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**On Microeconomics Efficiency and Entrepreneurship under Bounded Rationality**

By

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and

Bradford Barham*

Abstract: We present a dynamic model of economic behavior of an owner-operated firm under bounded rationality, and develop the implications for the assessment of economic efficiency and the understanding of entrepreneurship. Under bounded rationality, information about technology and market conditions is not perfectly known, creating the possibility for learning. Uncertainty is represented using a general state-contingent approach. Efficiency analysis is explored using a certainty equivalent measure, which is the sum of three parts: expected net income, a conditional value of information, and a risk premium (measuring the implicit cost of private risk bearing). The analysis yields critical insights about the role of learning in technical efficiency, allocative efficiency, and their implications for an improved understanding of entrepreneurship. These insights also provide micro-theoretic grounding to some classic debates (e.g., Leibenstein and Stigler) about what contributes to efficiency and technological progress in an economy.

Keywords: bounded rationality, firm efficiency, information, learning, risk, technology.

JEL: D2, D8, D9, L2

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**On Microeconomic Efficiency and Entrepreneurship**

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## 1. Introduction

As a primary measure of welfare outcomes, the concept of efficiency dominates neoclassical economic inquiry. Efficiency at the producer level involves assessing whether agents are operating on the production frontier (technical efficiency) and at the optimal point on the frontier (allocative efficiency). Following Debreu (1951), Koopmans, Farrell and others, economists have examined both types of efficiency outcomes.[1] However, theoretical and empirical analyses of efficiency have typically assumed perfect information.[2]

Most economic agents (e.g., entrepreneurs, farmers, prospectors, innovators) confront imperfect information about technology, production conditions, and/or market possibilities. In particular, economic agents may find it difficult or costly to gather and process information about technology, production, or market conditions, in which case their decisions can be modeled as ones made under bounded rationality (Simon; Conlisk).[3] The basic challenge that incorporating bounded rationality creates for the standard concept of efficiency is that it becomes a "conditional measure", one that depends on the agent's knowledge of the underlying (and potentially evolving) technology, production conditions, and market possibilities. And, because knowledge acquisition is a process the producer can manage through learning, conventional efficiency (and productivity) concepts may need to be modified to incorporate the role of learning under conditions of bounded rationality.

Learning is a dynamic process (one that generally takes time and effort) and a heterogeneous one (agents vary in their capacity to learn and their ways of learning). Under bounded rationality, the dynamic nature of learning may call into question unconditional, static statements about firm efficiency outcomes, because learning activities in the current period have the potential to shape productivity outcomes in future periods. The heterogeneous nature of learning also raises questions about standard measures of efficiency, because individual differences in human capital shape the capacity of economic agents to acquire knowledge about technologies, production conditions, and market possibilities (Schumpeter, 1934, 2005; Schultz, 1975, 1981; Nelson and Winter).

More explicit attention to the role of learning under bounded rationality has the potential to broaden our understanding of efficiency and entrepreneurship. This paper pursues that objective by developing a single-agent model of an owner-operated firm facing uncertainty and bounded rationality. Uncertainty and bounded rationality are interwoven: a critical aspect of imperfect information is the degree of knowledge that an agent has about future production and marketing conditions. They are integrated here to include risk preferences, which also play a role in dynamic decisions under uncertainty (e.g., entrepreneurial activities). In that sense, this paper extends the seminal work of Kihlstrom and Laffont on the role of risk and risk preferences in entrepreneurship by incorporating the role of learning. The explicit focus on learning adds a micro-foundation to the general insights of Schumpeter (1934, 2005), Schultz (1981), Lazear, Kirzner, and others regarding the role that differences in human capital play in the success of entrepreneurs.

Learning is treated in a general fashion in the model, which incorporates both "learning-by-doing" (Arrow, 1962) and investment decisions under uncertainty (Arrow and Fisher, Henry, and Pindyck and Dixit). Uncertainty is represented using a general state-contingent approach (Debreu, 1959). In complex economic environments, bounded rationality reflects the fact the evaluation of state-contingent decisions becomes difficult, forcing the decision maker to use crude decision rules. In this context, we show that the decision maker's problem can be formulated as one of maximizing his/her "certainty equivalent" (CE). We then use this CE to study efficiency outcomes. Under bounded rationality using the CE formulation, firm-level economic efficiency can still be decomposed into two parts: technical efficiency, and allocative efficiency.[4] Efficiency outcomes are further decomposed into three parts: expected net income, a risk premium (measuring the implicit cost of private risk bearing), and a conditional value of information.  .

The first two components of this efficiency measure are not novel, but the third one about the conditional value of information apparently is. Assuming heterogeneity of human capital among economic agents, we are able to develop several potentially valuable insights into the role of learning in efficiency and productivity assessment. One insight is that previous measures of technical inefficiency are likely to be biased upwards because of their omission of the role of learning under bounded rationality. Another is that these learning processes about production technologies may be the micro-theoretic underpinnings of what Leibenstein labeled as X-inefficiency and Stigler critiqued for its lack of clear microeconomic foundations. A third concerns the microeconomics of entrepreneurship. We analyze the dual role of entrepreneurs, both in discovering new technologies (yielding productivity gains) and in improving allocative efficiency through more judicious choice of inputs and products mix. Finally, building on Schumpeter (1934, 2005) and Schultz (1975, 1981), our investigation emphasizes the fundamental role of human capital in shaping learning processes, productivity change, and entrepreneurial success. The paper closes with reflections on how extending the model to a multi-agent setting might deepen our capacity to understand the roles of markets, institutions, and contracts in the process of productivity change and entrepreneurial activities.

## 2. The Model

Consider an economic agent making decisions for a firm over time. For simplicity, we focus our attention on a two-period model with production/investment decisions in both periods. Using the netput notation (where outputs are positive and inputs are negative), denote by $\mathbf{x}_1 = (x_{11}, \ldots, x_{nt}) \in \Re^n$ the n-vector of netputs chosen at time t = 1 and by $\mathbf{x}_2 = (x_{12}, \ldots, x_{m2}) \in \Re^m$ the m-vector of netputs chosen at time t = 2. The agent faces uncertainty which comes from the production technology as well as market conditions. Production uncertainty, represented by R possible states, r = 1, …, R, reflects all uncertain factors related to the production process, ranging from imperfectly understood aspects of the technology to stochastic factors (such as unforeseen weather effects, possibility of strikes, or equipment breakdown). Market uncertainty, represented by S possible states, s = 1, …, S, reflects all factors that generate uncertainty about market conditions and future market prices. Note that the number of states can be quite large. For example, if production uncertainty is generated by 10 random variables, each one taking one of 10 possible values, then $R = 10^{10}$. Dealing with a large number of states can be quite difficult and problematic for the agent and the economic analyst (e.g., Simon). This "curse of dimensionality" motivates the introduction below of a certainty equivalent approach for the assessment of efficiency. In addition, the model assumes incomplete risk markets, where risk exposure cannot be transferred entirely to other agents.

The economic agent is assumed here to be an owner-operator (e.g., entrepreneur). While period-two decisions can depend on the information that becomes available about the states of nature, all period-one decisions are made *ex ante*.

At time t, the agent has a fixed amount of time T to allocate between leisure $L_{et}$, labor input in the firm $L_{at}$, and wage activities $L_{wt}$ spent working outside the firm and earning a wage rate $p_{Lt}$. At time t, the time constraint is

$$T = L_{at} + L_{wt} + L_{et}, \tag{1}$$

with $L_{at} \geq 0$, $L_{wt} \geq 0$, and $L_{et} \geq 0$, t = 1, 2.[5] At time t, the agent also chooses a consumption good $c_t$, t = 1, 2. He/she faces price $p_{ct} > 0$ for consumption $c_t$, a wage rate $p_{Lt}$ for wage labor $L_{wt}$, and prices $\mathbf{p}_{xt} \equiv (p_{x1t}, p_{x2t}, \ldots)$ for netputs $\mathbf{x}_t$, where $p_{xit} > 0$ is the market price of $x_{it}$, t = 1, 2. Being the residual claimant, the owner receives the period-one firm profit $(\mathbf{p}_{x1} \cdot \mathbf{x}_1)$.[6] In period one, the agent also chooses to invest an amount I into an asset yielding a unit return of $[1 + \rho(s)]$ in period two. It follows that the period-one budget constraint is

$$p_{c1}\, c_1 \leq w + p_{L1}\, L_{w1} + \mathbf{p}_{x1} \cdot \mathbf{x}_1 - I, \tag{2a}$$

where w denotes initial wealth, $(p_{L1}\, L_{w1})$ is wage income, and $(\mathbf{p}_{x1} \cdot \mathbf{x}_1)$ is the firm profit at time t = 1.

At time t = 2, the agent chooses netputs $\mathbf{x}_2$, consumption good $c_2$, along with the time allocation $L_{e2}$, $L_{a2}$ and $L_{w2}$. Under market conditions, he/she faces market price $p_{c2}(s) > 0$ for $c_2$, a wage rate $p_{L2}(s)$ for $L_{w2}$, and prices $\mathbf{p}_{x2}(s)$ for netputs $\mathbf{x}_2$. Being the residual claimant, he/she receives the period-two firm profit ($\mathbf{p}_{x2} \cdot \mathbf{x}_2$). Denote by $c_2(r, s)$, $\mathbf{x}_2(r, s)$ and $L_{w2}(r, s)$ the period-two decision for $c_2$, $\mathbf{x}_2$ and $L_{w2}$, respectively, under state (r, s). It follows that the agent's period-two budget constraint is

$$p_{c2}(s)\, c_2(r, s) \leq p_{L2}(s)\, L_{w2}(r, s) + \mathbf{p}_{x2}(s) \cdot \mathbf{x}_2(r, s) + [1 + \rho(s)]\, I. \tag{2b}$$

Substituting (2a) into (2b) gives the overall budget constraint

$$\begin{aligned} p_{c2}(s)\, c_2(r, s) \leq\ & [1 + \rho(s)][w + p_{L1}\, L_{w1} + \mathbf{p}_{x1} \cdot \mathbf{x}_1 - p_{c1}\, c_1] \\ & + p_{L2}(s)\, L_{w2}(r, s) + \mathbf{p}_{x2}(s) \cdot \mathbf{x}_2(r, s). \end{aligned} \tag{3}$$

The period-two decisions for consumption and leisure under state (r, s) are respectively $c_2(r, s)$ and $L_{e2}(r, s)$). The associated decision rules under all possible states are $\tilde{c}_2 \equiv (c_2(1,1), \ldots, c_2(R, S))$ and $\tilde{L}_{e2} \equiv (L_{e2}(1,1), \ldots, L_{e2}(R, S))$. Using a state-contingent approach, the agent's preferences are represented by the *ex ante* utility function $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$. Note that this includes as a special case the expected utility (EU) model. Indeed, under the EU model, $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2}) = \sum_{r=1}^{R} \sum_{s=1}^{S} \Pr(r, s)\, U(c_1, L_{e1}, c_2(r, s), L_{e2}(r, s))$, where $\Pr(r, s)$ is the probability of facing the state (r, s) and $U(c_1, L_{e1}, c_2, L_{e2})$ is a von Neumann-Morgenstern utility function representing the agent's risk preferences. However, the state-contingent utility $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ applies under conditions much broader than the EU model (Debreu, 1959; Chambers and Quiggin). For example, it includes as special cases weighted utility (Chew), rank-dependent expected utility (Quiggin), and general smooth preferences (Machina). Unlike the EU model, this allows for preferences that are not linear in the probabilities. And more generally, the state-contingent approach does not even require that the agent formulates a probability assessment of the states (Debreu, 1959). Throughout, we assume that $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ is strictly increasing in $(c_1, \tilde{c}_2)$.

As noted above, the decisions made at time t = 1 (i.e., $\mathbf{x}_1$, $L_{a1}$, $L_{w1}$, $L_{e1}$, $c_1$ and I) are chosen *ex ante*. This means that they do not depend on the states (r, s). However, the decisions made at time t = 2 can depend on the states. This includes the period-two consumption and leisure decisions $\tilde{c}_2 \equiv (c_2(1,1), \ldots, c_2(R, S))$ and $\tilde{L}_{e2} \equiv (L_{e2}(1,1), \ldots, L_{e2}(R, S))$. The nature of state-contingency reflects the amount of the agent's learning about his economic environment. Below, we assume that the period-two consumption/leisure decisions ($c_2$, $L_{e2}$) are made *ex post*. It means that $c_2(r, s)$ and $L_{e2}(r, s)$ can be different across each state (r, s). However, we want to capture the role of the learning process for other period-two decisions. This includes the netput decision $\tilde{\mathbf{x}}_2 \equiv (\mathbf{x}_2(1,1), \ldots, \mathbf{x}_2(R, S))$, and the labor decision $\tilde{L}_{a2} \equiv (L_{a2}(1,1), \ldots, L_{a2}(R, S))$ and $\tilde{L}_{w2} \equiv (L_{w2}(1,1), \ldots, L_{w2}(R, S))$. Let $\mathbf{z}_2 \equiv (\mathbf{x}_2, L_{a2}, L_{w2}) = (z_{12}, \ldots, z_{m+2,2}) \in \Re^{m+2}$, with $\tilde{\mathbf{z}}_2 \equiv (\mathbf{z}_2(1,1), \ldots, \mathbf{z}_2(R, S)) \in \Re^{(m+2)RS}$. We allow the decisions $\mathbf{z}_2$ to reflect different amounts of learning by considering different partitions of the state space $P \equiv \{1, \ldots, R\}\times\{1, \ldots, S\}$. Let $P_i$ be a partition of P, i.e. a collection of disjoint subsets of P whose union is P. Assume that $z_{i2}$ (the i-th decision variable in $\mathbf{z}_2 \equiv (\mathbf{x}_2, L_{a2}, L_{w2})$) is chosen based on the information partition $P_i$ such that

$$z_{i2}(r, s) = z_{i2}(r', s') \text{ if } (r, s) \text{ and } (r', s') \text{ are in the same element of } P_i, \tag{4}$$

$i = 1, \ldots, m+2$. Equation (4) means that, when choosing $z_{i2}$, the agent cannot distinguish between states that are in the same elements of the partition $P_i$. This can represent different amount of information available. At one extreme, perfect information corresponds to $P_i = P^+ \equiv \{(1, 1), \ldots, (R, S)\}$, where $P^+$ has RS elements with each element corresponding to a state (r, s). Then, $P_i = P^+$ implies that the agent chooses $z_{i2}$ *ex post*. At the other extreme, no information corresponds to $P_i = P^- \equiv \{P\}$, where $P^-$ has only one element. Then, $P_i = P^-$ implies that the agent chooses $z_{i2}$ *ex ante*. And partial learning corresponds to intermediate situations where the number of elements in $P_i$ is greater than 1 but less than RS.

Denote by $\mathbf{P} = (P_1, \ldots, P_{m+2})$ the information structure supporting the second-period decisions $\mathbf{z}_2 \equiv (\mathbf{x}_2, L_{a2}, L_{w2}) = (z_{12}, \ldots, z_{m+2,2})$. To investigate the role of the learning process, we allow $\mathbf{P}$ to be endogenous. That is, we consider situations of active learning, where the agent uses the resources he/she controls to obtain information about his/her economic environment.

For a given information structure $\mathbf{P}$, represent the firm technology by the feasible set $F(\mathbf{P}) \subset \Re^{n+2+(m+2)RS}$, where $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})$ means that netputs $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ are feasible under the information structure $\mathbf{P}$. Note the generality of this characterization. It guarantees feasibility for ($\mathbf{x}_1$, $L_{a1}$, $L_{w1}$, $\tilde{\mathbf{x}}_2$, $\tilde{L}_{a2}$, $\tilde{L}_{w2}$) across all possible states. It allows for jointness between choosing ($\mathbf{x}_1$, $L_{a1}$, $L_{w1}$, $\tilde{\mathbf{x}}_2$, $\tilde{L}_{a2}$, $\tilde{L}_{w2}$) and learning (the choice of **P**) about both technology (represented by the states r = 1, …, R) and market conditions (represented by the states s = 1, …, S). As such, it can represent situations of active learning (including learning-by-doing; see Arrow, 1962). Under active learning, we assume that $F(\mathbf{P}) \subset F(\mathbf{P'})$ for any information structure **P'** that is at least as fine as **P**. Then, $F(\mathbf{P'})$ - $F(\mathbf{P})$ represents the set of resources required to learn so as to replace **P** by **P'**. And the benefits obtained from the new information are associated with equation (4) (which becomes less restrictive). The feasible set $F(\mathbf{P})$ also allows for the possibility that labor activities outside the firm ($L_{w1}$, $L_{w2}$) can affect the productivity of labor within the firm ($L_{a1}$, $L_{a2}$). And it can reflect contractual and institutional restrictions imposed on labor choices both within and outside the agent's firm. Finally, the characterization allows the amount of learning to be specific to each decision $z_{i2}$. This can represent situations where information processing requires the use of resources but with a learning process that varies across $z_{i2}$'s. [7]

Let $\mathbf{L}_t \equiv (L_{at}, L_{wt})$, t = 1, 2. Under economic rationality, the agent's decision is represented by the optimization problem:

$$\text{Max } \{u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2}) \text{: equations (1), (2a), (2b) and (4);}$$

$$(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})\}. \tag{5}$$

The utility function $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ being strictly increasing in ($c_1$, $\tilde{c}_2$), the budget constraint (3) is always binding under each state (r, s). Below, we assume for simplicity that leisure is always positive, with $L_{e1} > 0$ and $L_{e2}(r, s) > 0$. Then, after substituting (1) and (3) into the utility function, the optimization problem (5) can be alternatively written as

$$\text{Max}_{c_1, \mathbf{x}_1, \mathbf{L}_1 \geq 0, \tilde{\mathbf{x}}_2, \tilde{\mathbf{L}}_2 \geq 0, \mathbf{P}} \ \{u[c_1, T - L_{a1} - L_{w1}, \ldots,$$

$$[w + p_{L1} L_{w1} + \mathbf{p}_{x1} \cdot \mathbf{x}_1 - p_{c1}\, c_1][1 + \rho(s)]/p_{c2}(s)$$

$$+ [p_{L2}(s) L_{w2}(r, s) + \mathbf{p}_{x2}(s) \cdot \mathbf{x}_2(r, s)]/p_{c2}(s), \ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \ldots]:$$

$(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}$, (6)

where $Z(\mathbf{P}) \equiv \{(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$: equation (4) evaluated at $\mathbf{P}$; $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})\}$ denotes the feasible set, and $(\tilde{\mathbf{x}}_2^*, \tilde{\mathbf{L}}_2^*, \mathbf{x}_1^*, \mathbf{L}_1^*, c_1^*, \mathbf{P}^*)$ are the optimal decisions in (6).

## 3. Certainty Equivalent

Under incomplete risk markets, the agent cannot transfer his/her risk exposure entirely to other agents. Thus, risk exposure and related information are expected to affect the welfare of the agent. If so, how do risk and information affect production decisions? This section first explores under what conditions period-one netputs would be chosen in a way consistent with standard profit maximization and then modifies the approach to a certainty equivalent so as to allow a way of dealing with the "curse of dimensionality."

First, we consider whether profit maximization applies to period-one netputs? With the utility function $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ being strictly increasing in $(c_1, \tilde{c}_2)$, note that the optimization with respect to $\mathbf{x}_1$ in (6) implies the profit maximization problem

$$\pi(\mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P}) = \text{Max}_{\mathbf{x}_1} \{\mathbf{p}_{x1} \cdot \mathbf{x}_1:$$

$$(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}, \quad (7)$$

where $\mathbf{x}_1^{\pi}(\mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P})$ is the optimal solution for $\mathbf{x}_1$, and $\pi(\mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P})$ is a restricted profit function. The profit function $\pi(\mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P})$ is homogenous of degree one and convex in $\mathbf{p}_{x1}$. Equation (7) is thus a standard profit maximization problem conditional on period-two state-contingent decisions $(\tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ and on the information structure $\mathbf{P}$. However, the conditionality on $(\tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ has important implications. The state-contingent choices $(\tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ control for the distribution of risk across all possible states. This control for risk exposure is the key reason why risk preferences do not play any role in (7) and conceptually can be seen as a significant advantage of (7): it applies irrespective of risk preferences. However, making equation (7) empirically tractable can be quite challenging, because it requires identifying the decisions ($\mathbf{x}_a$, $L_{a2}$, $L_{w2}$) under all possible states. When the number of states is large, identifying these decisions becomes very demanding or perhaps infeasible. This "curse of dimensionality" is the main reason why the state-contingent approach has not been used much in the analysis of production decisions under risk, and we are left with three important implications. First, equation (7) shows that the maximization of profit remains a valid motivation for a firm under very broad conditions. Second, the problem with profit maximization under risk is not in its conceptual validity but rather in its empirical tractability. Third, an alternative approach is needed to move forward, and one way is to introduce a "certainty equivalent" formulation with an explicit treatment of risk preferences in the agent's initial decision choice.[8]

We start the certainty equivalent approach by introducing $\mathbf{q}_2(s)$ as the discounted prices for $\mathbf{x}_2$ under state s, and writing it as: $\mathbf{q}_2(s) = \mathbf{p}_{x2}(s)/[1 + \rho(s)]$, s = 1, …, S, with $\tilde{\mathbf{q}}_2 = (\mathbf{q}_2(1), \ldots, \mathbf{q}_2(S))$. Let

$$
\begin{aligned}
&v(\ldots, w + \mathbf{p}_{x1} \cdot \mathbf{x}_1 + \mathbf{q}_2(s) \cdot \mathbf{x}_2(r, s), \ldots, \cdot) \\
&\quad \equiv u[c_1, T - L_{a1} - L_{w1}, \ldots, [w + p_{L1} L_{w1} + \mathbf{p}_{x1} \cdot \mathbf{x}_1 - p_{c1} c_1][1 + \rho(s)]/p_{c2}(s) \\
&\qquad + [p_{L2}(s) L_{w2}(r, s) + \mathbf{q}_{x2} [1 + \rho(s)] \cdot \mathbf{x}_2(r, s)]/p_{c2}(s), \ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \ldots], \qquad (8)
\end{aligned}
$$

where "·" denotes other arguments that are suppressed to simplify the notation. Using equation (8), define the certainty equivalent as the sure monetary value CE which satisfies

$$
v(\ldots, w + \mathbf{p}_{x1} \cdot \mathbf{x}_1 + \mathbf{q}_2(s) \cdot \mathbf{x}_2(r, s), \ldots, \cdot) = v(\ldots, w + CE, \ldots, \cdot), \qquad (9)
$$

where the other arguments "·" are being held constant (including $c_1$, $L_{a1}$, $L_{w1}$, $\tilde{L}_{a2}$ and $\tilde{L}_{w2}$). Denote by $CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot)$ the solution of (9) for CE. It is conditional on the period-one netput $\mathbf{x}_1$ and on the state-contingent period-two netputs $\tilde{\mathbf{x}}_2$. Starting from a situation with zero profit, the certainty equivalent CE measures the agent's *ex ante* willingness to pay for the state-contingent discounted profit

($\mathbf{p}_{x1} \cdot \mathbf{x}_1 + \mathbf{q}_2(s) \cdot \mathbf{x}_2(r, s)$) across all states. With the utility function $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ being strictly increasing in ($c_1$, $\tilde{c}_2$), it follows that $v(\cdot)$ in (9) is strictly increasing in w. Comparing equations (6) and (9) generates the following result.

<u>Proposition 1</u>: The production/investment decisions ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$) in (7) satisfy

$$CE^*(w, \mathbf{p}_{x1}, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) = \text{Max}_{\mathbf{x}_1, \tilde{\mathbf{x}}_2} \{CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}, \tag{10}$$

Equation (10) shows that the certainty equivalent $CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot)$ along with the feasible set $Z(\mathbf{P})$ provide all the information necessary for the production/investment decisions ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$). Note this representation is very general. It applies under any specification of risk preferences and learning process. It applies even if the agent decides to work only in the firm, i.e. if he/she chooses $L_{wt} = 0$, t = 1, 2. And it also applies irrespective of the feasible set for $\mathbf{L}_t = (L_{at}, L_{wt})$, t = 1, 2. This allows for situations where labor contracts are not flexible and impose restrictions on the choice of ($L_{at}$, $L_{wt}$).[9] As such, the certainty equivalent given in (9) provides a broad characterization of the factors affecting period-one netput decisions $\mathbf{x}_1$.

Proposition 1 implies the following result that will prove useful in our analysis, as it makes explicit the role of information $\mathbf{P}$.

<u>Corollary 1</u>:

$$CE^*(w, \mathbf{p}_{x1}, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) = \text{Max}_{\mathbf{x}_1} \{CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot), \cdot)\}, \tag{11}$$

where $\tilde{\mathbf{x}}_2^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) \in \text{argmax}_{\tilde{\mathbf{x}}_2} \{v(\ldots, w + \mathbf{p}_{x1} \cdot \mathbf{x}_1 + \mathbf{q}_2(s) \cdot \mathbf{x}_2(r, s), \ldots, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\} \equiv \text{argmax}_{\tilde{\mathbf{x}}_2} \{CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}$.

## 4. Economic Efficiency under Bounded rationality

How much learning typically takes place? When the economic environment of the firm is simple, assessing the uncertainty facing the agent may be reasonably easy. Under such circumstances, obtaining perfect information may be attainable (provided that the decisions maker is willing to spend resources in the learning process). However, the economic environment of firms can be complex especially early in the development of a given sector or during periods of significant technological, market, or institutional change. This complexity provides many opportunities for learning, and could arise naturally in situations where the number of states R and S is large. In this context, information acquisition and processing may prove difficult. When R and S are large, we define bounded rationality as any situation where $F(\mathbf{P}^+) = \emptyset$ where $\mathbf{P}^+$ represents perfect information. This means that making all period-two decisions *ex post* is not feasible. Under such circumstances, while extensive learning remains feasible, perfect learning becomes impossible (Simon). In that context, our analysis provides a basis to investigate the economics of bounded rationality.

Proposition 1 provides a basis for conducting efficiency analysis for the firm under bounded rationality. It indicates that the certainty equivalent can be used as a measure of welfare for the agent and in the evaluation of the efficiency of production/investment decisions. For any $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$, equation (10) suggests the following measure of firm economic efficiency

$$EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) = CE^*(w, \mathbf{p}_{x1}, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) - CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot) \geq 0. \tag{12}$$

$EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (12) is the welfare loss facing the firm when the choice of $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ is not efficient under information $\mathbf{P}$. It states that economic efficiency for the firm is satisfied at $(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P})$ if and only if $EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) = 0$. Alternatively, finding $EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) > 0$ means that $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ is not efficient under information $\mathbf{P}$.

Note that $EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (12) provides an overall measure of firm efficiency. In general, it will be of interest to obtain additional insights into the source of possible inefficiency. In this context, let $\mathbf{g} \in \Re^n_+ - \{\mathbf{0}\}$ be some reference bundle of period-one netputs. For $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$, define

$$D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \in \text{argmax}_\beta \{CE(w + \mathbf{p}_{x1} \cdot [\mathbf{x}_1 + \beta\, \mathbf{g}], \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot):$$

$$(\mathbf{x}_1 + \beta\, \mathbf{g}, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}, \tag{13}$$

$D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (13) measures the number of units of the reference bundle $\mathbf{g}$ that moves point $\mathbf{x}_1$ to the upper bound of the feasible set under information $\mathbf{P}$. Both $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ and $(\mathbf{x}_1 + D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{g}, \tilde{\mathbf{x}}_2)$ being feasible under information $\mathbf{P}$, equation (13) implies that $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \geq 0$. Note from (9) that $CE(w + \mathbf{p}_{x1} \cdot [\mathbf{x}_1 + D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{g}], \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot) = D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{p}_{x1} \cdot \mathbf{g} + CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot)$. For any $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$, it follows from (10) and (13) that

$$CE^*(w, \mathbf{p}_{x1}, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) \geq CE(w + \mathbf{p}_{x1} \cdot [\mathbf{x}_1 + D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{g}], \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot)$$

$$\geq CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot). \tag{14}$$

This suggests the following decomposition of economic efficiency.

<u>Proposition 2</u>: For $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$, the economic efficiency measure $EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (12) can be written as :

$$EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) = TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) + AE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot), \tag{15}$$

where

$$TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \equiv CE(w + \mathbf{p}_{x1} \cdot [\mathbf{x}_1 + D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{g}], \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot)$$

$$- CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot),$$

$$= D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{p}_{x1} \cdot \mathbf{g} \geq 0, \tag{16a}$$

and

$$AE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \equiv CE^*(w, \mathbf{p}_{x1}, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot)$$

$$- \mathrm{CE}(w + \mathbf{p}_{x1} \cdot [\mathbf{x}_1 + D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)\, \mathbf{g}], \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot) \geq 0. \tag{16b}$$

Equation (15) decomposes the economic efficiency measure EE(·) into two additive parts: TE(·) in (16a) reflecting technical efficiency; and AE(·) in (16b) reflecting allocative efficiency. Equation (16a) states that the technical efficiency measure $\mathrm{TE}(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ is proportional to $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ given in (13). Given $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$, note that $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (13) can be alternatively written as

$$D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) = \mathrm{Max}_\beta \{\beta: (\mathbf{x}_1 + \beta\, \mathbf{g}, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}. \tag{13'}$$

The function $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (13') is the directional distance function proposed by Chambers et al. It is also the negative of the shortage function proposed by Luenberger. Luenberger (p. 20-22) and Chambers et al. have provided a detailed analysis of the properties of $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$. In particular, $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ can be used to assess technical efficiency.[10] Some key properties of the distance function $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ are:

- $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$ implies that $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \geq 0$.
- Under free disposal in $\mathbf{x}_1$,[11] $Z(\mathbf{P}) = \{\mathbf{z}: D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \geq 0\}$.
- Finding $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) > 0$ implies that $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ is not technically efficient.
- When $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ is strongly decreasing in $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$,[12] then $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ is technically efficient if and only if $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) = 0$.

This indicates that $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ provides a convenient measure of technical efficiency. In addition, $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ is closely related to other measures of technical performance (e.g., Farrell; Shephard; Grosskopf) that have appeared in the literature (see Chambers et al.; Färe and Grosskopf), and have been used in empirical analyses of production efficiency (Coelli et al. review this literature). The formulation of D($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) in (13') includes as special cases most measures of technical efficiency that have been proposed in the literature.[13] Combining these results with (16a), we obtain:

Proposition 3:

- In general, TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) ≥ 0 if the point ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$) is feasible,
- TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) > 0 implies technical inefficiency,
- When D($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) is strongly decreasing in ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$), technical efficiency holds if and only if TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) = 0,

Proposition 3 shows that finding TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) > 0 implies that the firm is technically inefficient. In this case, TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) measures the increase in the certainty equivalent that can be obtained by moving from point ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$) to the production frontier under information $\mathbf{P}$.

Next, AE(w, $\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{q}}_2$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) ≥ 0 in (16b) provides a measure of allocative efficiency. It measures the improvement in the certainty equivalent that can be obtained starting from point ($\mathbf{x}_1$ + D($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}$, ·) $\mathbf{g}$, $\tilde{\mathbf{x}}_2$) under information $\mathbf{P}$. Since this point is always on the frontier of the feasible set, it follows that AE(·) measures the improvement in the certainty equivalent moving along the upper bound of the feasible set.

## 5. Implications for productivity assessment

The measurement of technical efficiency under bounded rationality can be challenging, because it requires a clear understanding of the information set available to economic agents and such information may vary across individuals due to heterogeneity in their learning capacity. In this section, we use two conjectures to guide our exploration of the implications of bounded rationality for productivity assessment beginning with a more careful look at technical efficiency. Along the way, we suggest that the learning process that emerges under bounded rationality provides useful insights on productivity assessments and technological progress. Since a large part of economic growth comes from technological progress (as identified by Solow), economists continue to seek to understand the origins of productivity improvements (e.g., Romer).

There is considerable evidence that productivity can vary a lot both over time and across firms. In this context, Leibenstein has presented two arguments: 1/ Technical efficiency can vary a lot across firms; and 2/ Improving technical efficiency is a major way of improving economic efficiency. The empirical evidence generally supports these arguments (e.g., Frantz; Coelli et al.). But why would any firm choose to exhibit technical inefficiency? As argued by Stigler and Schultz (1975, 1981), economic rationality suggests that producers typically make efficient choices given the information available to them. The missing link explored below is that, under bounded rationality, heterogeneous agents with disparate learning capacities might have different information available to them about efficient netput combinations, both in a static and dynamic sense. This disparity means that producers might be technically efficient given their human capital but not as productive as others with higher learning capacity. Such arguments also appear relevant in studying entrepreneurship. Indeed, successful entrepreneurs may be agents with high learning capacities about changing technologies and markets (Kirzner; Minniti and Bygrave, 1999, 2001) which allow them to play a leading role in productivity improvement.

In the evaluation of firm productivity, we start with the following conjecture:

<u>Conjecture C1</u>: Under bounded rationality, TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$, $\mathbf{P}^*$, $\cdot$) = 0, where ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$) are observed production/investment decisions made by the agent.

Conjecture C1 states that, under bounded rationality, the agent chooses production/investment decisions that are on the upper bound of his/her feasible set. From Proposition 3, choosing otherwise would imply technical inefficiency, i.e. a decline in the firm certainty equivalent. Conjecture C1 simply states that the agent would have no reason to choose ($\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$) in a way that would violate TE($\mathbf{p}_{x1}$, $\mathbf{x}_1$, $\tilde{\mathbf{x}}_2$,

$\mathbf{P}^*, \cdot) = 0$. This is consistent with Stigler's critique of Leibenstein's arguments. And it is consistent with Schultz's view on the role of human capital.

Under C1, bounded rationality implies restrictions on production/investment behavior. The key is that $TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}^*, \cdot) = 0$ in C1 applies only at $\mathbf{P}^*$, i.e. at the optimal information structure defined in (6). But, then what happens to $TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ when $\mathbf{P} \neq \mathbf{P}^*$? To explore this issue, we analyze what happens in the model when we neglect bounded rationality.

Our measurement of technical efficiency $TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (15) relied on the distance function $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ in (13) or (13') and is conditional on the amount of information $\mathbf{P}$ available for the period-two decisions. We have argued that, in complex situations, perfect information $\mathbf{P}^+$ is not feasible under bounded rationality. In this context, the information structure $\mathbf{P}$ must be coarser than $\mathbf{P}^+$. Consider the case where bounded rationality is ignored and $F(\mathbf{P}) = F(\mathbf{P}^+)$ for all $\mathbf{P}$. This amounts to assuming costless learning. Noting that $F(\mathbf{P}^+) = Z(\mathbf{P}^+)$, it follows that $F(\mathbf{P}) = F(\mathbf{P}^+) = Z(\mathbf{P}^+)$. With $Z(\mathbf{P}) \subset F(\mathbf{P})$ for all $\mathbf{P}$, this gives

$$Z(\mathbf{P}) \subset Z(\mathbf{P}^+).$$

Using (13) and (16a), this generates the following result.

<u>Proposition 4:</u> Neglecting bounded rationality implies that, for all $\mathbf{P}$,

$$TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \leq TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}^+, \cdot). \qquad (17)$$

Note that, in the absence of learning cost, the optimal information structure in (6) would be given by $\mathbf{P}^* = \mathbf{P}^+$. It follows from (17) that $TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \leq TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}^*, \cdot)$ for all $\mathbf{P}$. Thus, Proposition 4 states that neglecting learning costs (where $\mathbf{P}^* \neq \mathbf{P}^+$) implies that technical inefficiency $TE(\cdot)$ tends to be overestimated. In other words, neglecting bounded rationality tends to generate upward biased estimates of technical inefficiency.[14] Proposition 4 seems important to the extent that bounded rationality issues have typically been neglected in previous analyses of firm efficiency (as further discussed below).

Consider next the following conjecture:

Conjecture C2: The capacity of economic agents to manage information varies across individuals.

Conjecture C2 is motivated by the complexity and limitations of cognitive processes associated with human learning, as documented in psychology and neuroeconomics (e.g., Camerer et al.). Combined with Conjecture C1, C2 implies that technical efficiency in a given sector is expected to vary according to the range of managerial capacity to gather and process information. Thus, from C1, two producers could be technically efficient (each with their own optimal $\mathbf{P}^*$) using the same resources. Yet, under bounded rationality, one may produce more output(s) than the other due to a differential ability to learn about technology and/or market conditions under C2. During stable times or in a mature economic sector, similar $\mathbf{P}^*$'s might be attainable by most agents. However, during periods of technological or structural changes, from C2, the optimal $\mathbf{P}^*$s could vary significantly across agents. Then, as argued in Schultz (1975), agents with higher learning capacities would be the ones more successful at identifying what will prove to be better combinations of inputs to produce given outputs (or higher outputs for given inputs). This has two important implications. First, if bounded rationality were ignored, agents with poor learning ability would appear to technically inefficient (from Proposition 4). However, under C1, this appearance of technical inefficiency is false. To the extent that learning abilities are not easily transferable across agents, the apparent technical inefficiency is due to differing learning abilities. In this context, estimating high levels of technical inefficiency simply reflects that the ability to process information varies across individuals. In other words, the prevalence of technical inefficiency reported by Leibenstein, Frantz and others can be interpreted as evidence supporting conjecture C2. Second, under C1 and C2, improvements in learning abilities can contribute to improving productivity. Such improvements can come from various sources: improved learning-by-doing, information diffusion across agents, and/or entrepreneurial activities.

These arguments provide a micro-theoretic basis for demonstrating how information processing and learning are likely to be an integral part of productivity improvements. They are novel, because previous theoretical and empirical studies of firm efficiency either have assumed full information and no learning (e.g., Farrell) or lacked explicit attention to the role of bounded rationality (Leibenstein, Schultz, 1975). Attempts have been made to relax the assumption that the production frontier is readily attainable (Coelli et al.). Among these attempts, most popular is the "stochastic frontier analysis" proposed by Aigner et al. and reviewed by Kumbhakar and Lovell. In the econometric estimation of the production technology, the method decomposes the error term into two statistically independent components, a measure of technical efficiency and a "measurement error.". This decomposition provides a statistical method for identifying "unobserved" heterogeneity in production possibilities. As shown by Jondrow et al., it also provides an empirical framework to measure technical efficiency. However, this approach raises some important questions. One issue is the ability to identify technical inefficiency from measurement errors. Doing so requires imposing *a priori* restrictions on the distributions of these two terms. Typically, econometric identification is achieved by assuming a symmetric distribution for measurement errors but a skewed distribution for technical inefficiency (see Aigner et al.; Kumbhakar and Lovell). Unfortunately, there does not seem to be strong theoretical justification for such assumptions.[15] Yet such assumptions are crucial to achieve identification of the technical inefficiency term.[16] In addition, under conjecture C1 (and as argued by Stigler and Schultz), the error term reduces to measurement error (e.g., reflecting unobserved heterogeneity in learning abilities). In this case, the error term decomposition used in the stochastic frontier approach appears inappropriate. In the absence of good *a priori* motivations for this decomposition, the stochastic frontier model would likely be misspecified, thus raising questions about the validity of the resulting inefficiency estimates. Finally, stochastic frontiers have not explicitly incorporated risk[17] or learning dynamics. As such, they fail to reflect the role of bounded rationality and learning in innovations and productivity improvements.

## 6. A decomposition of efficiency

Our proposed measures of firm efficiency given in (15)-(16) rely on the certainty equivalent CE($w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot$) defined in (9). In this section, we examine a decomposition of CE and the implications for efficiency. We focus our analysis on the case where the $\tilde{\mathbf{x}}_2$ decisions are made optimally, with $\tilde{\mathbf{x}}_2{}^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) \in \text{argmax}_{\tilde{\mathbf{x}}_2}$ $\{v(\ldots, w + \mathbf{p}_{x1} \cdot \mathbf{x}_1 + \mathbf{q}_2(s) \cdot \mathbf{x}_2(r, s), \ldots, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\} \equiv \text{argmax}_{\tilde{\mathbf{x}}_2}$ $\{CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}$, as stated in Corollary 1. Define

$$v^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) \equiv \max_{\tilde{\mathbf{x}}_2} \{v(\ldots, w + \mathbf{p}_{x1} \cdot \mathbf{x}_1 + \mathbf{q}_2(s) \cdot \mathbf{x}_2(r, s), \ldots, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}. \tag{18}$$

Note that, with $\tilde{\mathbf{q}}_2 > 0$, the solution $\tilde{\mathbf{x}}_2{}^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot)$ of (18) is necessarily on the upper bound of the feasible set $\{\tilde{\mathbf{x}}_2: (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}$. And when $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ is strongly decreasing in $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$, the point $(\mathbf{x}_1, \tilde{\mathbf{x}}_2{}^c)$ necessarily exhibits technical efficiency (as discussed above), with $TE(\mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{x}}_2{}^c, \mathbf{P}, \cdot) = 0$. This is consistent with conjecture C1. In addition, using (15) implies that $EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2{}^c, \mathbf{P}, \cdot) = AE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2{}^c, \mathbf{P}, \cdot)$. This means that the firm economic efficiency measure $EE(\cdot)$ reduces to the allocative efficiency measure $AE(\cdot)$. This section investigates the determinants of $EE(\cdot)$ by proposing a decomposition of $EE(\cdot)$ (or equivalently a decomposition of $AE(\cdot)$ under C1, when $TE(\cdot) = 0$). Under bounded rationality and learning, this will provide new insights on the factors and economic rationale affecting the choice of a particular point on the frontier technology.

### 6.1. The value of information

Consider the case where the $\tilde{\mathbf{x}}_2$ decisions are made without learning. Let the corresponding information structure be $\mathbf{P}^0 = (P^-, \ldots, P^-, P_{m+1}, P_{m+2})$. With $P^-$ having only one element, this means that the period-two production/investment decisions $\mathbf{x}_2$ are made ex ante, while the period-two labor decisions $L_{a2}$ and $P_{w2}$ are based on the information partitions $P_{m+1}$ and $P_{m+2}$, respectively. With $\mathbf{P} = (P_1, \ldots, P_m, P_{m+1}, P_{m+2})$, it follows that $\mathbf{P}$ is at least as fine as $\mathbf{P}^0$, with $F(\mathbf{P}) \subset F(\mathbf{P}^0)$ and $Z(\mathbf{P}) \subset Z(\mathbf{P}^0)$.

Using (18), a monetary evaluation of the change from $\mathbf{P}$ to $\mathbf{P}^0$ is given by the conditional selling price of information V that satisfies[18]

$$v^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) = v^c(w + V + \mathbf{p}_{x1} \cdot \mathbf{x}_1, , \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}^0, \cdot). \tag{19}$$

Since $v(\cdot)$is strictly increasing in w, it follows that $v^c(\cdot)$ in (19) is also strictly increasing in w. Denote by $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ the solution of (19) for V. It is the smallest amount of money the agent is willing to receive *ex ante* to give up the information structure $\mathbf{P}$ and replace it by $\mathbf{P}^0$. From (11b), $V(\cdot)$ is the sure amount of money paid to the agent to induce her to make the $\tilde{\mathbf{x}}_2$ production/investment decisions *ex ante*. It is a conditional value of information since it depends on the decision $\mathbf{x}_1$ and on information structures $\mathbf{P}$ and $\mathbf{P}^0$. Note that, in general, $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ can be either positive or negative. In the case where learning is costless, then $F(\mathbf{P}) = F(\mathbf{P}^0)$ and $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) \geq 0$, implying that the conditional value of information is non-negative. However, under active learning where information is costly, then $F(\mathbf{P}) \subset F(\mathbf{P}^0)$, and $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ can become negative when the resources used in learning $[F(\mathbf{P}^0) - F(\mathbf{P})]$ are significant.

Note that equation (19) applies for any risk preferences and any feasible information $\mathbf{P}$. The properties of the conditional value of information $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ provide useful insights on the role of the period-one production/investment decisions $\mathbf{x}_1$. Of special interest are the effects $x_{i1}$ (the i-th element of $\mathbf{x}_1$) on $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$. If $x_{i1}$ has a positive effect on $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$, then the i-th netput would increase the value of information. This can happen under two scenarios: 1/ under active learning, $x_{i1}$ is part of the firm's information gathering activities; or 2/ the use of $x_{i1}$ increases the options for the firm in adjusting its period-two decisions in response to new information. Note that this latter effect can be present with or without active learning. Alternatively, if $x_{i1}$ has a negative effect on $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$, then the i-th netput would decrease the value of information. This can happen under two scenarios: 1/ using $x_{i1}$ has adverse effects on the learning process; or 2/ the use of $x_{i1}$ decreases the options for the firm in adjusting its period-two decisions in response to new information. This latter effect would arise when $x_{i1}$ is an irreversible decision that cannot be easily undone either because reversing the decision is not feasible (see Henry, and Arrow and Fisher) or because it is costly (e.g., due to sunk costs; see Pindyck and Dixit). For example, when $\mathbf{x}_1$ involves choosing between a reversible and an irreversible decision, the associated change in the value of information $V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ reduces to Arrow and Fisher's "quasi-option value" under the reversible scenario.

### 6.2. Risk premium

In the right-hand side of equation (19), the agent makes the decisions $\mathbf{x}_2$ under limited information (as given by $\mathbf{P}^0$) while being compensated for it (through $V(\cdot)$). However, the agent still faces price uncertainty. Assume that the agent has a subjective probability assessment of the uncertainty, with $\Pr(r, s)$ denoting the probability of facing state (r, s). Let $\bar{\mathbf{q}}_2 = \sum_{s=1}^{S} \sum_{r=1}^{R} \Pr(r, s)\, \mathbf{q}_2(s)$ be the expected discounted price of $\mathbf{x}_2(s)$.[19] Consider the case of profit insurance, which would replace the period-two discounted prices $\mathbf{q}_2(s)$ by their expected value $\bar{\mathbf{q}}_2$. Using (11) and (19), the <u>risk premium for profit insurance</u> is defined as the sure amount of money Q which satisfies[20]

$$v^c(w + V(\cdot) + \mathbf{p}_{x1} \cdot \mathbf{x}_1, , \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}^0, \cdot) = v^c(w + V(\cdot) - Q + \mathbf{p}_{x1} \cdot \mathbf{x}_1, , \mathbf{x}_1, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}^0, \cdot). \quad (20)$$

Let $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ be the solution of (20) for Q. Equation (20) defines the risk premium $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ as the smallest amount of money the agent is willing to pay *ex ante* to replace period-two discounted prices $\tilde{\mathbf{q}}_2$ by their expected value $\bar{\mathbf{q}}_2$. Note that the $\tilde{\mathbf{x}}_2$ decisions are made *ex ante* (i.e. based on $\mathbf{P}^0$) in the right-hand side of (19). It follows that $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ measures the willingness to pay to eliminate profit risk. In general, the risk premium $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ is conditional on the period-one decisions $\mathbf{x}_1$. And its sign can be used to characterize the nature of the agent's risk preferences: the agent is said to be risk averse, risk neutral, or risk lover with respect to profit risk when $Q(\cdot) > 0$, $= 0$, or $< 0$, respectively. Under risk aversion, the risk premium $Q(\cdot)$ measures the implicit cost of risk bearing for profit risk.[21]

The properties of the risk premium $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$ provide useful insights on the role of the period-one netputs $\mathbf{x}_1$ in risk management. Of special interest are the effects $x_{i1}$ (the i-th netput in $\mathbf{x}_1$) on $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$. If $x_{i1}$ has a positive effect on $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$, then the i-th netput would increase the implicit cost of risk bearing. For a risk averse decision maker (with $Q(\cdot) > 0$), this means that the i-th netput is <u>risk increasing</u>. In this case, the agent has an incentive to reduce the use of $z_{i1}$ so as to reduce its risk exposure and lower the risk premium. Alternatively, if $x_{i1}$ has a negative effect on $Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot)$, then the i-th netput would decrease the cost of private risk bearing. For a risk averse decision maker (with $Q(\cdot) > 0$), this means that the i-th netput is <u>risk decreasing</u>. In this case, the agent has an incentive to increase the use of $z_{i1}$ so as to reduce its risk exposure and lower the risk premium.

### 6.3. Decomposition

Proposition 1 shows that the choice of $\tilde{\mathbf{x}}_2$ is consistent with the maximization of $CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \cdot)$. Using Corollary 1, let

$$CE^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) \equiv \text{Max}_{\tilde{\mathbf{x}}_2} \{CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}, \tag{21}$$

which has for solution $\tilde{\mathbf{x}}_2{}^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot)$ as stated in equation (11). Combining equations (6), (9), (11), (19), (20) and (21), we obtain the following result.

<u>Proposition 5:</u> The certainty equivalent $CE^c(\cdot)$ in (21) can be written as

$$CE^c(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \cdot) = M(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) + V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) - Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot), \tag{22}$$

where $M(w, \mathbf{p}_{x1}, \mathbf{x}_1, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) \equiv \mathbf{p}_{x1} \cdot \mathbf{x}_1 + \bar{\mathbf{q}}_2 \cdot \tilde{\mathbf{x}}_2{}^c[w + V(\cdot) - Q(\cdot) + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}^0, \cdot]$ is expected discounted profit.

Proposition 5 decomposes the certainty equivalent into three additive terms. Equation (22) shows that $CE^c(\cdot)$ equals the expected discounted profit $M(\cdot)$, plus the conditional value of information $V(\cdot)$, minus the risk premium $Q(\cdot)$. In addition to expected profit, $M(\cdot)$, this shows that both the value of information $V(\cdot)$ and the cost of private risk bearing $Q(\cdot)$ affect the welfare of the firm and its owner-manager. The former has a positive effect, stressing the importance of information processing in managerial decisions. And under risk aversion, the latter has a negative effect: it provides risk-averse agents an incentive to reduce their risk exposure.

Combining equations (12), (21) and (22), we can decompose economic efficiency as follows:.

Proposition 6:

$$
\begin{aligned}
&EE^c(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot) \\
&\quad \equiv EE(w, \mathbf{p}_{x1}, \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2{}^c[w + V(\cdot) - Q(\cdot) + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \mathbf{x}_1, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}^0, \cdot], \mathbf{P}, \cdot) \\
&\quad = EE_M(\cdot) + EE_V(\cdot) + EE_Q(\cdot) \geq 0,
\end{aligned} \tag{23}
$$

where

$$EE_M(\cdot) \equiv M(w, \mathbf{p}_{x1}, \mathbf{x}_1{}^c, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) - M(w, \mathbf{p}_{x1}, \mathbf{x}_1, \bar{\mathbf{q}}_2, \ldots, \bar{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot), \tag{24a}$$

$$EE_V(\cdot) \equiv V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1{}^c, \mathbf{x}_1{}^c, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) - V(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1{}^c, \mathbf{x}_1{}^c, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot), \tag{24b}$$

$$EE_Q(\cdot) \equiv -Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1{}^c, \mathbf{x}_1{}^c, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot) + Q(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1{}^c, \mathbf{x}_1{}^c, \tilde{\mathbf{q}}_2, \mathbf{P}, \mathbf{P}^0, \cdot), \tag{24c}$$

with $\mathbf{x}_1{}^c \in \text{argmax}_{\mathbf{x}_1} \{\text{Max}_{\tilde{\mathbf{x}}_2} \{CE(w + \mathbf{p}_{x1} \cdot \mathbf{x}_1, \tilde{\mathbf{q}}_2, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot): (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})\}\}$.

Equation (23) decomposes firm economic efficiency $EE^c$ into three additive parts: a part related to expected profit, $EE_M$; a part related to the value of information, $EE_V$; and a part related to the risk premium, $EE_Q$. The first term (expected profit) is a standard part of efficiency analysis for a neoclassical firm. However, the second and third terms are perhaps less common. Our analysis shows that they are integral parts of the welfare assessment of the firm in any setting where information is not perfectly known across all states of nature and time. In particular, the information term $EE_V$ reflects the importance of information and learning in efficiency analysis. And the third term $EE_Q$ reflects the role of risk exposure.

Proposition 6 identifies three sources of possible inefficiency: low expected profit, low value of information V, or high risk premium Q. Each measure depends on the first-period production/investment decisions $\mathbf{x}_1$. In other words,, the choice of $\mathbf{x}_1$ is inefficient when $\mathbf{x}_1 \neq \mathbf{x}_1^c$. Then, equations (23)-(24) show the choice of $\mathbf{x}_1$ can be inefficient in three possible ways: through a sub-optimal expected profit, through a sub-optimal value of information, or through a sub-optimal risk premium. Alternatively, the decomposition shown in Proposition 6 offers three possible paths to improve firm efficiency.

Proposition 6 shows that risk and risk preferences play a role in the evaluation of firm allocative efficiency. This suggests a need to incorporate such factors in the empirical analysis of firm efficiency. Kumbhakar has proposed an econometric methodology for doing so. Under the expected utility model, he incorporates the role of risk and risk aversion into a stochastic production function approach. As noted above, the error term decomposition used in the stochastic frontier approach appears problematic. In particular, under conjecture C1, the error term would be due entirely to measurement errors, raising questions about the validity of the resulting econometric estimates.[22]

The decomposition of efficiency outcomes developed in Proposition 6 provides a useful contrast with Kumbhakar. First, unlike Kumbhakar's approach, our model is presented in a dynamic rather than a static context. Including dynamics highlights the role of learning, including the prospect of learning about risk exposure or state contingent production possibilities in efficiency analysis. Second, the model also reveals the fundamental role that learning and risk play for agents in their pursuit of both technical and allocative efficiency. And, it does so without imposing arbitrary assumptions on the structure of risk, risk preferences, production technology, or market conditions, let alone the agent's knowledge about his or her human capital capacities.

Finally, the decomposition of allocative efficiency presented in Proposition 6 suggests how issues such as organizational skills, marketing expertise, and product-service differentiation might be more explicitly captured in an economic model of an agent or a firm's behavior. The three components of firm allocative efficiency (learning, risk, and profit) provide a basis for investigating behavioral heterogeneity that may arise along the frontier technology because of differences in human capital, risk preferences, and/or state contingent possibilities.

## 7. Implications for the economic analysis of entrepreneurship

Our explicit treatment of bounded rationality in a single-agent dynamic decision-making model provides three types of advances in economic analysis. The first is to expand the concept of efficiency beyond the standard terrain of expected profit and risk management to include the role of information and learning. As argued above, this expansion has important implications for the assessment of technical efficiency and our understanding of productivity improvements (including the potential for bias in measuring technical efficiency outcomes when the costs of learning and differential capacities of producers to learn are ignored). The second contribution relates to the role of risk and learning in firm allocative efficiency, and suggests how human capital in the form of better organizational skills, marketing, or improved product-service differentiation might also be captured by this integrated modeling approach. While the role of expected profit (and to a lesser extent of risk) is well understood, the importance of information and learning has received less attention.

The role of information and learning is particularly illuminating when we focus our attention on entrepreneurship. Successful entrepreneurs can be identified as those with superior information processing abilities (Kirzner; Minniti and Bygrave, 1999; Lazear). In turn, such entrepreneurs play an important role shaping efficiency and technological change outcomes in the economy. Kihlstrom and Laffont provide a useful take-off point for evaluating what our model adds to formal economic modeling of entrepreneurship. Kihlstrom and Laffont investigate how risk aversion can affect the choice to be an entrepreneur. They show how in the presence of heterogeneous risk preferences, the less risk-averse individuals tend to self select into becoming entrepreneurs because their lower risk aversion gives them an advantage in facing the risk associated with being residual claimants. In contrast, more risk-averse agents choose to be wage workers. Such results are consistent with our analysis: under risk aversion, the risk premium Q provides a disincentive to face risky outcomes.

Under bounded rationality and dynamic choice options, our model extends the analysis of entrepreneurial choice to incorporate explicitly the learning challenges that decision makers face and manage in dynamic and risky situations. One obvious conclusion from the model is that agents with higher capacities to acquire and process information are more likely to be entrepreneurs (agents discovering better combinations of inputs in a technical and allocative sense) because of their capacity to secure a more refined information structure at a lower cost. Under learning, this means that both information and residual risk exposure now become subject to management. While this does not eliminate the role of risk (e.g., as analyzed by Kihlstrom and Laffont), this stresses the importance of information. We suspect that the most important characteristic of successful entrepreneur may not be their risk preferences per se but their capacities to more fully assess the profitability and uncertainty they face (Minniti and Bygrave, 2001). In turn, such superior abilities may induce entrepreneurs into taking particular risks. However, attributing this risk-taking behavior to lower risk aversion may be inappropriate. To a large extent, entrepreneurs' risk-taking behavior may simply reflect their superior learning abilities about both technology and market conditions. In summary, once the dynamics of risk and learning are integrated in a model of bounded rationality, they interact and shape entrepreneurial choices in ways where learning might substitute for what would otherwise appear to be risk-taking behavior. Capturing the role of entrepreneurs in the process of innovating requires uncertainty, dynamics and learning. And bounded rationality also becomes a requirement if we want to provide a realistic representation of the complexity of innovation processes. In this context, our analysis stresses the role of information. This is just another way to say that the ability to process information is a critical if not <u>the</u> defining characteristic of a good entrepreneur.

The discussions of technical efficiency and allocative efficiency in sections 5 and 6 also have implications for how we think of entrepreneurship. In the former, the capacity of certain agents with higher learning capacities to discover more efficient combinations of inputs highlights the role that entrepreneurs can play in terms of helping to drive the process innovation side of technological change. In the latter, the capacity of certain agents to find better places on the frontier technology can be associated with improved management of differentiated goods and services, improved organizations, and institutional innovations. Fundamental to both results though is that the leading agents of efficiency improvements are ones with the human capital or learning capacity to discover the better states and to manage the full range of learning dynamics involved from learning-by-doing to refined investment choices. The very general formulation of our model incorporates this range and provides a structure for integrating it with standard profitability and risk considerations. It provides a general framework to examine in more detail the actual dynamics of particular situations where economic agents are making choices under bounded rationality.

## 8. Conclusion

This article develops a model of economic behavior of a firm owner-manager under bounded rationality, and explores the implications for the assessment of economic efficiency and entrepreneurship. Our primary modeling strategy is to use the certainty equivalent measure to study the optimal behavior of the agent, which in turn allows explicit treatment of the risk and learning components that, in addition to standard profitability outcomes, can influence the behavior of agents in a world of uncertainty and bounded rationality. Providing explicit attention to the role of learning in efficiency measurement and productivity assessment is at a theoretical level the novel contribution of this paper, though integrating it with risk management provides additional reflections on the potential interactions among these two components. At a broader level, the core contribution of this article is its effort to recast the microeconomic analysis of efficiency and entrepreneurship in a fashion that integrates standard profit analysis, risk management, and learning in a coherent and flexible fashion.

Four major insights emerge from this theoretical and conceptual foray. One is that learning plays a fundamental role in how we assess technical efficiency. In particular, because of fundamental differences in learning capacities (or human capital) across agents, information likely varies across agents. This focuses attention on a key feature of good entrepreneurs: their high capacity for learning about technology. A second and related insight is that empirical analyses of technical efficiency that fail to allow for the role of learning under bounded rationality are likely to be biased. This applies to most previous empirical analyses of firm efficiency (e.g., stochastic frontier analysis and Kumbhakar's recent integration of risk preferences into technical efficiency analysis). This insight also provides a way to reconcile the X-inefficiency idea of Leibenstein, the empirical evidence supporting it, and the critiques levied by Stigler and Schultz questioning why agents would choose to be technically inefficient.

The other two insights concern the analysis of entrepreneurship once we explicitly integrate learning and risk into standard efficiency (profitability) analysis. Just as learning and risk interactions can affect the design of productivity assessment, they can also affect how entrepreneurs are understood. Entrepreneurs are frequently viewed as agents with a higher propensity for taking risks. While our model incorporates that explanation in its decomposition of efficiency outcomes, it also underscores the potential for agents with higher learning capacities to better uncover the return and risk structure associated with certain activities. The superior capacity of good entrepreneurs to acquire and process information allows them to make more refined assessments of their uncertain economic environment. What looks like risky entrepreneurial behavior may just reflect better information about the distribution of returns associated with particular actions. Finally, the model provides a micro theoretic basis (through the allocative efficiency decomposition) for a common argument that successful entrepreneurs are also economic agents who are better at discovering how to organize, market, or differentiate their product-service so as to locate at more desirable points on the frontier technology.

While providing new directions for improved efficiency analysis, our approach raises a number of empirical challenges. For example, estimating technical efficiency of firms under uncertainty and learning can be problematic when using cross-section data. First, at a given point of time, each decision maker may have access to different information about technology and/or market conditions. Second, even if access to information is the same, the ability to process it can vary across individuals (depending on education, experience, etc.). When using panel data, the prospects for assessing the complexities of human learning improve: following particular decision makers over time can provide the measurements needed to examine how individuals learn. Such measurements should help refine our understanding of the factors contributing to firm efficiency and productivity growth.

Note that our analysis could be extended in several directions. First, when applied to multi-output firms, our approach to risk and information issues can be used to investigate the economics of specialization and diversification, with an emphasis on developing insights into the economics of industry structure, mergers, and divestitures. Second, introducing agency issues (such as separation of ownership and control or interactions in learning within a team as in Radner) would be valuable for understanding the role of contracting and other institutions that shape efficiency and other welfare outcomes in multiple agent contexts under bounded rationality. Third, it would be useful to expand our two-period analysis to a multi-period context. Finally, further investigation of the implications of risk and information for economic efficiency in a general equilibrium context could enhance the modeling of endogenous growth by focusing on the role and value of learning under bounded rationality. These appear to be good topics for future research.

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Footnotes

[1] See Coelli et al. and Kumbhakar and Lovell for an overview of this literature.

[2] A notable exception includes Kumbhakar, which will be discussed below.

[3] Compared to "full rationality", bounded rationality has been found to provide a better representation of human behavior (e.g., Gabaix et al.).

[4] Note that such a decomposition of firm-level efficiency is not new. It was first proposed by Farrell under "full rationality." Our contribution is to show how to extend it under uncertainty and bounded rationality.

[5] Thus, wage income at time t is $p_{Lt}\,L_{wt} = p_{Lt}\,(T - L_{et} - L_{at})$. It follows that when $L_{wt}$ is positive, the wage rate $p_{Lt}$ measures the unit opportunity cost of both $L_{et}$ and $L_{at}$.

[6] In our notation, "·" denotes the inner product, with $\mathbf{p}_{x1} \cdot \mathbf{x}_1 \equiv \sum_{i=1}^{n} p_{xi1}\, x_{i1}$.

[7] How individuals process information is complex. While neuroscience is making significant progress on this issue (e.g., Camerer et al.), developing a scientific understanding of how the brain processes information and makes decisions remains a very challenging task. In this context, our state-contingent approach is interpreted simply as a reduced-form representation of individual learning.

[8] Note that the certainty equivalent approach is not new. For example, it has been at the heart of the analysis of risk behavior under the expected utility model (Arrow, 1965; Pratt; Sandmo) and under a state-contingent approach (Quiggin and Chambers). The analysis presented here applies in the more general case of learning under bounded rationality.

[9] In the special case where labor contracts are flexible and the owner-manager chooses $L_{w1} > 0$, then both firm labor $L_{a1}$ and leisure $L_{et}$ would have an opportunity cost measured by the wage rate $p_{L1}$. This corresponds to the specification analyzed by Becker. However, note that our model applies under more general conditions: it does not require that the opportunity cost of firm labor $L_{a1}$ and leisure $L_{et}$ be equal to the wage rate $p_{L1}$.

[10] For any $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$ under information structure $\mathbf{P}$, the point $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ is said to be technically efficient if there is no $(\mathbf{x}_1', L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2', \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$ satisfying $(\mathbf{x}_1', \tilde{\mathbf{x}}_2') \neq (\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ and $(\mathbf{x}_1', \tilde{\mathbf{x}}_2') \geq (\mathbf{x}_1, \tilde{\mathbf{x}}_2)$.

[11] $Z(\mathbf{P})$ exhibits free disposal in $\mathbf{x}_1$ if, for any $(\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$, then $(\mathbf{x}_1', L_{a1}, L_{w1}, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in Z(\mathbf{P})$ for any $\mathbf{x}_1' \leq \mathbf{x}_1$. Note that this implies that $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ is non-increasing in $\mathbf{x}_1$.

[12] $D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ is strongly decreasing in $(\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ if $D(\mathbf{x}_1', \tilde{\mathbf{x}}_2', \mathbf{P}, \cdot) < D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot)$ for any $(\mathbf{x}_1', \tilde{\mathbf{x}}_2') \neq (\mathbf{x}_1, \tilde{\mathbf{x}}_2)$ satisfying $(\mathbf{x}_1', \tilde{\mathbf{x}}_2') \geq (\mathbf{x}_1, \tilde{\mathbf{x}}_2)$.

[13] First, consider the case where the reference bundle **g** is defined such that inputs in **g** are zero and outputs are equal to the period-one outputs. Then, with a focus on period-one outputs, Shephard's output distance function is obtained as $1/(1 + D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot))$ (Shephard; Färe and Grosskopf). Second, consider the case where the reference bundle **g** is defined such that outputs in **g** are zero and inputs are equal to the period-one inputs. Then, with a focus on period-one inputs, Shephard's input distance function is obtained as $1/(1 - D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot))$ (Shephard; Chambers et al.). And Farrell's measure of technical efficiency is obtained as $(1 - D(\mathbf{x}_1, \tilde{\mathbf{x}}_2, \mathbf{P}, \cdot))$ (as Farrell's measure is the inverse of Shephard's input distance function)

[14] Using a state-contingent approach, O'Donnell and Griffiths present supporting empirical evidence.

[15] Also there does not seem to be strong theoretical justification why the two components of the error term would be independently distributed.

[16] For example, technical inefficiency would be underidentified if the inefficiency term and the measurement error terms are both normally distributed.

[17] One exception is the approach proposed by Kumbhakar that will be discussed below.

[18] Equation (19) is presented under a state-contingent approach. It includes as a special case the value of information analyzed under the expected utility model (e.g., LaValle).

[19] This assumes that the subjective probabilities Pr(r, s) have been assessed. In cases where such assessment proves difficult and probabilities are not known, our analysis would still apply but with $\bar{\mathbf{q}}_2$ being interpreted simply as a measure of "central tendency" of $\mathbf{q}_2(s)$, s = 1, …, S.

[20] Equation (20) is presented under a state-contingent approach. It extends the analysis of the risk premium developed by Arrow (1965) and Pratt under the expected utility model.

[21] Note that there are alternative ways of defining the risk premium. For example, in the context of income insurance, the risk premium could be defined at the sure amount of money Q' that the decision maker is willing to pay to replace the period-two discounted state-contingent income $[p_{L2}(s)\, L_{w2}(r, s) + p_{x2}(s) \cdot x_2(r, s)]/[1 + \rho(s)]$ by its expected value. Note that, except in the case where $p_{L2}(s) = p_{L2}$ and $\rho(s) = \rho$ for all s, the risk premium Q' would in general differ from Q defined in (20).

[22] In addition, Kumbhakar (in his footnote 3) assumes that technical inefficiency is unknown to the agent. Assuming that the agent is uninformed about his/her managerial abilities seems peculiar.