WHO BENEFITS FROM AGRICULTURAL RESEARCH: COMMENT

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We have had the opportunity to read the paper by Jarrett and Lindner [3] on Scobie’s note [4] and have rewritten this note to avoid unnecessary duplication of comments. In general, however, we agree with Jarrett and Lindner that the reasons for the different results obtained from the various measurement formulae can be summarized as follows:

(i) Different specifications of the initial supply and demand functions;

(ii) different assumptions about the nature of the shift of the supply function; and

(iii) different mathematical methods used to calculate the producers’ and consumers’ gains (losses), viz. point versus arc elasticities.

The last-mentioned difficulty arises because of the different mathematical approaches taken to this problem—sometimes graphical, sometimes approximately finite, sometimes (exact) calculus. The different methods adopted make it an extremely difficult task to reconcile the results obtained from the different formulae. It seems desirable though that future workers in this area should be aware of the ways in which the results they will obtain are dependent upon the initial assumptions made—the nature of this dependence has been spelt out by Jarrett and Lindner.

In the interests of providing an overall framework for analysis of the problem of the allocation of productivity gains we have sought to develop a generalized specification of a market model, and from a horizontal shift of the supply curve to develop more general formulae for consumers’ and producers’ surplus. From these we derive the generalized responses to the questions which Scobie asked of each different specification of consumers’ and producers’ surplus.

(i) Rightwards Shifts in the Supply Curve

Suppose \( X \) (output) is a dependent variable, and \( P \) (price) is an independent variable. Then a supply function can be represented generally as

\[ X_S = S(P). \]

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For comparison purposes a new supply function will be denoted as $S^*(P)$.

A *proportionate* shift to the right in supply (in either linear or non-linear form) is defined by

$$X' = kS(P),$$

where $k > 1$ and constant. In this case supply increases by a constant proportion for all prices.

For a *parallel* rightwards shift in the supply function the supply of $X$ increases by an equal absolute amount for all prices.

For example, in a linear supply function a parallel shift is specified as follows

$$X_s = a + bP$$
$$X' = a' + bP$$

where $a' > a$.

For a combination of these two kinds of shift we can postulate, for example, a rotation of a linear function, where

$$X_s = a + bP$$
$$X' = a' + b'P$$

where $b' > b$.

This will give the same result for all $P$ as in the proportionate shift case *if, and only if*, $a = 0$.

For a generalized specification of a rightwards shift in the supply function we define,

$$X' = \{(P, X) | \text{ for all } P; \ X' > X\}$$
$$= \{(P, X) | \text{ for all } P; \ S^*(P) > S(P)\}.$$ 

This can be called a "strong" shift, i.e., for all prices $X' > X$.

If we define $X' = \{(P, X) | \text{ for all } P; \ S^*(P) > S(P)\}$, this can be called a "semi-positive" shift, i.e., for some prices $X' > X$.

Finally, for purpose of completeness, if we define

$$X' = \{(P, X) | \text{ for all } P; \ S'(P) = S(P)\}$$

this can be called a "weak" shift. In this case $X' = X$ can satisfy the "weak" inequality above.

(ii) Definition of Producers’ Surplus

In the general case, assuming $X_s$ is integrable and the inverse function $X_s^{-1}$ exists (i.e., $X_s$ is one-to-one and onto), producers’ surplus $PS$ will be equal to

$$PS = P_0X_0 - \int_0^{X_0} S^{-1}(X)dX,$$

where $P_0$, $X_0$ represents any price/quantity co-ordinate on the supply function.
(iii) Comparative Static Effect of a Change in $k$

To derive the effect on producers' surplus from a proportionate, parallel or rotational shift in the supply curve we move to a market specification and assume supply and demand functions, as follows:

$$X_S = S(P, k)$$

and

$$X_D = D(P),$$

where in equilibrium

$$X_S = X_D$$

and where $D(P) = S(P, k) = 0$. (1)

Assuming that this equilibrium exists, there is some implicit function $g$ [2] such that

$$P^* = g(k),$$

$$X^* = S(P^*, k) = D(P^*),$$

where $P^*$ and $X^*$ are equilibrium price and quantity respectively.

From the implicit function theorem we find the effect on the equilibrium of a "small" change in $k$. Equation (1) is an identity in the equilibrium values of the variable $P^*$.

Hence, $D(P^*) - S(P^*, k) = 0$ (4)

Assuming the demand and supply curves are differentiable, we differentiate (2) with respect to $k$ which gives:

$$dP^*/dk = S^*_{k} / (D^*_{p^*} - S^*_{p^*})$$ (5)

where $S^*_{k} = (\partial S^*/\partial k)$; $S^*_{p^*} = (\partial S^*/\partial P^*)$;

and $D^*_{p^*} = (\partial D^*/\partial P^*)$; where $*$ denotes evaluation at equilibrium.¹

We can write result (5) in terms of elasticities if we multiply numerator and denominator by $P^*/X^*$. This gives,

$$dP^*/dk = (P^*/X^*)S^*_{k} / (\sigma^* - \eta^*),$$ (6)

where $\sigma^* = (P^*/X^*) D_{p^*}$ (price elasticity of demand and

$\eta^* = (P^*/X^*) S^*_{p^*}$ (price elasticity of supply).

Using these results we can now evaluate a change in producers' surplus consequent on a small change in $k$.

¹ For example from (3) $S^*_{k} = S(P^*)$ and $S^*_{p^*} = kS'(P^*)$, where $S'(P) = dS(P)/dP$

Substituting into (5) gives $dP^*/dk = S(P^*) [D_{p^*} - kS'(P^*)]$ which is < 0 assuming

$X(P^*) > 0$, $D_{p^*} < 0$ and $kS'(P^*) > 0$, i.e., that an increase in $k$ increases supply, and that the supply and demand curves have the usual slopes.
Given certain conditions, the equation for producers’ surplus can be differentiated with respect to \( k \) as follows (where all derivatives are evaluated at the equilibrium, and noting that \( X^* \) is a function of \( k \) from (3) and using (6)):

\[
dPS/dk = P^*(dX^*/dk) + X^*(dP^*/dk) - d/dk \int_o^{X^*} S^{*-1}(X)dX
\]

\[
= P^*(dX^*/dk) + X^*(dP^*/dk) - \int_o^{X^*} [(\partial S^{*-1}(X))/\partial k]dX \]

\[
= [P^* - S^{*-1}(X^*)](dX^*/dk) + X^* (P^*/X^*)[S_k^*/(\sigma^* - \eta^*)]
\]

\[
- \int_o^{X^*} [(\partial P^*(X))/\partial k]dX
\]

\[
= (P^* - P^*)(dX^*/dk) + [(P^* S_k^*/(\sigma^* - \eta^*))]
\]

\[
- \int_o^{X^*} (-S_k^*/S_p^*) dX
\]

\[
= [(P^* S_k^*/(\sigma^* - \eta^*))] + \int_o^{X^*} (S_k^*/S_p^*) dX
\]

Equation (7) shows that the change in the producers’ surplus as a result of a small rightwards shift of the supply curve is the net effect of:

(i) \([P^* S_k^*/(\sigma^* - \eta^*)] = X^*(dP^*/dk)\),
which is the loss in revenue resulting from the fall in the equilibrium price due to the rightwards shift in supply; and

(ii) \[\int_o^{X^*} (S_k^*/S_p^*) dX = \int_o^{X^*} (-\partial P/\partial k) dX\],
which is the aggregate of the reduction in marginal costs (i.e., fall in total costs) as a result of the adoption of, for example, a cost-reducing technology.

This is the familiar result, as expressed in Figure 1 below, that the change in producers’ surplus from a shift of the supply curve is the difference between the total change in consumers’ and producers’ surplus \( ee_{1fg} \) and the change in consumers’ surplus \( pee_{1p1} \).

\footnote{Sufficient conditions for differentiating an integral with respect to a parameter (say \( x \)) is given by [1]:

Proposition:

(i) If \( f \) and \( f_x \) are continuous in \((x, t)\) on \([a, b] \times [a, b] \) and \( A \leq t \leq B \) and:

(ii) \( A \) and \( B \) are continuous in \( x \) on \([a, b] \times [a, b] \),
then \[d/dx \int_A^B f(x, t)dt = \int_A^B f_x (x, t)dt + B'(x) f(x, B) - A'(x) f(x, A)\].}
**Figure 1: Rightwards Shift of Supply Curve**
(iv) Change in Consumers' Surplus

Consumers' surplus \( CS \) is defined as follows:

\[
CS = \int_o^{X^*} D^{-1}(X) dX - P^* X^*
\]

If we also define \( P = g(X) \equiv D^{-1}(X) \), then

\[
CS = \int_o^{X^*} g(X) dX - P^* X^* \tag{8}
\]

Differentiating (8) with respect to \( k \), gives the change in consumers' surplus in response to a small change in \( k \), which is

\[
\frac{dCS}{dk} = g(X^*) \left( \frac{dX^*}{dk} \right) - P^* \left( \frac{dX^*}{dk} \right) - X^* \left( \frac{dP^*}{dk} \right)
= -X^* \left( \frac{dP^*}{dk} \right) \text{ since } P^* = g(X^*)
\]

and substituting (6) into the above gives,

\[
= - \left[ \frac{\partial (P^* S^* k)}{\partial (X^*)} \right]
\tag{9}
\]

Equation (9) can now be compared with equation (7) to see that the change in consumers' surplus resulting from a shift of the supply curve is equal to the negative of the first term of equation (7) which is the change in producers' total revenue as a direct result of the change in price. Other familiar results can also be observed such as for example when \( \sigma = \infty, 0 < \eta < \infty \); \( \sigma = 0, 0 < \eta < \infty \).

(v) Comparisons with Scobie's Results

We can now provide generalized responses to the questions which Scobie posed.

(a) Under what conditions will consumers gain more than producers, i.e., when is \( (dCS/dk) > (dPS/dk) \)?

This will be true when

\[
-X^* (\frac{dP^*}{dk}) > X^* (\frac{dP^*}{dk}) + \int_o^{X^*} (- \partial P/\partial k) \, dX
\]

i.e., \( -X^* (\frac{dP^*}{dk}) > \frac{1}{2} \int_o^{X^*} (- \partial P/\partial k) \, dX \), which is true when the consumers' gain is more than one-half the producers' gain from the reduction in marginal costs; in terms of Figure 1, when \( p_{ee}, p_{i} \) is greater than one-half of the difference between the triangles \( p_{1}c_{1}f, f \) and \( peg \).

(b) Under what conditions will the change in producers' surplus be positive, i.e., \( dPS/dk > 0 \)?

This will be true when,

\[
X^* (\frac{dP^*}{dk}) + \int_o^{X^*} (- \partial P/\partial k) \, dX > 0
\]
i.e., when $\int_0^{X^*} (-\partial P/\partial k) \, dX > -X^* \cdot (dP^*/dk)$, which is true when the producers' gain from the aggregate of the reduction in marginal costs is greater than their transfer of surplus to consumers; in terms of Figure 1 when $(p_1e - peg) > pee_1p_1$.

Finally, it should be noted that other formulae reduce to these after taking into account approximations necessary for larger discrete parameter changes (e.g., $\Delta k$ rather than $dk$).

REFERENCES


