THE THEORY OF SPATIAL EQUILIBRIUM AND OPTIMAL LOCATION IN AGRICULTURE: A SURVEY

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In this article the development and present state of the field of spatial equilibrium theory and analysis is reviewed, particularly with respect to its application to the location of agricultural production. After surveying the historical aspects of location theory and the relationships between classical and modern theory, three major model forms are developed in some detail, viz., standard equilibrium models using supply and demand relations, activity analysis models using production functions, factor endowments and demand relations, and dynamic (recursive) models incorporating behavioural restrictions into the standard programming form. Next the study of the location of agricultural processing industry is surveyed, and finally problems of practical application, including aggregation problems and questions of data availability, are reviewed.

Problems of the following kind are considered in the theory of spatial equilibrium and optimal location:

(*a*) the optimal location of a firm with a given production programme;

(*b*) the optimal production at a given location;

(*c*) the exchange of goods and factors between regions (locations);

(*d*) the difference in prices and factor earnings between regions (locations).

It is obvious that the solutions to problems (*b*), (*c*), and (*d*) are mutually dependent and that problems (*a*) and (*b*) are different sides of the same coin. All are only different aspects of the major problem of the general spatial equilibrium of production. Nevertheless they have been treated almost independently during a long period of the history of economic thought.

The optimal location of a firm has been considered in the theory of industrial location characterized by the work of Adolf Weber [207] and Launhardt [190] at its earlier stages. The determination of optimal production at a given location is the classical problem in the theory of the firm, viz. calculation of the optimum level of operation. At the aggregate level this problem has been considered in the pure theory of trade and in agricultural location theory, both fields differing in the assumptions on factor mobility and transportation costs from Ricardo and von Thünen until almost the present day.

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The integration of the various theories with the general theory of economic equilibrium was prepared by the work of Ohlin [60], Palander [68], Christaller [16], and Lösch [56] in the 1930's. It has been continued and more or less completed by Isard [41], Lefeber [54], von Böventer [11], and Kuenne [51] who applied elements of the general theory of economic equilibrium as well as modern analytical tools like interregional input-output analysis and interregional programming to the theory of location and spatial equilibrium.

This work has had a large impact on agricultural location theory in recent years. Modern agricultural location theory can be developed directly from the general equilibrium theory as a special case. However it may also be presented as a generalization of the traditional theory of the von Thünen type. We prefer the latter for historical reasons.

1 TRADITIONAL LOCATION THEORY

Agricultural location theory originates in von Thünen's work, which has continued to influence the development of the theory over time. In his famous *Isolated State with respect to Agriculture and National Economy*, von Thünen investigates the following problem:

"Imagine a very large town, at the centre of a fertile plain which is crossed by no navigable river or canal. Throughout the plain the soil is capable of cultivation and of the same fertility. Far from the town, the plain turns into an uncultivated wilderness which cuts off all communication between this State and the outside world.

There are no other towns on the plain. The central town must therefore supply the rural areas with all manufactured products, and in return it will obtain all its provisions from the surrounding countryside. . . . The problem we want to solve is this: What pattern of cultivation will take shape in these conditions?; and how will the farming system of the different districts be affected by their distance from the Town?"[1]

In the terminology of traditional location theory, von Thünen investigates the influence of a varying distance from the market on land use, keeping all other factors constant which affect the organization of land use. In the terminology of the theory of general economic equilibrium, he investigates the spatial equilibrium of agricultural production under the following assumptions:

(*a*) There exists only one central market. Prices at the central market or the demand functions are known for all agricultural goods.

(*b*) There are no imports. The demand must be satisfied from the production of the surrounding plain.

(*c*) The production functions for all of the n goods which can be produced are identical at all geographical points of the State.

[1] From the translation of von Thünen in Hall, *ed.*, [84], pp. 7–8.

(*d*) All production factors except land are perfectly mobile and divisible. They are available at every location at constant prices which may differ between locations as a function of the distance from the market.

(*e*) The transportation costs per unit may or may not differ between the goods but for each good they are a linear function of the distance from the market.

Dunn [23] has shown that this problem can be formulated as a special problem of the general theory of equilibrium. We use the following notation:

R_{ik} = rent per unit of land at a distance k from the market resulting from the production of good i;
E_i = yield per unit of land for the good i;
P_i = market price per unit of good i;
I_i = input per unit of land for the good i;
a_i = market price per unit of input I_i;
f_i = transport rate for the yield E_i per unit of distance;
f_{Ii} = transport rate for the input I_i per unit of land;
k = distance from the market.

If we assume given prices at the central market the total supply of agricultural goods may be derived as follows:

The land rent at a distance k resulting from the production of a good i is determined by

$$R_{ik} = E_iP_i - I_ia_i - (E_if_i + I_if_{Ii})k \qquad (1)$$

The land rent is a linear function of the distance from the market (figure 1). At a hypothetical distance of zero the land rent R_{i0} at market prices is

$$R_{i0} = E_iP_i - I_ia_i \qquad (2)$$

If n goods $G_1\ G_2\ .\ .\ .\ G_n$ are to be produced numbered in decreasing order of R_{i0}, one can establish the spatial equilibrium for a good i.

The rent for total agricultural production is maximized by expanding the production of each good in both directions until the marginal rent of expansion equals marginal opportunity costs.

Hence the nearest distance to the market, k_{ic}, at which a good i is produced is given when

$$R_i = R_{i-1} \qquad (3)$$

and the greatest distance k_{id} is given when

$$R_i = R_{i+1} \qquad (4)$$

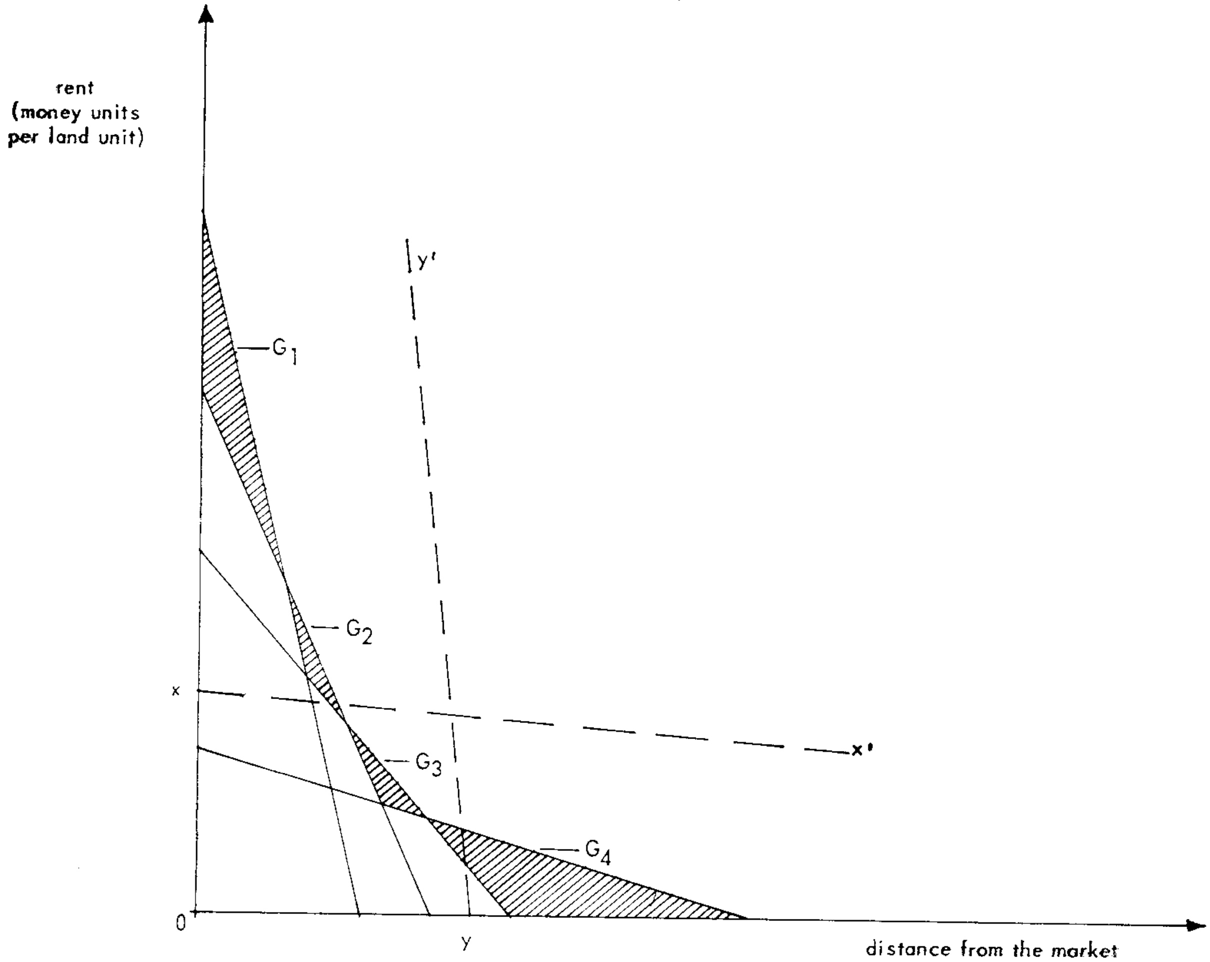
as shown in figure 1.

Figure 1 shows that two conditions are necessary for a good to be produced at all:

(*a*) The distance from the market, k, at which $R_i = R_{i-1}$ must be smaller than the value k for an intersection of R_i with all industries having a lower R_{i0} (land rent at market prices) than $i - 1$.

(*b*) The distance from the market, k, at which $R_i = R_{i+1}$ must be greater than the value k for an intersection of R_i with all industries having a higher R_{i0} (land rent at market prices) than $i + 1$.

Figure 1



If these conditions are not met, the price relations must be changed. For example in figure 1 the price of a good with the rent function $x\ x'$ must be decreased until it is lower than the price of G_4. The price of a good with the rent function $y\ y'$ must be decreased until its intersection with the rent function of G_1 is nearer to the market than the intersection of the rents from G_1 and G_2.

The supply of good i, S_i, is given under the special circumstances defined by the following assumptions:

$$S_i = E_i(k_{id}^2 - k_{ic}^2) \tag{5}$$

The supply for the n goods is then given by:

$$\begin{aligned} S_1 &= E_1(k_{1d}^2 - k_{1c}^2) \\ &\vdots \\ S_n &= E_n(k_{nd}^2 - k_{nc}^2) \end{aligned} \tag{6}$$

If demand functions are substituted for the prices of the n goods, a simple equilibrium model is obtained. Demand is given by:

$$\begin{aligned} D_1 &= F_1(Y_1, Y_2, \ldots Y_s;\ P_1, P_2, \ldots P_n) \\ &\vdots \\ D_n &= F_n(Y_1, Y_2, \ldots Y_s;\ P_1, P_2, \ldots P_n) \end{aligned} \tag{7}$$

where

$D_1\ D_2 \ldots D_n$ = aggregate demand for goods 1, 2, . . ., n;
$P_1\ P_2 \ldots P_n$ = market prices for the goods 1, 2, . . ., n;
$Y_1\ Y_2 \ldots Y_s$ = income of the s households.

We know from equation (1) that the rent which results from the production of a good n at a distance k is a function of the yield E_i, the price P_i, the inputs I_i, their price a_i, and of the transportation costs for goods and inputs f_i and f_{Ii}. Hence we can write an expression for k_{id}, the greatest distance from the market at which each good is produced, as follows:

$$\begin{aligned} k_{1d} &= f(E_1 I_1 P_1 a_1 f_1 f_{I1}, \ldots, E_n I_n P_n a_n f_n f_{In}) \\ &\vdots \\ k_{nd} &= f(E_1 I_1 P_1 a_1 f_1 f_I i, \ldots, E_n I_n P_n a_n f_n f_{In}) \end{aligned} \tag{8}$$

Finally supply must equal demand in equilibrium:

$$\begin{aligned} S_1 &= D_1 \\ &\vdots \\ S_n &= D_n \end{aligned} \tag{9}$$

Since the n values for E, I, a, f, and f_I are given parameters we have to determine the following endogenous variables:

n prices;
n quantities demanded;
n boundaries for the production;
n quantities supplied;

With equations (6) to (9) we have $4n$ equations.

Von Thünen's restrictive model permits generalizing and approaching reality in three directions:

(*a*) Generalizing the assumptions at the micro level by introducing different production functions and considering economies and diseconomies of scale.

(*b*) Including in the analysis the other location factors which affect the structure of production.

(*c*) Generalizing the restricting equilibrium model and approaching the real nature of the production process and the structure of demand by considering simultaneously the interaction of all factors which influence the spatial equilibrium of production. This is the approach of the modern theory based both on the results of research in directions (*a*) and (*b*), and on the application of modern tools like input-output analysis and linear and non-linear programming.

1.1 GENERALIZING THE ASSUMPTIONS AT THE MICRO LEVEL

Von Thünen's consideration of the nature of the production process at the micro level rests on the assumptions of the Walras-type equilibrium models. Each good can be produced only by one activity with given input-output relations, with the exception of grain, for which two activities (the *Koppelwirtschaft* and the *Dreifelderwirtschaft*) are taken into account. Each farm produces only one good for the market since all factors are perfectly mobile and since the requirements to keep the soil fertility in balance are met by the special nature of the defined activities.

Dunn [23] and Isard [41] have introduced neoclassical assumptions by assuming non-linear production functions and multiple product enterprises with continuous non-linear substitution functions. Their analysis leads to three major modifications of the von Thünen results:

(*a*) the intensity of land use decreases continuously with an increasing distance from the market and hence the land-labour-capital ratio changes continuously too;

(*b*) the rent-function becomes non-linear following the assumptions of non-linear production functions; and

(*c*) the quantity ratio of goods produced changes continuously within certain limits following the assumptions of non-linear substitution curves. Corresponding modifications occur if the assumptions of modern linear production theory are introduced, allowing for a limited number of activities with given input-output relations for each good and assuming explicit limits with respect to the maximum proportion of land a single crop is permitted to occupy for crop rotation reasons. The rent-distance function would become a curve made up of linear segments, with changing slopes between segments. Each location is likely to produce more than one crop because the rotational restrictions limit the expansion of a single crop. The proportion of land devoted to each crop changes abruptly at critical distances from the market, as in the von Thünen model.

The main results remain unchanged, however, if the restricting assumptions on market structure and on the differentiating effect of other location factors such as natural conditions are maintained. That is,

(*a*) the organization of land use (intensity and crops grown) is identical at all points at an equal distance from the central market;

(*b*) the intensity of land use decreases and hence the land rent decreases with increasing distance from the market; the shape of the rent-distance function is determined by the shape of the underlying production functions;

(*c*) no statement is possible on the relations between farm size and distance from the central market since the micromodels are point-models and all factors are assumed to be perfectly divisible.

1.2 FARM SIZE AND DISTANCE FROM THE MARKET

The relation between farm size (measured in land units) and distance from the market has been widely ignored in the literature. Its consideration requires the introduction of transportation and management costs at the farm level and of economies and diseconomies of scale resulting from an imperfect divisibility of factors and the existence of internal market economies.

Taking both factors into account one can divide production costs into three groups according to their behaviour with increasing farm size [86], [88].

(*a*) *Degressive costs*, which result from the imperfect divisibility of factors and the existence of physical economies of scale (costs of buildings, machinery, etc).

(*b*) *Proportional costs*, which result from the use of means of production which are perfectly divisible (fertilizer, variable costs of machinery, etc.) The costs for the use of land at a given rent fall into this category.

(*c*) *Progressive costs*, which result from the need for transportation on the farm and from the costs of management if farm size increases above a certain level.

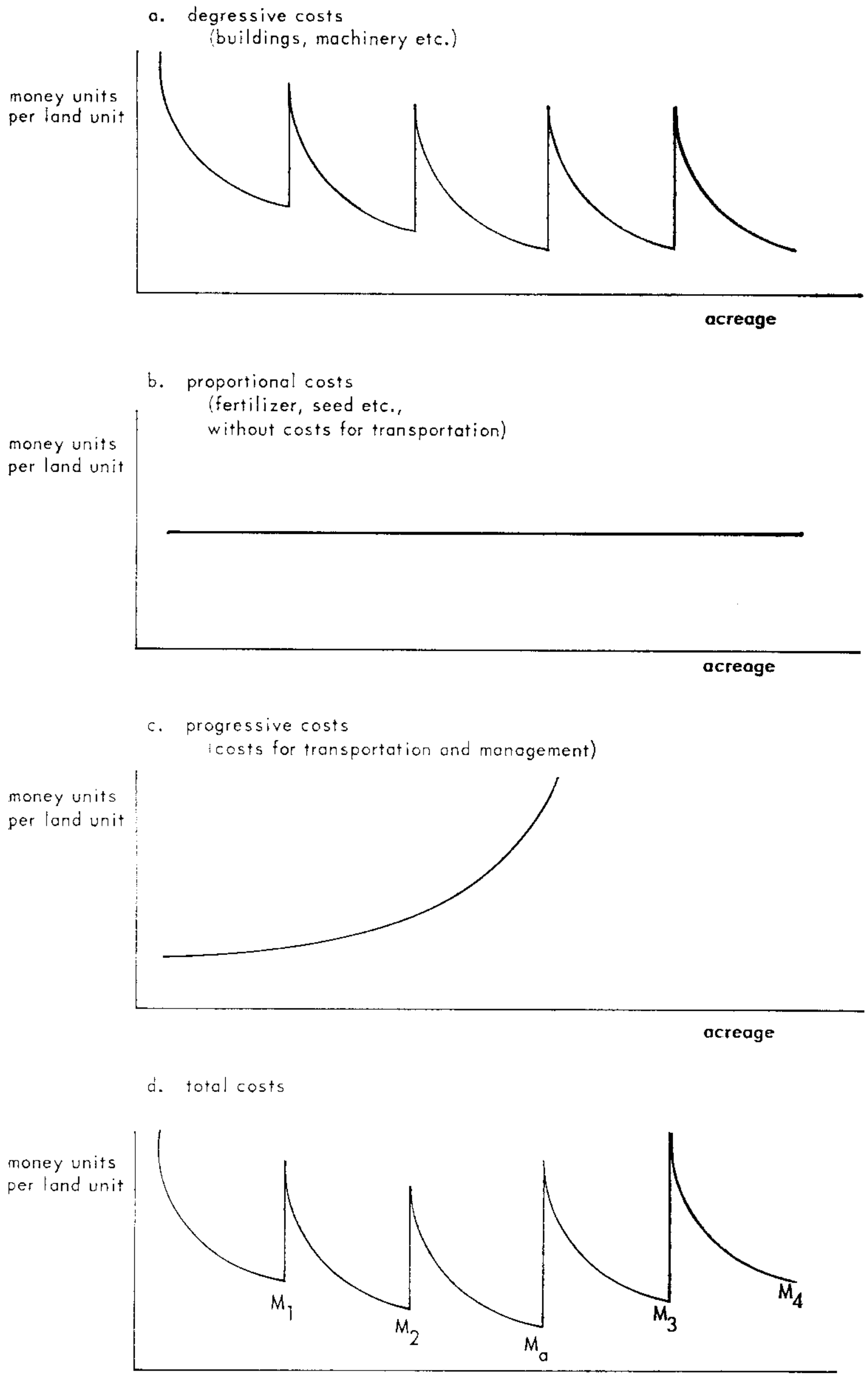
Figure 2 shows the behaviour of average costs with increasing farm size. Let us assume a given intensity of land use and a given proportion of crops per land unit. Figure 3 shows the average and the marginal cost curves with increasing farm size. The stepwise shape of the curve is smoothed out for simplicity. Point A_0 in figure 3 corresponds to point M_a in figure 2. All costs are included in the curves except for the costs of the use of land.

The shape of the cost curves corresponds to the shape of the cost curves in the neoclassical model under the simplified assumptions. The minimum of production costs (excluding the cost of land) is reached at the intersection of the marginal cost curve M_c and the average cost curve A_c. The farm size at the point of minimum average costs is S_0.

Since the intensity of land use and the proportion of crops per land unit are assumed to be constant, the marginal and the average revenue curves per land unit are identical. They are represented by the line *EE*.

Let us assume three types of farm with different farm size S_1, S_0 and S_2 located such that perfect land mobility between farms exists. The average revenue per land unit (*A*) and the marginal revenue (*B*) are shown in figure 3.

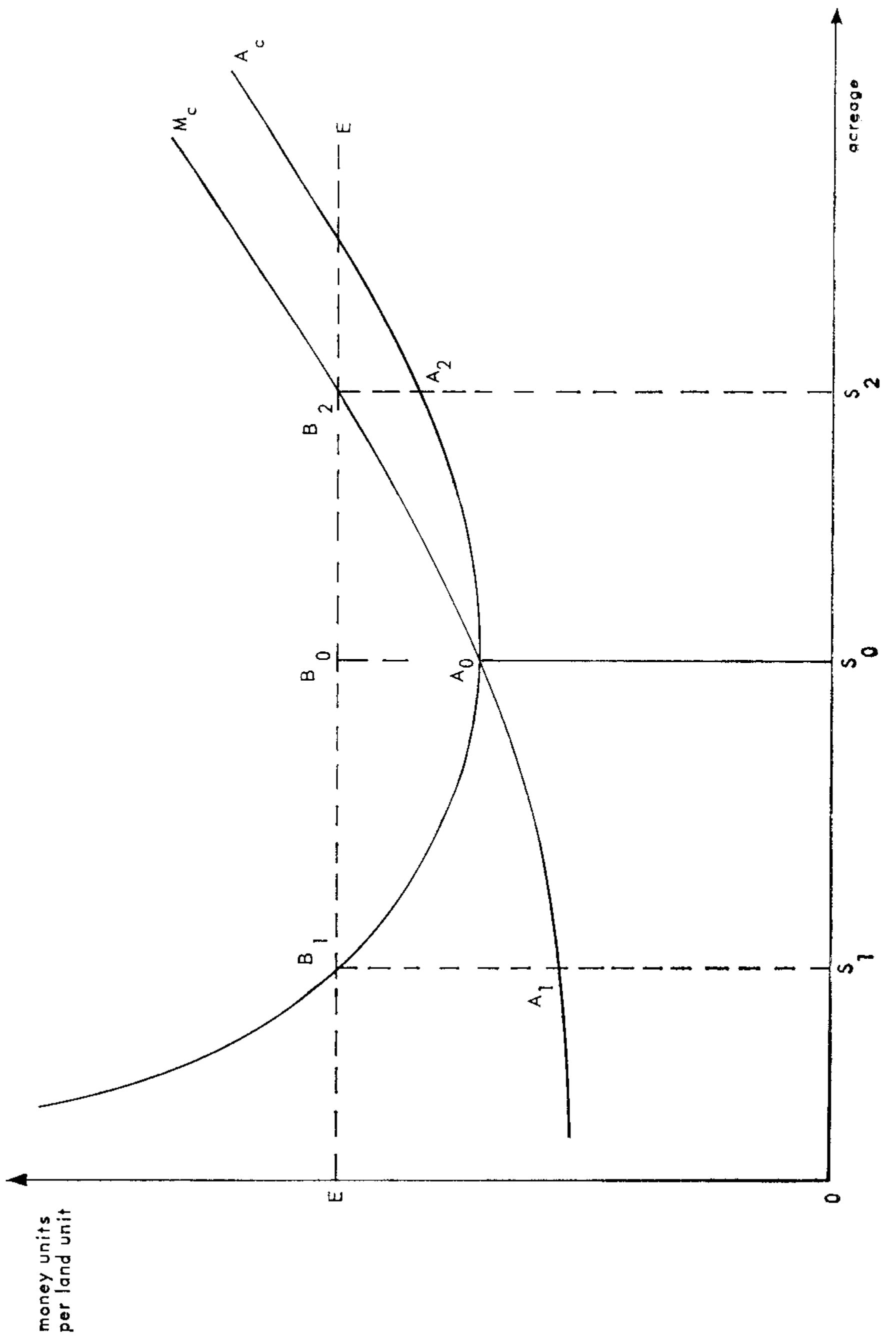
Figure 2



M_1 - M_4 (sub-) minima of average costs

M_a absolute minimum of average costs

Figure 3



For the farms of size S_1 they are

$A)$ $$0E - S_1B_1 = 0$$

$B)$ $$0E - S_1A_1 = A_1B_1$$

for the farms of size S_0 they are

$A)$ $$0E - S_0A_0 = A_0B_0$$

$B)$ $$0E - S_0A_0 = A_0B_0$$

for the farms of size S_2 they are

A) $\quad 0E - S_2A_2 = A_2B_2$

B) $\quad 0E - S_2B_2 = 0$

Thus the marginal revenues at different farm sizes are:

$A_1 B_1 > A_0 B_0 > 0$

Consequently the farmers with farms of size S_2 will sell land at any price higher than zero, while the farmers with farms of size S_1 will buy land at any price lower than A_1B_1.

The minimum selling price will increase from 0 to A_0B_0 as farms of size S_2 decrease their size from S_2 to S_0. The maximum buying price will decrease from A_1B_1 to A_0B_0 as farms of size S_1 increase from size S_1 to S_o. The equilibrium is established where price reaches A_0B_0 and farm size reaches S_0. S_0 and A_0B_0 are the equilibrium values at which no farmer has a reason to change his position as long as there are no changes in the prices for goods and factors (other than land), in production functions, and hence in the organization of land use.

Using the previous results the following conclusions may be drawn:

The equilibrium farm size is determined if the prices at the farm gate and the organization of land use (intensity and proportion of crops) are determined by demand functions, production functions, transport rates and distance from the market assuming perfect factor mobility and perfect mobility of land between farms at a given location.

All farms at an equal distance from the market are of equal size in a von Thünen system. Farm size is a dependent variable under the assumption of perfect factor mobility, determined by the cost curves of the equilibrium organization of land use. No direct relation between farm size and distance from the market exists as Dunn [23] has supposed. However there are indirect relations. One can assume that the optimal size (the size at which the farm produces at minimum average cost) is smaller the more intensive the land use, and hence one can assume (as did Isard [41] and von Böventer [13]) that farms are smaller the closer the distance to the market.

The neglect of farm size is a result of the assumption of perfect factor mobility. If this assumption is dropped, as is frequently done in modern location theory, one has to consider farm size as a location factor (i.e., as an exogeneous variable, in the terminology of modern theory).

1.3 GENERALIZING THE CONSIDERATION OF LOCATION FACTORS

Aereboe [1] and Brinkmann [15] who continued von Thünen's work at the beginning of this century refined von Thünen's description of the production process at the micro level and extended his analysis by inquiring into the effect of other location factors.

The refinement of the description of the production process is rather technical. In the writings of both scientists, especially of Brinkmann, the technical relations between enterprises within the farm with respect to the maintenance of soil fertility, the use of labour, and the balance of feed supply, all of which favour the diversification of the farm, are described as countervailing powers to the location factors, which favour specialization. The natural conditions, the stage of economic development and the personality of the farm operator are introduced as location factors in addition to the distance from the market. In describing their influence they use von Thünen's method of considering a single factor keeping all the others more or less constant and assuming perfect factor mobility.

Their work is an excellent example of descriptive economics but it is not operational in the sense of modern theory.

2 CLASSICAL AND MODERN THEORY

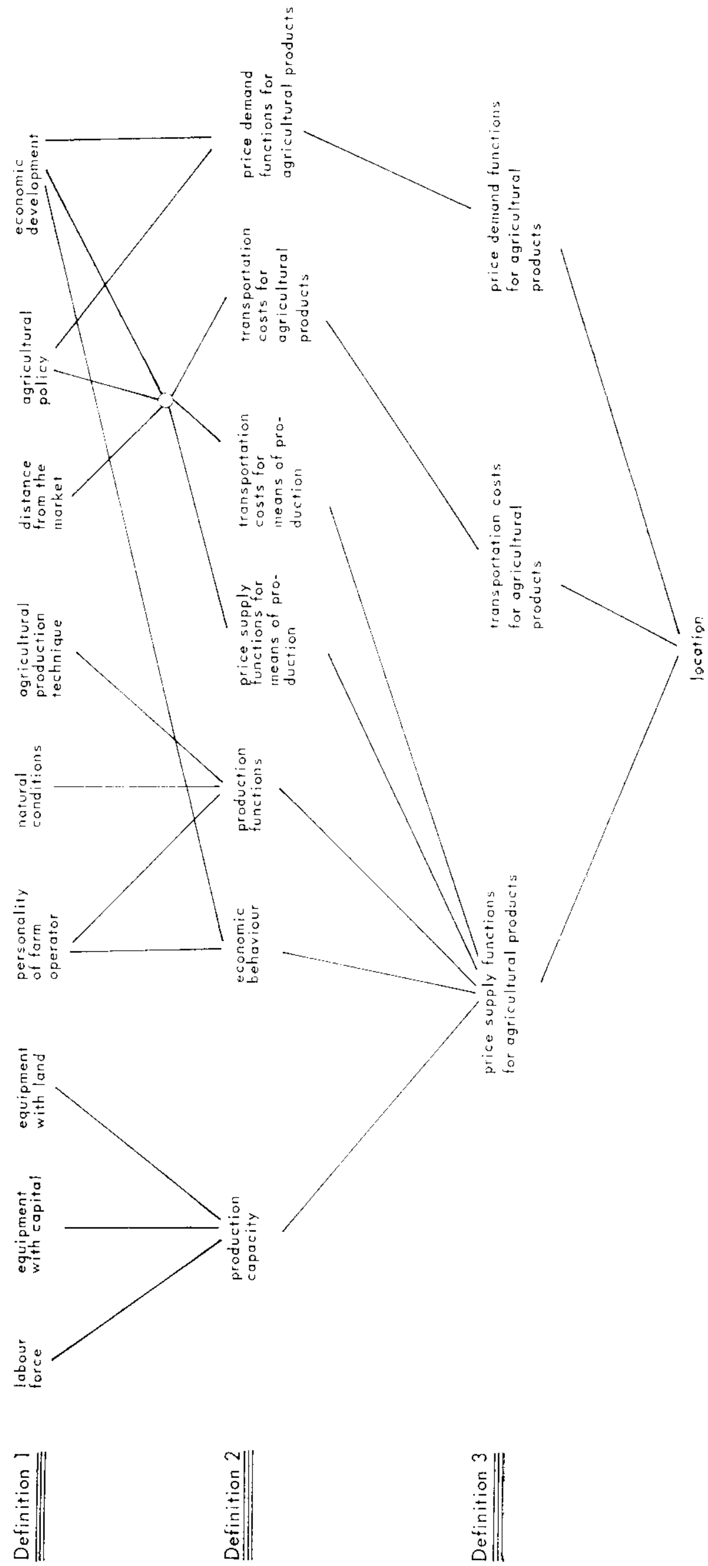
Classical and modern theory are basically distinguished by the goals of their analysis. The work of von Thünen, Brinkmann, Dunn, and others, aims at the description and explanation of the influence of single location factors on agricultural production. Their models are explanatory models of general validity under the given assumptions. In contrast to the classical theory, modern analysis aims at the determination of the spatial equilibrium of agricultural production in a special area (the national economy). Its explanatory models, which might be considered as generalized von Thünen models as well as partial equilibrium models of the Walras-Cassel-type, are not ends in themselves but a basis for the development of verification, decision, and prediction models which are operationally meaningful. The operational bias of modern analysis becomes obvious if the definition of location and the consideration of space are compared in classical and modern theory [91].

2.1 THE DEFINITION OF LOCATION IN CLASSICAL AND MODERN THEORY

The location factors of classical theory are the exogenous variables of modern analysis. If the assumption of perfect factor mobility is dropped, some of the dependent variables of the von Thünen model become exogenous too. Hence additional location factors must be added, such as farm size, labour force, equipment with buildings, etc., and location from the viewpoint of classical location theory must be defined as shown in figure 4, definition 1. This or a similar definition which is certainly sufficient to distinguish one location from another has been used in innumerable publications all over the world, in order to investigate the influence of a change in one location factor, keeping the others more or less constant. However it is hardly operational if one considers the interaction of *all* location factors aiming at the explanation and quantification of the spatial equilibrium of agricultural production in a given economy (area).

Figure 4

Definition of location and interaction of location factors



Therefore modern location theory defines a location either by its production capacity, the production functions, the supply functions for variable factors of production, etc. (definition 2 in figure 4) or, more comprehensively, by the supply and demand functions for agricultural goods and transportation costs (definition 3 in figure 4).

2.2 THE NUMBER OF LOCATIONS

All definitions given in figure 4 include variables which change continuously over space, such as the distance from the market, as well as variables which usually change discontinuously over space such as farm size or soil fertility. Therefore two assumptions are possible on the number of locations [11]:

(*a*) Space may be considered as a continuous phenomenon, allowing infinitely small changes of location, and thus leading to an infinite number of locations.

(*b*) The number of locations may be limited by considering only a small number of points, each of them representing a certain part of the whole area.

Von Thünen, who emphasizes the distance from the market as a continuously changing variable, uses the continuous approach logically. Aereboe, Brinkmann, Isard and Dunn, who maintain the von Thünen framework of analysis, basically use the continuous approach as well. Beckmann [5] has shown that an equilibrium solution exists for a generalized transportation problem using the continuous approach. Beckmann's model consists of a system of differential equations by which total transportation costs of a commodity flow can be minimized subject to a given programme of production. The solution yields the optimum flow and the efficient price structure in a competitive spatial market given the density functions for production and demand and the transportation costs independent of the direction of the flow.

The continuous approach becomes rather complicated if one includes in the analysis the relations between prices and supply, prices and demand, and the dependence of transportation costs on the direction of the commodity flow. At present, to the authors' knowledge, no solution exists for this problem using the continuous approach. Therefore modern location theory uses the discrete approach.

The area considered is divided into a finite number of regions. Each region is represented by a single point in space. These points are characterized either by definition 3 or by definition 2 of figure 4. In the first case the spatial equilibrium of production is determined on the basis of given regional supply and demand functions, in the latter it is determined on the basis of linear production models.

3 SPATIAL EQUILIBRIUM ON THE BASIS OF GIVEN SUPPLY AND DEMAND FUNCTIONS

The structure of the model used for the consideration of spatial equilibrium on the basis of given supply and demand functions may be described, after Enke [26], as follows:

The area considered (national economy) is divided into a finite number of regions, each of which is represented by a single point in space (location unit). Between the regions one or more agricultural goods are exchanged. For each product regional supply and demand functions are given as well as the transportation costs per unit between each pair of regions, which are assumed to be independent of the volume transported.

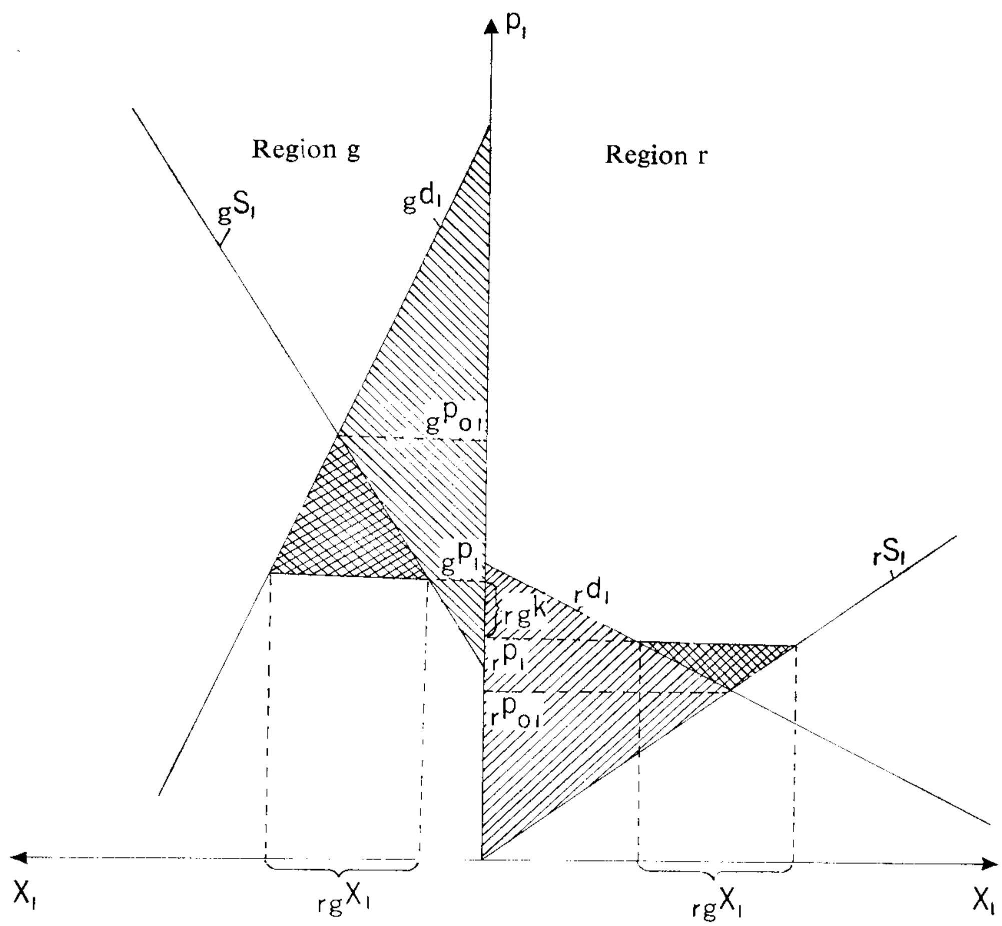
It is required to determine:

(*a*) regional equilibrium prices;
(*b*) regional supply and consumption;
(*c*) the flows of interregional exchange.

3.1 CONDITIONS FOR EXCHANGE EQUILIBRIUM

The conditions for the spatial equilibrium solution have been stated by Cournot: "It is clear that a good which is mobile will move from the market where its value is lower to the market where its value is higher, until differences of values are not larger than transportation costs" [17, p. 103]. In other words, the price differences between any pair of regions must not exceed transportation costs. They have just to equal transportation costs, if goods are actually exchanged in equilibrium.

Figure 5



From this it follows, for the regional price structure, that the price of a good in a region *r*, which delivers the good to another region *g*, is exactly lower by transportation costs than the price of the good in region *g*. In regions which are self sufficient the regional price is determined by demand and supply of that region alone.

These relationships might be demonstrated by the well-known textbook example for 2 regions and 1 good (figure 5). Without interregional exchange the price in region *r* is ${}_r p_{01}$ and in region *g* it is ${}_g p_{01}$. If the transportation costs are larger than ${}_g p_{01} - {}_r p_{01}$ interregional exchange will not pay. At transportation costs of ${}_{rg}k$ trade will be profitable. By exchange the price will rise in region *r* and fall in region *g*. As long as the price difference is larger than transportation costs, tradesmen will have a trading profit. This will vanish when price differences become equal to transportation costs. Then spatial equilibrium at prices ${}_r p_1$ and ${}_g p_1$ and interregional exchange of ${}_{rg}x$ is reached.

This spatial equilibrium problem is one of "descriptive economics" (Samuelson [70] p. 285). In the simple case of two regions the equilibrium solution can be determined without difficulty by graphical methods, and for a few regions the solution might be found by trial and error methods. A more elegant and direct solution has been proposed by Enke [26], who uses the analogy of an electric circuit. However it was recently shown by Samuelson [70] that the descriptive equilibrium problem can be cast mathematically into an optimizing problem. By this means the highly efficient mathematical tools of linear and non-linear programming become applicable to the solution of larger empirical problems.

3.2 TRANSFORMATION OF THE DESCRIPTIVE PROBLEM INTO AN OPTIMIZING PROBLEM

The basic idea of this transformation is to describe the equilibrium solution in some way by areas under the demand and supply functions, which have to be of maximum or minimum size in equilibrium. Samuelson started out by observing that the areas under the excess-demand-functions *minus* transportation costs ("net social pay-off") have to be maximized. Smith [75] has shown recently that in a similar way the equilibrium solution can be determined by minimizing the sum of producer and consumer rents (in the sense of Marshall) under certain restraints. We will follow the latter approach.

In figure 5 the single- and double-hatched areas describe the sum of consumer and producer rents in the equilibrium solution. It is easily seen that the sum of rents will be larger at all other prices and amounts of exchange subject to the condition that price differences must not exceed transportation costs. From figure 5 it can be further seen that the sum of rents of the regions in the case of isolation (the single-hatched areas) is constant at all possible constellations of prices (whereby the proportions of consumer and producer rents may change). Therefore it is only necessary to minimize the additional rents (the double-hatched areas).

For the case of t regions and one product[2] and linear supply and demand functions the spatial equilibrium problem can then be formulated analytically as follows:

Given are:

Regional demand functions

$$ {}_rd_1 = {}_r\alpha_1 - {}_r\beta_1\, {}_rp_{d1} \qquad (r = 1, 2, \ldots, g, \ldots, t) \tag{10}$$

with ${}_rd_1$ as volume of demand and ${}_rp_{d1}$ as price for good 1 in region r and ${}_r\alpha_1$ and ${}_r\beta_1$ as parameters.

Regional supply functions

$$ {}_rs_1 = {}_r\theta_1 + {}_r\gamma_1\, {}_rp_{s1} \qquad (r = 1, 2, \ldots, g, \ldots, t) \tag{11}$$

with ${}_rs_1$ as volume of supply and ${}_rp_{s1}$ as price for good 1 in region r and ${}_r\theta_1$ and ${}_r\gamma_1$ as parameters.

Then the spatial equilibrium model can be written:
Minimize the sum of rents R

$$ R = \sum_{g=1}^{t} \int_{{}_gp_{01}}^{{}_gp_{s1}} ({}_g\theta_1 + {}_g\gamma_1\, {}_gp_{s1})\, dp - \sum_{r=1}^{t} \int_{{}_rp_{01}}^{{}_rp_{d1}} ({}_r\alpha_1 - {}_r\beta_1\, {}_rp_{d1})\, dp \tag{12}$$

under the restraints

$$ \begin{aligned} {}_gp_{s1} - {}_rp_{d1} + {}_{gr}k_1 &\geq 0 \\ {}_gp_{s1},\quad {}_rp_{d1} &\geq 0 \end{aligned} \qquad \begin{pmatrix} r = 1, 2, \ldots, t \\ g = 1, 2, \ldots, t \end{pmatrix} \tag{13}$$

where ${}_{gr}k_1$ denotes transportation costs for good 1 from region g to region r.

After integration one can write (12) and (13) as a programming problem with a quadratic criterion function and linear restraints:

Minimize

$$ R = \sum_{g=1}^{t} ({}_g\theta_1\, {}_gp_{s1} + \tfrac{1}{2}\, {}_g\gamma_1\, {}_gp^2{}_{s1}) - \sum_{r=1}^{t} ({}_r\alpha_1\, {}_rp_{d1} - \tfrac{1}{2}\, {}_r\beta_1\, {}_rp^2{}_{d1}) \tag{14}$$

under the restraints

$$ \begin{aligned} -\, {}_gp_{s1} + {}_rp_{d1} - {}_{gr}k_1 &\leq 0 \\ {}_gp_{s1},\quad {}_rp_{d1} &\geq 0 \end{aligned} \qquad \begin{pmatrix} g = 1, 2, \ldots, t \\ r = 1, 2, \ldots, t \end{pmatrix} \tag{15}$$

Takayama and Judge [80] were the first ones to write the spatial equilibrium problem in the form of a quadratic programming model.

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[2] The assumption of identical production and consumption regions is not necessary. It is introduced only for convenience of analytical description.

3.3 PROPERTIES AND SOLUTION OF THE MODEL FOR ONE PRODUCT AND MANY REGIONS

For the solution of the quadratic programming problem described in (14) and (15) quite a number of efficient procedures are available[3]. We will sketch a method, by which the quadratic programming problem is transformed into a linear programme, that can be solved by well-known algorithms. It is based on the Kuhn-Tucker-conditions[4], which reveal at the same time some properties of the model. It can be shown analytically that the programme formulated in (14) and (15) corresponds to our spatial equilibrium problem.

From (14) and (15) we derive the following generalized Lagrange-function:

$$F(p,x) = \sum_{g=1}^{t} ({}_g\theta_1\, {}_gp_{s1} + \tfrac{1}{2}\, {}_g\gamma_1\, {}_gp^2{}_{s1}) - \sum_{r=1}^{t} ({}_r\alpha_1\, {}_rp_{d1} - \tfrac{1}{2}{}_r\beta_1\, {}_rp^2{}_{d1}) + \sum_{g=1}^{t} \sum_{r=1}^{t} {}_{gr}x_1\,(-{}_gp_{s1} + {}_rp_{d1} - {}_{gr}k_1) \quad (16)$$

Here ${}_{gr}x_1$ $(g = r = 1, 2, \ldots, t)$ are the Lagrange multipliers, which can be interpreted in our problem as amounts of interregional exchange between regions. The variable ${}_{gr}x_1$ denotes one unit of the good 1, which is transported from g to r.

According to Kuhn and Tucker the following conditions for an optimal solution result:

$$\frac{\delta F(p,x)}{\delta_g p_{s1}} = {}_g\theta_1 + {}_g\gamma_1\, {}_gp_{s1} - \sum_{r=1}^{t} {}_{gr}x_1 \geq 0 \qquad (g = 1, 2, \ldots, t) \qquad (17a)$$

$$\frac{\delta F(p,x)}{\delta_g p_{s1}} \cdot {}_gp_{s1} = 0 \qquad (17b)$$

$$\frac{\delta F(p,x)}{\delta_r p_{d1}} = -({}_r\alpha_1 - {}_r\beta_1\, {}_rp_{d1}) + \sum_{g=1}^{t} {}_{gr}x_1 \geq 0 \quad (r = 1, 2, \ldots, t) \qquad (17c)$$

$$\frac{\delta F(p,x)}{\delta_r p_{d1}} \cdot {}_rp_{d1} = 0 \qquad (17d)$$

The conditions (17a) and (17c) say that demand must not exceed supply, and conditions (17b) and (17d) state that at positive prices (17a) and (17c) have to be equations, i.e., supply has to equal demand.

The other set of Kuhn-Tucker-conditions is:

$$\frac{\delta F(p,x)}{\delta_{gr} x_1} = -{}_gp_{s1} + {}_rp_{d1} - {}_{gr}k_1 \leq 0 \qquad (g = r = 1, 2, \ldots, t) \qquad (18a)$$

$$\frac{\delta F(p,x)}{\delta_{gr} x_1} \cdot {}_{gr}x_1 = 0 \qquad (18b)$$

$${}_gp_{s1}, \quad {}_rp_{d1}, \quad {}_{gr}x_1 \geq 0 \qquad (19)$$

---

[3] For a review see Künzi and Krelle [53].
[4] The Kuhn-Tucker conditions are generalizations of the classical Lagrange-method, which allows inequations in the set of restraints [52].

Condition (18a) requires that price differences between any pair of regions must not exceed transportation costs. From (18a) and (18b) together the result is obtained that if there is interregional exchange then price differences equal transportation costs, and, conversely, if transportation costs exceed price differences, then no exchange takes place.

In summary, the Kuhn-Tucker-conditions for the quadratic programme (14) and (15) are the same as the conditions for exchange equilibrium described above. The quadratic programme and the equilibrium problem are identical.

An alternative formulation of the Kuhn-Tucker-conditions, which has been proposed by Barankin and Dorfman [3], leads to the linear programming version of the problem.

We define

$$\frac{\delta F(p, x)}{\delta\ {}_{gr}x_1} = {}_{gr}v_1 \tag{20}$$

Further we assume for economic reasons that prices > 0.

Then the conditions (17a), (17c), (18a), and (19) can be written as follows (after some rearrangement):

$$\begin{aligned}
&{}_g\theta_1 + {}_g\gamma_1\ {}_gp_{s1} - \sum_{r=1}^{t} {}_{gr}x_1 = 0 \qquad (g = 1, 2, \ldots, t)\\
&-{}_r\alpha_1 + {}_r\beta_1\ {}_rp_{d1} + \sum_{g=1}^{t} {}_{gr}x_1 = 0 \qquad (r = 1, 2, \ldots, t)\\
&-{}_gp_{s1} + {}_rp_{d1} - {}_{gr}k_1 + {}_{gr}v_1 = 0 \qquad \begin{pmatrix} g = 1, 2, \ldots, t \\ r = 1, 2, \ldots, t \end{pmatrix}\\
&{}_gp_{s1},\quad {}_rp_{d1},\quad {}_{gr}x_1,\quad {}_{gr}v_1 \geq 0
\end{aligned} \tag{21}$$

It is easily seen that the further condition (18b) is fulfilled, if

$$\sum_{g}^{t} \sum_{r}^{t} {}_{gr}x_1\ {}_{gr}v_1 = 0 \tag{22}$$

Equations (21) and (22) are the starting point for some of the solution methods for quadratic programming (Frank and Wolfe [28], Barankin and Dorfman [3], Wolfe [92]). The variables p, x, and v have to be determined in such a way that the restraints (21) are fulfilled and that (22) becomes zero. This will be reached by minimizing (22) under the restraints (21):

$$\begin{aligned}
&{}_g\gamma_1\ {}_gp_{s1} - \sum_{r=1}^{t} {}_{gr}x_1 = -{}_g\theta_1 \quad (g = 1, 2, \ldots, t)\\
&{}_r\beta_1\ {}_rp_{d1} + \sum_{g=1}^{t} {}_{gr}x_1 = {}_r\alpha_1 \quad (r = 1, 2, \ldots, t)\\
&-{}_gp_{s1} + {}_rp_{d1} + {}_{gr}v_1 = {}_{gr}k_1 \quad (g = r = 1, 2, \ldots, t)\\
&{}_gp_{s1},\quad {}_rp_{d1},\quad {}_{gr}x_1,\quad {}_{gr}v_1 \geq 0
\end{aligned} \tag{23}$$

This programme can be solved by the simplex method (taking into account some additional rules) and, if a solution for the problem exists[5] leads in a finite number of iterations to $Z = 0$.

3.4 SPATIAL EQUILIBRIUM FOR MANY REGIONS AND MANY PRODUCTS

In the more general case of more than one good the interdependencies between goods on the demand and supply side resulting from cross-price elasticities have to be taken into account. But the corresponding spatial equilibrium model can be derived in a similar way. Then the sum of rents for all goods and regions has to be minimized under the restraint that price differences between any pair of regions do not exceed transportation costs for any good.

Given are[6]:

Price-demand functions for n products in t regions[7]

$$\mathrm{d} = \alpha - \mathrm{B}\mathrm{p}_d$$

Price-supply functions for n products in t regions

$$\mathrm{s} = \theta + \mathrm{G}\mathrm{p}_s$$

Transportation costs for each product and pair of regions are denoted by k.

The matrices and vectors are defined as follows:

Vector of quantities demanded:

$$\mathrm{d}' = ({}_1d_1, \ldots, {}_1d_n;\ {}_2d_1, \ldots, {}_2d_n;\ \ldots;\ {}_td_1, \ldots, {}_td_n)$$

Vector of quantities supplied:

$$\mathrm{s}' = ({}_1s_1, \ldots, {}_1s_n;\ {}_2s_1, \ldots, {}_2s_n;\ \ldots;\ {}_ts_1, \ldots, {}_ts_n)$$

Vectors of prices:

$$\mathrm{p}'_d = ({}_1p_{d1}, \ldots, {}_1p_{dn};\ {}_2p_{d1}, \ldots, {}_2p_{dn};\ \ldots;\ {}_tp_{d1}, \ldots, {}_tp_{dn})$$
$$\mathrm{p}'_s = ({}_1p_{s1}, \ldots, {}_1p_{sn};\ {}_2p_{s1}, \ldots, {}_2p_{sn};\ \ldots;\ {}_tp_{s1}, \ldots, {}_tp_{sn})$$

Vectors of parameters:

$$\alpha' = ({}_1\alpha_1, \ldots, {}_1\alpha_n;\ {}_2\alpha_1, \ldots, {}_2\alpha_n;\ \ldots;\ {}_t\alpha_1, \ldots, {}_t\alpha_n)$$
$$\theta' = ({}_1\theta_1, \ldots, {}_1\theta_n;\ {}_2\theta_1, \ldots, {}_2\theta_n;\ \ldots;\ {}_t\theta_1, \ldots, {}_t\theta_n)$$

---

[5] With supply and demand functions of "normal" slope a solution does always exist. For a discussion on the existence of a definite solution in the more general case of many products, see section 3.4.

[6] From this point onwards matrix formulation seems to be more appropriate, because proofs of existence of solutions depend on matrix analysis.

[7] Matrices and vectors are denoted by Roman letters. Capital letters denote matrices and small ones vectors. Transposed matrices and vectors are indicated by a prime.

Matrices of parameters:

$$B = \begin{bmatrix} {}_1B & & & & & & \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & {}_rB & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & {}_tB \end{bmatrix} \quad \text{where } {}_rB = \begin{bmatrix} {}_r\beta_{11} & \cdots & \cdots & {}_r\beta_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & \cdots & {}_r\beta_{ij} & \cdots \\ \vdots & & \vdots & \vdots \\ {}_r\beta_{n1} & \cdots & \cdots & {}_r\beta_{nn} \end{bmatrix}$$

$$G = \begin{bmatrix} G_1 & & & & & & \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & {}_rG & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & {}_tG \end{bmatrix} \quad \text{where } {}_rG = \begin{bmatrix} {}_r\gamma_{11} & \cdots & \cdots & {}_r\gamma_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & \cdots & {}_r\gamma_{ij} & \cdots \\ \vdots & & \vdots & \vdots \\ {}_r\gamma_{n1} & \cdots & \cdots & {}_r\gamma_{nn} \end{bmatrix}$$

Vector of transportation costs:

$k' = ({}_{11}k_1, .., {}_{11}k_n; {}_{12}k_1, .., {}_{12}k_n; ..; {}_{1t}k_1, .., {}_{1t}k_n; ..; {}_{tt}k_1, .., {}_{tt}k_n)$

Further we define:

Vector of amounts of interregional exchange:

$x' = ({}_{11}x_1, .., {}_{11}x_n; {}_{12}x_1, .., {}_{12}x_n; ..; {}_{1t}x_1, .., {}_{1t}xn; ..; {}_{tt}x_1, .., {}_{tt}x_n)$

Vector of slack variables:

$v' = ({}_{11}v_1, .., {}_{11}v_n; {}_{12}v_1, .., {}_{12}v_n; ..; {}_{1t}v_1, .., {}_{1t}v_n; ..; {}_{tt}v_1, {}_{tt}v_n)$

and

$$E = [E_s\, E_d] = \left[\begin{array}{cccc:cccc} -I & & & & I & & & \\ -I & & & & & I & & \\ \vdots & & & & & & \ddots & \\ -I & & & & & & & I \\ & -I & & & I & & & \\ & -I & & & & I & & \\ & \vdots & & & & & \ddots & \\ & -I & & & & & & I \\ & & \ddots & & \vdots & & & \\ & & & -I & I & & & \\ & & & -I & & I & & \\ & & & \vdots & & & \ddots & \\ & & & -I & & & & I \end{array}\right] \quad \begin{array}{l} \}\ t \text{ times} \\ \}\ t^2 \text{ times} \end{array}$$

In matrix E we define I as a unit matrix of dimension nn. In other contexts I is a unit matrix with an appropriate number of elements.

The spatial equilibrium problem can now be formulated (corresponding to (14) and (15)) as a quadratic programming problem:

Minimize

$$R = [\theta' \mid -\alpha'] \begin{bmatrix} p_s \\ p_d \end{bmatrix} + [p'_s \mid p'_d] \frac{1}{2} \left[\begin{array}{c|c} G & O \\ \hline O & B \end{array}\right] \begin{bmatrix} p_s \\ p_d \end{bmatrix} \tag{24}$$

subject to the restraints

$$[E_s \mid E_d] \begin{bmatrix} p_s \\ p_d \end{bmatrix} \leq k$$

The Kuhn-Tucker conditions can be written (corresponding to (21)) as follows:

$$\begin{gathered} [E_s \mid E_d] \begin{bmatrix} p_s \\ p_d \end{bmatrix} - I\,v = k \\ \left[\begin{array}{c|c} G & O \\ \hline O & B \end{array}\right] \begin{bmatrix} p_s \\ p_d \end{bmatrix} + \begin{bmatrix} E'_s \\ E'_d \end{bmatrix} x = \begin{bmatrix} -\theta \\ \alpha \end{bmatrix} \\ x\,v = 0 \\ x,\ v,\ p_d,\ p_s \geq 0 \end{gathered} \tag{25}$$

Then the following linear programme results (corresponding to 23)):

Minimize

$$Z = [o' \ o' \ x' \ v'] \, [p_s \ p_d \ v \ x]'$$

subject to

$$\begin{bmatrix} E_s & E_d & O & -I \\ G & O & E'_s & O \\ O & B & E'_d & O \end{bmatrix} \begin{bmatrix} p_s \\ p_d \\ x \\ v \end{bmatrix} = \begin{bmatrix} K \\ -\theta \\ \alpha \end{bmatrix} \tag{26}$$

$$p_s,\ p_d,\ x,\ v \geq 0$$

The solution to this programme gives the regional equilibrium prices and interregional amounts of exchange for the different goods.

The solution method described (as well as other methods for solving quadratic programming problems) are only applicable if the criterion function and the restraints are convex. If this is the case, a spatial equilibrium solution will exist. If convexity is not ensured, an equilibrium solution may exist, but not necessarily. But in either case, it is not certain that quadratic programming procedures will lead to a global optimum (and hence to the spatial equilibrium solution).

It is easily seen that the constraints (24) of our spatial equilibrium problem will always be convex. Therefore it is only necessary to check whether the criterion function will also be convex. This depends on the properties of the matrices G and B. If the quadratic form

$$[p'_s \mid p'_d] \frac{1}{2} \left[\begin{array}{c|c} G & O \\ \hline O & B \end{array}\right] \begin{bmatrix} p_s \\ p_d \end{bmatrix}$$

is positive-semidefinite, the criterion function will be convex too (see Frank and Wolfe [28], p. 96). In the one-good model of section 3.3, with "normal" sloped supply and demand functions, this is always the case, because elements ${}_r\beta_{ij}$ and ${}_r\gamma_{ij}$ appear in matrices G and B on the diagonals only and will be equal to or greater than zero. In the case of many goods, matrix coefficients depend on the relation between the goods with respect to complementarity and substitution, and they may be greater than or less than zero. In this case the quadratic form can be positive-semidefinite or indefinite. This may be tested by construction of the eigenvalue of the matrix (Allen, [2], p. 477).

3.5 SPECIAL CASES AND ITERATIVE PROCEDURES

In the special case of completely price-inelastic demand and supply functions (fixed amounts of demand and supply for the regions) interdependencies do not exist. The spatial equilibrium problem reduces to the well-known transportation problem (Hitchcock [37], Koopmans and Reiter [49], Orden [66], von Böventer [9]). It can be solved by minimizing the sum of transportation costs under the restraints that regional supply is not exceeded and regional demand is fulfilled, i.e.:

Minimize

$$z = \sum_r \sum_g {}_{gr}x_i \, {}_{rg}k_i$$

subject to

$$\sum_r {}_{gr}x_i \leqslant {}_g x_i$$

$$\sum_g {}_{gr}x_i \geq {}_r d_i$$

where ${}_{gr}x_i$ = transport activity for good i from region g to region r;
${}_{gr}k_i$ = transportation costs for good i from region g to region r (with ${}_{rr}k_i = 0$);
${}_g x_i$ = supply of good i in region g;
${}_r d_i$ = demand for good i in region r.

It is easily seen[8] from the conditions for exchange equilibrium derived above that the resulting solution of the minimization problem corresponds to the solution of the spatial equilibrium problem under perfect competition. The primal solution shows the equilibrium amounts of interregional exchange and the dual solution shows the corresponding regional price differences.

Transportation models can also be used in some iterative procedures to solve the more general spatial equilibrium problem with price-elastic demand and/or supply functions. They are based on the solution of a sequence of transportation models determined by certain rules that do ensure a step-by-step approach to the spatial equilibrium solution (Fox [27], Judge and Wallace [137], Henrichsmeyer [36]). The main difficulty in establishing such rules is to deal with the problem of self-sufficient regions. The advantage of iterative procedures is that their demand for computer-capacity is small in comparison to quadratic programming.

[8] Or shown formally; see von Böventer [9].

3.6 EMPIRICAL APPLICATION

Regional demand and supply functions have to be known for the empirical application of this type of spatial equilibrium model, except in special cases where fixed amounts of supply and demand are assumed. We shall not discuss here the great difficulties of estimating these functions statistically. But in summary it can be stated that, despite a large amount of work at different methodological levels, our knowledge of agricultural supply response is very limited. Therefore empirical application has been limited to some narrow problems, especially short-run analysis and experimental comparative studies.

(*a*) In the short-run it can be assumed for many agricultural products that supply is largely determined by exogenous factors. When agricultural production processes are already initiated, there is usually little opportunity for changing production and hence supply. The outcome depends mainly on weather. Therefore a special version of the spatial equilibrium model described above, in which the expected amounts of regional supply are estimated exogenously, is often applied to short-run market analysis (Fox [111], Fox and Taeuber [112], Judge [136], Judge and Wallace [137]). The aim of these studies is to predict amounts of interregional exchange and price-structure in response to short-run variations in supply.

(*b*) Another area of application has been research on the competitive position of different regions in relation to long-run regional specialization of agricultural production, effects of changes in foreign trade, etc. This has been done by analysing the sensitivity of interregional exchange and regional price structures for alternative hypothetical assumptions on the supply and demand side (King and Henry [147], Henry and Bishop [126], Stemberger [161]).

(*c*) Other models of this kind have been used to evaluate the efficiency of actual transport organization. For this purpose actual and optimal exchange structures have been compared *ex post* (Gülicher [116]). Differences show the gains from reallocation of transportation facilities and give some insights into the relation between actual and optimal behaviour of traders.

4 SPATIAL EQUILIBRIUM ON THE BASIS OF PRODUCTION MODELS FOR LOCATION UNITS

The spatial equilibrium problem on the basis of production models can be described as follows. The whole area considered is subdivided into several demand and production regions, which are represented by single points in space. The same assumptions with respect to demand and transportation are introduced as in the previous section, viz. demand is represented by price-demand functions, the regions are separated by transportation costs which are given and independent of volume transported, while transportation costs within regions are neglected. But the production side now is represented by production models for groups of farms. In each production region one or more agricultural location units are distinguished consisting of a group of more or less homogeneous farms which are assumed to act like single decision-making units. They are characterized by:

(*a*) production functions for agricultural products;
(*b*) resources of land, labour, and capital;
(*c*) supply functions for industrially-produced factors of production; and
(*d*) economic behaviour of producers.

Production functions are assumed to be linear and homogeneous for each group of farms and the technology is described by a set of linear production processes. But this does not exclude scale-effects, if farms move from one group to another, e.g., when they rent or buy land.

Factors of production may be exchanged between location units as well as between regions. It is useful to distinguish the following four groups:

(*a*) factors of production which are fixed for location units (buildings);

(*b*) factors of production which are regionally fixed but exchangeable between location units within a region (land);

(*c*) factors of production which are fixed for the whole economy, but exchangeable between regions and location units (skilled agricultural workers); and

(*d*) variable factors of production which are assumed to be fixed neither for location units, nor for regions, nor for the economy as a whole (industrially-produced inputs in advanced economies).

It is not possible to classify different agricultural production factors in these groups generally; rather it depends on the technical and economic status of a country and on the objective of research (length of the time-horizon considered).

4.1 CONDITIONS FOR SPATIAL EQUILIBRIUM

The special and partly simplifying assumptions of the agricultural model described above make it possible to neglect many of the intricate problems resulting from economies of scale and aggregation effects, which have been analysed by Isard [41], Lefeber [54], von Böventer [11], and Kuenne [51], considering the conditions of spatial equilibrium under fairly general assumptions. The model becomes operational under the assumptions outlined and the conditions for spatial equilibrium can be easily stated:

(*a*) Conditions for *production equilibrium* of location units naturally correspond to the conditions known from microeconomic linear production theory. The monetary marginal product (at regional prices) of a production process must not exceed marginal costs (at regional prices). Marginal product is equal to marginal cost if a process is actually used in equilibrium, it is smaller than marginal cost if a process is not used.

(*b*) The conditions for *exchange equilibrium* of goods have already been described in section 3.1.

(*c*) The conditions for *equilibrium of factor use* depend on the assumptions on factor mobility. If factor supplies are assumed to be given for the location units in a short-run analysis, they are rewarded at their marginal product at that location unit. Then factor earnings will usually differ from location to location. Specialization of production between locations will tend to equalize rewards, but a complete equalization will be reached only under very special conditions (see Samuelson [199], p. 163). Therefore remaining differences of factor rewards will lead to an exchange of factor services between location units and to migration of production factors, if this is allowed by assumptions under a more long-run analysis.

Factor movements will be influenced by many sociological, psychological and institutional factors. But if non-economic influences are excluded, equilibrium conditions for factor migration (including the commuting of labour) can be stated in a way formally similar to conditions for exchange equilibrium for a good; i.e., differences in remuneration for factor services (differences in discounted values for all expected real factor earnings) must be not larger than costs for migration of factors. They must be equal if there actually is migration of factors in equilibrium.

4.2 LINEAR INTERREGIONAL PROGRAMMING MODELS

The mathematical structure of the models for the determination of the spatial equilibrium solution depends on the assumptions concerning:

(*a*) the shape of regional demand functions for agricultural products;

(*b*) the regional supply functions for production factors and industrial inputs in agriculture; and

(*c*) the mobility of resources.

First we introduce the simplifying assumptions that the demand and supply functions mentioned above are either perfectly elastic or perfectly inelastic. Specifically we assume:

(*a*) the regional demands for agricultural products are given;

(*b*) the regional resources of labour, the regional capacities of buildings, etc., are given; and

(*c*) the regional prices of industrial inputs are known.

Then the spatial equilibrium problem can be formulated as a linear programming model, the mathematical structure of which has been described elsewhere by Marschak and Beckmann [6], Isard [43], Stevens [77], and others, and which has been applied to agricultural problems by Henderson [34], Heady [118], Birowo and Renborg [97].

In order to make the specific structure of these interregional programming models evident, the problem will first be formulated for an example of three regions in which two location units are distinguished (see Table 1). The model is described by matrices and vectors. Lower indices in front of vectors and matrices denote the number of the region, upper indices denote the number of the location unit.

TABLE 1

Interregional equilibrium model for three regions and two location units per region

Production						Interregional exchange							
Region 1		Region 2		Region 3									
Locat. unit 1	Locat. unit 2	Locat. unit 1	Locat. unit 2	Locat. unit 1	Locat. unit 2								Production and transport acitivities
${}^{1}_{1}y$	${}^{2}_{1}y$	${}^{1}_{2}y$	${}^{2}_{2}y$	${}^{1}_{3}y$	${}^{2}_{3}y$	${}_{12}x$	${}_{13}x$	${}_{21}x$	${}_{23}x$	${}_{31}x$	${}_{32}x$		
${}^{1}_{1}c$	${}^{2}_{1}c$	${}^{1}_{2}c$	${}^{2}_{2}c$	${}^{1}_{3}c$	${}^{2}_{3}c$	${}_{12}k$	${}_{13}k$	${}_{21}k$	${}_{23}k$	${}_{31}k$	${}_{32}k$	$=$ Min.	Production and transport costs
${}^{1}_{1}Fg$												$\leq {}^{1}_{1}bg$	Production factors, fixed for location units.
	${}^{2}_{1}Fg$											$\leq {}^{2}_{1}bg$	
		${}^{1}_{2}Fg$										$\leq {}^{1}_{2}bg$	
			${}^{2}_{2}Fg$									$\leq {}^{2}_{2}bg$	
				${}^{1}_{3}Fg$								$\leq {}^{1}_{3}bg$	
					${}^{2}_{3}Fg$							$\leq {}^{2}_{3}bg$	
${}^{1}_{1}Fr$	${}^{2}_{1}Fr$											$\leq {}_{1}br$	Production factors, fixed for regions.
		${}^{1}_{2}Fr$	${}^{2}_{2}Fr$									$\leq {}_{2}br$	
				${}^{1}_{3}Fr$	${}^{2}_{3}Fr$							$\leq {}_{3}br$	
${}^{1}_{1}Fn$	${}^{2}_{1}Fn$	${}^{1}_{2}Fn$.	${}^{2}_{2}Fn$	${}^{1}_{3}Fn$	${}^{2}_{3}Fn$							$\leq bn$	Production factors, fixed for the whole economy.
${}^{1}_{1}A$	${}^{2}_{1}A$					$-I$	$-I$	I		I		$\geqq {}_{1}d$	Final demand.
		${}^{1}_{2}A$	${}^{2}_{2}A$			I		$-I$	$-I$		I	$\geqq {}_{2}d$	
				${}^{1}_{3}A$	${}^{2}_{3}A$		I		I	$-I$	$-I$	$\geqq {}_{3}d$	

Let

${}^{1}_{1}\mathrm{y}, {}_{1}\mathrm{y}, \ldots, {}^{2}_{3}\mathrm{y}$	= vectors of production activities of location units;
${}_{12}\mathrm{x}, {}_{13}\mathrm{x}, \ldots, {}_{32}\mathrm{x}$	= vectors of interregional exchange activities;
${}^{1}_{1}\mathrm{c}, {}^{2}_{1}\mathrm{c}, \ldots, {}^{2}_{3}\mathrm{c}$	= vectors of variable costs of production;
${}_{12}\mathrm{k}, {}_{13}\mathrm{k}, \ldots, {}_{32}\mathrm{k}$	= vectors of transportation costs;
${}^{1}_{1}\mathrm{F}_{g}, {}^{2}_{1}\mathrm{F}_{g}, \ldots, {}^{2}_{3}\mathrm{F}_{g}$	= matrices of input coefficients of production factors fixed for location units;
${}^{1}_{1}\mathrm{F}_{r}, {}^{2}_{1}\mathrm{F}_{r}, \ldots, {}^{2}_{3}\mathrm{F}_{r}$	= matrices of input coefficients of production factors fixed for regions;
${}^{1}_{1}\mathrm{F}_{n}, {}^{2}_{1}\mathrm{F}_{n}, \ldots, {}^{2}_{3}\mathrm{F}_{n}$	= matrices of input coefficients of production factors fixed for regions;
${}^{1}_{1}\mathrm{A}, {}^{2}_{1}\mathrm{A}, \ldots, {}^{2}_{3}\mathrm{A}$	= matrices of net output coefficients (output-coefficient minus input-coefficient);
${}^{1}_{1}b_{g}, {}^{2}_{2}b_{g}, \ldots, {}^{2}_{3}b_{g}$	= vectors of limitations for production factors fixed for location units;
${}_{1}b_{r}, {}_{2}b_{r}, {}_{3}b_{r}$	= vectors of limitations for factors fixed for regions;
b_{n}	= vector of limitations for factors fixed for the whole economy; and
${}_{1}d, {}_{2}d, {}_{3}d$	= vectors of regional demands.

Activities of the interregional programming model are the production processes of the location units and the interregional exchange activities for agricultural products. (For simplicity exchange activities for production factors are not introduced.[9])

The activities are valued in the criterion function by variable costs of production or transportation respectively. The objective of the model is to minimize total production and transportation costs under the restraints of regional demand fulfillment and of factor limitations.

The solution to the model yields the optimal spatial organization of agricultural production for a given set of natural, technical, and economic data, which is equivalent to static regional equilibrium assuming perfect competition and rational behaviour.[10]

Specifically, one obtains in the primal solution the levels of production processes for location units (and hence regional production and factor use) and the quantities of interregionally transported products. In the dual solution one obtains (for the demand restraints of the original problem) the marginal costs of production and transportation, which under perfect competition are equal to equilibrium prices of products. For the factor restraints one obtains marginal values which can be interpreted as equilibrium prices for factor services or rents respectively.

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[9] Exchange of intermediate products between location units and regions is allowed for in demand inequations. If a good is used exclusively as an intermediate product, the value of final demand in the restraint vector becomes zero.

[10] This will be shown formally under more general assumptions in the following section.

4.3 A MORE GENERAL STATIC EQUILIBRIUM MODEL

So far the interregional equilibrium model has been derived under the simplifying assumption that the exogenously given supply and demand functions are either perfectly inelastic or perfectly elastic. Now a more general model with price elastic demand functions will be described. (In a similar manner price-elastic functions for factor supply can be introduced, but this will not be done here, to simplify presentation).

One way to solve a problem of this kind is to apply an iterative procedure, which has been described in detail elsewhere by one of the present authors (Henrichsmeyer [36]). Another way is to follow Samuelson's proposal of maximizing some value of net social pay-off and solving the problem directly by quadratic programming (Takayama and Judge [81]). In the following analysis the direct quadratic programming solution will be described. To simplify presentation no location units are distinguished within regions and no factors fixed at the national level are assumed. The assumptions on the production side do correspond to the assumptions of the linear model of the previous section.

We define the following notation:

Variables:

Let ${}_r\mathrm{p}_i$ = price of good i in region r;

$\mathrm{p}' = ({}_1\mathrm{p}_1, {}_1\mathrm{p}_2, \ldots, {}_1\mathrm{p}_n;\ {}_2\mathrm{p}_1, \ldots, {}_2\mathrm{p}_n;\ \ldots;\ {}_t\mathrm{p}_1, \ldots, {}_t\mathrm{p}_n)$;

${}_r\mathrm{y}_k$ = production process k in region r;

$\mathrm{y}' = ({}_1\mathrm{y}_1, {}_1\mathrm{y}_2, \ldots, {}_1\mathrm{y}_z;\ {}_2\mathrm{y}_1, \ldots, {}_2\mathrm{y}_z;\ \ldots;\ {}_t\mathrm{y}_1, \ldots, {}_t\mathrm{y}_z)$;

${}_r\mathrm{c}_k$ = variable costs of process k in region r;

$\mathrm{c}' = ({}_1\mathrm{c}_1, {}_1\mathrm{c}_2, \ldots, {}_1\mathrm{c}_z;\ {}_2\mathrm{c}_1, \ldots, {}_2\mathrm{c}_z;\ \ldots;\ {}_t\mathrm{c}_1, \ldots, {}_t\mathrm{c}_z)$;

${}_r\mathrm{b}_h$ = capacity of factor h in region r;

$\mathrm{b}' = ({}_1\mathrm{b}_1, {}_1\mathrm{b}_2, \ldots, {}_1\mathrm{b}_m;\ {}_2\mathrm{b}_1, \ldots, {}_2\mathrm{b}_m;\ \ldots;\ {}_t\mathrm{b}_1, \ldots, {}_t\mathrm{b}_m)$;

${}_r\mathrm{q}_h$ = price of factor h in region r;

$\mathrm{q}' = ({}_1\mathrm{q}_1, {}_1\mathrm{q}_2, \ldots, {}_1\mathrm{q}_m;\ {}_2\mathrm{q}_1, \ldots, {}_2\mathrm{q}_m;\ \ldots;\ {}_t\mathrm{q}_1, \ldots, {}_t\mathrm{q}_m)$.

Transport activities ${}_{gr}\mathrm{x}_i$ and transportation costs ${}_{gr}\mathrm{k}_i$ are as already defined in section 3.3;

u, v, and w, are vectors of slack variables.

Coefficients:

Let ${}_r\mathrm{A}$ = matrix of net output coefficients ${}_ra_{ik}$ in region r:

$$\mathrm{A} = \begin{bmatrix} {}_1\mathrm{A} & & & & \\ & \ddots & & & \\ & & {}_r\mathrm{A} & & \\ & & & \ddots & \\ & & & & {}_t\mathrm{A} \end{bmatrix}$$

$_r\mathrm{F}$ = matrix of input coefficients for factor services $_rf_{hk}$ in region r:

$$
\mathrm{F} = \begin{bmatrix} {}_1\mathrm{F} & & & & \\ & \ddots & & & \\ & & {}_r\mathrm{F} & & \\ & & & \ddots & \\ & & & & {}_t\mathrm{F} \end{bmatrix}
$$

Also define matrix E as follows:

$$
\mathrm{E} = \left[\begin{array}{cccc|cccc|c|cccc}
\multicolumn{4}{c|}{\overbrace{-\mathrm{I}\ -\mathrm{I}\ \ldots\ -\mathrm{I}}^{t-1 \text{ times}}} & \mathrm{I} & & & & \ldots & \mathrm{I} & & & \\
\mathrm{I} & & & & \multicolumn{4}{c|}{-\mathrm{I}\ -\mathrm{I}\ \ldots\ -\mathrm{I}} & & & \mathrm{I} & & \\
 & \mathrm{I} & & & & \cdot & & & & & & \cdot & \\
 & & \ddots & & & & \ddots & & & & & & \cdot \\
 & & & & & & & & & & & & \mathrm{I} \\
 & & & \mathrm{I} & & & & \mathrm{I} & & \multicolumn{4}{c}{-\mathrm{I}\ -\mathrm{I}\ \ldots\ \mathrm{I}}
\end{array}\right] \left.\vphantom{\begin{array}{c} a\\a\\a\\a\\a \end{array}}\right\} \begin{array}{l} t-1 \\ \text{times} \end{array}
$$

Demand functions are as already defined in section 3.3.

We define net social pay-off as the sum of the algebraic areas under the price-demand curves *minus* the sum of factor earnings. Then maximization of net social pay-off under the marginal costs—marginal revenue and regional price-difference—transportation costs conditions leads to the spatial equilibrium solution[11].

Maximize

$$
\begin{aligned}
Z &= \sum_r^t \sum_i^n \int_0^{_rp_i} \Big({}_r\alpha_i - {}_r\beta_{ii}\, {}_rp_i - \tfrac{1}{2} \sum_{j \neq i}^n {}_r\beta_{ij}\, {}_rp_j\Big)\, d_rp_i - \sum_r^t \sum_h^m {}_rb_h\, {}_rq_h \qquad (27) \\
&= \sum_r^t \sum_i^n \Big({}_r\alpha_i\, {}_rp_i - \tfrac{1}{2}{}_r\beta_{ii}\, {}_rp^2{}_i - \tfrac{1}{2} \sum_{j \neq i}^n {}_r\beta_{ij}\, {}_rp_j\, {}_rp_i\Big) - \sum_r^t \sum_h^m {}_rb_h\, {}_rq_h
\end{aligned}
$$

subject to

$$
\begin{aligned}
&\sum_i^n {}_r\alpha_{ik}\, {}_rp_i - \sum_h^m {}_rf_{hk}\, {}_rq_h - {}_rc_k \leq 0 \qquad \begin{pmatrix} r = 1, 2, \ldots, t \\ k = 1, 2, \ldots, z \end{pmatrix} \qquad (28) \\
&-{}_gp_i + {}_rp_i - {}_{gr}k_i \leq 0 \qquad \begin{pmatrix} g \neq r = 1, 2, \ldots, t \\ i = 1, 2, \ldots, n \end{pmatrix} \\
&{}_rp_i,\ {}_rq_h \geq 0
\end{aligned}
$$

This quadratic maximization problem can be converted into a linear programme and solved by an extended simplex algorithm as has been shown in section 3.3. The structure of the linear programme can be derived easily from the Kuhn-Tucker conditions for the constraints in (28) and it can also be shown that the maximization problem and the spatial equilibrium problem are equivalent.

---

[11] That this is true can be shown by the fact that the Kuhn-Tucker conditions of this model (derived below) correspond to the complete set of spatial equilibrium conditions.

From the Lagrange function for equation (27)

$$Z = \sum_r^t \sum_i^n ({}_r\alpha_i\, {}_rp_i - \tfrac{1}{2}\, {}_r\beta_{ii}\, {}_rp_i^2 - \tfrac{1}{2} \sum_{j \neq i}^n {}_r\beta_{ij}\, {}_rp_j\, {}_rp_i) - \sum_r^t \sum_h^m {}_rb_h\, {}_rq_h -$$

$$\sum_r^t \sum_k^z {}_ry_k \,(\sum_i^n {}_r\alpha_{ik}\, {}_rp_i - \sum_h^m {}_rf_{hk}\, {}_rq_h - {}_rc_k) \sum_{g \neq r}^t \sum_{r \neq g}^t \sum_i^n {}_{gr}x_i\,(-{}_gp_i + {}_rp_i\, {}_{gr}k_i) \qquad (29)$$

the following Kuhn-Tucker conditions for optimal solution result:

$$\frac{\delta \Sigma}{\delta_r\, p_i} = {}_r\alpha_i - {}_r\beta_{ii}\, {}_rp_i - \tfrac{1}{2} \sum_{j \neq i}^n {}_r\beta_{ij}\, {}_rp_j - \sum_k^z {}_r\alpha_{ik}\, {}_ry_k - \sum_{g \neq r}^t {}_{gr}x_i + \sum_{g \neq r}^t {}_{gr}x_i \leq 0 \qquad (30a)$$

$$\frac{\delta Z}{\delta_r\, p_i} \cdot {}_rp_i = 0 \qquad \begin{pmatrix} r = 1, 2, \ldots, t \\ i = 1, 2, \ldots, n \end{pmatrix} \qquad (30b)$$

Demand *plus* export of a product must in every region be equal to production *plus* imports (if we exclude non-positive prices for goods).

$$\frac{\delta Z}{\delta_r\, q_h} = -{}_rb_h + \sum_k^z {}_rf_{hk}\, {}_ry_k \leq 0 \qquad \begin{pmatrix} r = 1, 2, \ldots, t \\ h = 1, 2, \ldots, m \end{pmatrix} \qquad (31a)$$

$$\frac{\delta Z}{\delta_r\, q_h} \cdot {}_rq_h = 0 \qquad (31b)$$

In every region the demand for factor services must not exceed factor supply and must be equal to it at positive factor-earnings.

$$\frac{\delta Z}{\delta_r\, y_k} = \sum_i^n {}_r\alpha_{ik}\, {}_rp_i - \sum_h^m {}_rf_{hk}\, {}_rq_h - {}_rc_k \leq 0 \qquad \begin{pmatrix} r = 1, 2, \ldots, t \\ k = 1, 2, \ldots, z \end{pmatrix} \qquad (32a)$$

$$\frac{\delta Z}{\delta_r\, y_k} \cdot {}_ry_k = 0 \qquad (32b)$$

Marginal revenue must not exceed marginal costs and if the process is actually used, these must be equal.

$$\frac{\delta Z}{\delta_{gr}x_i} = -{}_gp_i + {}_rp_i - {}_{gr}k_i \leq 0 \qquad \begin{pmatrix} r \neq g = 1, 2, \ldots, t \\ i = 1, 2, \ldots, n \end{pmatrix} \qquad (33a)$$

$$\frac{\delta Z}{\delta_{gr}x_i} \cdot {}_{gr}x_i = 0 \qquad (33b)$$

It is evident that these conditions for solution of the maximization problem correspond to the conditions for solution of the equilibrium problem.

The transformation of the problem into a linear programme is based on the following reasoning. We define slack variables for the constraints marked "*a*" above and express the "*b*" conditions by these slack variables. For example we define for condition (31a) the slack variable

$$\frac{\delta Z}{\delta_r q_h} = -{}_ru_h$$

Hence we obtain

$$-{}_rf_h + \sum_k^z {}_rf_{nk}\, {}_ry_k + {}_ru_h = 0 \qquad (31a)$$

$${}_ru_h\, {}_rq_h = 0 \qquad (31b)$$

Replacing in this way every pair of conditions and remembering that we excluded non-positive prices for goods (which indicates a zero level for the slack variables in (30)), we get thef ollowing set of conditions instead of the original "*a*" conditions (changing to matrix notation for simplicity).

$$\begin{bmatrix} B & O & A & E & O & O & O \\ O & O & F & O & I & O & O \\ A' & -F' & O & O & O & I & O \\ E' & O & O & O & O & O & I \end{bmatrix} \begin{bmatrix} p \\ q \\ y \\ x \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \alpha \\ b \\ c \\ k \end{bmatrix} \tag{34}$$

The "*b*" conditions can be written as follows:

$$[p' \quad q' \quad y' \quad x'] \; [o \quad u \quad v \quad w]' \tag{35}$$

Now we can formulate a linear programme in which (35) is minimized subject to the restraints of (34); i.e.,

Minimize

$$Z = [o' \quad u' \quad v' \quad w' \quad q' \quad y' \quad x'] \; [p \quad q \quad y \quad x \quad u \quad v \quad w]'$$

subject to

$$\begin{bmatrix} B & O & A & E & O & O & O \\ O & O & F & O & I & O & O \\ A' & -F' & O & O & O & I & O \\ E' & O & O & O & O & O & I \end{bmatrix} \begin{bmatrix} p \\ q \\ y \\ x \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \alpha \\ b \\ c \\ k \end{bmatrix} \tag{36}$$

It is easily seen that the conditions (34) and (35) are satisfied if in the solution of the linear programme $Z = 0$. By this means the spatial equilibrium solution is reached.

4.4 EMPIRICAL APPLICATION

In recent years a large number of empirical investigations based on the concept of regional production models have been carried out, of which only a few typical examples can be cited in this survey[12]. We will not cite the many applications for single or small numbers of regions. At the national level two groups of investigations can be distinguished, viz., models for single or few branches of production, and models which cover the main branches of agricultural production simultaneously.

At the beginning of this research about ten years ago work started out with partial models for single branches of production. In one of the first studies Heady and Egbert (119) analysed the competitive position of regional grain production in the United States. In a similar way the locational advantages of most branches of production have been analysed in the United States (see, e.g., Schrader and King [157], Snodgrass and French [159], Buchholz and Judge [99]). This research centred around the following questions:

[12] A more complete list of literature is given in the reference list on pages 60-70.

(*a*) the efficiency of the actual regional distribution of production in relation to optimal allocation;

(*b*) the quantification of the competitive advantage of production regions under different sets of assumptions;

(*c*) the analysis of regional adjustments of agricultural production under the influence of technical progress and development of demand; and

(*d*) the analysis of the effects of different measures of agricultural policy.

But the application of partial models for single branches of agricultural production is limited. They can be applied in a reasonable way only in special cases, when the interdependences with other branches of production are of no great importance. This will tend to be the case in countries and regions with a relatively high degree of specialization (hence the many studies in the United States) and for branches of production which are relatively independent of soil production, such as hogs and poultry.

Therefore the construction of more general spatial equilibrium models has been attempted, in which the interdependences between the main branches of agricultural production are taken into account simultaneously. A research group at Ames in Iowa under the direction of Heady is hoping to integrate the different partial studies into a more comprehensive model of agricultural production [118], [124]. In Sweden, Renborg, Birowo and Folkesson have established different interregional competition models which have been used for policy purposes [97]. Bassjuk has used a regional model for short-run planning and another one for long-term perspective planning in the Soviet Union [95]. A French team has applied a regional programming model on the basis of calculations at the farm level, which will be used for medium-term planning [150], [163]. Finally, the present authors have constructed a spatial equilibrium model in Hohenheim, which starts out from a consistent balancing model of the input-output type for a base year and in which elements of recursive programming are introduced.

5 DYNAMIC APPROACHES TO INTERREGIONAL COMPETITION AND PLANNING

In the static models so far described equilibrium solutions have been determined under the assumptions that producers, traders, etc., have perfect knowledge of technical possibilities and market conditions and act perfectly rationally. The results describe the optimal structure of production and transportation which might be interpreted under certain conditions as a system of quantitative goals for economic policy, but their use for explanation of actual changes and hence for forecasting and analysing the actual effects of agricultural policy measures is limited. The results show at best the probable direction of change of variables. Further, the adjustments to be explained are not unique adjustments to a certain set of data, but successive adjustments to constantly changing data, whereby the decisions met in a period t lead to results which are again data for decisions in the period $t + 1$.

There have been several proposals to overcome these limitations by the introduction of some kinds of behaviour functions, especially with respect to speed of production change, saving and investment behaviour, and price expectations[13]. The combination of these elements with programming models leads to a recursive procedure, which has been called "recursive programming" (Day [20]).

5.1 INTRODUCTION OF FLEXIBILITY CONSTRAINTS AND SAVING FUNCTIONS

Flexibility constraints were first introduced into linear programming models for the analysis of changes in supply of agricultural products in response to changes in prices. (Henderson [34], Day [20].) The concept is based on the empirical knowledge that, in general, regional production structure changes in the direction of the optimal organization, but that the extent and speed of adjustment depend on several factors such as risk, traditional behaviour of producers, financial limitations etc. By introducing flexibility constraints it is assumed that the compound influence of these factors (all factors which are not considered explicitly) can be taken into account by limiting the maximal change of production and investment variables from one time period to the next. The flexibility constraints can be estimated from time-series data on certain model variables. In the simplest case one can assume, as Henderson and Day have done in their empirical work, that the average yearly change of variables in successive time periods of increasing or decreasing production (investment) is an estimate for the expected maximal change in the future. Then restraints for variable x can be written:

$$x(t) \leq \beta x\,(t-1) \qquad (\beta > 1) \text{ and}$$
$$x(t) \geq \alpha x\,(t-1) \qquad (\alpha < 1),$$

where α and β are the so-called "flexibility-coefficients" which determine maximal increase or decrease from period $t - 1$ to period t. They are specified for every product (investment process) and region.

Thus the feasible area for a period t in which the production and investment variables may vary is limited by flexibility constraints as well as by physical capacity constraints. The results of the spatial equilibrium model for period t are then used as data for the model in period $t + 1$. By this recursive connection and solution of the model for successive time periods, time paths of variables can be computed.

The concept has been widened by Heidhues [34], who explicitly considered the financial sphere of farm enterprises. He introduced liquidity restraints for each period t which are determined by the income of the period $t - 1$, saving behaviour of farm households, and the conditions for bank credit and investment outside of agriculture in period t.

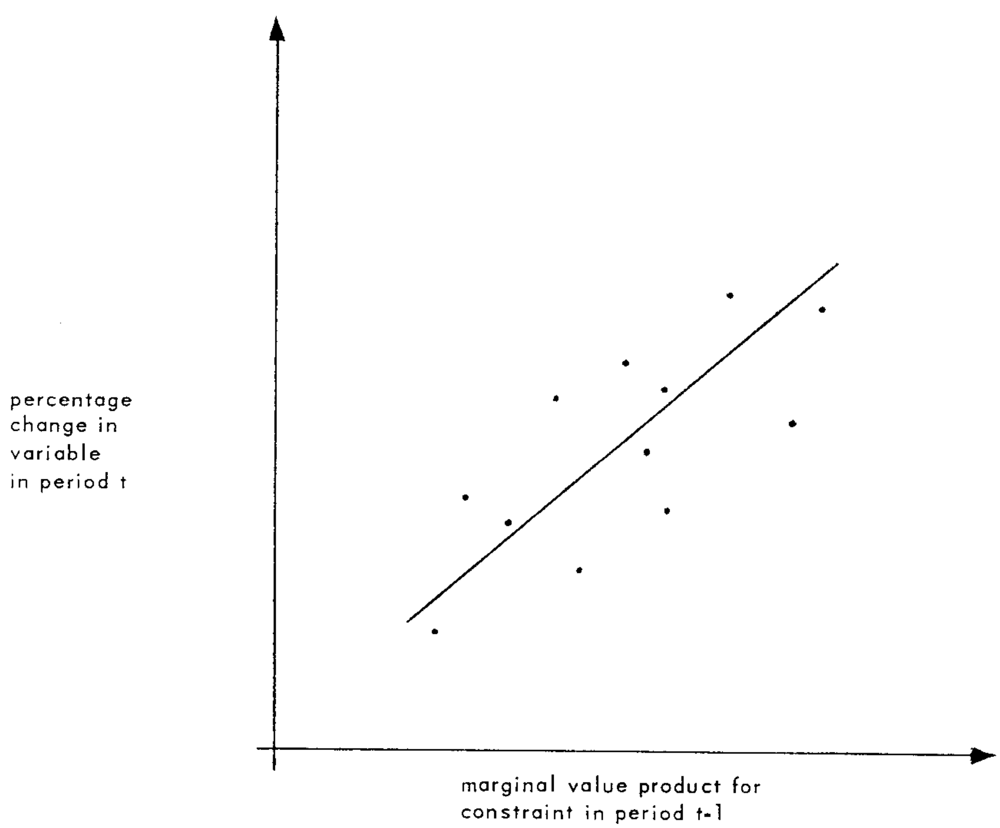
[13] The corresponding problem of optimal resource allocation over time has received little attention in the literature so far. Initial research has been done, but this is still in the sphere of methodological discussion.

The time paths of variables computed in recursive models are determined to a large extent by the numerical coefficients for the flexibility constraints. Thus the value of the concept for empirical research depends on the quality of the estimated values of these coefficients.

Let us consider now the formulation of more general flexibility constraints. Until now very little has been known about these relationships. The first empirical estimates of Henderson and Day were based on the simple assumptions explained above. As first starting points for empirical research these investigations have been valuable, but one might envisage much more general behaviour functions in which maximal increase and decrease in production, investment, migration, etc., in a certain period are related to the main economic variables responsible for their rate of change. This was pointed out very clearly by Day [20, p. 119 f] who put the above recursive programming concept into a much broader theoretical framework. As specific steps for further empirical research it might be useful to test the following hypotheses:

(*a*) On the basis of theoretical considerations, one would expect that the value of a flexibility coefficient for a time period t is not independent of the marginal value for the corresponding restraint in time period $t-1$ (which indicates the profitability of changing the constrained variable).

Figure 6



Instead one would expect some meaningful relationship to exist between the two variables. This relationship can be analysed by solution of interregional programming models for several successive time periods, in which activities are bounded by restraints exactly to the actual level of the time period. Then one can contrast the resulting marginal values for the restraints and actual percentage change in the following period, as is schematically done in figure 6. Preliminary empirical results support the hypothesis of a meaningful relationship between these two variables, but further empirical research is needed.

(*b*) There might exist a similar relationship between price-expectations (or the ratio of expected to current prices) for a product and the corresponding flexibility coefficient. This problem of price expectations will be taken up in a more general framework in section 5.2.

(*c*) If certain variables expand or contract over time in the above recursive programming concept, they will do so in certain cases on a geometric path until final physical restraints (capacity of buildings, crop rotation, etc.) are reached. This tends to be the case especially when a crop with relatively small acreage substitutes for a number of other crops. Then absolute changes will be the largest just before the ultimate restraints are reached, which will generally be an asymptotic approach. This might be empirically analysed by comparison of static normative equilibrium results and the actual development of variables over time and estimation of corresponding behaviour functions.

In summary, much empirical work will be necessary before we can judge and exhaust the possibilities of recursive programming in interregional models of agricultural production.

5.2 COMBINATION OF INTERREGIONAL EQUILIBRIUM WITH PRICE EXPECTATION MODELS

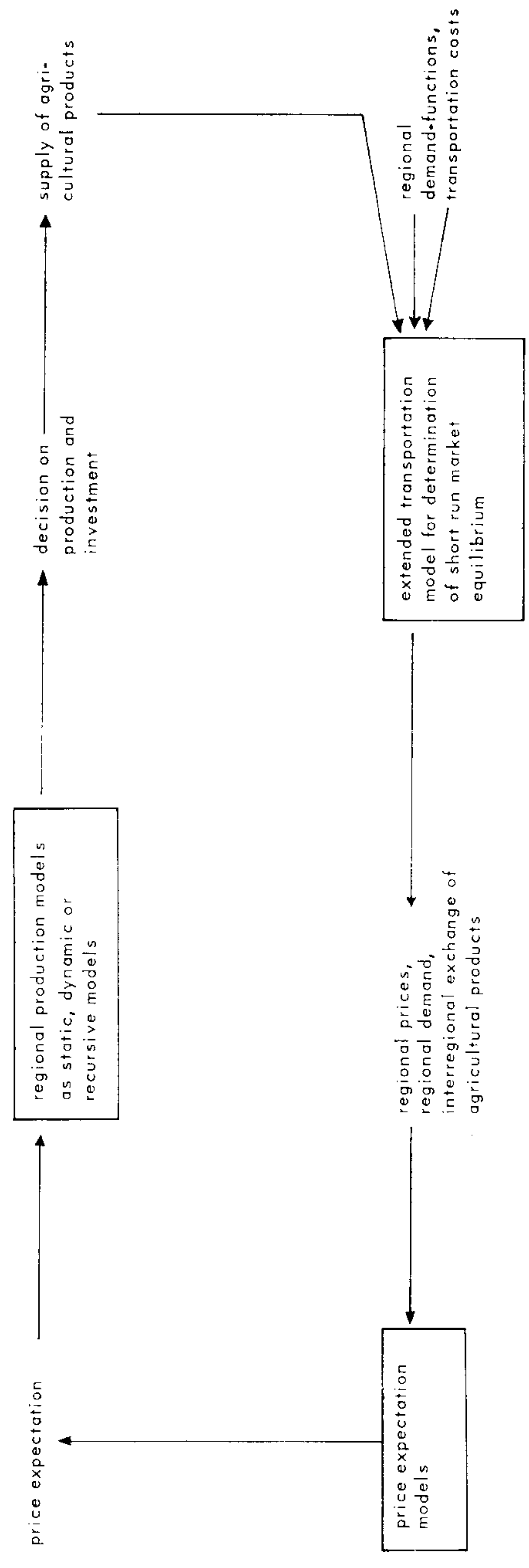
In the models so far described it has been assumed that price expectations of producers do coincide with equilibrium prices of the interregional models. Investigations into the relationships between price and supply response have shown that it is often reasonable to base the analysis on price expectations instead of on existing market prices (Nerlove [63], Weinschenck [90]).

It is true that in recursive programming models differences between expected and equilibrium prices have been indirectly accounted for together with other influences in the flexibility constraints, but they have not been introduced explicitly. Strictly speaking the prices determined in recursive models are equilibrium prices for given behavioural constraints, resulting from the uncertainty of price expectations, amongst other things.

For explicit introduction of prices and price-expectations it is necessary to distinguish between the interregional equilibrium of production at given price expectations and short-run market equilibrium at given production. The simultaneous consideration of production and marketing in the models discussed so far has to be given up and replaced by a sequence of production and marketing models. A procedure of this kind was first described by Day [22] and is graphed schematically in figure 7.

Figure 7

Combination of interregional equilibrium and price expectation models



Computation starts in period t with the determination of regional production, given the price-expectations prevailing at this period. Production leads to supply of agricultural products. From regional supply of agricultural products, regional price demand functions and interregional transportation costs, short-run market equilibrium, and especially short-run equilibrium prices are computed. Short-run equilibrium prices are considered to be identical with prices actually obtained and they form the basis for the estimation of price expectations. Then corresponding computations for period $t + 1$ can start.

The method of dynamic coupling has to be considered mainly as a complement to the models described in the previous sections for the analysis of short-run problems, in which supply fluctuations of the cobweb type are to be explained.

6 THE LOCATION OF AGRICULTURAL PROCESSING INDUSTRY

In the spatial equilibrium models described so far, demand has been represented by regional demand functions for agricultural products. This demand consists of the demand of final consumers as well as that of processing industries and traders. In highly industrialized countries only a small part of agricultural production is consumed directly by consumers, by far the largest part becomes processed by various processing industries. So the question of optimal location and, under conditions of economies of scale, the question of optimal size of these plants arises.

The problems of the regional organization of agricultural production and of the location of processing industries are mutually dependent. On the one hand the optimal location and size of a processing plant are influenced by the regional distribution of agricultural production because assembling costs are a function of the density of supply of agricultural products. On the other hand organization of agricultural production is influenced by the location of processing industries, because the spatial price structure is determined by it. Thus in a more general concept the problems of spatial organization of agriculture, location of processing industries and transportation of raw materials and final goods can be seen as one problem, the parts of which must be solved simultaneously.

The problem can be characterized generally as follows. Given are:

(*a*) the regional distribution of final demand of consumers, either by assumption of a continuous distribution function or by discrete points in space, at which price demand functions are given;

(*b*) the regional distribution of agricultural supply or of production functions and production factors which determine agricultural supply. On the production side, too, there might be assumed either a continuous distribution as in the von Thünen model, or discrete production points, as assumed in the previous sections;

(*c*) the technology of the processing industries, which is characterized in most cases by considerable advantages of mass production. Processing costs are assumed to be a given function of production; and

(*d*) the transportation costs per unit for raw products and final goods, which are assumed to be independent of volume transported as in the models of the previous sections.

Formally this more general problem could be written in a similar way to the spatial equilibrium models for agricultural production discussed in section 5. Only the technological matrices for processing industries and some additional transportation activities would have to be introduced. But two problems arise. Firstly, economies of scale in the processing industries make it necessary to introduce non-linear cost (production) functions and to consider a network of potential sites of firms in discrete models. This is no difficulty in principle, at least if one approximates discontinuous functions by linear segments. But in actual computation simultaneous formulation of the whole problem leads to rather large problems. Therefore quite a number of iterative procedures have been proposed, some of which will be described below.

Secondly, the existence of the economies of scale makes it necessary to reconsider the conditions of regional price differentiation according to transportation costs which is based on the assumption of perfect competition. If economies of scale lead to large sizes of processing firms, at which entrepreneurs do not take prices as given but are confronted with a price demand function, the problem of monopolistic behaviour and monopolistic price differentiation arises.

Therefore it is useful to distinguish two types of spatial equilibrium models:

(*a*) models in which the optimal (cost minimizing) structure of production and distribution is determined from the viewpoint of a central planning board; and

(*b*) models which are based on decentralized decision-making and simulate the market mechanism. In these models it is assumed that single firms make their decisions, without knowledge of optimal location, on the basis of the given economic data at the time.

6.1 MODELS OF CENTRALIZED LOCATION PLANNING

The models which minimize transportation and processing costs differ with respect to the assumption as to the continuity or discontinuity of space and the number of firms to be taken into account.

With respect to the continuity or discontinuity of space, one can distinguish:

(*a*) models, in which the continuous approach is used to determine the optimal market area of firms and, indirectly, the location of firms after arbitrary choice of the location of one single firm;

(*b*) models in which the continuous and the discrete approaches are combined by assuming a finite number of given markets for supply and demand but an infinite number of possible locations;

(*c*) models using the discrete approach by assuming a finite number of markets as well as a finite number of possible locations.

(a) The continuous approach

The simplest case is that where one can assume that it is possible to restrict consideration on one side of the market and to find a function in which production volume, market area and transportation costs are related to each other (Cobia and Babb [170], Olson [195], Williamson [208]).

In this case we have

$$A_c = \frac{P_c + T_c}{V} \tag{37}$$

where

V = production volume;

A_c = average costs of production;

P_c = total production costs; and

T_c = total transportation costs.

Substituting the cost function for P_c and the function of transportation costs for T_c in equation (37), the volume with lowest average costs can be determined by taking the first derivative of (37), setting it to zero, and solving for V. The optimum number of firms is computed by dividing the total market output by V. Since only the distance between firms is determined, one has to select the location of one firm arbitrarily in order to determine the location of all other firms.

Practical application of this model-type is restricted by difficulties in the determination of a transportation function for a given (continuous) area and by the neglect of one side of the market (either supply for a raw material or selling of final products).

The restriction of consideration to one side of the market is removed in the following model.

(b) Combination of discrete and continuous approaches

Given are:

(i) a finite number of markets;

(ii) the possibility of locating the processing plants everywhere in the area, i.e., an infinite number of potential locations;

(iii) the cost functions of the processing firms, which are assumed to be identical over the whole area under consideration;

(iv) the transportation costs which are assumed to be a function of the distance from the market, measured as the crow flies.

It is required to determine the number and location of firms which minimizes processing and transportation costs. Geometrical solutions for problems of this kind were first developed by Launhardt [190] and Weber [207] who restricted consideration to three markets. A more general iterative method developed by Kuhn and Kuenne [189] will be described.

Let us first restrict consideration to one firm and assume the supply of raw materials and demand for final products to be given, and expressed in the same quantity units.

The weights w_i of the different market points i which are spatially determined by their co-ordinates (x_i/y_i), are calculated by multiplying the total quantity m_i of each market point by the transportation costs r_i, i.e.,

$$w_i = m_i \,.\, r_i \tag{38}$$

It is required to determine the co-ordinates (x_p/y_p) of the location P, at which is minimized the function Φ of the total transportation costs:

$$\Phi = \sum_i (w_i \,.\, s_{pi}) \tag{39}$$

The distances s_{pi} between the market points $i(x_i/y_i)$ and the location $P\ (x_p/y_p)$ are:

$$s_{pi} = \sqrt{(x_i - x_p)^2 + (y_i - y_p)^2} \tag{40}$$

The minimum is obtained where:

$$\frac{\delta\Phi}{\delta x_p} = \sum_i \frac{w_i}{s_{pi}} \,.\, (x_i - x_p) \approx 0 \text{ and } \frac{\delta\Phi}{\delta y_p} = \sum_i \frac{w_i}{s_{pi}} (y_i - y_p) \approx 0 \tag{41}$$

The problem can be solved by an iterative procedure in several steps.

FIRST STEP:

A first approximation starts by calculating the centre of gravity of the weights w_i:

$$x_p^1 = \frac{\sum_i (w_i \,.\, x_i)}{\sum_i w_i} \text{ and } y_p^1 = \frac{\sum_i (w_i \,.\, y_i)}{\sum_i w_i} \tag{42}$$

SECOND STEP:

By reformulating equation (41) we obtain:

$$x_p = \frac{\sum_i \left(\frac{w_i}{s_{pi}} \,.\, x_i\right)}{\sum_i \frac{w_i}{s_{pi}}} \text{ and } y_p = \frac{\sum_i \left(\frac{w_i}{s_{pi}} \,.\, y_i\right)}{\sum_i \frac{w_i}{s_{pi}}} \tag{43}$$

As the s_{pi} are unknown the x_p^1 and y_p^1 are used for a first estimation of the s_{pi}.

$$s_{pi}^1 = \sqrt{(x_i - x_p^1)^2 + (y_i - y_p^1)^2} \tag{44}$$

If $\frac{w_i}{s_{pi}}$ in equation (43) is replaced by

$$w_i^1 = \frac{w_i}{s_{pi}^1} \tag{45}$$

a second estimation of the co-ordinates of P can be obtained:

$$x_p^2 = \frac{\sum_i (w_i^1 \cdot x_i)}{\sum_i w_i^1} \text{ and } y_p^2 = \frac{\sum_i (w_i^1 \cdot y_i)}{\sum_i w_i^1} \tag{46}$$

FOLLOWING STEPS:

The new estimated values of $P(x_p^2/y_p^2)$ are used, as described in the second step, for evaluating s_p^2 and w_i^2, and a further estimation (x_p^3/y_p^3) is calculated. This procedure is reiterated and approaches an optimal solution after only a few iterations.

The optimal location is found if $x_p^{n-1} - x_p^n$ and $y_p^{n-1} - y_p^n$ are not significantly different from zero. In this case the condition expressed in equation (41)

$$\sum_i \frac{w_i}{s_{pi}} (x_i - x) \approx 0 \text{ and } \sum_i \frac{w_i}{s_{pi}} (y_i - y) \approx 0$$

must be satisfied.

Let us assume separated markets for raw materials and final products and assume linear supply and demand functions in order to generalize the problem.

We assume i markets $(i = 1 \ldots I)$ for raw materials with the supply function

$$p_i = a_i \cdot m_i + b_i$$

and k markets $(k = 1 \ldots K)$ for final products with the demand function

$$p_k = -a_k \cdot m_k + b_k$$

In order to solve the problem, it is necessary to assume a certain behaviour of the firm with regard to pricing. In the following the firm is supposed to equalize costs and returns and to differentiate prices in proportion to transportation costs.

For a first estimation of $P(x_p/y_p)$ it is necessary to assume a market price, i.e., volumes of the markets. A first estimate of $P\ (x_p^1/y_p^1)$ can then be obtained as described in step one above. The s_{pk}^1 and s_{pi}^1 are calculated, but different freight rates for raw materials r_i and final goods r_k must be considered. For this reason, the s_{pi}^1 and s_{pk}^1 are replaced in the following steps by t_{pi}^1 and t_{pk}^1, the relevant transportation costs per unit:

$$t_{pi}^1 = s_{pi}^1 \cdot r_i \text{ and } t_{pk}^1 = s_{pk}^1 \cdot r_k \tag{47}$$

Then the supply and demand functions at the location of the processing plant are given by:

$$p_i^* = a_i \cdot m_i + b_i + t_{pi}^1 \text{ and } p_k^* = -a_k \cdot m_k + b_k - t_{pk}^1 \tag{48}$$

Therefore the aggregate shifted supply and demand functions are as shown in (49a) and (49b) respectively.

$$p_A^* = \frac{\sum_i m_i}{\sum_i \frac{1}{a_i}} + \frac{\sum_i \left(\frac{b_i + t_{pi}^1}{a_i}\right)}{\sum_i \frac{1}{a_i}} \tag{49a}$$

$$p_N^* = \frac{\sum_k m_k}{\sum_k \frac{1}{a_k}} - \frac{\sum_k \left(\frac{b_k - t_{pk}^1}{a_k}\right)}{\sum_k \frac{1}{a_k}} \tag{49b}$$

The average processing costs PC are:

$$PC = \frac{FC}{m} + vc \tag{50}$$

where FC and vc indicate fixed and variable costs respectively. At the point of intersection of the total cost function and the returns function, the following must be satisfied:

$$\sum_{i=1}^{I} m_i = \sum_{k=1}^{K} m_k = m \tag{51}$$

From equations (49), (50), and (51), the following condition for the equilibrium of costs and returns is derived:

$$-\frac{m}{\sum_k \frac{1}{a_k}} + \frac{\sum_k \frac{b_k - t_{pk}^1}{a_k}}{\sum_k \frac{1}{a_k}} = \frac{m}{\sum_i \frac{1}{a_i}} + \frac{\sum_i \frac{b_i + t_{pi}^1}{a_i}}{\sum_i \frac{1}{a_i}} + \frac{FC}{m} + vc \tag{52}$$

Transportation costs enter equation (52) only implicitly because they are taken into account in the definition of the supply and demand functions in (48). From equation (52) together with equations (49) and (48) the volume of each market can be computed. These quantities are now used for a further estimation of the co-ordinates of the location P as already described in step two.

So far the consideration has been restricted to the location of one firm. Extension to simultaneous consideration of the optimal number and size of all firms in a given area requires an extension of the iteration process.

Starting with an estimated number of firms and an estimated or given location pattern, the first step is to determine the optimal market area for each firm. Given the market areas, the optimal location of each firm must be re-examined by the alogrithm mentioned above. For the new pattern of allocation the market areas must be corrected towards the optimal pattern. This procedure must be repeated until no further modification occurs for the given number of plants. This procedure must be followed for each relevant number of plants, from which the optimal solution is selected.

To conclude, under simplifying assumptions concerning the transportation costs and identical processing costs, the algorithm described above shows great efficiency in solving the problem of the optimal location of a single plant, since the relatively small number of simple calculations can be done even without an electronic computer. Its application to macro-economic problems of allocation depends on the possibility of developing an efficient way of determining the optimal demarcation of the market areas of the firms.

(*c*) *The discrete approach*

If one assumes a finite number of points in space for supply and demand markets as well as for potential sites of processing plants, the homogeneity assumptions with respect to the distribution of raw materials and final goods, the transportation costs and the processing costs, can be relaxed:

(i) at the markets any quantities of supply and demand or linear price-elastic functions respectively can exist;

(ii) the processing costs may differ from location to location, but it is assumed that the total costs are a linear function of the production volume and have a positive intersection with the ordinate (i.e., the fixed charges);

(iii) the transportation costs between every region and plant are calculated according to the actual transportation structure.

Two principal ways for solving problems of this type can be distinguished: the method of complete enumeration and some more systematic algorithms.

COMPLETE ENUMERATION

The simplest case is where the quantities of supply *or* demand are given and the optimal number, size and locations of the plants are to be determined either with regard to the supply or the demand exclusively, as described by Stollsteimer [204] and Hoch [178]. The problem has the following general algebraic formulation:

Minimize

$$TC = \sum_j vc_j \,.\, m_j \mid L_J + \sum_i \sum_j t_{ij} \,.\, m_{ij} \mid L_J + \sum_j F_j \mid L_J \tag{53}$$

subject to

$$\sum_i m_{ij} = m_j \tag{54}$$

where

TC = total processing and assembly costs;

vc_j = variable processing costs per unit in plant j located at L_j;

m_j = quantity of raw material processed at plant j;

m_{ij} = quantity of raw material shipped from origin i to plant j;

t_{ij} = costs of shipping a unit of material from origin i to plant j;

L_J = one locational pattern for J plants among the $\binom{L}{J}$ possible combinations of locations for J plants given L possible locations;

F_j = fixed charge of establishing and maintaining plant j, derived from the long-run cost function.

From $\binom{L}{J}$ possible location patterns it is required to find L_J for J plants for which the total processing and transportation costs are a minimum.

For each of the $\binom{L}{J}$ possible location patterns a separate matrix of the transportation costs plus the variable processing costs is formulated in which the plants form the column vectors and the markets the row vectors. The markets are attached to the plant which shows the smallest coefficient in the corresponding row of the matrix. Thus, having obtained the minimum of the transportation and variable processing costs, the total costs are evaluated by adding the fixed charges for the plants under consideration. The pattern with lowest costs can be determined by comparing the total costs for all $\binom{L}{J}$ possible location patterns.

If in a more general case the optimal structure of processing industry is to be determined given the spatial distribution of supply *and* demand, the assembling and distribution problem cannot be solved independently. It is necessary to investigate the optimal assignment of the markets for every pattern by means of a transshipment model.

If quantities are given for each market, the model can be solved by the well-known algorithm for transportation models and has the following algebraic structure (Orden [66]).

Construct the feasible flow which minimizes

$$\sum_j \sum_i \sum_k m_{ij} \cdot t_{ij} + m_{jk} \cdot t_{jk} \tag{55}$$

subject to
the supply constraints:

$$\sum_j m_{ij} = m_i \tag{56}$$

the demand contraints:

$$\sum_j m_{jk} = m_k \tag{57}$$

the intermediate node contraints:

$$\frac{1}{c} \cdot \sum_i m_{ij} = \sum_k m_{jk} \tag{58}$$

where

m_i = quantity of raw material produced by market i.

m_k = quantity of final goods demanded by market k;

m_{ij} = quantity of raw material shipped from market i to plant j.

m_{jk} = quantity of final goods shipped from plant j to market k;

t_{ij}, t_{jk} = variable costs of shipping and processing one unit from market i to plant j or from plant j to market k respectively;

c = input coefficient for raw material.

Since such a transportation model has to be solved for each possible combination of sites the computational effort increases considerably, thus limiting the practical applicability of this type of model.

If we introduce linear demand and supply functions instead of given quantities for each market, the problem of optimal assignment of the markets to a given pattern of locations becomes even more complicated. To overcome the problem of non-linear constraints which arise from introducing demand and supply functions, the concept of maximizing the consumer surplus can be used as described in section 3.

The application of this model is limited to small areas with only a few plants since a total enumeration of all possible or probable location patterns is necessary for the determination of the optimum solution. Therefore Manne [193] has described a dynamic programming technique which permits short-cutting the calculation procedure. This technique known as "steepest ascent one point move algorithm", abbreviated by Reiter and Sherman [198] to SAOPMA, is based on the fact that the total costs of processing and distribution form a hypercube with regard to all possible combinations of locations. The procedure starts with an arbitrarily chosen allocation pattern, i.e. a lattice point, and then, moving in one dimension to another lattice point, the process is continued until no further improvements can be made.

As Manne has pointed out SAOPMA can lead to a local optimum instead of to the global optimum. In such a case the process has to be reiterated with a different starting point.

Application of Linear and Non-linear Programming

Linear and non-linear programming procedures have been used so far only for the solution of sub-problems in which the markets for raw material and final goods are given. However there exist at least theoretical possibilities for solving the whole problem of planning the optimal location pattern by the application of linear and non-linear programming.

If predetermined quantities are given for each market, the problem can be formulated as a linear programming model. The transportation and processing activities for each potential location form the activities, the market quantities the restraints. Difficulties arise in considering non-linear degressive cost functions, which can be overcome by using integer programming if the total processing cost function is linear with a positive intersection with the ordinate. However, Vietorisz and Manne [206] have shown that the algorithm for this type of problem developed by Gomory [30] is not very useful for larger problems. Therefore the only way of introducing non-linear average processing costs in a linear programming model is to approximate the function by a sequence of steps of constant average costs, i.e. by a discontinuous "step-function".

Let d_{jn} be the costs of the n separated steps of constant average processing costs and e_{jn} the lower and f_{jn} the upper limit for which these constant costs are valid for the j-th plant. The linear programme can be formulated as follows:

Minimize

$$\sum_i \sum_j t_{ij} \cdot m_{ij} + \sum_j \sum_n \sum_i d_{jn} \cdot m_{ij} + \sum_j \sum_k t_{jk} \cdot m_{jk} \quad (59)$$

subject to

$$\begin{gathered} \sum_i m_{ij} = m_i \quad \text{and} \quad \sum_j m_{jk} = m_k \\ e_{jn} \leqslant m_{ij} > f_{jn} \\ \frac{1}{c} \cdot \sum_i m_{ij} = \sum_k m_{jk} \end{gathered} \quad (60)$$

There is little practical experience with regard to the efficiency of linear programming models applied to empirical problems of plant location. But, as far as we can see, the only difficulty is to determine the range of the steps by which the degressive average cost function is approximated and to estimate the deviation of the optimal pattern arising from this approximation.

If instead of given quantities for each market linear demand and supply functions are assumed, the problem changes to a quadratic programming model of the type described above. As in the case of the linear programming model, it is necessary to introduce all potential locations as activities and to approximate their degressive average cost functions by steps of constant processing costs.

6.2 MODELS OF DECENTRALIZED LOCATION PLANNING

The models of decentralized location planning demonstrate the way in which an equilibrium is attained if the location planning is realized by autonomous firms lacking information concerning the equilibrium pattern. A problem restricted to one side of the market can be described as follows. The number of plants, the location and the volume of production for each firm are to be determined. Only the influence of the selling markets is examined. Each firm can buy any quantity of raw material at constant prices. The average cost function is degressive and includes a reasonable profit rate. The demand functions are linear. The differentiation of the prices set by a given plant corresponds with the transportation costs. The prices ex factory are determined so that costs equal returns for each firm. A set of potential locations is assumed. The process of the model can be outlined as follows. We start with a first basic solution where all potential locations are realized; the demarcation of the market areas and the prices can be obtained by means of the model of centralized location planning described above. Then it must be determined whether the different plants can undercut the prices of their competitors. The possibility of a firm's undercutting the prices of its competitors in market k depends on its capability to underbid the prices on other markets, since the cost curve is degressive and returns have to equal cost by assumption. Hence the degressive cost function requires a criterion for simultaneous consideration of all markets at which the prices of the competitors can eventually be undercut. This is done by calculating the differences between the market prices of the basic solution and the transportation costs between each market and the plant under consideration. The resulting differences are arranged in descending numerical order. Then the quantity of each market is attached to the corresponding difference. By continuous accumulation of these quantities we form the cumulative function of the differences. This function must intersect the average cost function; if there is no intersection point the firm is eliminated. Usually two intersections will occur from which the one with the greatest quantity attached, i.e. with the lowest average costs, is selected. All markets are supplied by the plant under consideration for which the calculated differences are greater than the average costs at the point of intersection. The market prices are fixed by adding the corresponding transportation costs to the average costs, i.e. the price ex factory. In the next step another plant is considered; this process is reiterated until no further changes of the demarcation of the market areas is possible.

As the resulting equilibrium solution is not independent of the basic solution, it depends on the objectives of the calculation which basic solution is selected. The existing location pattern can be an important candidate for selection as the basic solution.

Another procedure of decentralized location planning can be formulated by SAOPMA, as Manne [193] has pointed out. The assumptions are the same as above except in respect of prices. Each plant tries to gain markets by setting the lowest price which is calculated by adding the transportation costs to the constant marginal costs. If the firm is lowest bidder at one market it will increase its price to the limit which is determined by the lowest price of its nearest competitor. The difference between this price and the marginal price is a contribution to its profit or to its fixed costs. The procedure of SAOPMA starts with a basic pattern determining two prices for each market, the marginal price of the lowest bidder and the realized price, i.e. the marginal price of the next lowest bidder. The profits and losses are calculated under these conditions. The possibilities for new firms to enter the market are investigated by calculating the profits and losses at locations not yet in use. The firm with highest profits entering the market is selected and the prices and profits are calculated for the resulting location pattern again. These steps are reiterated until no further firm can profitably enter the market and no firm already in the market is making a loss. By this procedure of decentralized location planning the monopolistic margin of locationally separated firms is considered. The assumption of pricing is such that each firm is maximizing its profit under regional competition.

The present authors do not know of any models considering both sides of the market.

7 PROBLEMS OF PRACTICAL APPLICATION

Though numerous investigations (some of which have been mentioned above) show the workability of the modern approach in principle, its common usage for the solution of practical policy problems faces a number of difficulties. Besides insufficient understanding of possible improvement in the basis for rational policy decisions, three mutually dependent problems of major technical importance may be mentioned:

(*a*) limited computer capacity;
(*b*) the aggregation problem; and
(*c*) the lack of adequate statistical data and the inappropriate processing of data which have been collected.

Limited computer capacity has been one of the most serious bottlenecks until a few years ago. At present one may say that most of these difficulties have been overcome by the rapid technical progress in the computer industry and by the development of the decomposition principle. Computer capacity still remains limited, of course, but the limitations arising from it are less serious than the aggregation and data problems.

7.1 THE AGGREGATION PROBLEM

The consideration of space as a system of points each of which represents a part of the total area, as used in all models described in the previous sections, rests on the assumption that the regions which are represented by a single point are homogeneous with respect to all exogenous variables defining a location (see figure 4 in section 2.1). Since this is not the case in most real situations, erroneous results may arise. The possible error originates mainly in the inadequate solution of two problems:

(*a*) the problem of separation of regions; and

(*b*) the problem of aggregation of all farms of a region into a limited number of decision-making units.

(*a*) *The problem of separation of regions*

Transportation costs change continuously with variation in geographical location in continuous equilibrium models. Hence volume and direction of commodity exchanges and the separation of surplus and deficit regions are determined in the equilibrium solution [5]. Using the discrete approach the regions have to be separated at an early stage in the model formulation. The transportation costs in the regions are assumed to be zero, the transportation costs between regions are actually transportation costs between points. As a consequence, transportation costs at the geographical borders of a region and hence between neighbouring farms change step-wise. The resulting inaccuracy concerning the flow of goods and the farm-gate prices at the borders of the regions may be tolerated. However, at least some of the more important results may be affected since the separation of surplus and deficit regions, and hence the flow of goods and the regional price differences in the equilibrium solution, depend partly on the separation of regions predetermined at an early stage of model formulation.

Careful prior investigations are necessary to reduce this source of error. In doubtful cases the effect of the separation of regions has to be assessed by repeated calculations with differently separated regions.

(*b*) *The aggregation error*

The aggregation error originates in the heterogeneity with respect to production capacity and production functions of farms which are aggregated into one decision-making unit. Recent research [21], [55], [57], [89] has shown that a certain degree of heterogeneity can be tolerated without causing aggregation error.

Correct aggregation is possible if the differences with respect to production capacities, input coefficients, and net revenues of the defined activities remain in the stability range of the corresponding optimum solution [57], [89]. Consequently one can derive the tolerable range of heterogeneity from the stability range of the optimum solution. We define the stability range of an optimum solution as the range in which capacities, input coefficients, and net revenues of activities may change without affecting the set of non-zero-variables in the optimum solution, although, of course the absolute values of these variables may change with a change of capacities or input coefficients.

It is obvious that models with strictly proportional differences of capacities and/or input coefficients and/or net revenues of activities contain identical sets of variables in the optimum solution though the absolute value of these variables is different. Hence strict proportionality in these data allows aggregation without error [21]. However the tolerable level of heterogeneity is greater. One can increase (decrease) the capacity of one factor of production within certain limits without causing an exchange of variables in the optimum solution. The critical relation at which a further change would result in an exchange of variables is given if degeneracy could arise at certain price relations; i.e., the critical relation is given if the maximum expansion of at least one activity is limited by at least two production factors at the same level.

For two factors of production c_1 and c_2 and two goods with the inputs $a_{11}a_{21}$ and $a_{12}a_{22}$, this will be the case if $\frac{c_1}{a_{11}} = \frac{c_2}{a_{21}}$ or $\frac{c_1}{a_{12}} = \frac{c_2}{a_{22}}$.

If we assume the input coefficients and the net revenue of activities to be homogeneous in the farms under consideration and if we assume $\frac{a_{11}}{a_{21}} < \frac{a_{12}}{a_{22}}$ we get the following groups in which farms can be aggregated without aggregation error:

Group 1: all farms for which $\frac{c_1}{c_2} < \frac{a_{11}}{a_{21}}$

Group 2: all farms for which $\frac{a_{11}}{a_{21}} < \frac{c_1}{c_2} < \frac{a_{12}}{a_{22}}$

Group 3: all farms for which $\frac{a_{12}}{a_{22}} < \frac{c_1}{c_2}$

Similar arguments hold for the relations of net revenues p_1 and p_2 and input coefficients.

The critical values for the relation of net revenues p_1 and p_2 are given by:

$$\frac{p_1}{a_{11}} = \frac{p_2}{a_{12}} \text{ and } \frac{p_1}{a_{21}} = \frac{p_2}{a_{22}}$$

and consequently the critical relations for the input coefficients are given by

$$\frac{c_1}{a_{11}} = \frac{c_2}{a_{21}}; \quad \frac{c_1}{a_{12}} = \frac{c_2}{a_{22}}$$

$$\frac{p_1}{a_{11}} = \frac{p_2}{a_{12}}; \quad \frac{p_1}{a_{21}} = \frac{p_2}{a_{22}}$$

The critical relations which limit the tolerable level of heterogeneity are given in a linear model with m rows and n columns by the following relations:

$$
\begin{array}{ll}
\dfrac{c_1}{a_{1j}} = \dfrac{c_i}{a_{ij}} & \begin{array}{l} i = 2, 3 \ldots m \\ j = 1, 2 \ldots n \end{array} \\
\dfrac{c_2}{a_{2j}} = \dfrac{c_i}{a_{ij}} & \begin{array}{l} i = 3, 4 \ldots m. \\ j = 1, 2 \ldots n \end{array} \\
\vdots \quad \vdots & \\
\dfrac{c_{m-1}}{a_{(m-1)j}} = \dfrac{c_m}{a_{mj}} & j = 1, 2 \ldots n
\end{array}
\tag{61}
$$

and

$$
\begin{array}{ll}
\dfrac{p_1}{a_{i1}} = \dfrac{p_j}{a_{ij}} & \begin{array}{l} i = 1, 2 \ldots m \\ j = 2, 3 \ldots n \end{array} \\
\dfrac{p_2}{a_{i2}} = \dfrac{p_j}{a_{ij}} & \begin{array}{l} i = 1, 2 \ldots m \\ j = 3, 4 \ldots n \end{array} \\
\vdots \quad \vdots & \\
\dfrac{p_{n-1}}{a_{i(n-1)}} = \dfrac{p_n}{a_{in}} & i = 1, 2 \ldots m
\end{array}
\tag{62}
$$

where

x_j = activities $(j = 1, 2 \ldots n)$;
p_j = net revenues of activities; $(j = 1, 2 \ldots n)$;
c_i = capacities $(i = 1, 2 \ldots m)$;
a_{ij} = input of factor i for the activity j. $(i = 1, 2 \ldots m)$ $(j = 1, 2 \ldots m)$;

The number of groups z which one has to form if one wants to avoid the aggregation error completely depends on the critical relations x in the following way:

$$z = 2^x$$

The number of critical relations depends on two factors:

(i) the heterogeneity of the single farms; and

(ii) the number of activities and capacities to be distinguished in the farm models.

If the farms to be aggregated show disproportional differences with respect to capacities, input coefficients, and net revenue, the maximum possible number of groups z is given by:

$$z = 2^{\frac{1}{2}nm\,[(m-1) + (n-1)]} \tag{63}$$

where

n = number of activities;

m = number of capacities (rows).

If there are only disproportional differences with respect to the capacities, (63) reduces to:

$$z = (n+1)^{\frac{1}{2}m(m-1)} \tag{64}$$

The maximum number of possible groups has been derived under the assumptions that all input coefficients are greater than zero and that the m capacities are mutually independent. This number will be reduced by the number of input coefficients which are zero and by the number of capacities which are dependent for technical reasons such as seasonal labour capacity of the permanent workers or crop rotation restrictions.

However the number of groups which have to be formed to avoid the aggregation error completely still remains prohibitively high in most practical cases. Thus a certain aggregation error seems unavoidable in spatial equilibrium analysis.

Several ways have been discussed in the literature for reducing the aggregation error:

(i) Aggregation of activities and capacities at the farm level [74]. This is a very efficient way to reduce the number of groups, since the latter increases rapidly with the number of farm level activities and capacities. However aggregation at the farm level requires prior judgments and may reduce the value of the final results. In practical cases one has to look for a compromise between reduction in the aggregation error and limitations on the interpretation of the equilibrium solution resulting from aggregation at the farm level.

(ii) Classifying farms with respect to the most important factor relations only. Sheehy and McAlexander [73] have shown that the aggregation error can be reduced considerably if one groups farms according to this criterion.

(iii) Reduction of capacities in the aggregate. The aggregation error resulting from adding the respective capacities of single farms originates in the assumption that all factors are perfectly mobile between the farms which are contained in the aggregate. The possible level at which factor capacities will be exhausted will be overestimated, and hence the production of the most profitable goods tends to be overestimated too, while the production of the less profitable goods tends to be underestimated.

This error will be reduced if one takes into account in the aggregate only that part of the capacities which is used in the optimum solution in the average of all farms [29]. The average use of capacities in the optimum solution has to be determined from a sample of farms.

(iv) Systematic aggregation of related groups. Lee [55] has shown that systematic aggregation of related groups can reduce the aggregation error to a minimum.

None of these ways is ideal. Each of them is a compromise whose applicability depends on the special conditions of the subject under study. Since we have to live with the aggregation error, it seems necessary to ask which of them is the "smallest evil" under various conditions, assuming we wish to obtain results for spatial equilibrium analysis, which might in the future become one of the most valuable basic tools for the formulation of rational agricultural policy in many countries of the world.

7.2 DATA REQUIREMENTS

The field of application of the models described in previous sections is largely determined by the available data. [134], [157], [166].

Definitions 2 and 3 of a location in figure 4 indicate the data requirements for interregional analysis. Following definition 2 and the description of models in section 4, we see that the same data are needed on the production side as are required for the calculation of the optimum at the farm level (factor capacity, potential activities and corresponding input-output coefficients, variable costs, etc.). Following definition 3 and the description of models in section 3, we observe that the same data are required on the production side as are necessary to determine regional supply functions. Data which permit the determination of regional demand functions and of transportation costs are needed in both cases.

Hence using the activity analysis approach, available information must allow

(*a*) the formulation of the existing and potential activities on the "regional farm" or "group farms" and the determination of the input coefficients, the gross returns and the variable costs;

(*b*) the determination of factor capacity of the regional farm or of group farms sub-divided by the capacities which are derived from the available land, labour and capital.

Using the supply-demand model approach the same data are needed if "normative" supply functions are to be used. If empirical supply functions are to be used, regional time series on the relevant production and input data and on prices are needed. In general two types of statistics are available for the determination of these data in most countries:

(*a*) Statistical data on current production (yields per land unit, acreage of crops, number of animals, etc.) and current inputs (fertilizer, seed, etc). They are usually collected on a yearly, quarterly or monthly basis.

(*b*) Statistical data on the farm structure with emphasis on size, the structure of the labour force, supplies of durable capital goods, and distribution of production between farm groups. These data are usually collected at an interval of several years and are here called "structural data" for simplicity.

Usually none of these categories meets the requirements mentioned above. The present statistical situation in most countries of the world is characterized by:

(*a*) an insufficient subdivision of the national economy into regions;

(*b*) a lack of co-ordination between current statistics and "structural statistics"; and

(*c*) limited collection of data at a regional level.

(*a*) *Division into regions and co-ordination of statistics*

The maximum size of a region is limited by the requirements of the approximate homogeneity of price-structure (see section 7.1) from the viewpoint of theory. The division of regions with homogeneous prices but heterogeneous natural conditions or factor relations into subregions can be replaced by separation of farm groups, provided that all data needed are collected on the basis of these groups.

Statistical data on production and current inputs are usually available only for administrative units (countries, states, shires, counties, etc). In most countries, with the exception of the centrally-planned economies, the amount of data available follows the general rule: the smaller the region the more incomplete is the statistical information. Administrative units for which the necessary minimum information from current statistics is available are more or less heterogeneous at least with respect to the natural environment and factor relations.

Statistical information on structural data is available for administrative units as well as for farm groups in at least some countries. The subdivision of the total number of farms into groups on the basis of farm systems and/or farm size aims at separation of relatively homogeneous "subregions" within the heterogeneous administrative units. The ideal number and hierarchy of groups to be distinguished depends of course on the size and extent of heterogeneity of the administrative units. In those with heterogeneous natural conditions and farm sizes, a subdivision according to natural conditions and factor relations is usually required to achieve a minimum of homogeneity.

In most countries the homogeneity required is not achieved because very often only one criterion (e.g. farm size *or* farm system) is used for the subdivision of groups while two or more criteria (e.g. farm size *and* farm system) are needed. If two criteria are used they are very often used in parallel instead in a hierarchic order. Moreover current statistics and structural statistics are often insufficiently co-ordinated because, as noted earlier, current data on production and non-durable inputs are available only for relatively heterogeneous aggregates.

(*b*) *Minimum requirements of statistical information*

In order to construct a descriptive interregional model which shows the regional structure of production and the interregional flow of goods and services as a starting point for explanatory and decision analysis, the following statistical information must be available as a minimum programme:

(i) From current agricultural statistics:

(*a*) yields per acre and area of crops;

(*b*) variable inputs which determine the variable costs of at least the major crops;

(*c*) numbers of animals and yields per head;

(*d*) use of feed concentrates per animal in different livestock enterprises; and

(*e*) the demand for agricultural goods.

(ii) From structural or current agricultural statistics: all inputs which determine the production capacity of the regional farms or the farm groups (land, labour, water resources, capital goods). In particular the most important machinery should be known in order to estimate the level of mechanization and hence the input coefficients for labour which usually cannot be observed directly.

(iii) From trade and traffic statistics: the flow of goods between regions and the transportation costs.

Most countries' statistics lack at least some if not all of this information on a regional basis except in countries with centrally-planned economies. The determination of descriptive interregional models (interregional input-output models) is therefore very difficult in the western world and has been limited to a few very exceptional cases, whereas in eastern countries it has become quite a common tool of planning [62], [39].

However the spatial equilibrium models described in sections 3 and 4 can be formulated with less direct statistical information since their character, which is at least partially "normative", permits the use of data which are derived from scientific investigations and book-keeping results. In most of the studies mentioned in previous sections this has been done for the determination of many inputs, especially labour and feed inputs.

For the implementation of models on the basis of this kind of information it is useful, too, to check the consistency of all data using regional balancing computations of the input-output type. For this purpose the formal framework of the spatial equilibrium model as described above might be used, binding the production activities to the levels observed in the base period.

However the necessary improvement of statistics is not only a problem of expanding the collection of data but of the organization of its presentation. The existing custom in most countries of presenting all data in the form of tables is the main reason for the high labour input and costs on the data collection side as well as the data use side.

The use of magnetic tapes for the storage of data in the central statistical offices would save labour and money for the presentation of data and for its use, since at present the transformation of data from tables to punch cards or magnetic tape requires the highest proportion of labour in interregional analysis.

8 EVALUATION OF RESULTS

Following the classification of previous sections three major groups of models can be distinguished:

(*a*) standard equilibrium models using demand and supply relations (section 3);

(*b*) activity analysis models using production functions, factor capacities and demand relations (section 4); and

(*c*) dynamic (recursive) models using production functions, factor capacities, behaviour restrictions and demand relations (section 5).

All model types contain normative and positive elements which determine the interpretation of results. Table 2 shows the combination of normative and positive elements in the various model-types.

TABLE 2

Assumptions on the behaviour of production distribution and demand in interregional models

Model Type	Production	Distribution	Demand	Interpretation of results
Equilibrium models on the basis of production models.	Rational	Rational	Given by empirical investigations.	Rational (minimum cost) satisfaction of demand.
Equilibrium models on the basis of:				
(a) normative supply functions.	Rational	Rational	Given by empirical investigations.	Rational (minimum cost) satisfaction of demand.
(b) empirically determined supply functions.	Given by empirical investigation.	Rational	Given by empirical investigations.	Equilibrium solution for empirically given supply and demand functions under the assumption of rational distribution.
Dynamic (recursive) models . .	Rational subject to empirically determined behaviour restrictions.	Rational	Given by empirical investigations.	Rational (minimum cost) solution subject to empirically determined behaviour restrictions.

Standard equilibrium models using normative supply functions and activity analysis models contain only normative elements with the exception of the demand functions which are empirically determined as in all other models. Hence the interpretation of the results is normative too. The solution indicates the most rational satisfaction of demand on the basis of given production techniques and assumed factor mobility. The minimum cost solution is of course identical with the equilibrium solution under the assumption of profit maximizing behaviour of producers in competitive markets. A different interpretation of results arises from the consideration of various time horizons and hence from different assumptions on the mobility of factors and the change of production technique.

If one assumes all inputs to be fixed and hence the production to be given the solution indicates the most rational distribution of the given production. If only current inputs are assumed to be variable and the production technique given, results indicate the most rational short-run solution.

The results indicate the most rational long-run solution at a point in time not explicitly determined if one assumes perfect factor mobility, predicted changes of factor price relations based on earnings from employment opportunities outside agriculture, and projected changes in production technique. However the results are not predictions despite the use of predicted or projected values for some exogeneous variables. The predictive potential of these models is very limited because of their normative character. The results may at best indicate the direction of future changes but do not permit any statement of the time path of change. The time path is considered in dynamic models. The recursive models mentioned in section 5 determine a time path of production using normative and empirical elements. For each point in time t_i which is one element of a sequence of points $t_1, t_2, \ldots t_n$ a rational solution is determined subject to behavioural restrictions which limit the possible change of production and factor input between t_{i-1} and t_i. Investments may or may not be restricted by capital constraints arising from borrowing, saving or consumption behaviour or from direct behavioural constraints which limit the amount of investment between two points in time. The profitability of investments is determined under similar assumptions to those used in static models. Constant price expectations are assumed for the lifetime of invested capital.

By the solution of recursive models it is not ensured that the resulting time-path of variables, especially of capital formation, is rational from a long-run viewpoint. The problem of optimal allocation of resources in time would be solved in principal by dynamic models, in which the interdependences between time periods are taken into account. In accordance with the two branches of macroeconomic growth theory, two assumptions in respect to saving (investment) behaviour could be introduced: one could start out either from given (empirically estimated) saving (investment) functions as in neoclassical growth theory, or from a utility function which expresses time preferences with respect to consumption as in the theory of optimal growth. In the second case the savings ratio is a variable which has also to be optimized. A comprehensive investigation of these problems in the context of disaggregated regional models has not yet been undertaken to the knowledge of the present authors, though some attempts have been made, e.g. [91].

Standard equilibrium models on the basis of empirically determined supply functions contain normative elements only with respect to distribution. Hence their results show an equilibrium solution for empirically determined relations between price and supply and price and demand under the assumption that distribution is organized in the most rational way. The short- or long-run character of the solution is determined by the short- or long-run character of the estimated supply and demand relations.

Only the recursive models and the standard equilibrium models based on empirical supply functions seem to promise realistic and useful results following the modern theory of science and the fashionable emphasis on behaviourism in some fields of economic theory. However one must not overlook the difficulties still confronting the determination of results, though the use of these models permits in principle explaining and predicting actual changes for production. Some of these difficulties have been mentioned in section 5.2. Others have been discussed elsewhere in the context of supply analysis [64], [90]. At present only pragmatic solutions are known. Their applicability in interregional models remains to be investigated. However normative models will keep their place beside positive (empirical) models, even if the difficulties which presently limit the application of empirical models can be overcome. Theoretical considerations do not favour the exclusive use of positive models, but rather the complementarity of positive and normative models, for policy analysis.

The formulation of rational policy consists of at least two steps:

(*a*) the formulation of quantitative goals for production and inputs which satisfy given welfare objectives in the most rational way; and

(*b*) the determination of policy measures to achieve the ends determined under (*a*) in the most rational way.

The activity analysis models using production functions can provide valuable help in decision-making on the quantitative goals for production and factor inputs. Their application will most certainly indicate that a redistribution of regional activity is desirable from the viewpoint of rational satisfaction of demand, if one assumes perfect or even limited factor mobility. This might conflict with other policy goals which consider, for example, a distribution or remuneration of production factors, especially labour, to be desirable which is not the most efficient one from a purely economic point of view.

However the activity analysis models are flexible. One can take into account almost any set of consistent goals by proper formulation. Furthermore the development of computer technique allows a wide range of simulation of goals and assumptions with relatively small additional input if one basic model has been established. Hence they can indicate the most rational solution to alternative sets of consistent goals. However they do not indicate the effect of policy measures in time to achieve the desired goals. Hence they contribute very little to the problem of rational choice of policy tools.

A rational choice of policy tools to achieve established goals can only be determined by empirical models described in sections 3 and 4. Since the application of these models at an interregional level still faces great difficulties, one might think of a hierarchic policy planning process at present. Long-run regional production goals might be established by simultaneous consideration applying activity analysis models. Independent regional development plans, taking into account the policy measures to achieve them, might be established on the basis of these goals. The consistency of the regional development plans might ultimately be considered by simultaneous consideration of the whole economy.

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