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SUPPLY AND DEMAND STABILIZATION AND ECONOMIC WELFARE IN AGRICULTURE: SOME DYNAMIC CONSIDERATIONS

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Much previous work on agricultural stabilization has emphasized price stabilization. Stabilization is given a different meaning here. Additive error terms are attached to linear supply and demand functions, and the effects on consumers' and producers' surpluses of reducing these error terms are studied. That is, the error terms are assumed to have their variances reduced, and the expected values of the surpluses are derived and compared with their expected values corresponding to the original error term variances. The results are not favourable, and suggest that, with the particular criterion used here, stabilization of supply and demand functions is not as desirable as generally believed. Further results, allowing for shifts in the functions and the introduction of risk aversion are less unequivocal, and require empirical knowledge for conclusions to be reached.

1 STABILIZATION ONCE AGAIN

The meaning, methods and implications of agricultural stabilization remain at the centre of economic and political thinking [3]. Turnovsky's 1974 paper [6] is a major contribution in the research, and opens up the dynamic aspects of the problem. His emphasis is on price stabilization in a one commodity, partial equilibrium framework, whereas we wish to emphasize another aspect, which may be termed the stabilization of the economic environment. To be as brief as possible, the aim is to concentrate on the effects of activities other than direct price stabilization often seen as desirable in the agricultural industry.

The mathematics is quite straightforward, with the only complication being despatched to an appendix. We believe the accompanying verbal arguments and justification of the problem adequately complement the algebra.

2 TIME DEPENDENCE AND EQUILIBRIUM

To introduce the analysis, a word is needed on the lagged supply response and market equilibrium. The traditional analysis of the price stabilization problem, following Waugh [7], is to compare the expected producers' and consumers' surpluses, where expectation is defined over price, or

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function parameters, with the value of these benefits at expected price or expected parameter levels. Turnovsky references the literature and there is no need to repeat the details here.

However, the textbook world in which an instantaneous movement in the demand function is matched by an instantaneous movement along the supply curve to a new equilibrium price is not a sufficient approximation to the agricultural situation, and so a more realistic depiction of supply behaviour is required. Instantaneous adjustments of consumption to changes in price are assumed as usual.

Without describing the market or any time details, we utilize the widely adopted distributed lag supply function (1) [5], which is typically used for estimates of short and long run supply elasticities [2]. The function is

$$(1) \qquad x(t) = a + b[\lambda p(t-h) + \lambda(1-\lambda)p(t-2h) + \lambda(1-\lambda)^2 p(t-3h) \ldots] + v(t),$$

where $x(t)$ denotes the amount of commodity supplied at time t, $p(s)$ is the market price at time s ($s = t - h, t - 2h \ldots$), h is a fixed time interval, a, b and λ are constants ($a \geqslant 0$, $b > 0$, $0 < \lambda \leqslant 1$) and $v(t)$ denotes a stochastic disturbance. Assume that the family of random variables $v(t)$ is such that the function mapping t onto $v(t)$ can be differentiated.[1]

The term $v(t)$, with $E\ v(t) = 0$, is added to account for all those factors, technical and economic, which prevent suppliers, in aggregate, from exactly realizing their production plans. Included in $v(t)$ would be the prices of other commodities, both inputs and outputs, whose variation during the production period of $x(t)$ causes plans to change. For example, an unforeseen increase in seasonal wage rates at harvest time affects final production. Weather causes divergences between plans and reality, and biological pests destablize supply for any given price level.

The focus of attention here is the effect on economic welfare of those efforts designed to reduce the variance of $v(t)$. It is true that most technological changes move the supply function outward, and their secondary role is that of stabilizing the new supply function. Irrigation practices have this role. Nevertheless, some technological changes and economic factors only stabilize supply around a given function. A machine able to function in very wet conditions may be less effective on dry ground than a regular one, but enables a farmer to more fully realize his plans under all conditions. Government policy in stabilizing non-farm prices adds certainty to farmers' activities. Altering traditional planting times enables losses due to unusually early cold weather at flowering times to be reduced, but could reduce yields below those in normal weather years.

Later, the stabilization of demand is discussed. The forthright results in 3 are weakened in 4, at the cost of requiring empirical information to be usable.

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[1] Appendix 1 contains details on the admissibility of this.

We now return to (1), which can be written as

$$(2) \qquad x(t) = a\lambda + (1 - \lambda)\, x(t - h) + b\lambda p(t - h) + v(t) - (1 - \lambda)v(t - h).$$

The demand function at time t is described by

$$(3) \qquad p(t) = \alpha - \beta x(t) + u(t), \text{ or } x(t) = \alpha/\beta - p(t)/\beta + u(t)/\beta; \quad \alpha, \beta > 0.$$

It is plausible that $u(t)$, a disturbance term, be added to the demand function. Without loss of generality, we assume that $Eu(t) = 0$. And just as attempts are made to reduce the variation around the supply function, it is reasonable to describe a good deal of economic activity as an attempt to reduce the variance of $u(t)$. Efforts to smooth out seasonal demand fluctuations are of this nature.

One effort on the part of advertising firms is to reduce the elasticity of demand for their product. We have shown in particular cases (requiring linear demand functions) that the average elasticity of demand (the expectations being taken over $u(t)$) is greater than the elasticity of demand evaluated at the mean of $u(t)$, for any price. Hence it may be in the interest of advertisers to reduce the variation in consumption for any price level, even without attempting to shift the demand curve. Stabilization of complementary and competitive commodity prices, as part of an overall price stabilization policy, reduces the variance of $u(t)$.

As a result, it is believed that consideration of the effects of reducing the variance of $u(t)$ is a useful exercise. The prospect of moving the demand function for many agricultural commodities is unfavourable, but we will consider the implications of this in 4 as it entails no additional calculations.

Equation (3), with (2), yields

$$(4) \qquad x(t) = A + Bx(t - 1) + R(t),$$

where

$$(5) \qquad A = \lambda(a + b\alpha), B = 1 - \lambda - b\beta,$$

$$(6) \qquad R(t) = b\lambda u(t - 1) + v(t) - (1 - \lambda)v(t - 1),$$

and $h = 1$, the usual situation. Iteration of (4) yields

$$(7) \qquad x(t) = A\frac{1 - B^t}{1 - B} + B^t\, x(0) + \varepsilon(t),$$

where $x(0)$ is an initial supply condition, and

$$(8) \qquad \varepsilon(t) = R(t) + BR(t - 1) + \ldots + B^{t-1} R(1).$$

3 WELFARE AND STABILIZATION

(i) CONSUMERS

Corresponding to the demand function (3) is the consumers' surplus at quantity x and time t:

$$(9) \qquad CS(t) = \tfrac{1}{2}\beta x^2(t) = \tfrac{1}{2}\beta[K + \varepsilon(t)]^2,$$

where

$$(10) \qquad K = A\frac{1 - B^t}{1 - B} + B^t x(0).$$

Note that $u(t)$ in (3) does not appear in (9), because along with α, it is eliminated in the integration and calculation leading to (9).

Define $t = 0$ to represent the present time period, and consider the consumers' surplus at some future time t.

Suppose that the random variables $u(0), \ldots, u(t-1)$ and $v(0), \ldots, v(t)$ are replaced by the random variables $u'(0), \ldots, u'(t-1)$ and $v'(0), \ldots, v'(t)$ such that

$$(11) \qquad \text{var } u'(j) \leqslant \text{var } u(j) \text{ and } E[u(j)] = E[u'(j)],\ j = 0, \ldots, t-1$$

and

$$(12) \qquad \text{var } v'(k) \leqslant \text{var } v(k) \text{ and } E[v(k)] = E[v'(k)],\ k = 0, \ldots, t.$$

In particular, if nothing is done to reduce the randomness of supply behaviour until the last time period t, the following results remain true. The replacements in (11) and (12) distil the stabilization procedures outlined in 2.

Denote by $\varepsilon'(t)$ the expression (8) evaluated at $u'(0), \ldots, u'(t-1)$ and $v'(0), \ldots, v'(t)$. Assume that the random variables $u(0), \ldots, u(t-1)$, $v(0), \ldots, v(t)$ are mutually independent, as well as $u'(0), \ldots, u'(t-1)$, $v'(0), \ldots, v'(t)$.[2] Therefore, as $\varepsilon(t)$ is a linear function of $u(0), \ldots, u(t-1)$, $v(0), \ldots, v(t)$, a decrease in the variances of any subset of these random variables implies a decrease in the variance of $\varepsilon(t)$. That is,

$$(13) \qquad \text{var } \varepsilon'(t) \leqslant \text{var } \varepsilon(t).$$

It follows from (11) and (12) that

$$(14) \qquad E[\varepsilon(t)] = E[\varepsilon'(t)].$$

Denote by $CS'(t)$ the consumers' surplus evaluated at $\varepsilon'(t)$. Comparison of the expected value of $CS'(t)$ with the expected value of $CS(t)$ enables the benefits or otherwise of stabilization to be seen.

Letting $\triangle CS(t)$ denote the relevant difference, it is found that

$$(15) \qquad \begin{aligned} \triangle CS(t) &= E[CS'(t)] - E[CS(t)] \\ &= E[CS'(t) - CS(t)] \\ &= \tfrac{1}{2}\beta E[\varepsilon'^2(t) - \varepsilon^2(t)] = \tfrac{1}{2}\beta[\text{var } \varepsilon'(t) - \text{var } \varepsilon(t)] \leqslant 0, \end{aligned}$$

from (13) and (14), remembering that $\beta > 0$.

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[2] The mutual independence assumptions required for the simplicity of the results may not be quite valid, on the supply side more so than the demand side. Weather and biological factors are the major causes of randomness in supply response for given price levels, and these can be safely assumed to be independent through time, particularly if the commodity is produced over a wide geographical area. Prices of other commodities entering $v(t)$ and $v'(t)$ may be related through time on an individual basis, but this dependence would be reduced in the aggregate.

Therefore, in the situation where producers obey the lagged supply response function (1), a decrease of the variability in supply and/or demand functions would *harm* consumers.

(ii) PRODUCERS

The neat form of the demand function (3) enabled the consumers' benefit to be easily derived. On the supply side, however, (2) is not sufficient to derive the analogous producers' surplus. But (2) may be manipulated in a meaningful way to derive a timeless normative supply function, from which producers' surplus can be calculated.

Function (2) may be thought of as resulting from the response of farmers to an anticipated price, which is calculated from previous experience [5, p. 261]. The response to this anticipated price depends on how suppliers believe themselves best able to maximize profits at time t, being in possession of this anticipated price. That is to say, suppliers, as profit maximizers in the first instance,[3] are able to calculate how much *should* be supplied at *all* anticipated price levels. It is the production time lag h which necessitates the use of an anticipated price, but if this lag h were reduced to zero, the anticipated and actual price would coincide. Then, the relationship between production and market price would contain $x(t)$ and $p(t)$ alone.

To see this, observe that (2) yields

$$\text{(16)} \qquad \frac{x(t+h)-x(t)}{h} = (1-\lambda)\frac{x(t-h)-x(t)}{-h} + b\lambda\frac{p(t-h)-p(t)}{-h}$$
$$= \frac{v(t+h)-v(t)}{h} - (1-\lambda)\frac{v(t-h)-v(t)}{-h}.$$

Taking the limit of both sides of (16) as $h \to 0$,

$$\text{(17)} \qquad \dot{x}(t) = b\dot{p}(t) + \dot{v}(t),$$

the dot representing differentiation with respect to time. This, on integration with respect to t, yields the timeless (i.e., non-lagged) supply function

$$\text{(18)} \qquad p(t) = \psi + \delta x(t) + \delta[v(0) - v(t)],$$

where

$$\text{(19)} \qquad \psi = p(0) - \frac{x(0)}{b} \text{ and } \delta = \frac{1}{b}.$$

The constant a does not occur in (19) as the initial supply level $x(0)$ includes a, from (1).

It follows from (3) and (7) that

$$\text{(20)} \qquad p(t) = \alpha - \beta\, B^t\, x(0) - \beta A\,\frac{1-B^t}{1-B} - \beta\varepsilon(t) + u(t).$$

[3] *See* the next section where risk aversion is introduced.

Using (20), the producers' surplus is given by

$$\text{(21)} \qquad PS(t) = \tfrac{1}{2}\,\delta x^2(t) = \tfrac{1}{2}\delta[p(t) - \psi - \delta(v(0) - v(t))]^2$$
$$= \tfrac{1}{2}\,\delta[M + F(t)]^2,$$

where

$$M = \alpha - \psi - \beta B^t\, x(0) - \beta A\,\frac{1 - B^t}{1 - B} = \text{constant and}$$

$$\text{(22)} \qquad F(t) = u(t) - \beta\varepsilon(t) - (v(0) - v(t)).$$

It is clear that the random variable $F(t)$ is a linear function of the variables $u(t), u(t-1), \ldots, u(0)$ and $v(t), v(t-1), \ldots, v(0)$, from (6) and (8).

Define $F'(t)$ analogously to $\varepsilon'(t)$, noting now the inclusion of $u(t)$. It will be seen that if efforts are made to reduce the variances of $u(t)$ and $v(t)$ only in the final future period t, the following conclusions remain true.

Corresponding to (13) and (14) are

$$\text{(13')} \qquad \operatorname{var} F'(t) \leqslant \operatorname{var} F(t)$$

and

$$\text{(14')} \qquad E[F(t)] = E[F'(t)].$$

Defining $PS'(t)$ and $\triangle PS(t)$ in the same way as $CS'(t)$ and $\triangle CS(t)$, it is found that, using (21),

$$\text{(23)} \qquad \triangle PS(t) = E[PS'(t)] - E[PS(t)]$$
$$= E[PS'(t) - PS(t)]$$
$$= \tfrac{1}{2}\,\delta[\operatorname{var} F'(t) - \operatorname{var} F(t)] \leqslant 0,$$

as $\delta = 1/b > 0$.

Again, the result is not encouraging. If the economic environment is stabilized, producers lose, in terms of expected producers' surplus. It should be mentioned that the stabilization we use does not imply that price is constant—if it were, the foregoing analysis has nothing new to say. But the dynamic nature of (1), and $v(t)$ not always being zero mean that $p(t)$ and $x(t)$ depend explicitly on time.

Before attempting an intuitive explanation of these results, two matters raised in passing previously must be discussed.

4 SUPPLY AND DEMAND SHIFTS, RISK AVERSION AND STABILIZATION

(i) SUPPLY AND DEMAND SHIFTS

Suppose now that as the result of some activities the economic environment is stabilized, as shown in (13), (14), (13') and (14'). In addition, assume that some or all of these activities shift the supply and demand functions (1) and (3) such that a and α are replaced by a' and α'. Consequently, in (10) K becomes K' and in (22), M becomes M'.

As a result, expressions (15) and (23) are replaced by

$$(15') \qquad \triangle CS(t) = \tfrac{1}{2}\,\beta[(K'^2 - K^2) + (\text{var}\ \varepsilon'(t) - \text{var}\ \varepsilon(t))].$$

and

$$(23') \qquad \triangle PS(t) = \tfrac{1}{2}\,\delta[(M'^2 - M^2) + (\text{var}\ F'(t) - \text{var}\ F(t))].$$

The signs of (15′) and (23′) can only be determined with empirical aid. Such complications are similar to those faced by Turnovsky at one stage in his discussion of the benefits of price stabilization [6, p. 710]. The information required may not be as difficult to obtain as at first thought. Statistical estimates of supply and demand functions are often available, and if not, estimates of elasticities may be, which can be used to provide linear approximations of the functions over some range. On the other hand, the evaluation of the second term in the squared brackets in (15′) and (23′) is more difficult.

(ii) RISK AVERSION

The analysis of the effect on producers' surplus was carried out in the belief that farmers consider expected returns alone as their welfare criterion. This is widely believed to only approximate the truth. To account for the aversion to risk experienced by many decision-makers, we introduce a simple welfare function, which contains expected producers' surplus and the variance of producers' surplus as its arguments. To be manageable, assume the function is linear.

Precisely, let the welfare function W be

$$(24) \qquad W[E[PS(t)],\ \text{var}\ PS(t)] = E[PS(t)] - k\ \text{var}\ PS(t),\ k > 0.$$

By virtue of (21) and (23), if $F(t)$ in (22) is replaced by $F'(t)$, then (24) can be used to obtain

$$(25) \qquad \triangle W(t) = W(\text{var}\ F(t)) - W(\text{var}\ F'(t)) = \frac{1}{4b^2}\{[\text{var}\ F'(t) - \text{var}\ F(t) \\ [2b + k(\text{var}\ F'(t) + \text{var}\ F(t) - 4M^2)] \\ - 4Mk(u'_3 - u_3) - k(u'_4 - u_4)]\},$$

where $b = 1/\delta$ and u_n and u'_n denote the n-th moments of the random variables $F(t)$ and $F'(t)$ respectively.

Let us now assume that $F(t)$ and $F'(t)$ are normal variables, which is true if $u(0), \ldots, u(t), v(0), \ldots, v(t)$ are normal. Then, $u_3 = u'_3 = 0$, $u_4 = 3(\text{var}\ F)^2$ and $u'_4 = 3(\text{var}\ F')^2$, and (25) becomes

$$(26) \qquad \triangle W(t) = \frac{1}{2b^2}\,[\text{var}\ F'(t) - \text{var}\ F(t)]\,[b - k(\text{var}\ F'(t) + \\ \text{var}\ F(t) + 2M^2)].$$

If stabilization is possible, var $F'(t)$ − var $F(t) < 0$, and so producers' welfare will increase only if var $F'(t)$ + var $F(t) > \frac{b}{k} - 2M^2$, using the second square bracketed term in (26). This inequality is in a convenient form; as k approaches zero, the inequality must eventually be violated, implying that benefits are unobtainable, the result we obtained in the previous section, where $k = 0$. Conversely, as k increases, benefits are more likely to be obtained by reducing var $F(t)$ to var $F'(t)$, indicating the preference for smaller variation in output. Again, empirical aid is needed to test for benefits or otherwise.

5 INTERPRETATION AND CONCLUSIONS

How can these results be given an intuitive interpretation? The explanation which is felt most satisfactory lies in the very nature of our linear demand and supply functions.

From (9), it is seen that consumers' surplus increases with consumption, $CS(t) = \frac{1}{2}\beta x^2(t)$. Hence, $\frac{dCS(t)}{dx(t)} = \beta x(t) > 0$ and $\frac{d^2CS(t)}{dx^2(t)} = \beta > 0$.

That is, as consumption rises, the level of consumers' surplus rises at an increasing rate. Or, to put it another way, consumers are more than compensated by reductions in consumption by corresponding rises in consumption around any given level. The downward sloping linear demand function guarantees this. Hence, a variable level of consumption is preferable to a stable consumption. The same argument can be applied to producers, remembering the risk aversion qualifications.

It must be pointed out that the foregoing results relied heavily on the linearity of the functions. The conclusions in the last paragraph, for example, are incorrect for a constant elasticity of demand function, constant for all $x(t)$ and $p(t)$. Typically, however, our interest is upon only a small range of $x(t)$ and $p(t)$, for which linear approximations serve well, enabling the conclusions to remain intact.

The virtues of price stabilization have long been extolled by many involved in agriculture, and despite some mild views to the contrary [4][4], it remains a well respected objective of agricultural policy. Turnovsky's recent work adds to the favourable view of price stabilization.

By looking at agricultural stabilization as a set of policies not necessarily leading to price stabilization, but rather as an attempt to reduce the uncertainty of outcomes associated with any price level, a rather pessimistic set of results has been obtained. Attempts to stabilize the agricultural environment as we have described would appear at first sight to lead to price stability, but the dynamic nature of the supply function precludes this. As a result, there is a clear distinction between price stabilization *per se*, and attempts to stabilize prices by reducing production and consumption variability as we have described. These widely practised attempts may be worse than doing nothing at all in a dynamic framework, but, as shown in 4(ii), the existence of risk aversion can modify this conclusion.

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[4] The point made there is that, at times, prices should change markedly for some social goals. But in general, the argument is for price stability.

APPENDIX I

Consider the set X of all real functions defined on a set S. The random variables introduced in 2 may be thought of as elements of X if S is the set of elementary events.

Define the addition of elements of X and multiplication of elements of X by the real numbers in an ordinary way, i.e., if $x, y \varepsilon X$, $r \varepsilon R$, define $(x + y)s = x(s) + y(s)$ and $(r \,.\, x)s = r \,.\, x(s)$. Thus X is a linear space. Define the norm in X by $\| x \| = \sup_{s \varepsilon S} |x(s)|$.

Suppose now that v is a function defined in an interval (t_1, t_2) and having values in X. Thus v is a mapping from an open subset of reals into a normed linear space and so the limit

$$\lim_{h \to o} \frac{v(t + h) - v(t)}{h}$$

is well-defined. This limit may or may not exist. If it exists, it is the derivative of v at the point t and is denoted by $\dot{v}(t)$. Thus if $\dot{v}(t)$ exists, it is an element of X. In particular, if S is the set of elementary events, $\dot{v}(t)$ is a random variable itself. In 3(ii) the assumption that v is differentiable is required.

Formula (17) may be rewritten as

(17′) $\quad \dot{\psi}(t) = 0,$

where $\psi(t) = x(t) - b\, p(t) - v(t)$ is a random variable. Thus, (cf [1], p. 155), $\psi(t) = \text{constant} = \psi(0)$. That is, equality (18) holds.

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