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The 84th Annual Conference of the Agricultural Economics Society

Edinburgh, Scotland

29th March to 31st March 2010

Risk and the decision to produce biomass crops: a stochastic analysis

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Abstract

There is increasing interest in biomass crops as an alternative farm enterprise. However, given the relatively low uptake of these crops in Ireland, there is limited information concerning the risk associated with their production and its potential impact on returns. The uncertainty surrounding risky variables such as the costs of production, yield level, price per tonne and opportunity cost of land make it difficult to accurately calculate the returns to biomass crops. Their lengthy production lifespan may only serve to heighten the level of risk that affects key variables. A stochastic budgeting model is used to calculate the returns from willow and miscanthus. The opportunity cost of land is accounted for through the inclusion of the foregone returns from selected conventional agricultural activities. The potential for bioremediation to boost returns is also examined. The NPV of various biomass investment options are simulated to ascertain the full distribution of possible returns. The results of these simulations are then compared using their respective CDF's and the investments are ranked using Stochastic Efficiency with Respect to a Function (SERF).

Keywords: Biomass, Bioremediation, Stochastic Budgeting, NPV, SERF

1. Introduction:

While risk is an intrinsic component in decision-making in all businesses it is viewed as being particularly important in agriculture, an industry that is especially exposed to variability (Thorne and Hennessy 2005; Richardson 2006). Farmers face uncertainty about the economic consequences of their actions due to the difficulties of predicting the outcome of factors such as weather, prices and biological responses to different farming practices all of which will have an impact on their level of returns earned. In recognising this feature of farming, studies addressing risk and uncertainty are common in the agricultural economics literature (Pannell et al. 2000), with risk widely seen as an issue of critical importance to farmers' decision making and to policies affecting those decisions (e.g. Anderson et al., 1977; Boussard, 1979; Anderson, 1982; Robison and Barry, 1987). Hence, risk should be explicitly considered in studies of agricultural production choices (e.g., Reutlinger, 1970; Hardaker et al.,

2004a, Richardson et al., 2006). From the perspective of the analysis of economic returns, risk refers to the potential variability of outcomes from a decision alternative (Anderson 2003). Similar to Hardaker et al. (2004a), we define uncertainty as imperfect knowledge and risk as uncertain consequences, particularly exposure to unfavourable consequences.

Despite being used as an energy source for centuries (Rosillo-Calle et al. 1999), it is only recently that the potential of biomass as an alternative to fossil fuels and conventional agriculture has been examined. The Net Present Value (NPV) approach to calculate the returns generated by perennial biomass crops such as willow and miscanthus has been prevalent in the literature (e.g., Goor et al., 1999; Heaton et al., 1999; Toivonen and Tahvanainen, 1998; Rosenqvist and Dawson, 2005; Styles et al., 2008; Clancy et al., 2009). These papers used sensitivity analysis to examine the effects of variation in key parameters such as yield level, discount rate, price per tonne, harvest cycle and subsidies. The literature indicates that the economics of growing biomass crops is marginal. Consequently, Rosenqvist and Dawson (2005) suggest that any value which can be added to the crops by giving them a dual function would greatly enhance their economic sustainability. Increasing interest is being given to the concept of disposal of agricultural and municipal wastes on energy crops. This potentially provides organic matter and nutrients needed for crop growth at a low cost (ADAS 2002). The ability to attract a gate fee for the recycling of wastes also has a significant effect on the returns that can be expected from biomass crops (Dawson 2007). Therefore, the use of bioremediation as a means to boost biomass crop profitability and reduce the risk of generating a negative investment return is also examined in this analysis.

However, perennial energy crops may have a greater number of uncertainties than exist with conventional agricultural activities (Meijer et al., 2007). The question marks which remain over the agronomic characteristics and economic returns of willow and miscanthus make them risky alternatives compared with long established farm enterprises such as beef or cereal production. For example, the lack of information regarding the crops' suitability to country-specific soil and climate conditions means that yield levels are difficult to predict. Moreover, the lengthy

production lifespan and extremely long payback periods to recoup establishment costs in biomass crops serves to heighten the level of risk associated with key parameters. Uncertainty about critical variables such as yield level and price per tonne make it difficult to accurately calculate the returns of such investments (Clancy et al. 2009).

It has been argued that uncertainty is a key barrier to the successful uptake of emerging renewable technologies such as bioenergy (Kemp et al., 1998; Foxon et al., 2005), principally because it hinders the fulfilment of entrepreneurial activities (Jacobsson and Bergek, 2004). In order for entrepreneurs to act, motivation needs to outweigh perceived uncertainty and so identifying dominant sources of uncertainty can deliver valuable insights (Meijer et al., 2007). Various studies of farmers' attitudes to risk have generally found that farmers are risk averse (e.g., Brink and McCarl 1978; Chavas and Holt 1990; Schurle and Tierney 1990; Pope and Just 1991) so given the uncertainty that exists over the returns from biomass, an analysis of the risk from adopting biomass crop production needs to be conducted.

In this paper a stochastic budgeting model, including stochastic costs, yields and prices is used to calculate the financial returns from willow and miscanthus. The opportunity cost of land is accounted for through the inclusion of foregone returns from a conventional agricultural activity, such as spring barley, winter wheat or store to finished beef. The NPV of the cash flow from the stochastic biomass returns minus the stochastic superseded enterprise gross margins is used to simulate the financial performance of alternative investment options over a 16 year planning horizon. The results of this simulation are then compared using their respective CDF's and the enterprises are ranked using Stochastic Efficiency with Respect to a Function (SERF).

The following section of the paper gives a background to the area of stochastic modelling and risk ranking, with the method of analysis and the data and assumptions employed described in section 3. Section 4 details the results of the analysis while a discussion of these is contained in section 5. Finally, section 6 summarises the research findings and draws conclusions from the results.

2. Background:

Existing empirical analyses of land use conversion typically assume deterministic decision making based on the NPV of returns to alternative land uses (Schatzki 2003). Traditionally, a deterministic approach which uses a set of predefined parameters as certain input data is formulated (D'Ovidio and Pagano 2009). Estimates of these parameters, typically the mean, must be used as the values which will actually occur are not known with certainty. Regardless of the estimate selected, in reality many of the events and conditions planned for will not turn out as assumed, leading to an estimated result significantly different to the one actually experienced (Milham 1998). Therefore, the deterministic approach is not adequate to consider model uncertainties, as the nature of several model parameters is random (D'Ovidio and Pagano 2009). Sensitivity analysis, while a valid and useful technique for determining the range of feasible outcomes from a model, does not give any indication of the likelihood of particular results being achieved (Milham and Hardaker, 1990).

Instead, evaluation under uncertainty or a stochastic environment should be conducted, where stochastic modelling of the future prices and costs from all management activities plays an important role (Yoshimoto and Shoji 1998). Milham (1998) argued that since farm financial planning decisions are made in a dynamic and uncertain environment, attempts to model such decisions should be conducted in a stochastic framework. Risk assessment is a process for identifying adverse consequences and their associated probabilities (McKone, 1996). D'Ovidio and Pagano (2009) observed that in recent years probabilistic approaches have been widely used to characterize the inherently random nature of renewable energy sources. Therefore, in order to examine the economic viability of biomass as energy resource in an unpredictable context, a stochastic approach can be formulated.

Stochastic budgeting is an improvement on the traditional deterministic approach as it involves attaching probabilities of occurrence to the possible values of the key variables in a budget, thereby generating the probability distribution of possible budget outcomes. A stochastic budget will typically have a deterministic equivalent in the form of a conventional budget under assumed certainty (Hardaker et al. 2004a).

Stochastic budgeting involves developing a model that mimics the operation of a business and provides projections of financial performance while taking account of the uncertainty inherent in many aspects of the decisions (Milham, 1998).

After using a simulation model to analyse alternative enterprises, you are faced with the problem of which one is best (Richardson 2003). To evaluate the relative risks of alternative farming systems appropriately we need to consider the whole range of outcomes, good and bad, and their associated probabilities (Lien et al. 2007a). Risk ranking procedures such as mean variance, mini-max and the coefficient of variation rely solely on summary statistics, and so a superior procedure which utilises all simulated outcomes and so considers the full range of possible outcomes rather than just the mean or standard deviation is required (Richardson 2003).

To present the financial feasibility of alternative strategies, CDF's of the performance measure are informative (Lien 2003). A CDF contains all of the information on the output distribution of the risky prospects and, therefore, is useful for decision making (Evans et al. 2006). Therefore the CDF of the various biomass investment options are included in the results so as to highlight the likelihood of each of project being profitable, and to allow for a comparison between options. Although possible to graph the CDF's and visually pick the preferred choice (the one that is furthest to the right), this procedure lacks rigour and fails when the distributions cross (Richardson 2003).

According to expected utility theory, the decision maker's utility function for outcomes is needed to assess risky alternatives as the shape of this function reflects an individual's attitude to risk (Hardaker et al. 2004b). In practice however, this rarely holds true. Efficiency criteria allow some ranking of risky alternatives when the preferences for alternative outcomes of decision are not exactly known (Grove 2006). Risky outcomes are measured in terms of the probability distribution of the returns from each enterprise, defined here as the annual gross margin. To assess and compare the risk efficiency of willow and miscanthus relative to conventional agricultural enterprises we have used stochastic efficiency with respect to a function (SERF) (Richardson et al. 2000, Hardaker et al., 2004b, Lien et al. 2007a).

SERF is a procedure for ranking risky alternatives based on their certainty equivalents (CE). The certainty equivalent¹ values show the amount of money that the decision maker would have to be paid to be indifferent between the particular scenario and a no-risk investment. The value of the certainty for any given risky alternative is dependent upon the expected utility function of the decision maker and the decision maker's level of risk aversion. The principle is the same as that used with expected utility, i.e., more is preferred to less (Hardaker 2000). Lien et al. (2007a) used SERF to measure the risk efficiency of two alternative farming systems (organic and conventional farming systems) in terms of the probability distribution of current wealth from farming, defined as the NPV of farm equity at the end of the planning horizon. SERF has also been applied to analyse optimal farm strategies (tree planting on harvested land) for a specified range of attitudes to risk (Lien et al. 2007b). This method lends itself to the analysis of biomass crop production in Ireland where data on individual farmers risk preference is non existent.

3. Materials and Methods:

3.1 Stochastic Budgeting

Richardson (2003) outlines the steps for developing a production-based economic feasibility simulation model, which were used in this analysis. First, probability distributions for all risky variables must be defined, parameterized, simulated, and validated. Second, the stochastic values from the probability distributions are used in the accounting equations to calculate production, receipts, costs, cash flows, and balance sheet variables for the project. Stochastic values sampled from the probability distributions make the financial statement variables stochastic.

Third, the completed stochastic model is simulated many times (1,000 iterations) using random values for the risky variables. The results of the 1,000 samples provide the information to estimate empirical probability distributions for unobservable Key

¹ See Hardaker et al. (2004b) for a detailed description of how Stochastic Efficiency with Respect to a Function uses Certainty Equivalents to rank risky alternatives

Output Variables (KOV's), so potential adopters can evaluate the probability of success for a proposed enterprise. Fourth, the analyst uses the stochastic simulation model to analyze alternative enterprises, with the results provided in the form of probabilities and probabilistic forecasts for their respective KOV's (gross margins). The model was programmed in Microsoft Excel and simulated using the Excel Add-In, Simetar.

3.2 Estimation of Stochastic Biomass Returns

Hardaker et al. (2004a) stated that the selected variables to be added stochastically should comprise those that will have the largest effect on the level of risk of a certain outcome. Therefore, the stochastic variables included in the biomass model are costs, yields and prices, each of which can have a large effect on the returns from biomass crops as demonstrated by various studies (eg. Toivonen and Tahvanainen (1998), Rosenqvist and Dawson (2005), Styles et al. (2008), Clancy et al. (2009)). Another key issue in multi-parameter studies is to find appropriate mathematical methods to handle the determinations of the values of risk parameters (Anderson 2003), as beneficial simulations can only be achieved when meaningful estimates of the input stochastic variables are used in the model (Evans et al. 2006). This section describes the estimation of the stochastic variables used in the calculation of biomass returns in greater detail. Figure 1 depicts a flow chart of this section of the model, illustrating the major components and their interrelationships.

[INSERT FIGURE 1 ABOUT HERE]

3.2.1 *Stochastic Biomass Costs and Prices*

Both biomass direct costs² and prices received per tonne are assumed normally distributed around a stochastic time trend, with the hierarchy of variables approach (Hardaker et al. 2004a) used to account for this. This approach indirectly establishes the relationship between each pair of correlated variables (Milham 1998) and requires

the selection of a macro-level variable to which all types of costs or prices can be expected to be correlated (Lien 2003). The macro-level variable which biomass costs are assumed to be correlated to was the price index of agricultural inputs, while biomass prices are assumed to be correlated to the price index of solid fuels. These indices were produced by the Central Statistics Office over the period 1995 – 2008. Estimation of the stochastic costs and prices follow the same steps, so just the calculation of the stochastic costs are detailed here

A regression relationship for each of the costs against this independent variable is derived. Forecast values of the independent variable are then entered into these equations to provide estimates of the various costs (Milham 1998) in biomass production. Following the approach used by Lien (2003), the first step was to derive the time trend through the regression of the macro level price index, Y , against time, t :

$$Y_t = \beta + \delta_t + u_t \quad u_t \sim N(0, \sigma^2_Y), \quad t = (1, \dots, 14) \quad (1)$$

In the next step, Eq. (1) was used to predict the price index of agricultural inputs, Y , for every year in the biomass plantation lifespan. The predicted means from Eq. (1) [where $t = (15, \dots, 30)$ for the planning years 2009-2024] were assumed to be the means of normal distributions, with the standard deviations of error component, σ^2_Y , used as the standard deviation of the normal distribution. The price indices for agricultural services, Z_1 , seed, Z_2 , plant protection, Z_3 , and other costs, Z_4 , were regressed against Y :

$$Z_{it} = x_t + v_i Y_t + u_{it} \quad u_{it} \sim N(0, \sigma^2_{Zi}), \quad t = (1, \dots, 14), \quad i = (1, \dots, 4) \quad (2)$$

where i is the type of direct cost index, Z_i . Finally, the predicted stochastic time trend from the second step was used in Eq. (3) to forecast price indices of future costs for each i . The error component from Eq. (2) with mean zero and standard deviation, σ^2_{Zi} , was included to account for normally distributed costs for each i :

² To avoid problems of allocation of fixed costs associated with owned machinery, calculations assume contractor charges for all field operations. All machinery and labour costs are therefore assumed to be

$$Z_{it} = x_i + v_i Y_t + N(0, \sigma^2_{Zi})$$

$$Z_{it} = (x_i + v_i \beta_i) + v_i \delta_t + N(0, v_i \sigma^2_Y + \sigma^2_{Zi}), t = (15, \dots, 30) \quad (3)$$

From Eq. (3) observe that the predicted price index of each fixed cost i has a different constant term, a different drift term and different variance, despite the fact that these terms for each index depend partly on the predicted trend in the macro level variable, Y . The hierarchy of variables procedure implies an assumption that the stochastic time trend in the macro-level variable experienced in the historical reference period will continue, and that all stochastic effects derived from national data are applicable to the individual case farm (Lien 2003).

The figures in Table 1 are based on those used in a deterministic model by Clancy et al. (2009) and form the baseline on to which the stochastic trends calculated through the Hierarchy of Variables approach described above, are added.

[INSERT TABLE 1 ABOUT HERE]

3.2.2 Stochastic Biomass Yields

A mix of Irish (Clifton-Brown et al. 2007) and UK (Christian et al. 2008) data were used for miscanthus yield levels. Insufficient Irish yield level data resulted in the use of figures from the UK (DEFRA 2007) for willow. The similarity between the two countries in terms of the range of soils and climatic conditions means that data in one country should be applicable to the other. The trials from which the data was extracted were conducted over relatively long periods of time (12 – 14 years) in multiple plots distributed throughout both countries and with a mix of crop varieties used. Therefore it is assumed they approximate the full range of possible yield level values. Table 2 details the summary statistics of the biomass yield level data used in this analysis.

[INSERT TABLE 2 ABOUT HERE]

variable costs.

The unbalanced willow and miscanthus panel datasets were used to estimate the following fixed effects model for each crop:

$$x_{it} = \mu + \alpha_i + x_{it-1} + e_{it} \quad (4)$$

where x_{it} is yield on farm i in year t ($t = 1, \dots, 16$), μ is the general mean, α_i is the effect on yield due to farm i (variation between farms caused by different management practices, soil, etc.), x_{it-1} is the previous years yield lagged. The residual e_{it} is a random variable, which upon testing, was shown to follow a beta distribution for both crops. The resulting parameters were used in the estimation of stochastic yield levels for each harvest during the project lifespan.

Both willow and miscanthus have a yield building phase of growth, experienced in the first harvest, followed by a plateau during which yields are at their maximum level. In order to replicate the lower yield in the first harvest, the intercept was multiplied by a coefficient. The conservative 16 year lifespan assumed in this analysis insures that a third phase during which yields could potentially deteriorate is not an issue. There was no historical evidence of a link between their yields and therefore, no intra or inter-temporal correlation in yield levels among the biomass crops was assumed in this analysis.

3.2.3 Establishment Risk

Although establishment of willow may be slow on heavy clays (Tubby and Armstrong 2002), the growing of willow in Ireland since the mid-1970's (Rosenqvist and Dawson 2005, McCracken and Dawson 2007, Finnan 2010a) has resulted in a relatively stable establishment rate for the crop, assuming best practice management guidelines are adhered to (Finnan 2010b). However, for miscanthus field establishment has often been poor or uneven (McKervey et al. 2008). Poor establishment in miscanthus is generally due to poor over-wintering (Clifton-Brown and Lewandowski 2008), which can make the crop susceptible to both frost damage and disease attack (Thinggaard 1997). McKervey et al. (2008) have noted that although a few studies have looked at aspects such as the effects of rhizome size,

planting density and moisture status on establishment and early growth, overall, very little agronomy research has been carried out on the crop.

In order to address the added risk of miscanthus failing to establish, a discrete (Bernoulli) probability distribution was used to simulate the occurrence of crop establishment failure. This should improve the accuracy of modelling the downside risk of miscanthus. The effect of a failure on the budget was threefold: (1) a replanting cost, without the benefit of an establishment grant, was accrued, (2) the revenue from the first harvest was lost and (3) the lower yield level was applied in year 3, with the plateau yield level pushed back until year 4 of the project.

3.2.4 Bioremediation

From an agronomic viewpoint, treated sewage sludge from sewage or wastewater can improve soil structure and supply nutrients, therefore becoming a beneficial material for application to energy crops (Plunkett 2010). The application of sewage sludge provides energy crop fertilisation at little to no cost (Finnan 2010a). However, despite these benefits the low uptake of biomass crops necessitates other incentives from which additional economic benefits can accrue from growing willow and miscanthus (Bullard and Nixon 2002). In this context, the possibility of using bioremediation as a means to boost the returns from willow and miscanthus was investigated through the incorporation of a stochastic gate fee.

While theoretically a farmer accepting sludge from a treatment plant could be paid up to the alternative cost of conventional treatment, in practice it is likely society would try to profit from a new treatment system by lowering the payment for disposal of waste (Rosenqvist and Dawson 2005b). The market for bioremediation in Ireland is not yet fully established, and so the gate fee received per tonne of sludge could vary from a positive value in the case of escalating costs and legislative pressures for treatment plants to dispose of waste, to a negative value in the case of farmers purchasing sludge in order to reduce fertiliser costs and increase yields (Finnan 2010b). In this analysis, the uncertainty regarding the price per tonne received for the application of sludge was modelled using a triangular distribution. The minimum was

set as zero, with the maximum being the price offered by the leading bioremediation company operating in Ireland.

However, the spreading of sludge of municipal or industrial origin on agricultural land can impact on animal health, the environment and food safety, and consequently there are a number of legislative restrictions on the use of sludge in agriculture (Plunkett 2010). The extent of bioremediation usage per hectare is difficult to ascertain, as it is not appropriate or practical to provide examples of application rates for sludge's or effluents as each situation will be highly individual and dependent on the nature of the waste, the nutrients it contains and the soil and climatic conditions in which it is applied (Dawson 2007). In this analysis the application rate of sludge per hectare was assumed to be triangularly distributed between a range of 0 – 14 tonnes. These figures were based on the latest available best practice guidelines (Dawson 2007).

3.4 Estimation of Stochastic Returns from Superseded Enterprises

The opportunity cost of land is accounted for through the inclusion of foregone returns from a conventional agricultural activity, in this case land rental, spring barley, winter wheat and store to finished beef. The estimation of parameters of the probability distribution for the stochastic superseded enterprise gross margins (GM) was empirically based, with Irish National Farm Survey (NFS) data used to estimate historical GM variations of enterprises within farms between years. Each enterprises financial performance, measured as GM per hectare, was calculated from historical data from 1998 – 2008. This unbalanced panel data was then used to estimate the following fixed effects (FE) model for each enterprise:

$$g_{it} = c + b_i + g_{it-1} + r_{it} \quad (5)$$

where g_{it} is GM on farm i in year t ($t = 1, \dots, 16$), c is the general mean, b_i is the effect on GM due to farm i (variation between farms caused by different management practices, soil, etc.), g_{it-1} is the previous years GM lagged. The residual r_{it} is a random variable, and after testing was shown to be normally distributed for land rental and winter wheat, with the beta and double exponential distributions fitting best for spring

barley and store to finished beef respectively. The resulting parameters were used in the estimation of stochastic gross margins for each superseded enterprise in each year of the project lifespan.

3.5 Risk Ranking of Alternative Investment Options

Since we do not know farmers utility functions, an efficiency criterion that allows a partial ordering of the risky alternatives when the exact degree of risk aversion is not known must be used (Lien et al. 2007). Risk Aversion Coefficients (RAC's) are used to define groups of decision makers, so that within a certain range all decision makers will have the same preferences (Pratt, 1964). The use of risk aversion bounds allows one to draw inferences about how different groups might rank risky choices for business or policy purposes (Richardson 2003). However, there is a degree of difficulty in specifying the value of the RAC's (Richardson, 2000). McCarl and Bessler (1989) describe how the RAC bounds depend upon the coefficient of variation and the standard deviation for the distributions being analysed. Following one of their suggested methods, the RAC level is calculated as follows:

$$RAC = 5.0 / std. dev \quad (6)$$

To further refine the specification of the RAC's, Anderson and Dillon (1992) proposed a classification of RAC levels based on magnitudes about 1, with 0.5 being hardly risk averse and 4.0 being extremely risk averse. As suggested by Richardson (2000), in this analysis the Anderson and Dillon (1992) scale is used to define the relative levels of risk aversion about RAC's formulated through the method outlined in McCarl and Bessler (1989). So, for example, if Eq. (6) suggests a RAC of 0.04, then 0.02 is hardly risk averse and 0.16 is extremely risk averse.

The software computer program developed by Richardson et al. (2000) was used for the computational task of ranking the alternative investments using the SERF approach. The negative exponential utility function assumes constant absolute risk aversion and increasing relative risk aversion (Hardaker 2000). As stated earlier, various studies have generally found farmers to be a risk averse group and so under

this criterion, the negative exponential utility function is used in the implementation of the SERF procedure for this analysis. Figure 2 depicts a flow chart of this section of the model, illustrating the major components and their interrelationships.

[INSERT FIGURE 2 ABOUT HERE]

4. Results:

4.1 Baseline Results

The CDF's of the stochastic NPV's from the alternative biomass investment scenarios are presented in Figure 3. The graph shows that three of the four willow investment projects fail to generate a positive NPV as the entire CDF lies to the left of the point representing a return of zero. This suggests that these investments are not economically viable and so should not be considered. An investment in willow which supersedes a store to finished beef enterprise is also relatively risky, with only a 30 percent probability of generating a positive return. All things considered, the risk of making a loss is likely to be a major barrier to farmers planting willow, and may be a reason for the low adoption rates of this crop in Ireland thus far.

[INSERT FIGURE 3 ABOUT HERE]

Although an investment in miscanthus superseding a winter wheat enterprise has an extremely high probability of producing a negative return, the other miscanthus projects seem to be economically viable investments. High probabilities of making a profit are recorded on the remaining three investments, although the added risk as a result of uncertain establishment in Ireland does allow the possibility of negative returns being generated. The kink in the distribution of the returns from the various miscanthus investment options is as a result of the greater downside risk resulting from the possibility of the crop failing to establish. The effect is a much wider distribution, and subsequently increased risk, for miscanthus than willow.

However, as stated earlier, the CDF graph procedure does not always result in an unambiguous ranking of the alternative options as when the graphs cross there is no clear ranking (Richardson 2003). For example, in Figure 3 an investment project in which miscanthus supersedes winter wheat has an upper tail greater than and a lower tail less than that of projects in which willow supersedes spring barley or land rental. Therefore, which of the alternatives a farmer would prefer may depend on a comparison in terms of overall risk efficiency (Lien et al. 2007a), and so Stochastic Efficiency with Respect to a Function is used to produce a ranking of the alternative enterprises. A certainty equivalent chart, which shows the amount of money that the decision maker would have to be paid in to be indifferent between a particular scenario and a no-risk investment (Richardson 2003), is detailed in Figure 4.

[INSERT FIGURE 4 ABOUT HERE]

Farmers with a negative risk aversion coefficient are willing to take on risk in an investment if the returns from the investment are high enough. From the figure above, it is apparent that even these farmers would be unwilling to invest in a project in which willow superseded winter wheat, spring barley or land rental. The lack of interest in these investment options is magnified as you move to the right of the graph and farmers have a higher risk aversion coefficient, with this group also unwilling to invest in a willow project in which store to finished beef was superseded. Miscanthus fares marginally better, in terms of the certainty equivalent required to be indifferent with a no-risk investment, however most farmers with positive levels of risk aversion would not make an investment in miscanthus given the available returns.

4.2 Bioremediation

The effect of bioremediation on the investment returns of willow and miscanthus projects was also examined, with the CDF's presented in Figure 5. The positive effect of the gate fee was a shift to the right in each project CDF. However, the probability of generating a negative return remains extremely high for three of the four willow projects, with store to finished beef being superseding the only one with a high

probability of making a profit. The positive returns from miscanthus were further boosted by the incorporation of a gate fee from bioremediation, although this extra source of revenue was insufficient to make superseding winter wheat with miscanthus a viable investment. SERF was again used to rank the competing investment options and generate certainty equivalents. The results are presented in Figure 6.

[INSERT FIGURE 5 ABOUT HERE]

[INSERT FIGURE 6 ABOUT HERE]

The incorporation of a gate fee from bioremediation has a considerable effect on the willingness to invest in willow projects, as farmers with negative risk version coefficients have positive certainty equivalents for all but the option in which winter wheat is superseded. However, even the additional returns generated would not persuade farmers with a high risk aversion coefficient to invest in willow. A similar boost in interest is noted for miscanthus investment options. The miscanthus superseding store to finished beef option has a positive certainty equivalent for all levels for risk aversion, while miscanthus projects in which spring barley or land rental are superseded are only negative for those at the ‘very risk averse’ and ‘extremely risk averse’ levels. Miscanthus superseding winter wheat is deemed to be a worthwhile investment for those at the lower risk aversion levels, but has a considerable negative certain equivalent for those with higher levels of risk aversion.

5. Conclusions

Since farming is a risky business it is important to account for risk in planning. Information from an ordinary deterministic budgeting model done on the basis of point estimates of uncertain variables may inform investment and management decisions on a farm; however they fail to capture the effect that risk may have on the investment decision. The uncertainty surrounding the risky variables involved in

producing biomass crops, such as the yield level and energy price, make it difficult to accurately calculate the returns of such a project. A stochastic budgeting approach may give more realistic and more useful information about alternative decision strategies (Lien 2003) and so is used in this analysis to account for risk in biomass crop investment decisions.

This paper found that accounting for risk underlined the results of the baseline economics from Clancy et al. (2009), who found that under typical costing assumptions, miscanthus had a greater level of returns than willow. While the distributions of investment returns for miscanthus are wider than those of willow, implying greater uncertainty, the results from the SERF analysis show miscanthus generally has higher certainty equivalents. This indicates that farmers would be more likely to switch to miscanthus production rather than willow. Despite the wider distribution of returns, the SERF analysis suggests that a greater level of risk is associated with willow than with miscanthus. The results suggest that the potentially higher returns from miscanthus outweighed the downside risk associated with the possibility of the crop failure to establish. The disparity in the level of risk is likely due to the superior yield potential, annual production cycle and cash flow profile of miscanthus compared to willow. Evidence of this can be found in the fact there was much greater uptake of miscanthus than in willow in Ireland during the 2007 – 2009 period (Caslin 2010).

Accordingly, it can be expected that more risk averse farmers are unlikely to find willow an attractive alternative to conventional agricultural systems during the pioneer stage of the bioenergy market in Ireland. Ekboir (1997) noted that when decisions are irreversible and risky, investment by individual firms is known to be sporadic. Due to the level of risk involved in growing willow, widespread adoption in Ireland is only likely when the economic merits of these crops have been proven over an extended period (Clancy et al. 2009). Therefore, while miscanthus investments can generate positive returns, it is important to bear in mind that the risk associated with the extremely long payback periods necessary to recoup initial investment costs would be a concern to most investors.

The failure of willow to generate the same level of returns as miscanthus may be as a result of willow's multi-year harvest cycle, which takes longer to produce a positive net value in comparison to miscanthus with its annual harvest cycle. The length of the harvest cycle may also account for the lower level of risk associated with miscanthus. An annual income stream as opposed to a lump sum every two to three years would help reduce the variability in returns. The reduction of the harvest cycle length for willow is seen as fundamental in making it competitive with both conventional agricultural enterprises and miscanthus. Better crop management techniques developed as expertise in the area grows could potentially increase yield levels and thereby reduce the variability of these yields between harvest cycles, decreasing risk significantly.

Acknowledgements

The authors are grateful to the Irish Department of Agriculture, Fisheries and Food for providing financial support through the Research Stimulus Fund.

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7. Tables and Figures

Table 1: Model cost and revenue assumptions for willow and miscanthus

	Willow	Miscanthus
Maximum Production Period	16 Years	16 Years
Number of Harvest Cycle	7 Harvest Cycles	15 Harvest Cycles
Establishment Grant ³	€1450 (75% payable in year 1, 25% in year 2)	€1450 (75% payable in year 1, 25% in year 2)
Harvest Strategy	Stick harvested, naturally dried, stored outdoors then chipped	Baled harvest, naturally dried, stored outdoors
Moisture Content	25%	20%
Price per Tonne	€55	€60
Establishment Costs	€2575	€3130
Harvest Costs	€747	€216
Crop Removal	€562	€225

³ The establishment grant and costs are detailed on a per hectare basis

Table 2: Summary Statistics of biomass yield level data (dmt.ha⁻¹yr⁻¹)

	Willow Yield	Miscanthus Yield
Mean	8.92	12.63
Std. Dev	3.38	2.94
CV	37.87	23.31
Min	0.60	6.93
Median	8.58	13.53
Max	24.33	17.69

Table 3: Working capital released per hectare for the superseded enterprises

	Grazing Land Rental Value	Spring Barley	Winter Wheat	Store to Finished Beef
Working capital released ¹	-	€240	€295	€787

¹ Working capital released is the average capital tied up in stock and variable inputs for each enterprise

Figure 1: Flow Chart of Biomass Returns Calculation

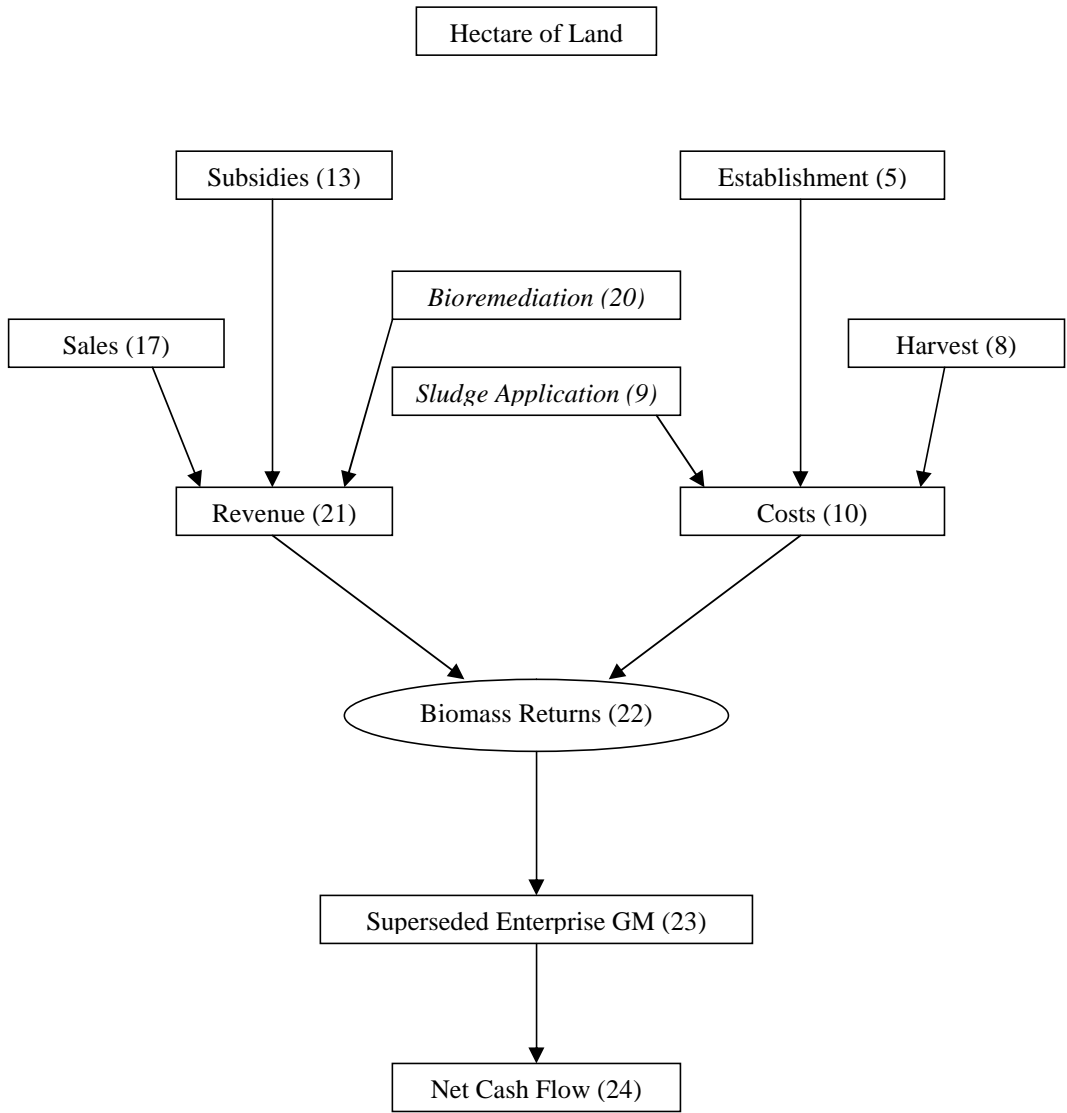


Figure 2: Flow Chart of Risk Ranking Procedure and Investment Decision

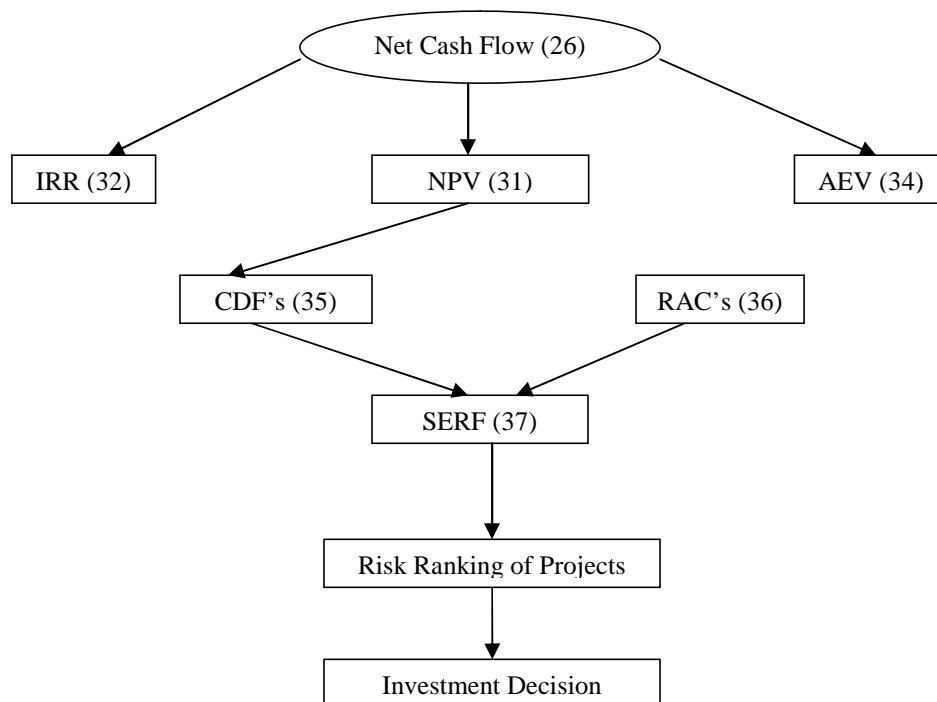


Figure 3: Cumulative distribution of alternative willow and miscanthus investment option returns

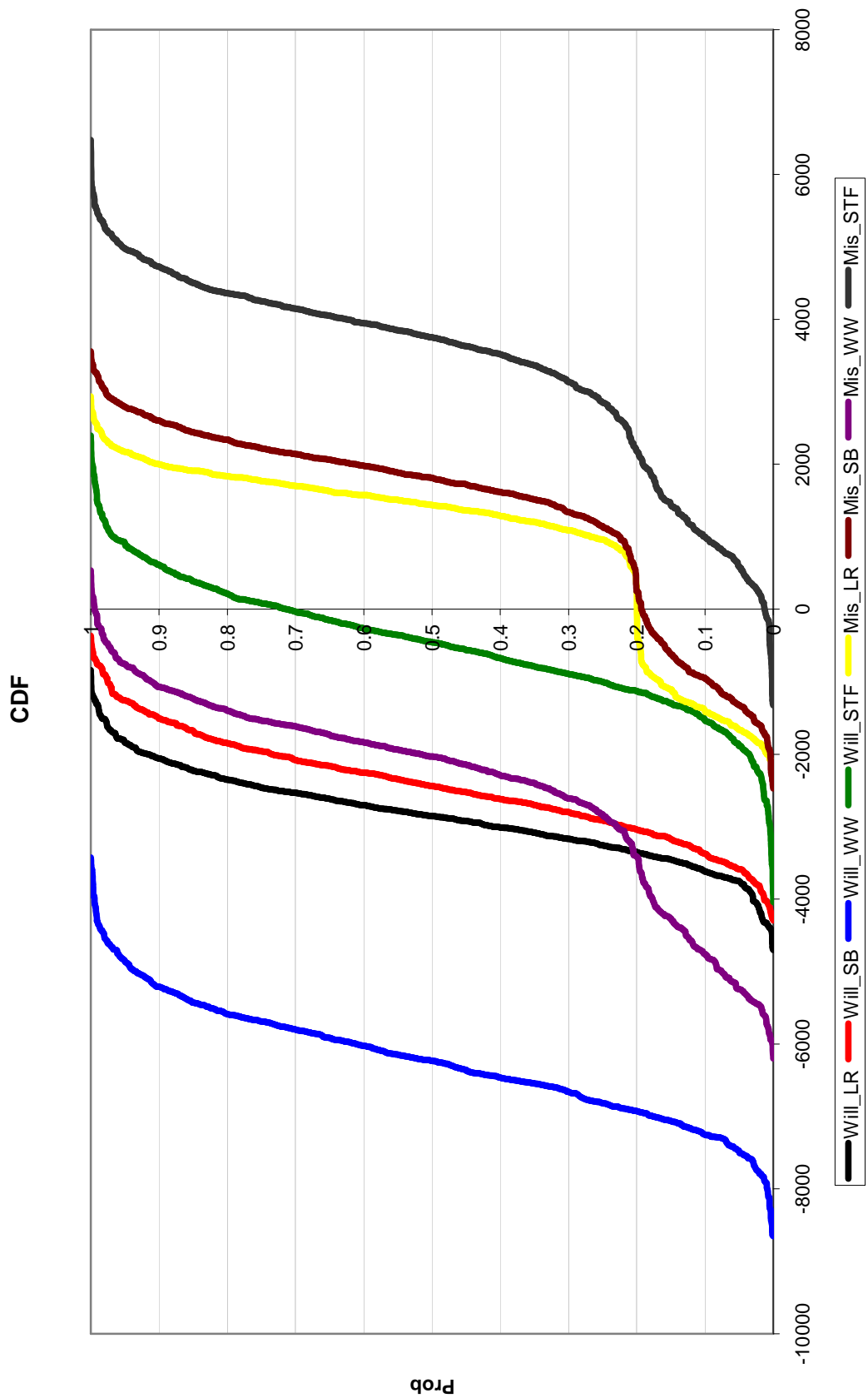


Figure 4: Certainty Equivalent's of alternative willow and miscanthus investment options for different degrees of risk aversion

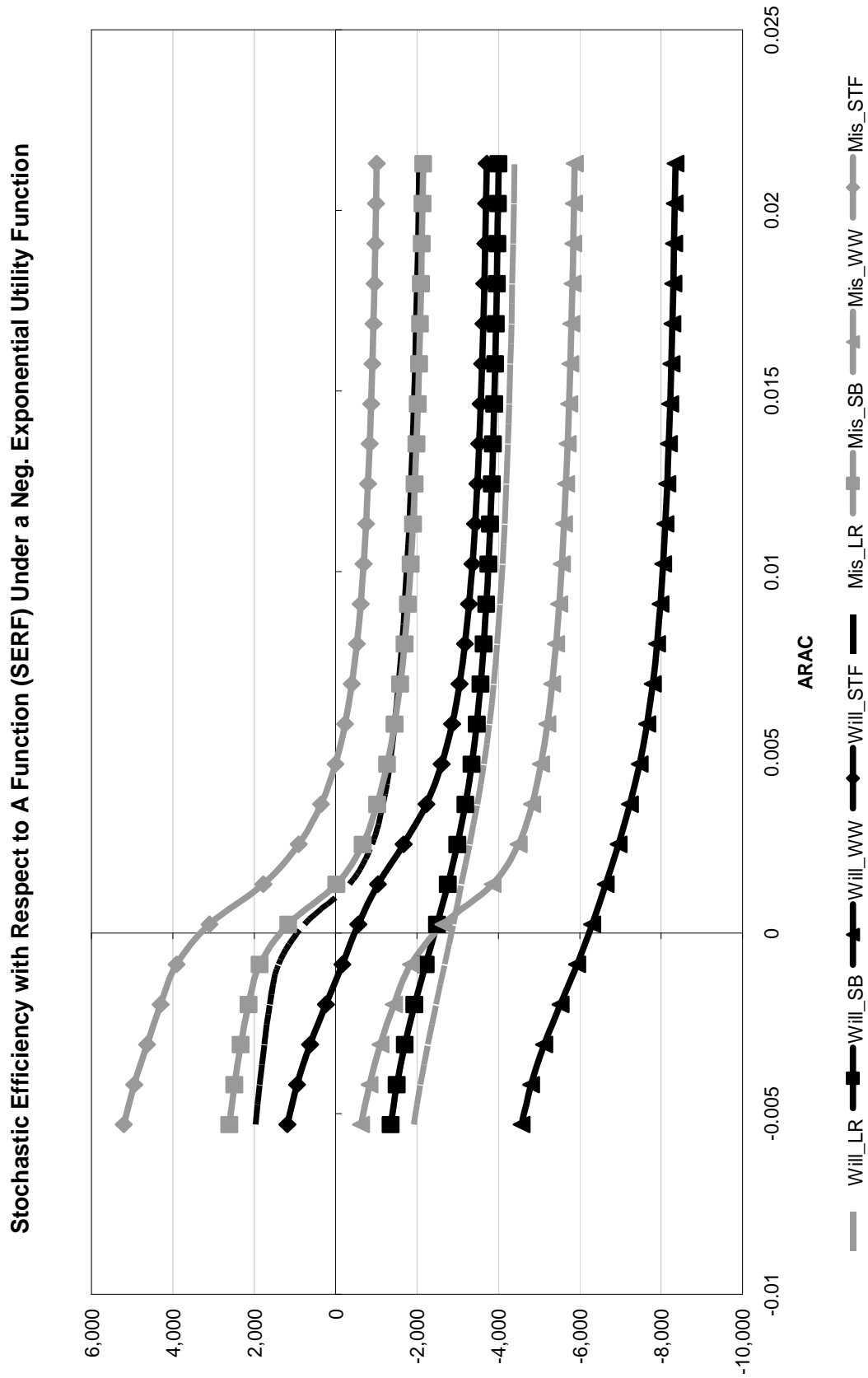


Figure 5: Cumulative distribution of alternative willow and miscanthus investment option returns with gate fee from bioremediation included

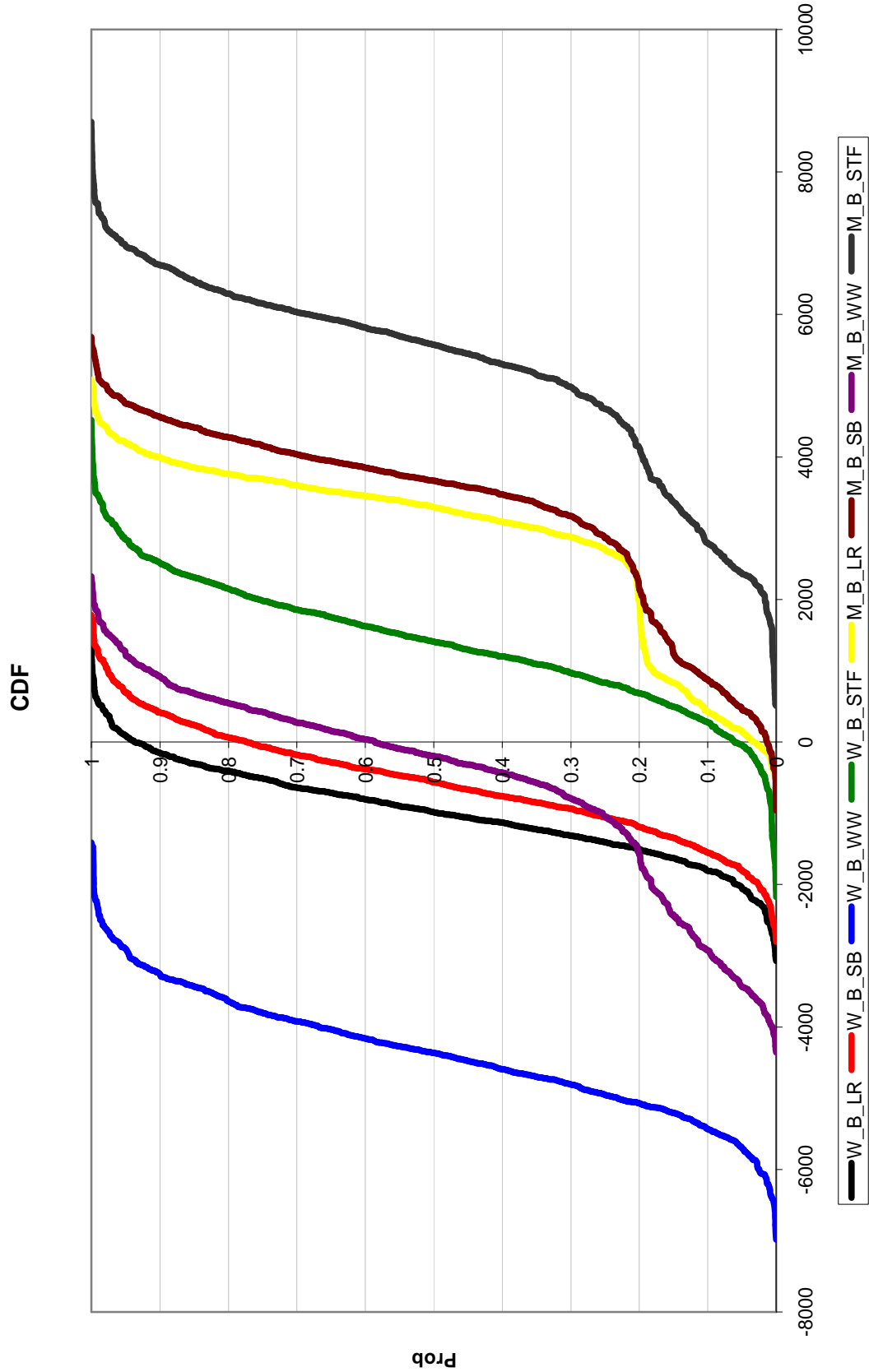
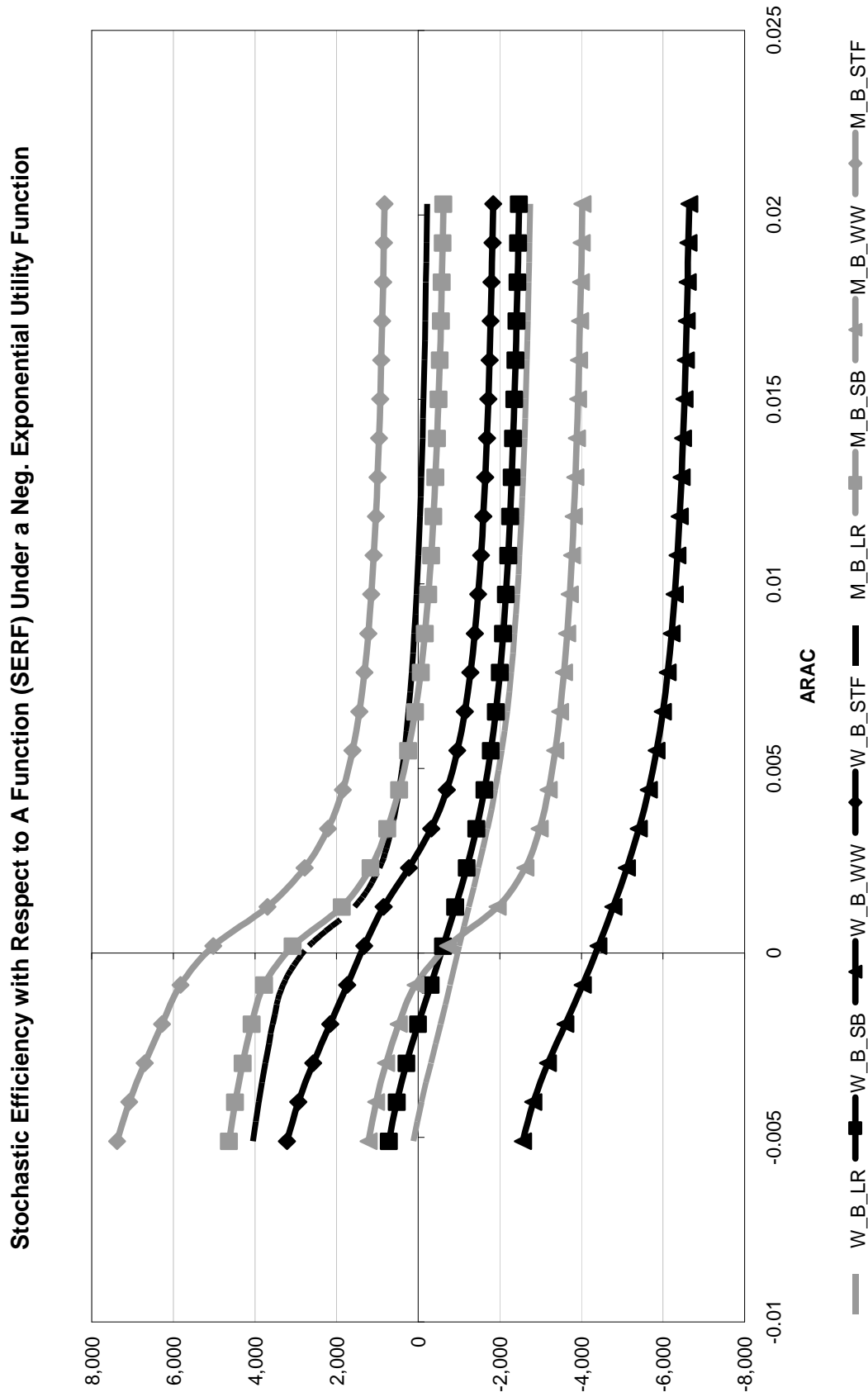


Figure 6: Certainty Equivalent's of alternative willow and miscanthus investment options with gate fee from bioremediation for different degrees of risk aversion



8. Appendix: Stochastic Variables and Equations for Biomass Investment Model

(Variables in bold are stochastic, or a function of a stochastic variable)

Costs

$$\text{Cultivation}_1 = \text{Cost}_1 * [\text{Price Index for Agricultural Services}_1] \quad (1)$$

$$\text{Cuttings/Rhizomes}_1 = \text{Cost}_1 * [\text{Price Index for Seed}_1] \quad (2)$$

$$\text{Spraying}_1 = \text{Cost}_1 * [\text{Price Index for Crop Protection}_1] \quad (3)$$

$$\text{Miscellaneous}_1 = \text{Cost}_1 * [\text{Price Index for Other Expenses}_1] \quad (4)$$

$$\text{Establishment}_1 = [\text{Cultivation}_1 + \text{Cuttings/Rhizomes}_1 + \text{Spraying}_1 + \text{Miscellaneous}_1] \quad (5)$$

$$\text{Harvesting/Mowing}_t = \text{Cost}_t * [\text{Price Index for Agricultural Services}_t] \quad (6)$$

$$\text{Dry/Transport}_t = \text{Cost}_t * [\text{Price Index for Agricultural Services}_t] \quad (7)$$

$$\text{Harvest}_t = [\text{Harvesting/Mowing}_t + \text{Dry/Transport}_t] \quad (8)$$

$$\text{Sludge Application}_t = \text{TRIANGULAR} [\text{min, max, mode}] \quad (9)$$

$$\text{Total Costs}_t = [\text{Establishment}_1 + \text{Harvest}_t + \text{Sludge Application}_t] \quad (10)$$

Revenue

$$\text{Grant}_1 = [\text{Grant} * 0.75] \quad (11)$$

$$\text{Grant}_2 = [\text{Grant} * 0.25] \quad (12)$$

$$\text{Subsidies} = [\text{Grant}_1 + \text{Grant}_2] \quad (13)$$

$$\text{Yield}_t = [(\text{Constant} * 0.6) - \text{Lagged Yield Coefficient} + \text{BETA} (\text{Error})] \quad (14)$$

$$\text{Yield}_t = [\text{Constant} - (\text{Lagged Yield Coefficient} * \text{Lagged Yield}_t) + \text{BETA} (\text{Error})] \quad (15)$$

$$\text{Price}_t = \text{Cost}_t * [\text{Price Index for Solid Fuels}] \quad (16)$$

$$\text{Sales}_t = [\text{Yield}_t * \text{Price}_t] \quad (17)$$

$$\text{Sludge Volume}_t = \text{TRIANGULAR} [\text{Min, Max, Mode}] \quad (18)$$

$$\text{Gate Fee}_t = \text{TRIANGULAR} [\text{Min, Max, Mode}] \quad (19)$$

$$\text{Bioremediation}_t = [\text{Sludge Volume}_t + \text{Gate Fee}_t] \quad (20)$$

$$\text{Total Revenue}_t = [\text{Subsidies} + \text{Harvest}_t + \text{Bioremediation}_t] \quad (21)$$

$$\text{Biomass Returns}_t = [\text{Total Revenue}_t - \text{Total Costs}_t] \quad (22)$$

Cashflow

$$\text{Superseded Enterprise GM}_t = [\text{Constant} + (\text{Lagged GM Coefficient} * \text{Lagged GM}_t) + \text{NORMAL or BETA or DOUBLE EXPONENTIAL} (\text{Error})] \quad (25)$$

$$\text{Profit/Loss}_t = [\text{Biomass Returns}_t - \text{Superseded Enterprise GM}_t] \quad (26)$$

In the Event of Miscanthus Establishment Failure

$$\text{Costs in Year 2: IF (BERNOULLI (0.2) = 1, (5), (8))} \quad (27)$$

$$\text{Harvest in Year 2: IF (BERNOULLI (0.2) = 1, 0, (17))} \quad (28)$$

$$\text{Yield in Year 3: IF (BERNOULLI (0.2) = 1, (14), (15))} \quad (29)$$

Key Output Variables

$$\text{Capital Invested}_1 = [\text{Establishment}_1 - (\text{Grant}_1 + \text{Bioremediation}_1)] \quad (30)$$

$$\text{Net Cash Flow}_t = [(- \text{Capital Invested}_1) + \text{Biomass}_t] \quad (31)$$

$$\text{Discount Factor} = [1 / (1 + \text{Discount Rate}) ^ \text{Year}] \quad (32)$$

$$\text{Present Value of Cash Flow} = [\text{Net Cash Flow}_t * \text{Discount Factor}] \quad (33)$$

$$\text{Net Present Value} = \sum_t [\text{Present Value of Cash Flow}] \quad (34)$$

$$\text{Internal Rate of Return} = [(\text{Net Cash Flow}_t / (1 + r)^t = 0] \quad (35)$$

$$\text{Amortization Factor} = [\text{Discount Rate} / (1 - (1 + \text{Discount Rate}) ^{-t})] \quad (36)$$

$$\text{Annual Equivalent Value} = [\text{Net Present Value} * \text{Amortization Factor}] \quad (37)$$

Risk Ranking

$$\text{Cumulative Distribution Function} = [1000 * \text{Net Present Value}] \quad (38)$$

$$\text{Risk Aversion Coefficient Bounds} = [5.0 / \text{std. dev}] \quad (39)$$

$$\text{Stochastic Efficiency with Respect to a Function} = [] \quad (40)$$