SUB-OPTIMAL PROGRAMMING METHODS FOR PRACTICAL FARM PLANNING

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This paper discusses the use of sub-optimal programming techniques in farm management. The scope for employing such techniques to derive farm plans which more closely conform to farmers' real objectives and preferences is reviewed. A method using conventional linear programming in this way is illustrated by means of some planning results for a group of farms in northern N.S.W.

1 INTRODUCTION

Over the last decade or so a considerable fund of experience has been accumulated in the use of linear programming for farm planning. This growing experience has resulted in practical farm planners tending to reject the idea of a single, unique optimal solution to a linear programming model of a particular farm, preferring instead to compute a number of solutions for the farmer's consideration. Disenchantment with the idea of a unique optimum has arisen because of a better appreciation of, first, the limitations of the technique itself, and second, the complex nature of the objectives which many farmers are striving to achieve.

Perhaps the most restrictive assumption underlying linear programming is that implied by the deterministic nature of the model. This is particularly important in Australia, where, as a result of rapidly changing technology, input-output relationships are often very imperfectly known, and where a high degree of climatic variability creates severe uncertainty in farm planning. In such a situation it is clearly desirable to study the effect on the solution obtained of varying the assumptions concerning the most uncertain components of the model.

The second important limitation of the technique is that linear programming permits only one (linear) objective function to be considered at once—normally maximum (expected) profit. However, the importance of the higher order moments of the profit criterion is now well recognized. For example, Officer, Halter, and Dillon [10] have indicated the importance of attitudes to risk in farmers' decisions. Other farm planners, for example Daw [3] have suggested further factors that farmers regard as important in choosing a farm plan, such as compatability with the present farm pattern, or the farmer's own preferences about activities

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to be included or excluded. Thus, the farmer's utility function may be multi-dimensional, perhaps non-linear, or even impossible to represent adequately or accurately in the linear programming framework. Furthermore, farmers may have difficulty in articulating their objectives sufficiently precisely to be incorporated into any formal model, but may be able to determine which of a set of plans suits them best. Finally, it has been found, for example, in the study by Renborg [12], that the solution space is often relatively "flat" in the region of the optimum. This will often mean that solutions exist which would enable the farmer to satisfy better some secondary objective at the expense of relatively little reduction in expected profit.

This paper briefly reviews methods of computing sub-optimal solutions for farm planning purposes, and shows how linear programming can be used to generate a range of sub-optimal solutions of interest to the farmer.

2 SUB-OPTIMAL PROGRAMMING METHODS

There are two methods of generating sub-optimal solutions for a particular linear programming matrix by making minor additions to the computer program. Firstly, upon solution of the initial problem each non-basic activity in turn may be forced into the basis. In this way a number of solutions are obtained, most of which will be sub-optimal[1]. While such manipulation of the final matrix can be done by computer, Tyler [17] has elaborated on this method using hand calculations to show how simultaneous changes in resource restraints may be analysed, and how activity vectors can be combined permitting a number of non-basic activities to be forced into the basis simultaneously. Rickards and Musgrave [13] used a similar procedure to find a sub-optimal integer solution which was a practicable farm plan, compatible with restraints imposed by soil types, paddock sizes, and paddock boundaries.

A second method of obtaining an array of sub-optimal solutions to a given linear programming problem is to arrange for the solutions generated at each iteration of the calculation to be printed out. Of course the solutions which happen to lie on the simplex path within a certain area below the optimum are only a fraction of the total number of (vertex) solutions in this area and the fact that they lie on this path constitutes no special reason for preferring them over any other solutions in this area. Using the Product Form of the Inverse (Revised Simplex) Method it is possible to confine the sub-optimal solutions obtained in this manner to those which yield a profit within some specified percentage of the optimal profit.

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[1] Some of these alternative solutions may yield the same profit as the initial optimum. This will occur if the marginal value product of one of the non-basic activities is zero [5, p. 70]. A non-unique optimal solution also exists if the basic solution includes an activity at zero level. Such an activity may be replaced in the basis by a non-basic activity, also at zero level, providing the pivot coefficient is non-zero [14, p. 43].

These two methods generate sub-optimal solutions quickly and easily. However, many of these plans are likely to exhibit trivial differences from the optimum. Even when important differences do exist, the sub-optimal plans may be of no particular interest to the farmer. There exists an infinite number of sub-optimal (vertex and non-vertex) solutions to any linear programming problem and it would be largely a matter of chance how well solutions generated in these ways happened to conform to the farmer's own objectives and preferences.

In contrast to the methods described above which are based on the derivation of sub-optimal solutions for a given initial matrix, it is possible to explore the consequences of adding further constraints to the original problem, or of varying existing constraints. Such adjustments can be designed to incorporate additional or secondary objectives of the farmer into the analysis. Two alternative approaches are possible.

(a) Maximize the primary objective function (income) subject to the secondary objective(s) as (a) constraint(s). The level of the constraint is assigned various values, or is varied parametrically over the relevant range, thereby generating a set of solutions[2]. While the decision maker may be prepared to forego some income to achieve a secondary objective, there will be some limit to the financial sacrifice he is prepared to make. Thus, this method has the disadvantage that the constraint may be set at such levels that some of the resultant solutions are of little interest because of the low level of income. This risk may be reduced by using parametric procedures, but for efficient computation and analysis of solutions, upper and lower bounds may have to be set on the constraint, which could result in the exclusion of the solution most preferred by the farmer.

(b) Convert the income objective function into a minimum income constraint, and optimize some other objective. Again, by varying the minimum income level parametrically, a set of solutions is obtained. This method has the advantage that all solutions lie within the range of acceptable incomes. However the information on shadow prices, cost ranges, etc. is not directly interpretable in terms of income because of the changed objective function. This loss is not too serious because the stability of any solution may be ascertained by judicious comparison of solutions, or by performing a few further simple calculations.

Miller and Nauheim [8] used this latter method to minimize the outlay on variable costs subject to a minimum income constraint. They generated a set of solutions indicating for each specified level of income, the minimum required outlay on variable costs. Insofar as risk attaches to the outlays on variable costs, this is a simplified approach to minimizing risk.

Method (b) is also used in quadratic risk programming, where the objective is to minimize variance subject to an income constraint. The usual

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[2] Although solutions are obtained only when the composition of the basis changes, in fact an infinite number of solutions may be obtained by interpolation between adjacent solutions.

method is to use a parametric procedure to generate the "efficient" set of minimum variance solutions for each specified level of expected income. The technique, which has not been widely used for farm planning in Australia[3], depends upon an assumption that the risk associated with alternative farm plans can be adequately represented by a variance-covariance matrix attached to the activity net revenues. However, where a high degree of uncertainty is attached to the b_i or a_{ij} coefficients, the variance-covariance matrix may not be independent of the activity levels in the solution. More complete specification of the problem to achieve this independence will encounter practical problems such as very large matrices, and difficulties of adequately estimating the variance-covariance matrix. Thus, the technique can be most appropriately used for farm planning when variance of product prices or cash crop yields comprises the main source of risk [2], [6].

An example of method (a) is found in Boussard and Petit [1]. A number of constraints are used to incorporate Shackle's concept "focus loss" into the linear programming formulation. The primary objective is to maximize income subject to a secondary objective of "negligible risk of ruin", i.e. negligible risk of an income insufficient to cover unavoidable costs, including fixed costs and minimum living requirements. While this is a useful contribution to the formulation of more complex farmer objectives into the linear programming model, there are difficulties associated with estimation of some of the parameters, for example the "focal loss" for each activity. Furthermore, it has not been shown that the objectives used in this study are widely held by farmers.

A somewhat similar approach to sub-optimal programming has been developed by McInerney [7] who used game theory concepts to find the maximin level of income for a farm operating under uncertainty. The model is composed of two parts: (i) the set of physical farm constraints; and (ii) the set of constraints to find the maximin solution of a game against nature. The maximin criterion has been shown to be an unsatisfactory rule for decision making as it implies that Nature is a conscious adversary when in fact she is neutral. Thus, this criterion will define a conservative farm plan, choosing the best strategy given that the worst state of nature will occur [9]. For example, if the worst state of nature results in negative returns for every set of activities, a not impossible situation in Australia's variable physical environment, the solution would be to do nothing in all years. This is clearly an unsatisfactory farm plan.

An alternative approach to the derivation of sub-optimal solutions to a farm planning problem lies in the use of Monte Carlo programming methods [4], [16]. The method involves the selection of activities and activity levels at random, subject to the condition that the solution so obtained must lie within the feasible set[4]. Monte Carlo programming has

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[3] One Australian study of this type was undertaken by Sturgess [15].

[4] The Monte Carlo programming method developed by Thompson [16] includes a procedure which ensures that all the solutions generated are feasible. A sub-optimizing routine is also incorporated to take advantage of the more obvious opportunities for profit improvement.

certain important advantages over linear programming. For example, integer constraints are readily incorporated and it is possible to arrange for more than one objective criterion to be evaluated for each solution. The disadvantages lie in the limited size of the problem which can be handled and the computer time and storage needed to generate and analyse the requisite number of solutions. Further developments in computers and of the technique itself should enable Monte Carlo programming to play an increasing role in solving farm planning problems.

3 AN EMPIRICAL APPLICATION

In order to provide some indication of the value of sub-optimal programming methods under Australian conditions the authors have applied one such method as part of a farm planning study of a group of farms in the Warialda area of northern New South Wales. The method adopted was that of maximizing a secondary objective function subject to a parametrically varied minimum income constraint. The use of this technique had three important advantages.

(a) The technique is a convenient and speedy one with well developed, easily used computer routines. This is in contrast to such alternatives as quadratic risk programming and Monte Carlo programming.

(b) Linear programming is now widely understood by farm advisers who know of its advantages and limitations, in contrast to some of the alternative techniques.

(c) By considering a number of secondary objective functions in turn it was possible to generate sets of solutions emphasizing selected aspects of the farm planning problem.

Five case study farms in the area were selected and programmed solutions obtained for each using a variety of secondary objective functions with a minimum income constraint. For each farm the minimum income constraint was varied parametrically between 85 per cent and 100 per cent of the maximum net farm income. The objectives considered were not necessarily the actual objectives held by the farmers studied, but were chosen to be of general interest to farmers in the area. A selection of the results is discussed below[5].

Because of possible adverse agronomic effects of continued grain cropping, the effects of introducing a lucerne rotation were analysed by computing plans to maximize the acreage of lucerne subject to a minimum income constraint. For example, with net farm income for Farm *A* at 92·5 per cent of the optimal level, 20·5 per cent of the arable area could be sown to lucerne, while at 85 per cent of optimal net farm income, 40·7 per cent of the arable area was under lucerne, permitting a four-year wheat, three-year lucerne rotation to be used. Plans for all five farms were computed in this way. The results are shown in table 1.

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[5] A full discussion of the area, the farms studied, and the results may be found in Powell and Hardaker [11].

TABLE 1

*Lucerne Areas for Sub-optimal Income Levels**

Farm	Percentage of arable area under lucerne	
	85 per cent optimal income	92·5 per cent optimal income
A	40·7	20·5
B	31·8	16·7
C	33·3	18·2
D	29·1	15·2
E	50·6	25·6

* The optimal income plans for all five farms did not include any lucerne.

These results indicate the possibility of using a wheat-lucerne rotation given that a lower net farm income is acceptable. Furthermore, by interpolation from adjacent solutions, it is possible to formulate a maximum lucerne farm plan for any level of income in the range considered. Similar results would be obtained for the maximization or minimization of any activity or group of activities subject to a parametrically varied minimum income constraint. The information obtained clearly indicates the behaviour of the farm plan over the relevant range of incomes and provides a better basis for deriving sub-optimal farm plans than the use of shadow prices obtained from any single solution, which are relevant only for marginal (generally small) changes in activity levels.

A second set of solutions was computed for Farm *A* designed to minimize labour requirements. Farm *A* of 3,700 acres has 1,600 acres suitable for grain cropping. The remaining 2,100 acres is natural pasture suitable for either cattle or sheep grazing. Table 2 shows the optimal income plan for Farm *A*. The minimum labour results appear in table 3.

TABLE 2

Optimal Income Plan, Farm A

Activity	Unit	Level
Wheat	Acre ..	1,333
Grain oats*	Acre ..	267
Natural pasture	Acre ..	2,100
Merino ewes	Head ..	2,632
Net farm income	Dollar ..	28,194

* The oats also provide winter grazing.

TABLE 3

Farm Plans to Minimize Labour Requirements

Activity	Unit	Optimal income	85 per cent optimal income
Wheat	Acre ..	1,333	1,600
Grain oats	Acre ..	267	..
Natural pasture	Acre ..	2,100	2,100
Merino ewes	Head ..	2,632	..
Beef cows	Head ..	..	150
Labour required	Mandays ..	1,000	380
Net farm income*	Dollar ..	28,194	23,965

* Labour costs have been included as part of overhead costs which are assumed to be constant for all farm plans. Therefore, net farm income for the sub-optimal plan has not been adjusted to account for the possibility of reducing the farm labour force. In this case, a saving of labour costs of about $4,850 would be possible.

The farmer's objective in this instance may be to maximize his leisure time while earning a specified level of income, to minimize the worry of supervizing employed labour, or to restrain the input of a resource the cost of which is rising more rapidly than that of other inputs. The solutions in table 3 indicate the possibility of adopting a simpler, less labour intensive farm plan. The sub-optimal income plan could be operated by two full time men, compared to four for the optimal income plan. Thus, if the farmer is able to vary the number of workers employed a substantial saving in labour costs could be achieved.

TABLE 4

Farm Plans to Maximize Minimum Income

Activity	Unit	Optimal income	85 per cent optimal income
Wheat	Acre ..	1,642	1,400
Grain oats	Acres ..	158	..
Sorghum	Acre ..	..	400
Natural pasture	Acre ..	655	..
Merino ewes	Head ..	952	..
Average net farm income*	Dollar ..	23,025	18,744
Maximin net farm income*	Dollar ..	—30,653	—24,679

* After deduction of overhead costs, assumed to be constant over all farm plans, of $13,583.

Farm *B* is a property of 2,445 acres of which 1,800 acres are arable leaving 655 acres of natural pasture. On this farm the secondary objective of maximizing minimum income was investigated. The minimum activity gross margins were estimated on the assumption that crops were sown but failed and that livestock had to be hand-fed for a ten-month period. This minimum income was then maximized subject to the constraint that the expected income should not fall below some specified level. The optimal income plan and sub-optimal income plan which maximizes minimum income subject to a minimum income constraint of 85 per cent of the maximum are shown in table 4.

These results indicate the possibility of reducing losses should a severe drought occur. While this method of analysis provides useful results, it must be regarded as only a crude approximation to the problem of planning farms in an uncertain environment. However, the results highlight those activities which are likely to incur large losses should a severe drought occur, in this case merino ewes, and enable the farmer to weight the risk of large losses against the extra income obtained from undertaking those activities.

4 DISCUSSION AND CONCLUSIONS

Although the analysis described above produced results which were of interest to the farmers concerned, an objective assessment of the value to the farmers of these results is impossible, primarily because the study was not performed in an advisory context, i.e. the farmers did not ask to have their farms planned. In using linear programming as an advisory aid it would normally be necessary to tailor the analysis much more closely to the individual farmers' requirements. Nevertheless, the study does demonstrate that there often exists a number of farm plans which differ considerably in their technical characteristic while resulting in only small differences in financial result. This corresponds with the conclusion of Renborg [12] who made an extensive study of this aspect of farm planning in Sweden. Given the finanical similarity of a number of farm plans, it is likely that the farmer will select a particular plan on the basis of some other objective(s). For example, he may select a plan because of a negligible risk of low income, or because he prefers cattle to sheep, etc. Thus, in using linear programming as an advisory aid it may be important to generate some of these sub-optimal solutions based on the significant preferences expressed by the farmer.

The various ways of deriving such alternative solutions were discussed in the first part of the paper. While no single method is ideal in all cases, this study of the Warialda farms demonstrates that standard linear programming methods can be used to obtain the kind of results needed for practical farm planning. The procedure quickly and easily generates a set of solutions which highlight aspects of the farm plan as may be indicated by the farmer. From the set of solutions, the farmer is able to select a farm plan which most satisfactorily corresponds to his real planning objectives.

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