindgren [1] have completed such an analysis showing the relation between net profit and a measure of risk. Clearly, these classifications help to create a picture of the planning environment and the feasible operational structure within this environment (Renborg [12]).

2.2 SIMULATION BY LINEAR PROGRAMMING AND MONTE CARLO PROGRAMMING

It has been pointed out by many authorities (for example Hare [8]) that simulation is essentially a two stage process. In stage one the model which is an abstraction of the real situation is developed. The second stage may be considered an "experimental" stage in which the outcomes from varying the input combinations in the model are determined.

If, as is generally accepted, the model of the farm can be represented by the matrix of a linear programme, then the work both of Powell and Hardaker [11] and of Donaldson and Webster [6] can be broadly considered to be simulation. One of the severe problems in the experimental stage of a simulation study is the handling and interpretation of the bulk of computer output which results (Conway [3]). The classification of plans in the two ways mentioned above represents a way of facilitating the interpretation of the results of this form of simulation and is therefore a marked improvement in concept.

The models and the resultant classification of plans from the models have so far been involved with the static aspects of farm planning. For some planning work temporal aspects must be introduced in order to express adequately the real situation (Harle [9]). It is possible, however, by suitable construction of the model in the form of a single stage linear programme or a Monte Carlo programme to introduce a dynamic element into the planning procedure. This dynamic element may then be transferred to the criterion classification as suggested by Dent [4] in order to form a basis for interpretation of the results of the experimental phase of the simulation. This phase can include a consideration of the capital investment alternatives open to the farmer.

The rest of this paper attempts to indicate the extent to which simulation with this dynamic element may be applied by both linear programming and Monte Carlo programming methods to the problems of farm development planning.

3 DEVELOPMENT PLANNING

The essential difference between development planning and static planning is that in the former case injections of capital are envisaged at various stages. This in turn will influence outputs and subsequent decisions over a number of years. This capital may be partly endogenous (i.e. generated as profit from the productive activity of previous years) or it may be exogenous (i.e. new capital borrowed from outside the farming system). Generally, the return from this capital will accrue over a number of years of operation of the farm business. To assess the worth of any particular capital investment the returns must be discounted and expressed as a present value.

Investment for development planning purposes may conveniently be divided into two types. The first type will refer to that form of investment which immediately yields returns and which for practical purposes, and apart from maintenance charges, yields the same level of return each year of its productive life. This arises when there is a constant physical input/output ratio over time, such as investment in buildings and machinery. For ease of exposition this type will be referred to as investment in *equipment*. On the other hand, there is that type of investment which once made involves a stream of physical input/output relationships which change from period to period during its productive life. Such is investment in pasture improvement. For ease of exposition this type will be referred to as investment in *land development* although there may be other unrelated investments that take this form.

For the development plan the relevant form of classification of simulated output is the "criterion analysis". The primary criterion may be farm profit in terms of net present value. When presenting the criterion analysis in graphical form the primary criterion will be the dependent variable while the quantity of capital invested (the secondary criterion) will be the independent variable).

3.1 INVESTMENT PLANNING—EQUIPMENT INVESTMENT

The concept for the "criterion analysis" for investment decisions in machinery, buildings, and other fixed equipment has been developed by Thompson [15]. The general structure of the Monte Carlo model needed to produce the required output is given in figure 1.

Figure 1 may be more readily interpreted in the light of a specific example. A beef feed lot enterprise is considered where the alternative capital investment activities are viewed as extra units of the various types of

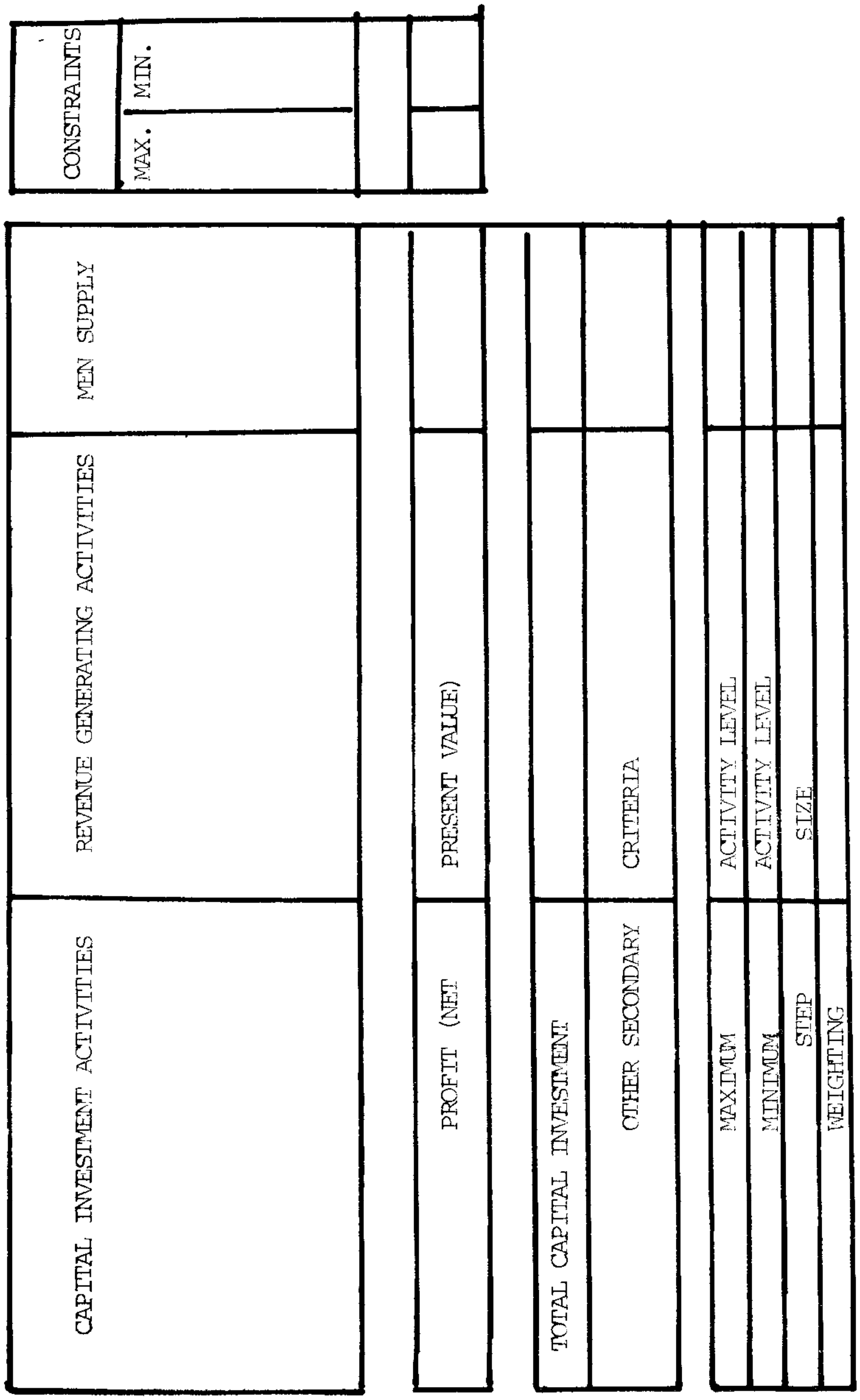

FIGURE 1
Schematic Layout of Monte Carlo Matrix for Equipment Investment Mode

building needed for expansion. The inclusion of these activities gives a "permit" for an increase in the revenue generating activities (various methods of beef production in the feedlot) over the limits imposed by the

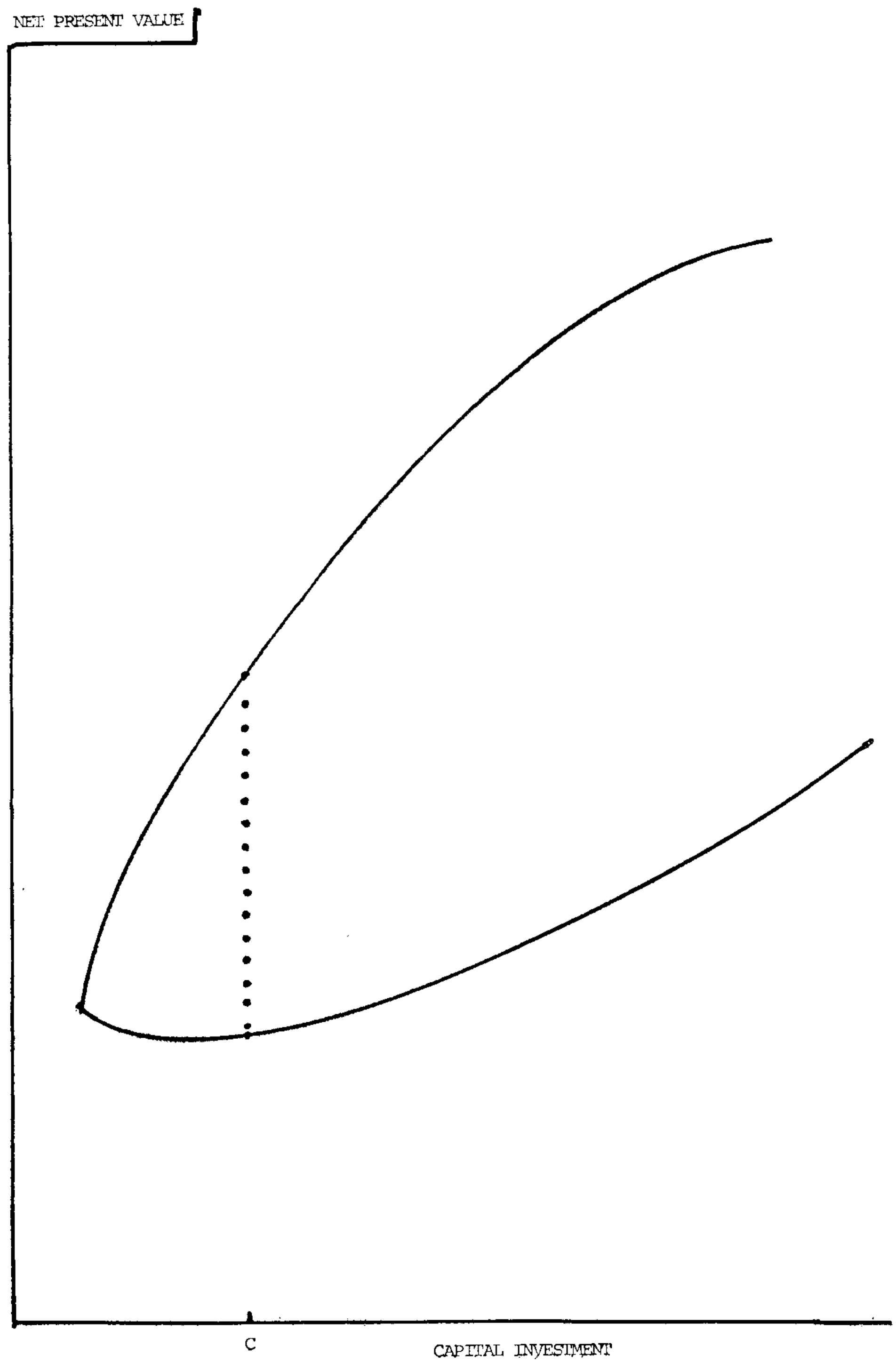

FIGURE 2

Criterion Analysis: Influence of Level of Capital Investment on Profit maximum constraint row. Hence, the capital investment activities take the form of supply activities in a conventional linear programme.

The primary criterion row has positive elements for the revenue generating activities. These elements are expressed in terms of the present value of their respective cash flows over a period of time. This period of time may most conveniently be the expected life of the investment activities or some multiple of this time. Postulated changes in product prices over time may be incorporated into the calculations of the cash flows if this is appropriate. Similar calculations are needed to supply the negative element in the primary criterion row for number of permanent employees.

The elements in the primary criterion row for the capital investment activities represent the present value of the capital outlay per unit and bear a negative sign. Associated with the elements already described may be some fixed overhead cost related generally to the new feedlot project. It may, for example, be necessary to undertake extra drainage works which are different in design for the various types of building but which once completed will suffice for any size of operation. In this situation the flexibility of the *criterion function* allowed by equation (2) is essential for the correct specification of the model. The summed primary criterion row will represent the profit of the farm expressed as net present value.

Among the secondary criteria is a row representing capital outlay per unit for the capital investment activities. The elements in this row are precisely the same as the ones in the primary criterion row but now they bear a positive sign. All other coefficients in this row are zero.

The Monte Carlo procedure now selects at random a sample of possible organizations within the limit set on the amount of new capital invested. A figure generally similar to figure 2 may be obtained directly as computer output showing the financial implications from the whole range of investment amounts. Capital investment at a level of, say, C will, as indicated in figure 2, support a series of different enterprise organizations each directing this amount of capital in different ways, and each yielding a specific return for the enterprise. The plan which gives the greatest return on capital can be located for each level of capital invested. This profit level may then be plotted on a graph showing the best possible returns to be expected from any amount of capital invested. A lower boundary may be similarly derived to enclose the feasible area of investment.

The integer nature of the investment alternatives for this type of development planning is an important factor in the correct specification of the problem. Investment activities here represent complete buildings, or at least sizeable modules of buildings, and possibly whole machines. In addition, the labour force must be comprised of an exact number of men. Quantities of such items cannot sensibly be considered as non-integers and therefore an integer programming method is essential. The fact that the matrix elements may take the form indicated in expression (1) further improves the realism with which the true situation can be modelled given this form of problem. These two features of the Monte Carlo matrix make this method greatly superior to a linear programming approach to this particular problem.

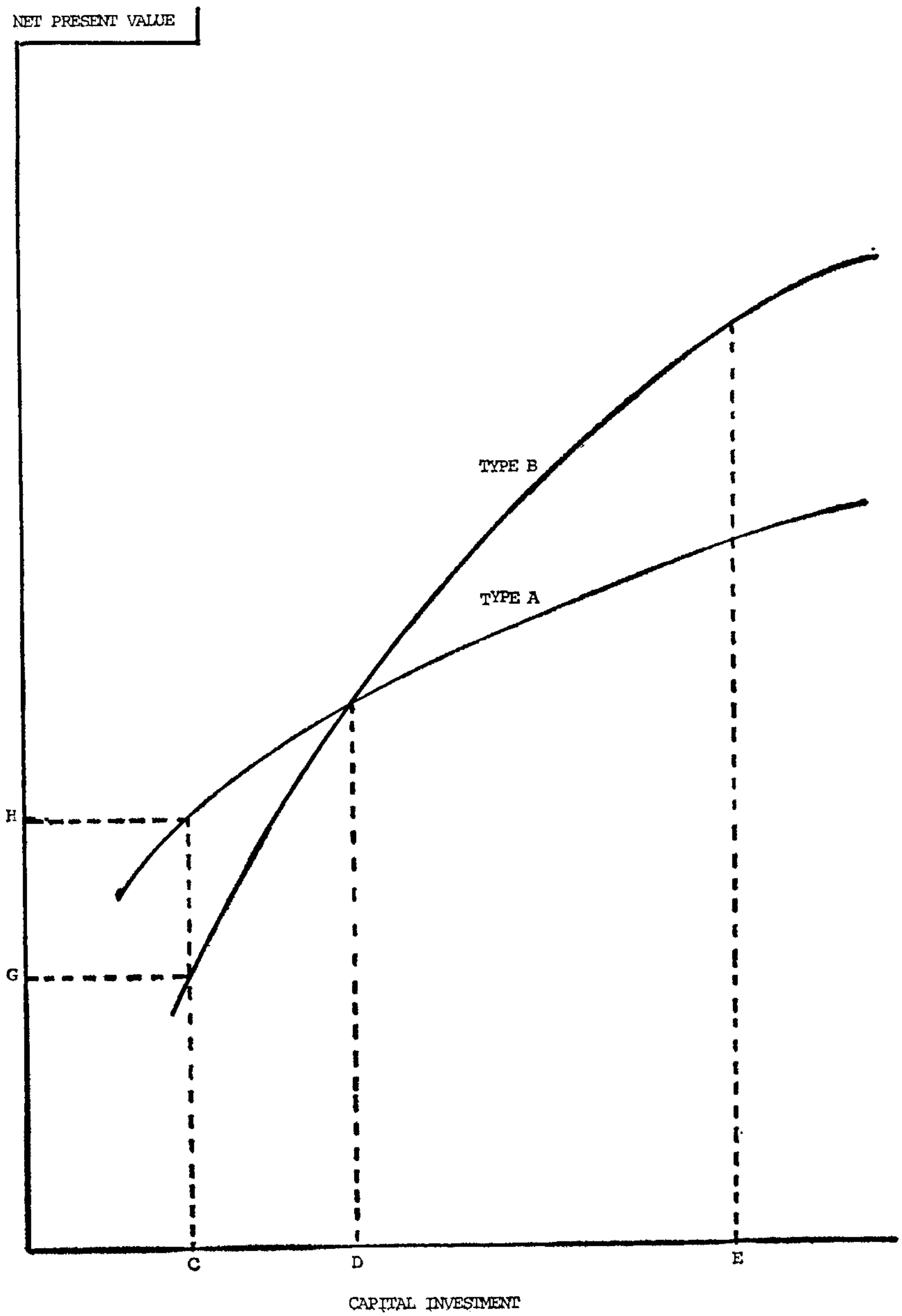

FIGURE 3
Influence of Level of Capital Investment in Two Building Types on Farm Profit

The general response to extra capital investment shown in figure 2 may now be examined in more detail, to expose the changes in the physical structure of the plans associated with various points on the curve. Suppose, for example, that there are two types of building arrangements whereby the postulated beef feed lot could be developed. A more detailed form of figure 2 would indicate the relationship between capital investment and profit for building type A and building type B. This is shown in figure 3. For low levels of capital investment building type A yields higher profit though with subsequent larger investments building type B becomes more profitable. If an initial investment of $\$C$ were made the best return on this investment would clearly be from the purchase of type A buildings. Should a final total investment of $\$D$ be envisaged for the feedlot, the initial investment in type A buildings would be compatible with a finally desired state which would maximize profit and consist entirely of type A buildings. On the other hand if, in the long run, an investment greater than $\$D$ is likely there would need to be some specific non-monetary utility associated with type A buildings before the above line of development was pursued.

Assuming, as seems likely to be the case, that this form of investment remains "locked-up" in the enterprise because of its permanent nature, investment of the initially available sum of $\$C$ in type A buildings would be incompatible with a finally desired investment of $\$E$ in type B buildings. Had the initial investment $\$C$ been invested in type A buildings a total of $\$(C + E)$ would be required to attain the profit level associated with $\$E$ investment as shown in figure 3. In this case the investment of $\$C$ in type A buildings would finally be wasted. The alternative strategy would be to invest the $\$C$ in type B buildings. This would involve a short-run sacrifice in profit of $\$(H - G)$ but would be compatible with the most profitable investment when the total of $\$E$ is available. The strategy to adopt can be decided by calculating the net present values as well as assessing the non-monetary aspects of each.

This approach is possible because the investment is essentially additive in nature, regardless of the time period over which it occurs. After the initial investment of, say, $\$C$, further investment may take place when sufficient capital is forthcoming from farm profits or loans to permit a worthwhile increase in the size of operation. This further investment proceeds from precisely the point on figure 3 at which the system had previously been operating.

3.2 INVESTMENT PLANNING—LAND DEVELOPMENT

While the Monte Carlo programming approach is ideally suited to handle the dynamic aspects of investment in such items as farm buildings, some problems are encountered when a similar approach is made to investment in land improvement. The format of the basic model for this type of problem is given in figure 4. The actual matrix of the example to be considered is presented in the appendix.

While this model is essentially the same as that in figure 1, it is clearly more complex. This is because in the land improvement model investment at one point in time and in one direction releases a stream of nonconstant fodder production which lasts over *n* years. In addition, for some forms of improvement there is the need for a stream of extra capital investment in succeeding years. Within any year there may be transfer of fodder from season to season and a similar transfer must be available between years.

Profit may be generated from livestock activities or from cropping activities incorporated within the improvement activities. The profit row is expressed in terms of net present value for all the activities. As can be

LAND IMPROVEMENT ACTIVITIES	LAND USE ACTIVITIES	LIVESTOCK ACTIVITIES	FEED TRANSFER	CONSTRAINT MAX.	CONSTRAINT MIN.
FEED SUPPLY	YEAR 1	FEED REQUIREMENTS YEAR 1			
FEED SUPPLY	YEAR 2	FEED REQUIREMENTS YEAR 2			
FEED SUPPLY	YEAR N	FEED REQUIREMENTS YEAR N			
PROFIT	(NET PRESENT	VALUE)			
TOTAL	CAPITAL	OUTLAY			
YEAR 1	CAPITAL	OUTLAY			
YEAR 2	CAPITAL	OUTLAY			
YEAR N	CAPITAL	OUTLAY			
MAXIMUM	ACTIVITY	LEVEL			
MINIMUM	ACTIVITY	LEVEL			
STEP	SIZE				
WEIGHTING					

FIGURE 4

Schematic Layout of Monte Carlo Matrix for Land Development Model seen from figure 4 and the appendix table, some of these have negative values. For the improvement activities profit represents the discounted cash flows of the fertilizer, seed, and machinery costs for pasture development, and any saleable crop yield. A terminal value of the improved land is estimated and this too is incorporated in the profit figure. For the livestock activities the profit is the sum of the discounted cash flows of the gross margins and terminal value of the animals less their initial cost.

Total capital investment for the land improvement activities is the present value of the monetary outlay on fertilizer, seed, and machinery costs for improvement. In the particular example to be considered the cost of the annual maintenance application of fertilizer is not included. For the livestock activities the capital investment figure represents the present value of the outlay for the animals purchased or the present value of the receipts from any sold. These activities must be arranged in such a way that the number of animals present on the property can be increased or decreased in any year. Since capital for both livestock and improvement may take place over a number of years it is useful apart from the summed discounted value of this investment, to examine the actual commitment for each year at the time of investment. This may be indicated as in the last rows of the model in figure 4 and appendix I.

At this point it should be noted that two of the features of Monte Carlo programming which made it so attractive in studying the equipment type of investment (viz. the provision of integer answers and the possibility of coefficients being in the form of equations (1) and (2)) are largely irrelevant to the present problem. The generation of a large number of feasible plans by the Monte Carlo method, however, is still useful in permitting the "criterion analysis". It is quite possible for a form of this analysis to be made available from a linear programming simulation of the same problem. In this case the secondary criteria are incorporated within the main body of the linear programming matrix. The quantity of capital available for investment over the *n* years is varied parametrically, so that the upper boundary of the criterion analysis is defined. For this situation the two methods are broadly equivalent except that with linear programming it is possible to generate relatively few plans. It will become clear that this is one of the main limitations of the linear programming approach for this type of problem.

In the example as presented in the appendix there are four land types three of them appearing as rows in the matrix. The other land type is heavily timbered country and has no immediate productive value. Since there is only one activity that can be carried out on this land its level of operation can be restricted simply by using the maximum inclusion row facility of the Monte Carlo method. For each of the three other land types there is one activity which indicates the feed available if this land is not improved as well as a number of opportunities for improvement (investment) on these land types: two improvement methods are possible on the first two land types and one method on the other type.

The role of the land use and improvement activities is to supply feed into quarterly feed "banks". These banks are utilized by the sheep and/or cattle grazing activities. As mentioned earlier, the other group of

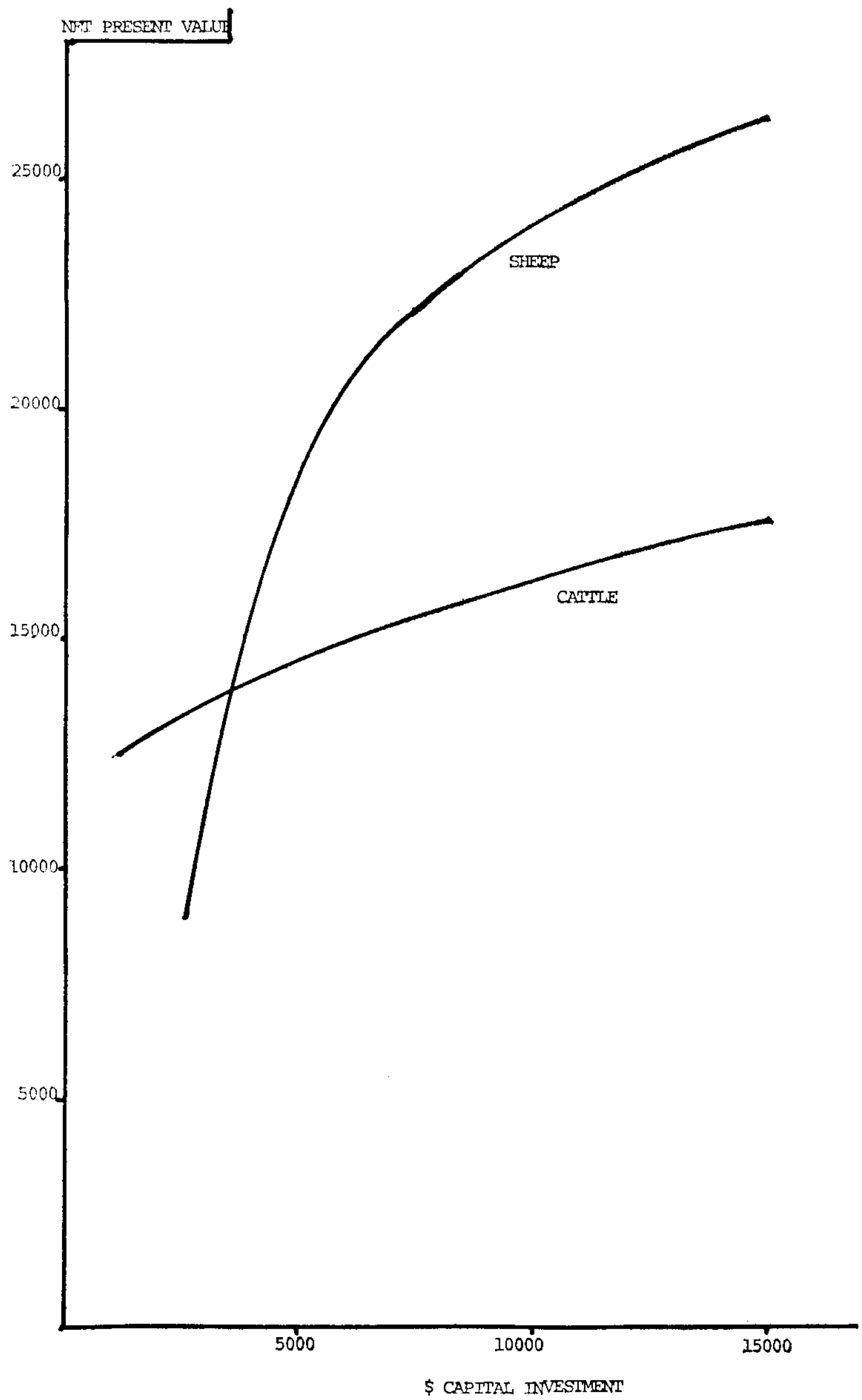

FIGURE 5

Influence of Investment in Land Improvement on Farm Profit Showing the Effect of Choice of Livestock Enterprise activities comprises the feed transfer activities. These permit standing feed to be saved from one quarter to the next, except that no feed is saved from the winter to the spring quarter.

The nature of the investment opportunities in land improvement means that these activities are by no means mutually exclusive. In fact, it is unreasonable to suggest that feed supplies may originate from only one type of land since some feed is available in any case from the different types of non-improved land. At any level of investment in improvement there will therefore be a large number of possible combinations of "feed supplying" activities. This is especially true since integer specification of the land improvement activities is not necessary. Because of this, classification of plans according to the land improvement type is not relevant.

However, it may well be of some value to classify output from the simulation in terms of the type of livestock involved: all sheep, sheep and cattle combinations, or all cattle. This is illustrated in figure 5. For each type of livestock farming and for each level of capital investment there will be, of course, many different combinations of land improvement activities. Since a linear programming approach will only yield the upper boundary of the classification, only one of these combinations will be provided. It will be that combination which gives the greatest profit from the dual investment in land improvement and livestock.

Having made one investment the next injection of capital must be in a combination of land improvement and livestock activities that are compatible with the present state of the farm system. The livestock activities in this respect are no problem, since the numbers of each type of animal can easily be increased or decreased. It remains, however, a most important point with regard to the land improvement activities, for it will not always follow that subsequent investments along the boundaries shown in figure 5 will indicate new plans which are compatible in terms of the areas of land of various types to be improved. Figure 5 shows cattle grazing to be more profitable than the sheep enterprise at lower levels of

TABLE 1

Relationship Between Capital Investment and the Profit of the Example Farm with Either a Sheep or Cattle Enterprise

Capital investment (present value)	Profit (net present value)		Improvement activities in most profitable plan*	
	Sheep	Cattle	No. 3	No. 5
$	$	$	ac.	ac.
3,000	10,780	13,557	0	105
6,000	20,426	14,660	1·9	2·2
9,000	23,018	15,763	58·1	67·2

* Improvement activity No. 3 is the establishment of perennial grasses on basalt soil. Activity No. 5 is the establishment of a lucerne pasture on the arable slate.

capital investment. In this stage of development if only one type of livestock were desired it would be logical in profit terms to have a cattle enterprise. Later, at higher levels of capital investment, sheep become more profitable. Table 1 indicates the relative profitabilities of cattle and sheep at various levels of capital investment as well as the area of different types of land to be developed for the most profitable enterprise.

At $3,000 investment, cattle systems are more profitable than sheep and require all of the second land type (arable slate) to be improved. When a further $3,000 are invested sheep systems are most profitable and these then require only 2·2 acres of the arable slate and 1·9 acres of the first land type (basalt) to be developed. Thus, in terms of land improvement a transition from the "best" plan given $3,000 to invest (cattle) to the "best" plan given $6,000 to invest (sheep) would be impossible.

If sheep systems were preferred in spite of the low profit at $3,000 investment, a smooth development transition could be effected as the investment is increased through to $9,000. In this case the dynamics of the investment procedure would be exactly similar to those for the equipment type of investment. The important thing is that compatability must be established before a movement from one state to another is prescribed. Compatibility between different states according to different level of investment may not occur along the boundary curves of the "criterion analysis" graph as shown in figure 5. Hence, in this case the linear programming output is inadequate for assessing different sequences of investment. Using Monte Carlo programming, however, it will usually be possible to locate compatible plans for future investments of any desired amount. In this case the first investment is made based on the profit and the physical nature of the possible plans for that level of investment. Then the vertical axis of the graph is moved to the right to the level of investment made. Future investment opportunities now lie to the right of this vertical axis. The actual opportunities for a given further investment may frequently be below the boundary curves. Unfortunately there is no systematic method available to locate such compatible plans if they do not lie on the boundary curve. It is simply a question of sorting through all the plans randomly generated for the level of capital investment involved.

Although it is feasible, this method becomes somewhat cumbersome to apply. It becomes worse as the number of non-mutually exclusive investment opportunities increases. Nevertheless, it does provide a basis upon which a large number of investment sequences can be investigated with more flexibility than would be available by using methods such as poly-period linear programming. For this latter technique various assumptions would need to be investigated in separate runs of the model.

4 FUTURE DEVELOPMENT

It is clear that for the two investment problems examined Monte Carlo programming does offer some very useful features not available when using linear programming. This is true because of the large number of answers generated by the method, even when integer answers are not required and overhead costs are not relevant.

There is a difficulty in the application of Monte Carlo programming, however, in that presently available computer programmes [1, 7, 14] limit the size of the matrix that can be handled even when using large computers. It has previously been explained that some savings in matrix size can be made over a linear programming matrix specification [5], but nevertheless present limitations are quite severe. There is no doubt, however, that computer programming improvements would greatly increase the size of problem that can be solved. For example, the present programmes operate entirely within the computer core store. Storage of feasible plans on magnetic disc or tape before transferring back to the core store of the computer for classification would enable larger matrixes to be designed.

Even the particular model layout considered which involved a total of 41 activities was close to the capacity of the available computer programme. The possibility of larger matrices would permit the second type of investment problem to be investigated as a number of randomly selected complete strategies. This could be accomplished by specifying the matrix in a similar way to a poly-period linear programming matrix. For any total amount of capital to be invested a number of strategies would be generated from which a choice could be made at least partly on subjective grounds. Alternatively, a Monte Carlo routine may be designed to select a plan for the first time period, using the typical Monte Carlo approach. The plan selected would then determine the plans that are feasible for the next time period from which a further plan may be selected in the manner analagous to that previously described. The relative efficiencies of these two approaches cannot be fully established at this time. Further work is required to develop computer routines and explore the use of these potentially important methods.

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APPENDIX

DESCRIPTION OF ACTIVITIES

1. Use Basalt. Use current pastures on basalt soil. No improvement is carried out but superphosphate is applied annually at a cost of $1.50 per acre. The net present value of $1.50 over five years is $6.70 as indicated in the PROFIT row. The unit for all land use and improvement activities is one acre.
2. Establish Lucerne. Establish a lucerne pasture on basalt soil following a grazing oats crop. PROFIT is the sum of the discounted increase in land value less discounted pasture establishment and fertilizer costs.
3. Establish Grasses. Establish perennial grasses on basalt following an oats crop. The oats is grazed and then harvested for seed which is sold.
4. Use Slate (arable). Use current pasture on the arable slate soil.
5. Establish Lucerne. Establish a lucerne pasture on the arable slate soil.
6. Establish Grasses. Establish perennial grasses on the arable slate.
7. Use Slate (semi-arable). Use current pastures on the semi-arable slate soil.
8. Establish Grasses. Establish perennial grasses on the semi-arable slate soil.
9. Kill Green Timber. Green timber is poisoned so that clover pastures can be established.
10. Buy Sheep 1. Purchase sheep at the start of year one. The activity implies that these are kept for the remaining five years but the existence of selling activities means that they can be kept for a shorter period.
11. Buy Sheep 2. Purchase sheep at the start of year two. These are kept for the remaining four years.
12. Sell Sheep 2. Sell sheep at the start of year two. A combination of activities 10 and 12 permits some or all of the sheep to be run for one year only.
13. Buy Sheep 3.
14. Sell Sheep 3.
15. Buy Sheep 4.
16. Sell Sheep 4.

17. Buy Sheep 5.
18. Sell Sheep 5.
19–27. Cattle. These activities are identical in structure to the sheep activities.
28–41. Feed Transfer.

ADDITIONAL NOTES ON MATRIX

Activity Max. and Min. Any activities are selected will enter the plan at some level between these specified limits. In this example the limits are generally not effective with the exception of activity number 9 which is restricted to a maximum of 230 acres.

Step Size. The step size specifies the size of the integer units for each activity. Since integer specification is not necessary in the above example the step sizes are small.

Weight. The weighting can be varied so that some activities have a higher probability of being selected. In this case land use and improvement activities are essential before any other activities are feasible. The former activities therefore have been given a higher weighting in order to improve the efficiency of the programme (see [5]).