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ON THE OPTIMAL OUTPUT OF FLOOD PLAINS†

Colin Aislabie*

A simple model is presented to demonstrate the point that allocative efficiency requires a suitable mix of flood-protection measures and this is not necessarily afforded by supplanting structural flood control measures with compulsory flood insurance schemes.

Claims in the literature by Krutilla [2] and Forsythe [1], among others, that “perverse” incentives exist for flood plain location due to the jointness of many flood mitigation measures and to an inability to exclude individuals and firms from enjoying their benefit have not been followed up by any detailed investigation of the impact that particular flood mitigation measures might have on decision-making by individuals and firms located on flood plains. It is true that it is recognized, for example, that flood insurance may reduce individual initiatives to limit flood damage and that structural flood measures can reduce the risk that flood insurance must meet. It is also recognized that flood insurance is not purchased both because it is not available and because the flood plain dweller lacks full information as to the risks of flooding. Nevertheless these matters have not received as much attention in the literature as proposals for compulsory flood insurance. Consequently, it is possible that valuable policy implications go unrecognized regardless of whether or not the most commonly recommended policy solution for flooding, compulsory flood insurance, is in force or not.

As a first step towards exploring these and related questions a simple model of resource allocation by flood plain micro-units is developed in this note. The model analyses the optimal flood plain output and mix of flood control measures.

The model is constructed on the assumption that the flood control measures are private rather than public goods. While the problems of the jointness of many flood mitigation measures is recognized and the inability to exclude individuals and firms from enjoying their benefit is acknowledged, it is argued that it is desirable to determine how flood plain micro-units would behave in a general equilibrium perfectly competitive framework so that the precise pattern of behaviour consistent with the existence of “perverse” incentives can be studied.

* Senior Lecturer in Economics, University of Newcastle.

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The analysis begins with a consideration of the form of the production function. Following the literature it would appear to be desirable to make a distinction between structural and non-structural measures. Furthermore, since the principal objection to current flood prevention practices appears to be that they encourage excessive investment in the flood plain, it is desirable to take into account both the current and the capital expenditures of the firm.

To make the analysis less abstract it will be assumed that we are concerned with the resource allocation decisions of a profit maximizing firm already located on a flood plain which treats all previous capital expenditures as sunk costs, whose output will vary with inputs of flood insurance premiums, x_2 , and structural flood prevention measures, X_4 , upon its own property.¹

Its production function is assumed to be:

$$f(q(t), x_1(t), x_2(t), X_3, X_4) = 0 \quad (1)$$

where $q(t)$ is the product, $x_1(t)$ is the current input (excluding flood insurance premiums) and X_3 is the capital input (excluding structural flood prevention measures). X_3 and X_4 are inputs made in period 0, while $q(t)$, $x_1(t)$ and $x_2(t)$ relate to input and output decisions in time t . It will be further assumed that the price of the product is p , those of the two current inputs, w_1 and w_2 , and those of the two capital inputs W_3 and W_4 . The discount factor is r .

For profit maximization to the time horizon, T , the firm has to maximize the following functional:

$$\pi = \int_0^T [pq(t) - w_1x_1(t) - w_2x_2(t)]e^{-rt}dt - [W_3X_3 + W_4X_4] \quad (2)$$

To have recourse to the calculus of variations, expressions (1) and (2) will be combined as follows:²

$$\pi = \int_0^T [pq(t) - w_1x_1(t) - w_2x_2(t) - \lambda f(q(t), x_1(t), x_2(t), X_3, X_4)]e^{-rt}dt - [W_3X_3 + W_4X_4] \quad (3)$$

where (3) is to be maximized with respect to $q(t)$, $x_1(t)$, $x_2(t)$, X_3 and X_4 .

The various first order conditions for a maximum³ are:

$$\frac{\partial \Pi}{\partial q(t)} = [p - \lambda f_1]e^{-rt} = 0 \quad (4)$$

¹ X_4 can also be interpreted as any expenditure on flood prevention measures of a capital nature.

² The analysis follows Smith [5; pp. 283–286 and pp. 290–291]. However, in line with the criticism made by Miller [3] of an earlier paper by Smith [4], a joint determination is made of inputs and outputs (rather than determining the minimum current and investment outlays to achieve a static output). Furthermore, Smith's discounting expression has been simplified and the analysis extended beyond Smith's consideration of the one current and one capital input case.

³ As Smith points out, "the well-known Lagrange Multiplier shadow price tricks of static theory apply to our class of calculus of variations problems" [5: p. 284n].

$$\frac{\partial \Pi}{\partial x_1(t)} = [-w_1 - \lambda f_2]e^{-rt} = 0 \quad (5)$$

$$\frac{\partial \Pi}{\partial x_2(t)} = [-w_2 - \lambda f_3]e^{-rt} = 0 \quad (6)$$

$$\frac{\partial \Pi}{\partial X_3} = -\int_0^T \lambda(f_4)e^{-rt}dt - W_3 = 0 \quad (7)$$

$$\frac{\partial \Pi}{\partial X_4} = -\int_0^T \lambda(f_5)e^{-rt}dt - W_4 = 0 \quad (8)$$

$$\frac{\partial \Pi}{\partial \lambda} = -\int_0^T f[q(t), x_1(t), x_2(t), X_3, X_4]e^{-rt}dt = 0 \quad (9)$$

where

$$f_1 = \partial f(q(t), x_1(t), x_2(t), X_3, X_4)/\partial q(t)$$

$$f_2 = \partial f(q(t), x_1(t), x_2(t), X_3, X_4)/\partial x_1(t)$$

$$f_3 = \partial f(q(t), x_1(t), x_2(t), X_3, X_4)/\partial x_2(t)$$

$$f_4 = \partial f(q(t), x_1(t), x_2(t), X_3, X_4)/\partial X_3$$

$$f_5 = \partial f(q(t), x_1(t), x_2(t), X_3, X_4)/\partial X_4$$

To study the trade-offs between the various kinds of inputs it is necessary to eliminate λ , taking expressions (5) to (8) two at a time. For example, the trade-off between flood insurance premiums and structural flood prevention measures may be studied with the aid of the following expression:

$$\int_0^T w_2 \frac{f_5}{f_3} e^{-rt}dt - W_4 = 0 \quad (10)$$

The expression indicates that structural flood prevention measures should be substituted for flood insurance premiums until the total cost of the premiums saved from a marginal investment in structural flood prevention measures equals the cost of these measures. In a similar manner the additional expressions which could be derived indicate that trade-offs should be made between structural flood prevention measures and the other two categories of inputs incorporated in the model. Furthermore, it will be realized that the optimal input of structural flood prevention measures will vary with changes in the prices of the other inputs as well as in the price of the product.

It will also be seen that the analysis is consistent with insurance being used to cover the residual damage potential inherent in any programme of structural flood prevention measures. Furthermore, it should be noted that there is nothing in the analysis to indicate, on *a priori* grounds alone, whether an unwillingness or inability to insure is responsible for a more inefficient allocation of resources than an inability to exclude individuals and firms from enjoying the benefits of many structural flood prevention measures. Consequently, on efficiency grounds alone,

it is not possible to determine, if a mix of policies cannot be implemented simultaneously, whether an insurance scheme should be implemented before a structural flood prevent measure, or vice-versa.

The question of a possible quantification of the marginal rates of substitution between the inputs in this model are beyond the scope of this note. However, it should be stressed that attempts to determine the ranges of these marginal rates of substitution could be of considerable policy significance since the validity of the claims in favour of compulsory insurance depend on assumptions which do not appear to have been explored in detail. For example, little appears to be known about the responsiveness of output to expenditures on insurance or the extent to which compulsory insurance would inhibit other flood protection expenditures at the micro-level.⁴ The significance of the lack of knowledge regarding the responsiveness of output to expenditures on insurance cannot be too strongly stressed. A compulsory flood insurance scheme may have no discernible effect on flood plain output and any secular growth in flood plain output (and flood damage) may arise from causes other than the lack of expenditure on flood insurance premiums. It can also be noted, in passing, that the claim that the desirable level of expenditure on structural flood prevention measures can be determined by examining the reductions these measures allow in flood insurance premiums, assumes that flood plain micro-units regard structural flood, prevention measures as a perfect substitute for all flood insurance (excepting that which covers the residual risk) and that output and profits cannot be more responsive to inputs of structural flood prevention measures than flood insurance.

Although this note represents no more than a first step to redressing an imbalance which appears in the literature, it does cast doubts on the view, which appears to be gaining ground, that structural flood prevention measurement must necessarily be a self-defeating folly.

⁴ The problem of "moral hazard".

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