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LEAST-COST FEED RESERVE USING DROUGHT PROBABILITIES DERIVED FROM A GRAZING MODEL*

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An inventory model is used to determine least-cost fodder reserves for sheep in the Hamilton area of Victoria. Unlike earlier studies a grazing model is used to generate feeding requirements and thus allows for the gradual onset of drought and the associated rise in feed prices. Compared with studies based on probabilities of effective rainfall, the approach used measures more accurately the way in which drought incidence is affected by the seasonal pattern of pasture production in relation to animal requirements. The grazing model facilitates a study of the relative severity of drought at a range of stocking rates and a number of probability distributions are identified. The refinements achieved by using a grazing model also allow a more realistic treatment of penalty costs than the usual prescription of a constant drought price for fodder. Furthermore it allows for the determination of least-cost fodder reserves for a range of stocking rates, and thus a comparison between stocking rates is possible because allowances can be made for the effect of different timing and level of feeding on feed costs. The sensitivity of the least cost reserve to changes in acquisition cost and rate or interest is examined for various stocking rates.

1 INTRODUCTION

Previous studies on fodder reserves have used one of two approaches to determine drought probabilities. Dillon and Lloyd [2] used distributions calculated by Verhagen and Hirst [8] which were based on effective rainfall, and Mauldon and Dillon [4] also used the probabilities of effective rainfall in their inventory model. However, Officer and Dillon [5] left the individual producer to estimate his subjective drought probabilities for his current state of property development because ". . . it puts the final responsibility for the choice of odds where it belongs—with the farmer".

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The use of a grazing model to calculate the supplementary feeding requirements is a third approach to estimated drought probabilities. The application of climatic data, derived either by generating a rainfall sequence or using an historical sequence, to such a model provides probability estimates for predefined stocking rate-pasture production regimes.

Implicit in the use of the effective rainfall criterion is the notion that pasture production is an all-or-nothing phenomenon, i.e., when rainfall is "effective" pasture growth occurs, and there is no attention to the other climatic variables which also determine the growth rate of pasture. This further implies a drought—non-drought dichotomy, whereas drought is a continuous rather than discrete phenomenon, and the onset of drought is frequently quite gradual. A grazing model can "grow" pasture at a rate determined by the weather and then add this to the standing pasture, whilst accounting for the effects of livestock activity and pasture decay. Further, a grazing model can account for the seasonal requirements of livestock.

2 THE GRAZING MODEL

The grazing model used simulates fine-woolled comeback wethers grazing on improved pastures in the Western District of Victoria, and has been fully described by Thatcher [7]. Briefly, it consists of a pasture production sub-model to which the endogenous variables are the cumulative June to October rainfall and the date of the autumn break, which is calculated from a soil moisture budget which takes into account the net effects of rainfall and evapotranspiration. The pasture is continuously grazed by sheep whose intake is dependent upon pasture availability and pasture digestibility. The intake is then converted to liveweight change and wool output through relationships dealing with maintenance requirements which vary through the year, and the efficiency of conversion of intake to liveweight and wool. The inter-relationships between variables in the grazing model are illustrated in figure 1. Where time appears as an independent variable it acts as a proxy for the various climatic variables which were not included in the model due to insufficient data, (for example, hours of sunlight, temperature).

The endogenous variables for the grazing model were calculated from a ninety year sequence of rainfall data from the Hamilton area. The pasture growth relationships were based on current species and pasture management, so that the output derived approximates what would be expected over this period with current technology.

The same sequence of rainfall data was used for each computer run because comparisons were being made between a number of stocking rate—pasture production regimes. Furthermore, the use of an historical sequence of rainfall data enabled the output for 1967–9 from any computer run of the grazing model to be validated against data from an experiment conducted on the Pastoral Research Station at Hamilton (P.R.S.). It was found that for a range of stocking rates the patterns

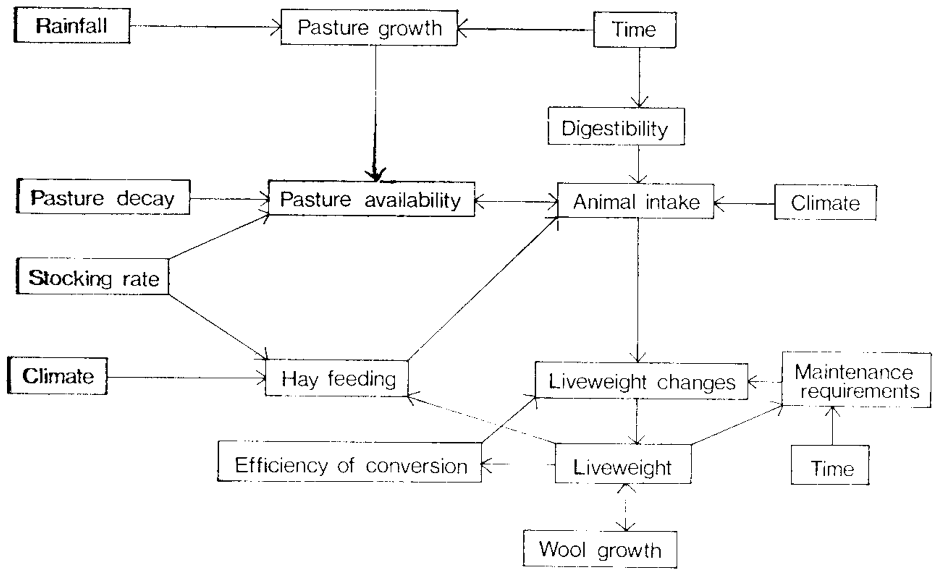

Figure 1: Relationship of variables included in the grazing model

of pasture availability, sheep liveweights and hay feeding requirements derived from the grazing model closely matched those of the Hamilton experiment [7].

Stocking rates between five and twenty wethers per hectare were run on three levels of pasture production, with the highest production equal to that achieved on the P.R.S. Levels of 80 per cent and 65 per cent were also specified to reflect the generally lower levels of production found on neighbouring properties. The assumption of a vertical shift in the pasture production curve holds true at the 80 per cent level because pastures would be based on the same species; however it is possible that at the 65 per cent level the reduced production would be accompanied by a small change in the pattern of production because native species would predominate the sward.

The results published in this paper refer to the 80 per cent level of pasture production, other results being presented elsewhere [6].

2.1 DROUGHT STRATEGY

Supplementary feeding is the only drought strategy included in the grazing model. Early in the drought hay is fed, but is replaced by wheat when its price exceeds that of wheat on a starch equivalent basis. The feeding programme is conducted subject to a series of rules based on common district practice. These took into consideration the pasture available, sheep liveweight, the rate of liveweight change and the minimum liveweight to which sheep were allowed to fall. The formulation of optimal feeding rules was not attempted, although these could be studied using the model.

The alternative strategies to counter drought, such as agistment or sale of stock, could be readily incorporated into the model provided realistic decision rules, perhaps concerning pasture availability and condition of sheep could be determined for each of these strategies. However considerable difficulties arise in evaluating the costs of the stock-selling strategy because of the paucity of data regarding price movements of sheep either during a drought or in "normal" times.

Dillon and Mauldon [3] showed how other strategies can be incorporated into within drought planning in an inventory model, as additional information about sheep prices, fodder prices and the likelihood of the drought continuing are taken into account.

2.2 DROUGHT FEEDING REQUIREMENTS

Drought was defined as any period of supplementary feeding. This definition avoids the conceptually vague distinction between drought feeding and "normal" year-to-year feeding, a distinction which depends on grazing pressure and the producers reasons for feeding (for example, to flush ewes prior to joining). The analysis was simplified by assuming that supplementary feeding is never conducted on 1st October (i.e., during the spring peak of pasture growth), thereby limiting the potential length of any drought to less than 12 months. Although this is a valid assumption in the high rainfall zone where Hamilton is located, it is an assumption which would limit the use of the model in the arid zone where droughts frequently last longer than twelve months.

The calculated quantities of hay fed in the October to September periods over the ninety years provided probability distributions of droughts of various severities for each stocking rate.[1] In the absence of prior knowledge of the distributions, the probabilities were retained in their unsmoothed state.

Obviously, drought probabilities are critically dependent on stocking rate. The probability (P) of requiring k units of hay on stocking rate s is Pk_s (where there are 80 units to the tonne after allowing for 10 per cent wastage in feeding out hay (Cocks [1]). If the greatest number of hay units ever required is w then

$$\sum_{k_s=o}^{w} Pk_s = 1 \qquad \ldots \quad (1)$$

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[1] The feeding requirements in the first year were ignored in order to allow the variables in the grazing model to reach equilibrium, i.e., the status of the variables in the model were allowed to reach a value independent of the artificial starting values given to them.

Other studies have recognized the need to include in the analysis the influence of grazing pressure on drought probabilities; however they did not attempt to directly quantify the problem.

Mauldon and Dillon [4] and Dillon and Lloyd [2] avoided the necessity of specifying a direct relationship between drought probabilities and stocking rate by asking the grazier to express stocking rate in terms of the number of "dry months" (i.e., months of non-effective rainfall) which could occur before hand-feeding is required. That is, the higher the stocking rate, the shorter the number of months before hand-feeding would be adjudged to be necessary. This requires a judgement by the grazier. The grazing model generates drought feeding requirements for a range of stocking rates. A grazier may, of course, alter these feed requirements to conform with his experience on his property, but he has an objective starting point for making his subjective estimates.

2.3 INFLUENCE OF STOCKING RATE ON SEVERITY OF DROUGHT

Table 1 illustrates how the distribution of drought probabilities varies with stocking rate, and table 2 shows the rankings of droughts in terms of severity. These rankings vary with stocking rate in a way which would not be revealed if the effective rainfall criterion was used. The effective rainfall approach gives misleading results because it takes an "all-or-nothing" approach to pasture growth, and fails to allow for carry-over of surplus feed into periods of non-effective rainfall. The proportion of this feed which will be wasted will depend on stocking rate. For example, 1953 is ranked only sixtieth at five wethers per hectare,

TABLE 1

Drought Probabilities based on 1879-1969 *Hay Requirements*

Hay Required (units/sheep)[a]	Stocking Rate (Wethers/Hectare)			
	5	10	15	20
0	·216	·125		
1	·534	·171		
2	·182	·261	·023	
3	·034	·193	·102	
4	·011	·205	·148	·023
5	·023	·023	·284	·102
6		·023	·193	·080
7			·171	·193
8			·057	·193
9			·023	·125
10				·114
11				·125
12				·034
13				·011

[a] Where there are 80 units to the tonne.

but at twenty wethers per hectare it is ranked sixth in severity. At the other extreme, 1963 is ranked sixth with five wethers per hectare and forty-first with twenty wethers per hectare. In 1953, pasture growth, though low, was well distributed throughout the year, and would have adequately supported a low stocking rate, but heavy feeding throughout the year would have been required at high stocking rates. In 1963, the peaks in pasture availability were pronounced; all stocking rates required feeding between the peaks, but there would have been more wastage of pasture at low stocking rates than at high stocking rates, compared with normal years.

TABLE 2

Rankings of Years based on the Severity of Drought

(Worst years of drought in order of severity)

Ranking	Stocking Rate (Wethers/Hectare)			
	5	10	15	20
1	1950	1967	1968	1915
2	1967	1950	1915	1968
3	1921	1968	1953	1945
4	1920	1920	1967	1967
5	1904	1921	1916	1894
6	1963	1910	1894	1953
7	1968	1904	1910	1897
8	1964	1915	1920	1910
9	1927	1927	1950	1918
10	1902	1891	1945	1950
Year	**(Ranking of some selected years)**			
1894	43	32	6	5
1904	5	7	12	15
1915	58	8	2	1
1921	3	5	27	28
1945	17	14	10	3
1953	60	29	3	6
1963	6	18	37	41

3 THE INVENTORY MODEL

The total annual cost (A) of acquiring a reserve of h units of hay is given by

$$A = ch(1 + r) \qquad \ldots \quad (2)$$

where c is the acquisition cost per unit of hay and r is the relevant rate of interest.

The salvage value (V) of a unit of fodder left unused at the end of a drought is given by

$$V = A(1 - d) \quad \quad . \; . \; . \; (3)$$

after allowing a rate of deterioration of d per cent.

The expected salvage revenue ($E(R)$) from all unused fodder is given by

$$E(R) = V \sum_{k_s=o}^{h} (h - k_s) \, . \, Pk_s \quad \quad . \; . \; . \; (4)$$

where $h > k_s$.

The penalty incurred from holding insufficient reserves is assumed to be the purchase of feed as required at drought-affected prices.

The gradual increase in feed requirements within a drought means that early-fed units of hay will be fed out over a longer period than late-fed units when full feeding is in progress. Thus, as hay prices increase during a drought, the price of a unit of hay will be higher than its predecessor by an amount determined by the time it took to feed out the preceding unit. This time period is determined by the grazing model.

This approach implies a policy whereby the producer buys fodder as he requires it, and although this is not common it is usual to buy fodder at various intervals during the drought. This approach accounts for the increasing market value of fodder being fed to sheep and gives a more realistic value of the costs of carrying stock through a drought.

The effects of stocking rate on penalty costs are firstly, to necessitate earlier feeding on higher stocking rate, and secondly, to hasten full feeding on higher stocking rates. This is illustrated for 1967 by the isofeed lines of figure 2, which show the time taken to feed out various quantities of hay over a range of stocking rates. The isofeed lines for 1967 become denser to the right and to the top of the graph as full feeding is invoked on the higher stocking rates.

On the basis of the behaviour of feed prices at Hamilton in 1967, the hay price is assumed to rise at a constant rate of \$3 per tonne per month (i.e., 3·75 cents per unit) from \$13 to \$32 per tonne over a period of six and a half months. The limit of \$32 per tonne of hay is equivalent to wheat costing \$63 per tonne on a starch equivalent basis, after allowing for a 10 per cent wastage in feeding hay, as reported by Cocks [1].

Table 3 shows the starting prices and price rises occurring on some stocking rates investigated. The total price paid for a unit is calculated by adding to the starting price all price increments occurring up to the level of feeding under consideration. In effect, the starting price reflects the time at which the drought commences on a particular stocking rate in relation to the increases in hay prices which have already taken place. For example, the starting price at five wethers per hectare is greater than for the higher stocking rates because drought feeding commences later in the drought.

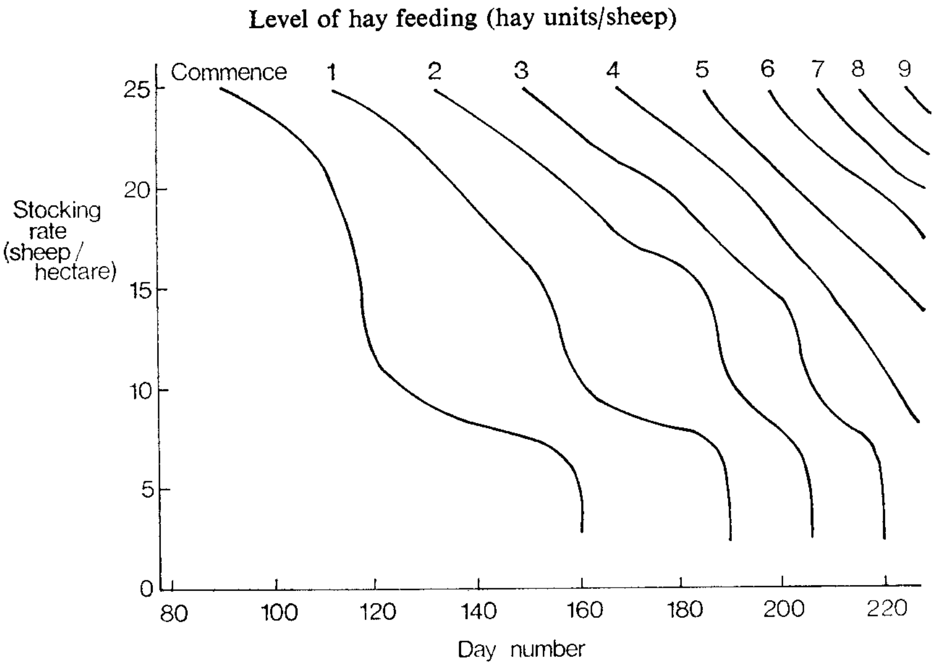

Figure 2: Isofeed lines for 1967

The price increases which occur reflect the level of feeding and thus the time it takes to feed out a unit of hay. The last price increment on five, ten and fifteen sheep per hectare is greater than its predecessor because it is fed after the break in season when some pasture was again available, so that the last unit of hay is fed over a longer period of time.[2] At twenty sheep per hectare the final price increment does not occur because the combined circumstances of pasture growth, sheep liveweight and feeding rules meant that hay prices reach their peak whilst the last unit of hay is being fed.

No comparison of the various levels of incremental rises can be made between stocking rates because the drought commences on each stocking rate at a different time. Furthermore, the liveweight of the sheep and the pasture available at the commencement of drought varies with stocking rate and therefore different stages of the hay feeding rules are invoked.

This treatment of penalty costs is clearly more realistic than charging a constant price for fodder purchased during the drought, regardless of its duration.

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[2] The hay prices continued to rise after the break in season during the 1967 drought, This may have been due to the lateness of the break, which occurred in early July. and probably caused expectations of a feed shortage after the spring. The continued high level of fodder prices was again evident after the drought in Victoria which ended in early 1973, but prices continued to remain high until April or May.

TABLE 3

Prices of Hay During Drought

	Stocking Rate (Wethers/Hectare)			
	5	10	15	20
Starting Price (cents per unit)[a]	24·38	19·06	18·19	18·13
Hay fed (units)[b]	Price increments occurring during the time necessary to feed out each unit of hay (cents/unit)			
1	5·00	5·00	5·00	4·37
2	2·50	4·37	3·75	2·50
3	2·50	3·75	2·50	2·50
4	1·88	1·88	2·50	2·50
5	3·75	1·88	1·88	2·50
6		4·07	1·88	1·88
7			1·88	1·88
8			2·44	1·25
9				1·25
10				1·25

[a] The price of hay at the commencement of hand feeding.

[b] There are 80 units of hay to the tonne.

The expected penalty cost $E(Y)$ is given by

$$E(Y) = \sum_{k_s = h+1}^{w} \left[\sum_{b=k_s}^{w} Pb \right] \left[z_s + \sum_{n=o}^{k_s - 1} i_n \right] \text{ where } k_s > h \qquad \ldots (5)$$

where i_n is the price increment occurring whilst the n^{th} unit of reserve is being fed out, and z_s is the starting price for hay feeding on stocking rate s.

Summarizing (2), (4) and (5) gives (6), in which $E(h)$ is the expected cost of a fodder reserve of h units.

$$E(h) = ch(1 + r) + \sum_{k_s = h+1}^{w} \left[\sum_{b=k_s}^{w} Pb \right] \left[z_s \sum_{n=o}^{k_s - 1} i_n \right] - V \sum_{k_s=o}^{h} (h - k_s)\, Pk_s \qquad \ldots (6)$$

The sensitivity of the minimum cost reserve for the stocking rates of five, ten, fifteen and twenty wethers per hectare is examined for the following parameter values:

A: 5·0, 12·5, 20·0 cents per hay unit

r: 7, 20, 50 per cent

d: 10 per cent per annum

4 RESULTS

The maximum reserve ever required is identified by the grazing model for each level of stocking. Then the expected cost of holding each level of reserve up to the maximum ever required is calculated for all combinations of parameter values, and the least-cost reserve is identified for each level of stocking under consideration. This is shown in table 4, together with the "reserve ratio", which is the ratio of the least-cost reserve to the largest reserve ever required. As expected, the reserve ratio increases as stocking rate increases for a given level of acquisition cost and interest rate. This reflects the fact that more intensely stocked sheep have lower sheep liveweights and they require rations closer to full maintenance levels to keep them at the minimum liveweight specified in the feeding rules. Sheep on lower stocking rates are able to build up greater body reserves in periods when pasture is readily available, and this, in association with the longer period when standing dry feed is available to supplement the hay fed, means that lower daily rations of hay are necessary.

The reserve ratio for any given stocking rate decreases with increasing acquisition costs. However, the sensitivity of the relationship varies with stocking rate; the lower the stocking rate, the more sensitive is the size of the least-cost reserve to changes in acquisition costs because the probabilities of using the last units of reserve are smaller.

The sensitivity of the least-cost reserve to changes in interest rate is obviously largely dependent on the amount of capital invested in acquiring the reserve. When the acquisition cost is 5 cents per hay unit, raising the interest rate from 7 to 20 per cent has little effect on the size of the least-cost reserve. However, when acquisition costs are 20·0 cents per hay unit, raising the rate of interest from 7 to 20 per cent significantly reduces the size of the least-cost reserve, whilst raising it to 50 per cent makes it uneconomic to hold any reserve. An interest rate of 50 per cent will be relevant for graziers facing severe capital constraints and therefore a high opportunity cost of investing funds in fodder reserves.

5 CONCLUDING REMARKS

The above approach offers a number of advantages. The model avoids the unrealistic "all-or-nothing" approach to drought feeding by prescribing varying levels of feeding during drought (reflecting sheep body weights and determined by various hay feeding rules) and by

TABLE 4

Size and Expected Cost of Minimum-Cost Hay Reserve

Acquisition Cost	Rate of Interest	Stocking Rate	Maximum Reserve ever required (h^{**})	Cost minimizing Reserve (h^{*})	$\frac{h^{*}}{h^{**}}$	Expected cost of Cost Minimizing Reserve $E(h^{*})$
cents/hay unit	per cent	wether/ hectare	hay/ wether (units)[a]	hay/ wether (units)		cents/ wether
5	7	5	5	4	·80	9·34
		10	6	5	·83	15·50
		15	9	8	·89	31·79
		20	13	12	·92	47·37
	20	5	5	3	·60	11·46
		10	6	4	·67	18·59
		15	9	8	·89	36·99
		20	13	12	·92	55·17
	50	2	5	2	·40	14·82
		4	6	4	·67	24·59
		6	9	7	·78	48·62
		8	13	11	·85	71·78
12.5 ..	7	5	5	2	·40	20·05
		10	6	4	·67	36·43
		15	9	7	·78	78·09
		20	13	11	·85	116·93
	20	5	5	2	·40	23·30
		10	6	4	·67	42·93
		15	9	7	·78	89·47
		20	13	11	·85	134·80
	50	2	5	1	·20	27·84
		4	6	3	·50	55·67
		6	9	6	·67	114·33
		8	13	10	·77	173·45
20.0 ..	7	5	5	1	·20	29·03
		10	6	4	·67	56·87
		15	9	7	·78	122·61
		20	13	11	·85	185·72
	20	5	5	0	0	30 63
		10	6	0	0	56 64
		15	9	6	·67	139·23
		20	13	9	·69	211·96
	50	2	5	0	0	30·63
		4	6	0	0	56·64
		6	9	0	0	144·50
		8	13	0	0	235·50

[a] There are 80 units of hay per tonne.

allowing for a gradual rise in feed prices over the drought. The model also permits a more satisfactory treatment of the relationship between stocking rates and drought probabilities than the use of effective rainfall or farmer "guesstimate" approaches.

Nevertheless there remain some practical and conceptual limitations. Having specified a model which allows for varying prices of drought feed, how does one specify the pattern of prices which will rule? The 1967 pattern which was assumed is one of a range of possibilities, and further model-building should allow for the stochastic nature of drought prices for supplementary feed.

Another limitation of the study is the possibility of being unable to conserve the required store of hay at the beginning of each 12-month planning period. To avoid great complexity, each year was treated as being independent of the previous year, which ignores the possibility of being unable to replenish the fodder reserve in a poor spring following a drought. This leads to some under-estimation of the least-cost reserve and its expected cost. On the other hand, some over-estimation results from the exclusion of agistment and sale of livestock from consideration in pre-drought planning. Despite these limitations, the model reported is more comprehensive and realistic than previous studies, and if in addition to the range of feeding rules, it was extended to incorporate agistment, and sheep sales, it would provide a realistic basis for predrought planning.

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