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Gönül Doğan, M.A.L.M. van Assen,
Arnout van de Rijt and Vincent Buskens

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Gönül Doğan and M.A.L.M. van Assen, *Department of Methodology and Statistics, Tilburg University*
Arnout van de Rijt, *Department of Sociology, Cornell University*
Vincent Buskens, *Department of Sociology, Utrecht University*

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The Stability of Exchange Networks

Summary

This paper develops a formal model of exchange network stability that combines expected value theory (Friedkin 1995) with the economic literature on network dynamics. We identify stable networks up to size 8 for varying costs and investigate whether they are Pareto efficient and egalitarian. Only a very small number of networks are stable. Odd cycles and networks consisting of dyads and at most one isolate are the only egalitarian, efficient, and stable networks for a large cost range. We show that some of these results are generalizable to networks of any size and are independent of using expected value theory.

Keywords: Exchange Networks, Stability, Efficiency, Equity, Social Dilemma

JEL Classification: D85

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Address for correspondence:

Gönül Doğan
Department of Methodology and Statistics
Tilburg University
PO Box 90153
5000 LE Tilburg
The Netherlands
E-mail: g.dogan@uvt.nl

1. INTRODUCTION

An exchange situation can broadly be defined as a situation involving actors who have the opportunity to collaborate for the benefits of all actors involved. While exchange has been intensively studied in economics for more than a century (e.g., Coddington 1968; Edgeworth 1881; Young 1975), exchange entered the fields of social psychology and sociology only in the second half of the twentieth century. Homans (1958: 606) introduced the idea that 'social behavior in an exchange of goods, material goods but also non-material ones, such as the symbols of approval and prestige'. The conception of social behavior as exchange was also used by other prominent social scientists in the same time period, such as Thibaut and Kelley (1959) and Blau (1964). After these important works research on exchange as a model of any social behavior gained a prominent position in social psychology and sociology.

Since Stolte and Emerson (1977) and Cook and Emerson's (1978) seminal study, sociologists have focussed on the effect of social structures on outcomes of exchange. The basic idea of this research is that social behavior is shaped by the social relations in which it occurs, which are in return conditioned by the structures within which they are embedded (Willer 1999: xiii). Where social behavior is conceived of as exchange, the social relation is dubbed an 'exchange relation' and the structure is denoted an 'exchange network'. If two persons have an exchange relation, this means that both persons have the opportunity to exchange, but they need not to do so. If they do not have an exchange relation, they have no opportunity to exchange. These opportunities and restrictions to exchange arise naturally in many real-life situations. Two of the most common causes for the absence of an exchange relation between two persons are natural barriers and non-matching preferences. Examples of barriers are not knowing each other, or not being able to contact or meet each other. And two persons also might not have an exchange relation because one of them has nothing to offer that is valuable enough to the other. An exchange network is a set of persons together with their exchange relations with persons in this set. Figure 1 depicts a simple example of an exchange network of four players, the Line4. In this network person A has a possibility to exchange with B, B additionally has a possibility to exchange with C, and C also with D.

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FIGURE 1 ABOUT HERE

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In research on exchange networks in sociology two assumptions were commonly made, that we also make in the present study (see, e.g., Willer, 1999; special issue Social Networks June 1992; special issue Rationality and Society January 1997). First, an exchange relation is represented as an opportunity to split a common resource pool of size 24. Exchange occurs if two connected actors can agree on a division, if they do not agree they obtain no payoff. Second, players can only engage in one exchange, the so-called one-exchange rule. Applied to the Line4, the one-exchange rule implies, e.g., that B can exchange either with A or C, but not with both.

In almost all theoretical and empirical studies on exchange in sociology the social structure was the independent variable, i.e., the effect of the network structure on outcomes of persons on different positions in the network was studied. The main result of this research on exchange networks is that the network structure has a huge impact on what actors earn in their exchange relations (e.g., Willer 1999; special issue Social Networks June 1992; special issue Rationality and Society January 1997). Since different positions in the network lead to vastly different payoffs, there is an enormous potential for actors to change the network. Important questions are therefore how these networks evolve in the first place, and which exchange networks are stable or resistant to change. That is, in our study the structure of the exchange network is the dependent variable: what does the structure look like if actors have the opportunity to change with whom they have an exchange relation?

The question of how exchange networks evolve has received little attention in the literature (Kollock 1994). Only a handful of theoretical studies and, to our knowledge, not a single empirical study on the evolution of exchange networks has been carried out whereas there exist extensive experimental investigations of static exchange networks (Willer and Willer 2000:252). In these experiments, networks were exogenously determined by the experimenters. Hence, by fixing the network, one of the most powerful tools to enhance outcomes of exchange is ignored: negotiating changes in the network itself (Leik 1992:309). Therefore, a desirable step in expanding the theory on network exchange is to formally incorporate an actor's (person's) potential to manipulate (i.e., to delete and add) his links (exchange relations), thereby enhancing his bargaining power in subsequent exchanges, thus indirectly increasing his expected payoff.

In the present study we allow actors to manipulate their links in order to answer three research questions. The questions are:

(i) Which exchange networks are stable?

(ii) Are stable exchange networks efficient?

(iii) Are stable exchange networks egalitarian and are egalitarian exchange networks stable?

The notion of stability reflects that no actor in the network is willing to change his links; it can be considered as an equilibrium concept. We use Pareto efficiency as an efficiency measure: a network is considered Pareto efficient if there is no other network in which no actor earns less and some actors earn more. A welcome result would be that stable networks are also Pareto efficient. If no stable network is efficient at a given cost, it represents a social dilemma: the actors always end up in a social structure that is inferior to what they could have obtained in another structure, but none of the actors has an incentive to change one of his own links. Finally, egalitarian exchange networks are networks in which all actors earn exactly the same payoffs. A welcome result would be that stable networks are also egalitarian. If a stable network is not egalitarian, it represents an unfair situation because all actors in the exchange situation have identical properties.

The structure of our paper is as follows. Related research and the assumptions underlying our analysis are discussed in Section 2. We also motivate our selection of Friedkin's (1992, 1993, 1995) expected value theory (EVT) in order to answer research questions (i) to (iii). In Section 3, we introduce our model of network evolution, and our research questions are explicated. In Section 4, the results of our analyses are presented. The first part of Section 4 contains the results on networks up to size 8 obtained with EVT. The second part contains the theorems for networks of any size and independent of the theory or payoff allocation function. We conclude with a discussion in Section 5.

2. THEORETICAL BACKGROUND

The basic assumptions underlying our analysis are in line with those employed in the literature on exchange networks (e.g., Leik 1992; Markovksy et al. 1988, Willer and Willer 2000). It is assumed that the number of nodes in the network is constant and only directly linked nodes can engage in exchange. All resource flows are dyadic and the joint profit for any exchange is constant. All actors are involved in at most one exchange (the 1-exchange rule).

As a baseline to analyze the dynamics of exchange networks, we add the assumptions that actors have complete and accurate information about all network links and they maximize their own payoffs. We also assume that actors act as if they use the same theory to predict the consequences of adding and deleting links. The last assumption can also be phrased as: "Actors add and delete links and act upon it *as if* they were applying EVT of Friedkin (1995)"[1].

Many theories of exchange networks have been developed and tested in the last three decades; power-dependence theory (e.g., Cook and Emerson, 1978; Cook and Yamagishi, 1992), exchange-resistance theory (e.g., Skvoretz and Willer, 1993), a graph analytic theory using the graph-theoretic power index (GPI) (e.g., Markovsky et al., 1988), core theory (e.g., Bienenstock and Bonacich, 1992), optimal seek theory (Willer and Simpson, 1999), identity theory (Burke, 1997), Yamaguchi's (1996; 2000) rational choice model, expected value theory (e.g., Friedkin, 1992), and non-cooperative bargaining models (Berg and Panther, 1998; Braun and Gautschi, 2006). Four theories have received much more attention in the literature than the other theories (Willer, 1999; special issue Social Networks June 1992; special issue Rationality and Society January 1997); core theory, power-dependence theory, expected value theory, and NET, which is the collection of exchange-resistance, GPI, and optimal seek theories.

A requirement for our analysis is that the exact effect of adding and deleting links on actor payoffs is computable. Hence unique point predictions are required. Since core theory and power-dependence theory do not provide unique point predictions they are not suitable for our investigation. We did not select NET either; there are several different versions of the theory, and the most recent version of NET advocated by its developers is not computerized yet (Emanuelson 2005; Emanuelson & Willer 2006). Consequently we select the remaining theory: Friedkin's (1992; 1993; 1995) expected value theory (EVT). Although uniqueness and existence

[1] '*As if*'-assumptions are common in the social sciences; one famous example being the assumption in economics that actors behave *as if* they are rational maximizers (Friedman, 1953).

for all networks is not proven, for every network up to 8 actors, the algorithm of EVT generates a point prediction.

Expected Value Theory (EVT)

Building upon the theory of social power proposed by French (1956), Friedkin (1986) first suggested the idea of using expected values to predict the outcomes in a power structure. Friedkin (1992; 1993) extended the idea of expected values to analyze outcomes in an exchange network. Friedkin's model predicts the probability with which each maximal exchange pattern occurs, and the distribution of outcomes in each one of these patterns. A maximal exchange pattern is maximal in the sense that no further feasible transaction exists between the actors that haven't exchanged yet. For example, the Line4 that has two maximal exchange patterns: {{*A*-*B*}, {*C*-*D*}}, and {{*B*-*C*}}. Using an iterative algorithm, each actor's expected payoff is calculated as the expected value of his payoffs over all possible maximal exchange patterns.

As opposed to what the name suggests, EVT is not a theory based upon actors rationally maximizing their payoffs. The algorithm generating the predictions assumes that both actors' claim of their share of the 24 points in their relation is increasing non-linearly in the probability that each of them is excluded in any exchange. Three rules determine the final allocation in the relation. Which rule is applied depends on the sum of both actors' claims and their claims relative to half of the resource pool (12 points). An inconvenience of the EVT model is the analytical intractability of the algorithm because of the non-linear function and the three rules embedded in it. See Friedkin (1995) for details of the EVT model.

Related Literature

There exist two other approaches relevant to the study of the dynamics of exchange networks. Firstly, Bonacich (2001) simulates exchange network evolution and finds that in equilibrium payoff differentials are small. In Bonacich's simulation, actors are myopic *satisficers*: They change the network if their earnings drop below a certain level. Our actors are myopic *maximizers*: they keep changing the network as long as marginal benefits outweigh marginal costs. Another difference is that instead of adding or deleting links, Bonacich' actors move to another cell on a checkerboard, where they can exchange with actors in adjacent squares. Bonacich (2004) provides some intuitions for how this approach can be extended to more general structures.

The second approach is a rapidly growing literature on network formation in economics (e.g., Dutta and Jackson 2003). Some networks considered in this literature are highly similar to exchange networks. Moreover, the concept of pairwise stability that we borrow from this literature derives from a model of network formation in which actors try to maximize their own payoff, and add and delete links one by one. In this work we use the stability concepts and the network formation model from economics. Thus, we explicitly bring research on network exchange in sociology and research on network formation in economics together.

3. STABLE EXCHANGE NETWORKS: APPROACH AND HYPOTHESES

A model of actor behavior is needed to study the stability of exchange networks. Our model assumes that for establishing a link between two actors mutual consent is required, while deletion of a link is unilateral. Since we also assume that both actors pay maintenance costs for a link between them, the assumption of mutual consent for link addition intuitively follows.

The payoffs in each exchange relation in the network are predicted with EVT (Friedkin 1995). For all exchange relations in each network we check if they are candidates for deletion by comparing the original EVT payoffs to the EVT payoffs after deleting the link. Similarly, for all pairs of actors without a direct link we check if the absent links are candidates for addition. Note that if two actors are both indifferent with respect to the absence or presence of a link, that link will neither be added to the network that does not contain that link nor be deleted from the network containing it. Hence, ‘not adding’ and ‘deleting’ are not the same.

Friedkin’s EVT is a system of many assumptions and equations that makes a formal analysis very difficult. Nonetheless we could prove some general stability results that concern networks of any size, which either depend partially or do not depend on the assumptions of the EVT. However, we could not prove general results on the density of stable networks and the possible (non)existence of stable strong power networks. In order to get more insight into the properties of stable networks we choose to analyze a large subset of all possible exchange networks: all 13,597 exchange networks of size 2 to 8 are investigated. Of these networks 12,112 are connected[2] and 1,485 are unconnected[3] (see sequence A001349 of Sloane’s Online Encyclopaedia of Integer Numbers). In Section 4 we will first present the results of the analysis of networks size 2 to 8 obtained with EVT, and second we will prove our general results for any size, all but one of which are independent of the payoff allocation function.

Pairwise stability

Stable networks are defined as exchange networks that are pairwise stable (PS). An exchange network is PS if (i) adding a currently *absent* link is costly to at least one of the two actors or leaves both actors equally well off, (ii) removing a *present* link does not benefit either of the two actors it currently connects. Jackson and Wolinsky (1996) introduced the pairwise stability concept, and in his survey on network formation Jackson (2003) argues that it might be

[2] A network is connected if a path exists from each node to every other node.
[3] We need to consider unconnected networks as well, because through link deletion networks can become unconnected. In later sections it will be demonstrated that many stable and efficient networks are unconnected.

considered as a necessary condition for network stability. It is the weakest notion of stability, allowing for link formation while providing narrow predictions about the set of stable networks.

As an example of how to utilize the notion of pairwise stability, consider an 8-actor network where actor *H* is connected to actors *A*,*B*,*C* but not to other actors, denoted by the 'adjacency row' 1110000. Then, with regard to actor *H*, PS holds if no single change from 1 to 0 (the deletion of one link) increases *H*'s expected payoff, and if no single change from 0 to 1 (the addition of one link) increases the expected payoff of *H* while not decreasing the expected payoff of the other actor. Hence, only *N*-1=7 changes of the network are considered for that actor. A network is PS if this condition holds for each actor in the network.

The pairwise stability concept perfectly fits our model of actor behavior; link deletion is unilateral and for establishing a link mutual consent is required. In addition, pairwise stability is the prominent stability concept employed in the game- theoretic literature on network formation (for an overview, see Dutta and Jackson 2003).

We analyze the stability of exchange networks as a function of link costs. In our analysis it is assumed that an *equal* cost is incurred for *both* actors involved in the link. Additionally, it is assumed that the cost of a link is independent of the number of links an actor has. Finally, the cost to *establish* a link is equal to the cost of *maintaining* a link. In our analyses the cost of a link varies from 0 to 12, where *each* actor involved in the corresponding link pays this cost. Note that link costs larger than 12 need not be considered. Since an exchange relation can only generate 24 points, there always exists an actor in the relation who wants to delete the link for costs higher than 12; only networks without links are then PS.

Concerning the stability of networks, we conjecture but cannot prove that with EVT payoffs all complete networks are PS when links are costless, and that there does not exist a PS network in which one actor earns a very high profit and other actors earn almost nothing (a so-called 'strong power' network). Moreover, we expect that the average degree of PS networks decreases in costs.

Refinements of pairwise stability

In our analysis we use two stability concepts that are refinements of pairwise stability: pairwise Nash and unilateral stability. Pairwise Nash is a refinement of pairwise stability, and unilateral stability is a refinement of pairwise Nash.

A network is *strongly pairwise stable* (Gilles and Sarangi 2004:13) or *pairwise Nash* (PN) (Calvó-Armengol & İlkılıç 2004:7) if (i) adding a presently *absent* link is costly to at least one of the two actors or leaves both actors equally well off; and (ii) removing a subset of an actor's *present* links does not benefit this actor. Note that (i) of PN is identical to condition (i) of pairwise stability. The difference between PS and PN is that PN allows for simultaneous deletions in (ii). Continuing our other example, consider again the 8-actor network with *H*'s adjacency row equal to 1110000. PN holds for *H* if each change of *one* 0 to 1 does not increase the payoff of *H* and the other actor is not worse off, and each change of a subset of 1s to 0s does not increase *H*'s payoff. Hence, in total $4+2^3-1 = 11$ changes are considered for that actor. A network is PN if this condition holds for each actor in the network.

An undesirable feature of PN is that it is asymmetric: it is concerned with the effect of the deletion of *one or more links* and of the addition of a *single link*. It also does not allow for simultaneous addition and deletion of links so that a network in which actors can only make themselves better off by replacing one relation by another relation is still considered stable. In *unilateral stability* (US), a refinement of PN, this asymmetry is resolved. A network is US if no actor can profitably reconfigure his links without objection by his *new* contacts. In our example, US holds for H if no adjacency row other than 1110000 simultaneously (i) increases H's payoff, *and* (ii) makes no actor connected to H in the new adjacency row, but not in the old one, worse off. Only the actors newly connected to H in the new configuration or adjacency row have to agree, because only for creating new links mutual consent is required. Note that H considers all possible $2^{N-1}-1$ = 127 reconfigurations of his links. A network is US if this condition holds for each actor in the network.

Egalitarian networks

Networks are defined as egalitarian if all actors in the network obtain the same payoff, after subtracting the costs for links actors have. Some examples of egalitarian networks are complete networks, cycles, and even sized networks consisting of only dyads. We will emphasize these networks in our results. We will categorize PS networks that are egalitarian and that are not.

It can be shown that EVT predicts equal outcomes for vertex transitive graphs for any cost level. A vertex transitive graph is a graph where all actor positions are equal (see Weisstein for a formal definition). There are 37 vertex transitive graphs from size 2 to 8. There are 7 other networks that are egalitarian only at cost 0 which are combinations of disconnected vertex transitive graphs of size 6 and 8. These are not included in our calculations.

Efficiency

With the introduction of link costs inefficient PS networks might arise. Efficiency here is defined as *Pareto efficiency*; there exists no other network in which no actor earns less and at least one actor earns more. To determine if a network is Pareto efficient, the actor payoffs for all exchange networks of the same size are ordered in a vector from large to small. A network is then Pareto efficient if no other network's payoff vector is strictly larger than that of the network under consideration. We will categorize both efficient PS and inefficient PS networks. We will investigate if there are cost levels for which no PS network of a given size is Pareto-efficient. If such cost levels exist, we say that there is a tension between efficiency and stability and say that a *social dilemma* exists.

4. RESULTS

4.1 Results for networks of sizes 2 through 8

The results for networks of sizes 2 through 8 include the analysis of PS networks, the analysis of PN and US networks, and the efficiency and equality of stable networks, respectively. All results are aggregated over size, but not over cost levels since cost is an important explanatory variable while size is not. Nonetheless, if different trends are observed for different network sizes then they are reported. Finally, in the appendix, we include the number of PS, PN, and US networks of each size at any cost level and whether they are egalitarian or efficient.

4.1.1 Pairwise Stable networks

To see which networks are PS with changing costs, we checked whether each network is PS and if PS then in which cost interval. Across cost intervals 0 to 12, in total, 180 different PS exchange networks exist, which is only 1.32 % of all 13,597 exchange networks up to size 8. The percentage of PS networks decreases with size as can be seen from the table in the appendix. At size 2, both the empty network and the dyad (100%) are PS whereas at size 8 only 0.8 % of networks are PS. Hence pairwise stability is a rare attribute of exchange networks of small size. We exclude from the analysis 9 networks consisting of dyads and at least two isolates that are only PS at cost 12.

When there are no costs of adding, maintaining and deleting a link there are 10 PS networks. As expected, all 7 complete networks are PS. For the networks of size 5 and 7 also networks consisting of two unconnected complete networks are PS. With 5 actors, a dyad and a triangle that are not connected is PS. Likewise, when there are 7 actors, the combination of the complete 3 and 4, and complete 2 and 5-actor networks are PS. These unconnected networks are PS because no one wishes to delete any link, and if the two parts in the network are connected, the payoff decreases for the actor in the larger sub-network that connects to the smaller sub-network. As an example consider the disconnected dyad and triangle (3-cycle). The actors cannot gain from deleting any existing link. As for adding a link, the expected payoff of the actor in the triangle connected to the dyad becomes 7.46, while before the addition it was 8 points.

With the introduction of costs, we see clear patterns of change in the set of stable networks. Starting at zero cost and gradually increasing cost, complete networks destabilize[4], PS networks become less dense and only unconnected dyads and at most one isolate (in case of networks of odd size) are stable when cost is in (6.551,12). We call the unconnected dyads and at most one isolate (in case of networks of odd size) *M*(inimal) networks. In Theorem 1 below, we show that given EVT payoffs an *M*-network of any size is US (and therefore also PN and PS) for any cost *c* in (3.48,12). We also show in Theorem 2 that M networks are the only PN and US networks for cost *c* in (6,12)[5] independent of the payoff allocation function.

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FIGURE 2 ABOUT HERE

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The gradual change in the number of stable networks across size as a function of cost level is shown in Figure 2. The x axis denotes the cost level; the y axis denotes the number of PS networks. Since at certain cost levels some PS networks cease to be PS and some networks enter as PS, the number of PS networks changes with increasing costs. The number of stable networks gradually increases up to cost 0.42, and fluctuates thereafter. The maximum number of 46 PS networks is obtained at cost 1.22. The minimum number of 7 PS networks is obtained at cost 6.55 and stays there until cost 12. A similar trend is observed in networks of each size separately from 4 to 8.

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FIGURE 3 ABOUT HERE

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The effect of cost on density is apparent from Figure 3. As expected, the average density of stable networks decreases with cost, although not monotonically. As an example consider the PS networks of size 4. For costs in the interval [0, 4.026] the complete network is PS. The Box (4-cycle), is PS for costs in [2.811, 5.138]. Two dyads are PS when cost is in [3.482, 12]. A triangle and an isolate is stable for costs in (4.114, 6.551). At cost 12 or higher, only the empty or null network is PS. Note that the often investigated Line4 network is never stable. Hence our analysis suggests that in real-life exchange settings, the Line4 network will not occur often. The networks of size 5 to 8 have a similar trend.

We also investigated the cost ranges for which networks are PS. A *cost range* is defined as the width of the interval a network is PS. For example, the cost range for which the triangle is stable is 6.551 since it is stable from cost level 0 to 6.551. When we look at the range of costs that networks are PS, we see that the median cost range is 0.250, and 62 PS networks are stable

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[4] The only PS complete network at cost 12 is the dyad with 2 actors. Of all complete networks, the 7-person network destabilizes at cost 0.395, the 5-person at cost 0.839, 8-person at cost 1.325, the 6-person at cost 2.104, the 4-person

for a cost range of 0.5 or more. There are only 24 networks which are PS in the cost range between 2.5 and 12. So, the majority of the PS networks are very sensitive to the cost level and destabilize even in small cost changes. Odd-sized cycles and networks consisting of dyads including the *M* networks have large stability ranges: between 4.973 and 12 with the median being 8.512. Thus, these networks are among the most stable networks.

Finally, an important observation on the payoff allocation of PS networks can be made. They are rather egalitarian. First, no PS network is a so-called 'strong power' network; there is no network up to size 8 with at least one actor earning almost all points. Also, a majority of the PS networks yield payoffs which are less than or equal to 12 before subtracting the cost of links. Out of 180 PS networks, 32 of them are weak power, that is, contain payoffs higher than 12 for some players, before subtracting costs. Among these 32 networks, 9 networks include players with payoffs between 13 and 14 and 1 network where one player earns 14.69. After subtracting the costs of links, all payoffs fall below 12-*c*, the payoff of actors in even *M* networks. No weak power network is PS for costs larger than 2.723, and none is PS for a cost range larger than 0.428. The median cost range of weak power networks is 0.097, which is substantially smaller than that of PS networks (0.250). Thus, weak power networks destabilize quickly. Among the network sizes investigated, only equal power networks are PS for a large cost range.

4.1.2. Refinements of Pairwise Stability

Pairwise Nash networks

There are 149 PN networks among the networks investigated. At cost 0, all 10 PS networks are also PN. For 103 of the 149 networks the cost interval for which it is PN is identical to that for which it is PS. For the other 46 networks the interval is smaller. The lower bound of the PS

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at cost 4.026 and the triangle at cost 6.551.

interval is the same as that of the PN interval because the lower bound concerns link addition. Hence differences in the intervals always are in the upper bound of the interval that concerns single link deletion (PS) versus multiple link deletion (PN).

The majority of the PN networks, 97 out of 149, are PN within a cost range of 0.5 or smaller. Only 17 networks are PN over a cost range of 2.5 or more, including *M* networks (7) and odd cycles (3).[6] All *M* networks are PN in the same cost range as they are PS. However, the odd-sized cycles -3, 5 and 7- are PN in smaller cost ranges than in PS. This obviously emerges from the fact that PN networks allow for multiple deletions of links, hence no network where players earn negative payoffs in a cycle can be PN. For example, the triangle is PS but not PN in the interval (4, 6.551).

The stricter stability notion of PN reduces both the proportion of stable weak power and the proportion of stable equal power networks. There are 26 PN networks with some players' payoffs larger than 12, and 9 of these networks contain payoffs larger than 13.

Unilateral Stable networks

There are 130 US networks among the networks investigated. At cost 0, only the complete networks of each size are US; the 3 unconnected networks that are PS and PN are not US. Unilateral stability cost intervals are smaller compared to PS in 48 of the US networks and are the same for the rest. The differences in the cost intervals between US and PS lie both on the upper bound and the lower bound.

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[5] We cannot prove a similar theorem for PS because of the complicated EVT payoff calculations.

[6] These are: M networks (7), odd cycles (3), 4 sized complete network, 3-cycle and dyad, 5-cycle and dyad, 5 sized complete network with a dyad, 3-cycle and 4 sized complete, octagon, and 2 disconnected 4 sized complete networks.

The majority of the US networks, 89 out of 130, are US within a cost range of 0.5 or smaller. Only 15 networks are US over a cost range of 2.5 or more including *M* networks (7) and odd cycles (3)[7]. All M networks are US in the same cost range as they are PS.

As PN, US does not eliminate weak power networks; there are 21 US weak power networks, where 8 of them contain payoffs larger than 13.

4.1.3. Stability and equality of exchange networks

Of all 13,597 networks, 37 networks are egalitarian, which is only a very small percentage (0.003%). Out of 180 different PS networks 35 exchange networks have equal payoffs within the cost range they are stable, which amounts to 19.44% of all PS networks. Hence except 2 egalitarian networks of size 8, all egalitarian networks are also PS at some cost (94.59%).

When we look at the cost range these networks are PS, we see that egalitarian networks tend to be PS over a larger range than non-egalitarian networks. 17 of the 35 egalitarian symmetric PS networks are stable in a cost range of 1 or more, and the median cost range is 0.925. In contrast, 117 of 145 non-egalitarian PS networks are PS in a cost range less than 1. Hence both weak power and non-egalitarian networks are very sensitive to small changes in cost and tend to destabilize quicker than egalitarian networks.

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FIGURE 4 ABOUT HERE

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[7] The other networks are: 4 sized complete network, 3-cycle and dyad, 5-cycle and dyad, 5 sized complete network with a dyad, and octagon

All egalitarian PS networks are also PN; hence there are 35 egalitarian PN networks. These egalitarian networks have the same properties as PS networks except 4 networks containing cycles where the PN ranges are substantially smaller than PS ranges. The cycles are 3, 5 and 7 cycles and a 6-player network consisting of two disconnected triangles. There are 34 egalitarian US networks. Almost all egalitarian networks have the same cost ranges as PN with the most notable exceptions of 3 odd-sized cycles. The only egalitarian network that is not US but PS is the 6-player network consisting of two disconnected triangles.

4.1.4. Stability and Pareto efficiency of exchange networks

Figure 4 summarizes results that concern the efficiency and equality of PS networks. Let a network be globally efficient if it is Pareto efficient in the cost interval in which it is PS, partially dominated if it is Pareto efficient in only a part of this interval, and completely dominated if it is never Pareto efficient within the interval. Then, of the 180 PS networks, 19 are globally Pareto efficient, 15 are partially dominated, and 146 are completely dominated in the cost interval within which they are PS. Of the 19 globally Pareto efficient networks, 7 networks are empty and 7 are *M*. The *M* networks are undominated for cost ranges of 8.518 to 12. The other 5 globally Pareto efficient networks are PS in a cost range of 0.383 or less.

Out of the 15 partially dominated networks only 7 are undominated for a cost range from 0.412 to 4.678. The least dominated PS networks are cycles, cycles combined with dyads, and a 7-person network which is difficult to describe[8]. The median stability range of these 7 networks is 5.826. So, in addition to the *M* networks the 3, 5 and 7 cycles, possibly in combination with dyads, are the most stable and efficient networks. Note also that the even *M* networks and odd

[8] These are: the triangle, the triangle with one dyad, the pentagon, the triangle with two dyads, the pentagon with the dyad, and the heptagon.

size cycles are egalitarian, which are in total 7 networks. The odd *M* networks and cycles in combination with one or more dyads are not egalitarian.[9]

If no stable network is efficient at a given cost level, we say a social dilemma exists: given any starting network, the actors never reach an efficient (stable) network . When we examine the undominated and partially dominated PS networks and their ranges of stability, we see that for size 4, 6 and 8 there exist social dilemmas. For size 4 there does not exist any undominated PS network between cost 0.004 and 3.4819; for size 6 there is no undominated PS network between cost 0.0021 and 3.4819, and for size 8 between cost 0.0013 and 3.4819. In these cost ranges all PS networks are dominated by an M network and the M networks are PS only after cost 3.4819. Although these results suggest that it holds for even networks of any size, we were not able to prove it.

The inefficiency of PS networks can be due to our limiting of the actors' action space. Pairwise stability considers only single link changes, while pairwise Nash allows for simultaneous deletions and unilateral stability allows for any reconfiguration of links. Thus PN and US networks might be more efficient than PS networks. As an example, the complete 3-actor network with link cost 5 is PS but not PN nor US. The expected payoff of each actor in the complete 3-actor network is $8 - 2\times5 = -2$, while in the empty 3-actor network, each actor earns 0. If an actor deletes one of his links, his expected payoff decreases from –2 to $1.45–5 = –3.55$. Only after deleting both his links does an actor's payoff increase to 0. However we found that neither pairwise Nash nor unilateral stability eliminates inefficient stable networks.

[9] We also checked Pareto efficiency within the set of 180 PS networks for each size. In the range that networks are PS, 38 networks are undominated, 54 networks are partially dominated, and 88 PS networks are totally dominated in this range by another network that is also PS in that range. This means that there might be a high probability that the outcomes of the end paths of the network evolution are inferior to that of other possible end paths, as well as to those in networks that are not an end path (in case of a social dilemma).

Only 23 PN networks are globally efficient; the 7 *M* networks, 7 empty networks, 6 cyclic networks which are 3, 5, and 7 cycles and dyads with these cycles, and 3 networks which are of size 7 and difficult to describe. Note that the cycles are not globally efficient in the range they are PS, but are globally efficient in the range they are PN. This is because the PS cycles are dominated by empty networks for larger costs, at which they are not PN. 117 of the 149 PN networks are totally dominated and a further 9 networks are partially dominated. However, among the partially dominated networks, with the exception of one network, the domination range covers almost all stability range. Hence, the PN concept seems to differentiate the Pareto efficient cycles and *M* networks more from the other stable networks than PS does.

There are 23 globally efficient US networks. These networks are the same as the undominated PN networks. 101 of the 130 US networks are totally dominated, a further 5 networks are partially dominated such that the undomination range is less than 0.004 and 1 network with an undomination range of 0.382.

4.2 General Results

The analysis of networks up to size 8 demonstrated that *M* networks are important: they were efficient, stable for a large cost range, and egalitarian if the network size is even. Here we prove some general results on the stability (theorems 1 and 2) and efficiency (theorems 3 to 5) of *M* networks of any size. The results of theorems 2 to 5 are also independent of the selection of theory or payoff allocation function used.

The analysis of networks up to size 8 demonstrated that *M* networks are US (and hence PN, and PS) for high costs. This result can be generalized to networks of any size with EVT payoffs, as theorem 1 shows.

Theorem 1: Given EVT payoffs, an *M* network of any size is US (and therefore also PN and PS) for any cost *c* in (3.48,12).

Proof of Theorem 1: Consider an odd sized *M* network of size 13 or larger. If the network is US, neither a reconfiguration of the ego network of the isolate nor of a member of a dyad is accepted. Let us first consider each possible reconfiguration of the ego-network of the isolate. The isolate has no links to delete and can maximally profitably add six links, because then $c \times 3.48 > 24$. If the isolate wants to add links to both actors in one dyad, EVT reveals that these actors reject his proposal because their payoff is lowered independent of which other links are added. If links are added to one actor of different dyads, the isolate obtains a disadvantageous position resulting in a low expected payoff that does not exceed the link costs. Consequently, the isolate does not want to add links in odd-sized *M* networks.

Let us now consider reconfigurations of the ego-network of an actor in a dyad. This actor cannot improve his payoff by adding more than four links, since then he earns $24 - 5 \times 3.48$ or less, which is less than the $12 - 3.48$ he earns in the *M* network. Three changes are possible; (i) If he proposes to connect to the isolate, the isolate always rejects the proposal. (ii) If he proposes to add links to actors of the same dyad, these actors reject the proposal. (iii) Finally, EVT reveals that if he connects to one or more actors of up to four different dyads, the other actors reject his proposal. Hence also in odd-sized *M*-networks actors in dyads do not want to change their ego-network. End of proof for odd sized networks. Note that the combination of (ii) and (iii) complete the proof for even sized networks of any size. Q.E.D.

Although the result of Theorem 1 is dependent on using EVT, one can construct alternatives of Theorem 1 using other theories of network exchange. In order to construct such an alternative one requires a theory of network exchange that predicts both probabilities of exchange patterns and outcomes of actors in each of these patterns. The two well-known theories of network exchange briefly discussed in the introduction, core theory and power-dependence theory, do not satisfy this criterion. The other and most employed theory of network exchange, NET, does. Three variants of NET, i.e., GPI-R, GPI-RD, GPI-l*2, allocate 13.5, 14.5, 14.6 points to the middle actors in the Line4, respectively, and 1 point to peripherals in the Line3 (Emanuelson 2005: 160). An alternative of Theorem 1 can be constructed and proven for each variant of NET with cost *c* in (*x*-12,12), with *x* equal to the payoff of the middle actor predicted by that variant.

The absence of any other PS networks than *M* networks at costs larger than 6.551 suggests that for any cost *c* in (6.55,12) the only PS networks of any size are *M* networks. Although we firmly believe this to be true, we were unable to prove it using Friedkin's EVT. If the conjecture is false, then there exists at least one network in which *each* link has a marginal benefit to *both* actors in the link larger than 6.55. Nonetheless, we can prove that with cost *c* in (6,12), the only PN and US networks are M networks regardless of the theory used..

Theorem 2: Given any payoff allocation function and assuming that each actor in a dyad gets half of the total payoff, the only PN (and US) networks with *c* in (6,12) are M networks.

Proof of Theorem 2: The networks with fewer links than *M* networks are not stable since link addition is profitable for two isolates. Hence it suffices to show that in any network with more links than an *M* network with *c* in (6,12) at least one actor wants to delete at least one of his links. Such a network has at least one component consisting of at least three actors. For the proof one only needs to consider such a component.

Let us assume the component has k actors with p links in total. Consider first that k is odd. Then the net total payoff cannot exceed 24×(k-1)/2 – 2pc, which is less than 0 if c > 6(k-

1)/p. Since the minimum value of p is k-1, the net payoff in the component is negative if c > 6. Then there exists at least one actor who obtains a negative payoff. This actor can improve her payoff and obtain 0 by deleting all of her links.[10]

Now, consider that k is even. The net payoff is less than 0 if c > 6k/p. Components with k or more links are not stable for c > 6 because then the net total payoff is negative and at least one of the actors will delete all his links. Consider the minimally connected component containing k-1 links. In such a component there is an actor *i* with 1 link. Actor *i* will maintain his link iff *i*'s payoff excluding link cost is larger than c so that $x_i \geq c$. Actor *i*'s neighbor *j* has at least two links and should get at least 12 – c, or else he would delete all his links except with *i*. Assume *j* has two links, then $x_j - 2c \geq 12 - c$, which yields $x_j \geq 12 + c$. However, if c > 6 then $x_i + x_j > 12 + 2c > 24$. Hence if 6 < c < 12 actor *j* will delete one of his links to end up in a dyad. If *j* has more than two links he will delete all links except to one of his neighbors who has only one link. Q.E.D.

It can easily be shown that many Pareto efficient networks exist at cost 0. Consider all networks in which always a maximal number of exchanges is completed for that size, called *F* networks in Theorem 3. This maximum, denoted by *FX*, is equal to *N*/2 if the number of actors *N* is even and to *N*/2 – ½ if *N* is odd, where *N* is the size of the network.

Theorem 3: Given any payoff distribution function, *F* networks are Pareto efficient at zero cost.

Proof of Theorem 3: In *F* networks a maximum sum of payoffs equal to 24×*FX* is divided among the *N* actors. Since the sum of payoffs to be divided in all these networks is equal, one of the networks cannot Pareto-dominate the other. Q.E.D.

[10] Since with pairwise stability one cannot delete more than one link, this proof cannot be generalized to PS networks. Note that there are PS networks in which all actors obtain negative payoffs, like the complete 3-actor network for cost (6, 6.551).

There are many *F* networks. However, there are only very few PS networks at zero costs. Consequently, at zero costs there are many efficient networks that are not PS.

When costs are introduced Theorem 3 no longer applies. To derive statements concerning the Pareto efficiency of PS networks another theorem is required.

Theorem 4: Given any payoff distribution function, an *M* network is Pareto efficient for any cost in the closed interval [0,12].

Proof Theorem 4: Exactly *FX* exchanges are carried out in the *M* network that consists of exactly *FX* links. Hence the total payoff to be divided in the *M* network is equal to $FX(24 - 2c) \geq 0$ if $c \leq 12$. Adding a link decreases the payoff to be divided with $2c$, deleting a link decreases the payoff to be divided with $24 - 2c \geq 0$ if $c \leq 12$. Since there is no network with a larger payoff to be divided, the *M* network is Pareto efficient. Q.E.D.

Note that M networks are the *only* efficient networks for costs in the interval (0,12) if efficiency of a network is defined as the maximum sum of actor payoffs across all networks of the same size.

At cost 0 all PS networks are Pareto efficient. From cost 6 to 12 (excluding 12 itself) only the seven *M* networks, one for each size, are Pareto efficient. This result can be generalized to networks of any size and to almost all payoff distribution functions.

Theorem 5: Given that the payoff distribution function never assigns 24 points to a player in an exchange, the only networks that are Pareto efficient at costs greater than or equal to 6 are *M* networks.

Proof Theorem 5: Theorem 4 indicates that an *M* network is Pareto efficient. It remains to be shown that an *M* network is the only Pareto efficient network for $c \geq 6$. If it can be shown that the theorem is true for $c = 6$, it is also true for all $c > 6$ because the actors' degree and hence their costs are minimized in the *M* network. The theorem is false if either (i) there is a network in which all actors obtain a positive expected payoff if the number of actors is odd, or (ii) there is a network in which one actor obtains more than $12 - c$ points. Let us first show that (i) is false. First assume the network is odd and of size $n + k$, with n even and k odd. All n actors form dyads, and the k actors form a connected network. k actors form a connected component, otherwise at least one of them does not obtain a positive payoff. The sum S of net actor payoffs in the connected k-actor component is at most $S = (k\text{-}1)\times 12 - t\times 12$, with t denoting the number of links. Note that $t \geq k - 1$, otherwise k actors cannot be connected. If $t > k - 1$ then $S < 0$ and there is at least one actor obtaining a negative payoff. If $t = k - 1$ then $S = 0$ and either all actors earn 0 net payoffs or some actors earn positive payoffs while at least 1 other actor earns negative. Hence (i) is false.

Consider part (ii). An actor can only earn more than 12 points if he has more than one link. If he has one link he obtains at most $12 - c$ when he is a member of a dyad. If he has three links or more his expected payoff is smaller than $12 - c$ because he can never gain 24 points excluding costs. Consider an actor having two links. The marginal value of the link for both of his partners must be at least 6. If one of his partners obtains at least 6, he can never get more than 18 in his exchanges with his partners. Consequently, he obtains $18 - 2c < 12 - c$. Q.E.D.

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5. DISCUSSION

Research on exchange in sociology has focused on the effect of the social structure on outcomes of exchange. In almost all of this research the exchange network was the independent variable. The main result of this research is that the network structure has a huge impact on what actors earn in their exchange relations. Because different positions in the network obtain different payoffs, the questions of how these networks evolve, and which networks are stable, arise. However, these questions have received little attention in the literature. In the current article the network structure is the dependent variable, i.e., we study what the network structure looks like if actors have the opportunity to change with whom they have an exchange relation. The questions investigated in the current article are: (i) which exchange networks are stable?, (ii) are stable exchange networks efficient?, (iii) are stable exchange networks egalitarian and are egalitarian exchange networks stable?

In answering these three research questions we employ the same assumptions mostly used in the sociological literature on exchange networks, including the 1-exchange rule and that the value of each exchange relation is the same. Additionally, we assume that actors can manipulate their links; for adding links mutual consent is needed, while link deletion is unilateral. Links are costly and the cost is the same for both actors in the link. To assess stability of an exchange network we employed three stability concepts from the economic literature on networks: pairwise stability, pairwise Nash, and unilateral stability. Networks were considered as efficient if they were Pareto efficient, while networks were considered egalitarian if all actors in the network earn the same payoff. To calculate the actors' payoff we used Friedkin's Expected Value Theory. First, we investigated all networks up to size 8, then we proved five general results. One result uses EVT but generalize to networks of any size, the other results not only generalize to networks of any size but are also independent of the payoff allocation function.

Focusing first on the results on small size networks, the percentage of PS networks is very small and decreases in network size. The density of the PS networks decreases in cost. While no strong power network is pairwise stable, some weak power networks are. Two types of networks are observed to have a large cost range in which they are stable: *M* networks which consist of only dyads and at most one isolate, and odd cycles. Only very few networks are egalitarian, but almost all of them are stable at a certain cost. Moreover, egalitarian networks are stable in a larger cost range than other stable networks. Even size *M* networks and odd cycles are egalitarian, and these networks are also efficient for a larger cost range than other networks. Finally, we observe social dilemmas in even size networks for low cost; that is, for low costs none of the stable networks is efficient. Using pairwise or unilateral stability does not change any of these results.

Despite the fact that we could not prove results on the relation between cost and density, the absence of strong power networks, the presence of social dilemmas for even size networks using EVT, we were able to derive some general results. We showed that, given EVT payoffs, an *M* network of any size is stable for cost in (3.48, 12). We also showed that this result can be generalized using the payoff allocation function of Network Exchange Theory, but the cost interval changes to (2.78, 12). We also found that, independent of the payoff allocation function, *M* networks are the only stable (PN and US) networks for cost (6, 12), they are efficient for any cost, and are the only efficient networks for cost (6, 12).

A welcome result was that almost all egalitarian networks up to size 8 are stable at a certain cost. Although actors are payoff maximizers and do not care for equity, they are "satisfied" in egalitarian networks. However, many unfair networks are stable as well. An extreme example would be the odd size *M* network where one actor is excluded, which is efficient, stable, and very unfair. An interesting empirical question is to what extent exchange networks would evolve into (un)fair exchange networks. Additionally, to what extent actors' preferences for equity *do* affect stability of networks, and whether these preferences are observed in the evolution of exchange networks remain to be examined. Preferences for equity can be incorporated in the payoff function as in Fehr and Schmidt (1999).

As for the efficiency of stable networks, the results were not as one might hope. Although some stable networks at a certain cost were also efficient, many stable networks were not. Hence coordination problems exist at many cost levels; some stable networks are efficient, but other stable networks are dominated at the same cost. Social dilemmas, which are coordination problems where none of the stable networks at a certain cost is efficient, were also found for even size and low cost. At cost levels with social dilemmas, the *M* network is efficient but not stable because each actor would like to form a link with one actor of another dyad. It is an interesting empirical question whether actors are able to resist this temptation and end up in the efficient but not stable *M* network. Similar to equality, the effect of actors' preferences for efficiency on the stability of networks, and whether these preferences are observed when studying the evolution of exchange networks in the lab remain to be examined. Efficiency, like equity, can also be incorporated in the payoff function as in Charness and Rabin (2002).

The focus of the present article was on the possible end paths of the evolution of exchange networks, that is, the stable networks. Investigating how to get to these possible endpoints, i.e., the evolution of the network, would be a natural extension. The probability that each stable network is reached would also be investigated. Factors that will determine these probabilities are the initial network configuration (e.g., empty [nobody knows each other], complete [everybody has access to and can exchange with each other], or random), costs, and preferences for equity and efficiency. It might also be that stable networks differ to the extent that they are sensitive to possible errors of actors. In one stable network one error might be enough to move to another stable network, while another stable network might still be stable even after one or more accidental errors. These issues remain to be investigated both in the lab and by simulation.

In our analysis we assumed that actors were myopic; they only took their immediate payoff into account, and not the consequences of their behavior on future payoffs. The justification of this assumption is not only that it makes the analysis much more tractable, but also that actor behavior in the lab is myopic as well. Most actors are found to think two or at most three steps ahead in many different experimental games (Camerer 2003, Ch 5). Allowing for farsighted actors in models of network evolution is nevertheless identified by Dutta and Jackson (2003:13) in their book on network formation "as perhaps the most important (and possibly the hardest) issue regarding modeling the formation of networks". Actors taking the future into account are likely to affect the evolution and stability of exchange networks. To mention two examples, taking the future into account gives opportunities to solve a social dilemma and the problem of inefficiency of stable networks as described above. The even M network which is efficient but not stable at low costs can be sustained as an equilibrium if the actors are rational and the 'network evolution game' is indefinitely repeated. The odd M network is efficient but neither stable at low cost nor egalitarian. Again, if the shadow of the future is sufficiently long the actors might coordinate on alternating who is excluded in the M network such that both efficiency and equity is satisfied.

REFERENCES

Berg, Christopher and Stephan Panther. 1998. "Network Exchange as a Non-Cooperative Bargaining Game." Unpublished manuscript.

Berninghaus, Siegfried K., Karl-Martin Ehrhart and Marion Ott 2005. "A Network Experiment in Continuous Time: the Influence of Link Costs." Working paper 05-02. http://www.sfb504.uni-mannheim.de/publications/dp05-02.pdf

Bienenstock, Elisa J.and Phillip Bonachich. 1992. "The Core as a Solution to Exclusionary Networks." *Social Networks 14*:231-43.

Blau, P.M. (1964). *Exchange and power in social life.* John Wiley and Sons, Inc.

Bonacich, Phillip. 1998. "A Behavioral Foundation for a Structural Theory of Power in Exchange Networks." *Social Psychology Quarterly 61*:185-98.

Bonacich, Phillip. 1999. "An Algebraic Theory of Strong Power in Negatively Connected Exchange Networks." *Journal of Mathematical Sociology 23*:203-24.

Bonacich, Phillip. 2001. "The Evolution of Exchange Networks: A Simulation Study." *Journal of Social Structure* 2 (5).

Bonacich , Phillip. 2004. "Social Capital in Exchange Networks: A Simulation Study of Network Evolution. Pp. 104-115 in Henk Flap and BeateVölker (Eds.), *Creation and Returns of Social Capital: A New Research Program*, London: Routledge.

Bonacich, Phillip and Elisa J. Bienenstock. 1995. When Rationality Fails: Unstable Exchange Networks with Empty Cores. *Rationality and Society 7*:293-320.

Bonacich, Phillip and Elisa J. Bienenstock. 1997. "Latent Classes in Exchange Networks: Sets of Positions with Common Interests." *Journal of Mathematical Psychology 22*:1-28.

Braun, Norman and Thomas Gautschi. 2000. "Who gets how much in which relation? A non-cooperative bargaining approach to exchange networks." Unpublished manuscript.

Burke, Peter J. 1997. "An Identity Model for Network Exchange." *American Sociological Review 62*:134-50.

Calvó-Armengol, Antoni and Rahmi İlkılıç. 2004. "Pairwise Stability and Nash Equilibria in Network Formation." Unpublished manuscript.

Camerer, Colin (2003). Behavioral game theory: experiments in strategic interaction. Russell Sage Foundation: Princeton University Press.

Charness, Gary and Matthew Rabin. 2002. "Understanding Social Preferences with Simple Tests," *Quarterly Journal of Econonomics*. 117:3, pp. 817–69.

Coddington, A. 1968. *Theories of the bargaining process.* London: Allen & Unwin.

Cook, Karen S. and Richard M. Emerson. 1978. "Power, Equity, and Commitment in Exchange Networks." *American Sociological Review* 43:721-39.

Cook, Karen S. and Toshio Yamagishi. 1992. "Power in Exchange Networks: A Power-Dependence Formulation." *Social Networks 14*:245-65.

Corominas-Bosch, Margarida. 2004. "Bargaining in a Network of Buyers and Sellers." *Journal of Economic Theory 115*:35-77.

Dutta, Bhaskar and Matthew O. Jackson. 2003. *Networks and Groups*. Berlin: Springer.

Edgeworth, F.Y. 1881. *Mathematical physics*. Fairfield: Kelley.

Emanuelson, P. 2005. "Improving the Precision and Parsimony of Network Exchange Theory: A Comparison of Three Network Exchange Models." *Contemporary Research in Social Psychology* 10:149-65.

Emanuelson, P., and Willer, D. (2006). *Testing Ten Theories*. Unpublished manuscript.

Emerson, Richard M. 1962. "Power-Dependence Relations." *American Sociological Review 27*:31-40.

Fehr, Ernst and Klaus Schmidt. 1999. "A Theory of Fairness, Competition, and Cooperation." *Quarterly Journal of Economics,* 114: 817-868.

Friedkin, Noah E. 1992. "An Expected Value Model of Social Power: Predictions for Selected Exchange Networks." *Social Networks 14*:213-29.

Friedkin, Noah E. 1993. "An Expected Value Model of Social Exchange Outcomes." *Advances in Group Processes 10*:163-93.

Friedkin, Noah E. 1995. "The Incidence of Exchange Networks." *Social Psychology Quarterly 58*:213-21.

Gilles, Robert P. and Sudipta Sarangi. 2004. *The Role of Trust in Costly Network Formation.* Working paper.

Girard, C. Dudley and Casey Borch. 2002. "Predicting the Pattern of Exchange in Economic Exchange Networks." Unpublished manuscript.

Homans, G.C. 1958. Social behavior as exchange. *American Journal of Sociology*, *62*, 597-606.

Jackson, Matthew O. and Asher Wolinsky. 1996. "A Strategic Model of Social and Economic Networks." *Journal of Economic Theory* 71:44-74.

Jackson, Matthew O. 2003. A Survey of Models of Network Formation: Stability and Efficiency, Chapter 1 in Demange G. and M. Wooders (eds), *Group Formation in Economics: Networks, Clubs and Coalitions*, Cambridge University Press, Cambridge U.K.

Kollock, Peter. 1994. "The Emergence of Exchange Structures: An Experimental Study of Uncertainty, Commitment, and Trust." *American Journal of Sociology 100*:313-45.

Leik, Robert K. 1991. "Strategic Timing in Network Processes." *Advances in Group Processes* 8:1-28.

Leik, Robert K. 1992. "New Directions for Network Exchange Theory: Strategic Manipulation of Network Linkages." *Social Networks* 14:309-24.

Markovsky, Barry, David Willer, and Travis Patton. 1988. "Power Relations in Exchange Networks." *American Sociological Review* 53:220-36.

Molm, Linda D. 1997. *Coercive Power in Social Exchange.* Cambridge: Cambridge University Press.

Schmeidler, David. 1969. "The Nucleolus of a Characteristic Function Game." *SIAM Journal on Applied Mathematics* 17:1163-1170.

Skvoretz, John and David Willer. 1993. "Exclusion and Power: a test of four theories of power in exchange networks." *American Sociological Review 58*:801-18.

Stolte, J.F. and Emerson, R.M. 1977. Structural inequality: position and power in exchange structures. In R.L. Hamblin, & J.H. Kunkel (Eds.), *Behavioral Theory in Sociology* (pp. 117-138). New Brunswick: Transaction Books.

Thibaut, J., & Kelley, H.H. 1959. *The social psychology of groups*. New York: John Wiley and Sons.

Van Assen, Marcel A. L. M. 2003. "Exchange Networks: An Analysis of All Networks Up To Size 9." *Advances in Group Processes* 20:67-103.

Van de Rijt, Arnout and Vincent Buskens. 2005. "A Nash Refinement for Myerson's Network Formation Game." Unpublished manuscript.

Wasserman, S., & Faust, K. (1994). *Social network analysis: methods and applications*. Cambridge: Cambridge University Press.

Weisstein, Eric W. "Vertex-Transitive Graph." From *MathWorld*--A Wolfram Web Resource. 6 March 2007. http://mathworld.wolfram.com/Vertex-TransitiveGraph.html

Willer, David. 1999. *Network Exchange Theory*. New York: Greenwood.

Willer, David and Brent Simpson. 1999. "A New Method for Finding Power Structures." Pp. 270-284 in David Willer (Ed.), *Network Exchange Theory*, Westport, CT: Praeger Press.

Willer, Robb and David Willer. 2000. “Exploring Dynamic Networks: Hypotheses and Conjectures.” *Social Networks* 22:251-72.

Yamaguchi, Kazuo. 1996. “Power in Networks of Substitutable and Complementary Exchange Relations: A Rational-Choice Model and an Analysis of Power Centralization. *American Sociological Review 61*:308-32.

Yamaguchi, Kazuo. 2000. “Power in Mixed Exchange Networks: A Rational-Choice Model.” *Social Networks 22*:93-121.

Young, O.R. (1975). *Bargaining: formal theories of negotiation.* Urbana: University of Illinois Press.

APPENDIX

Network Size	Number	Stable			Egalitarian	Egalitarian & Stable			Pareto Efficient & Stable		
		PS	PN	US		PS	PN	US	PS	PN	US
2	2	2 (1)	2 (1)	2 (1)	2	2	2	2	2 (1)	2 (1)	2 (1)
3	4	3 (0.750)	3 (0.750)	3 (0.750)	2	2	2	2	3 (2)	3 (2)	3 (2)
4	11	5 (0.455)	4 (0.364)	4 (0.364)	4	4	4	4	3 (1)	3 (1)	3 (1)
5	34	8 (0.235)	7 (0.206)	6 (0.176)	3	3	3	3	6 (4)	5 (3)	5 (3)
6	156	17 (0.109)	15 (0.096)	13 (0.083)	8	7	7	6	3 (1)	3 (1)	3 (1)
7	1044	40 (0.038)	34 (0.033)	30 (0.029)	4	4	4	4	14 (7)	13 (6)	10 (6)
8	12346	105 (0.008)	84 (0.007)	72 (0.006)	14	13	13	13	3 (1)	3 (1)	3 (1)

Table 1: Number of PS, PN, and US networks for costs 0 to 12. Number of stable networks do not include networks that are stable at only cost 12, and the parentheses indicate the percentage of networks that are stable. Number of egalitarian networks do not include networks that are egalitarian at cost 0 or at cost 12. The number of Pareto efficient networks include both partially dominated and undominated networks; the parentheses indicate the number of networks that are undominated in a range of 0.1 or more.

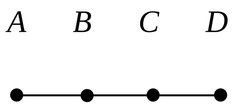

Figure 1: The Line4 exchange network.

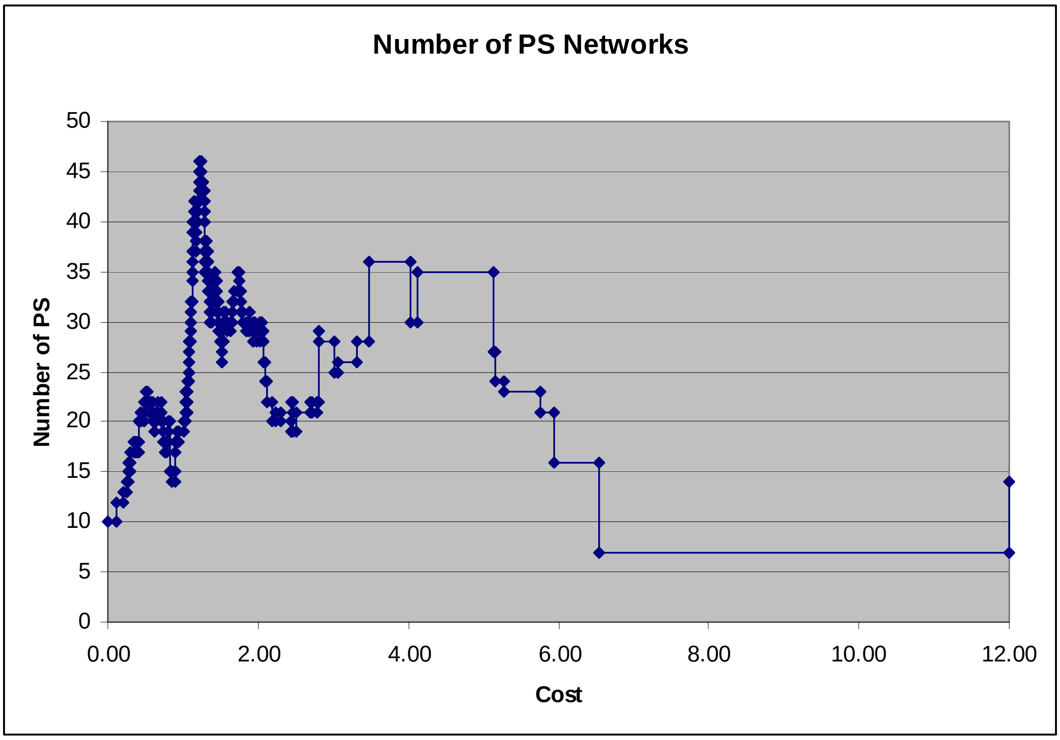

Figure 2: Number of PS networks in total from size 2 to 8 as a function of cost. The points refer to the cost levels where there is a new PS network emerging or a PS network dropping from the list or both.

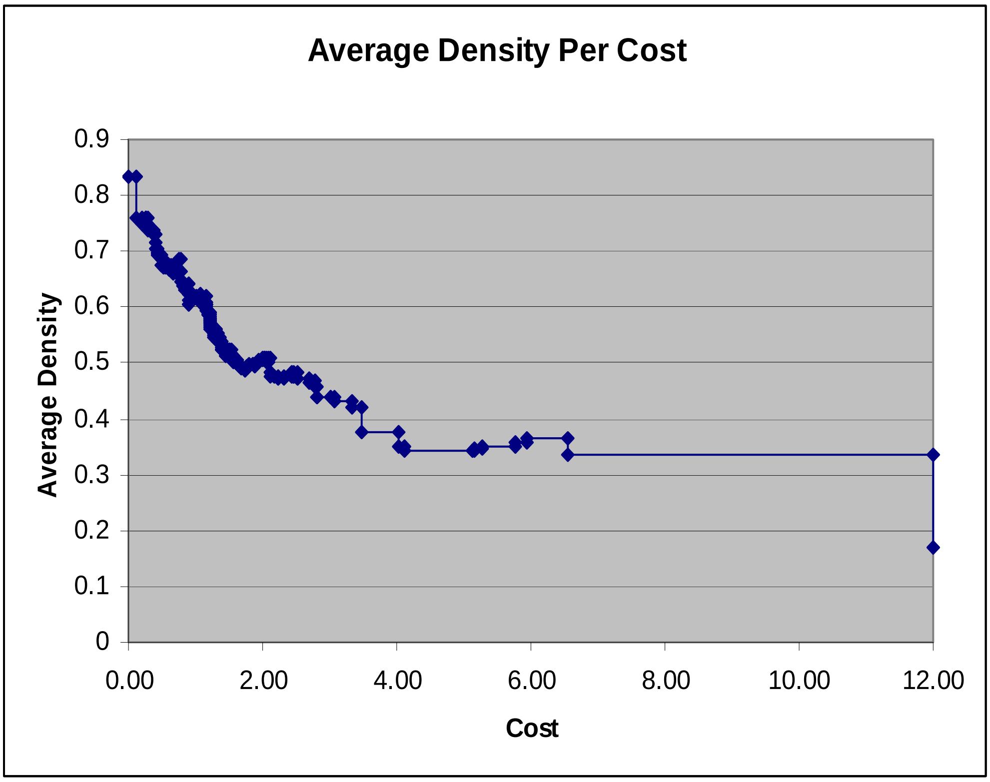

Figure 3: Average density of PS networks from size 2 to 8 as a function of cost. Density is defined as the proportion of links in the network to the number of possible links. At each cost level the average density is calculated as the sum of densities of PS networks divided by the number of PS networks. Note that a complete network has a density of 1 and an empty network has a density of 0.

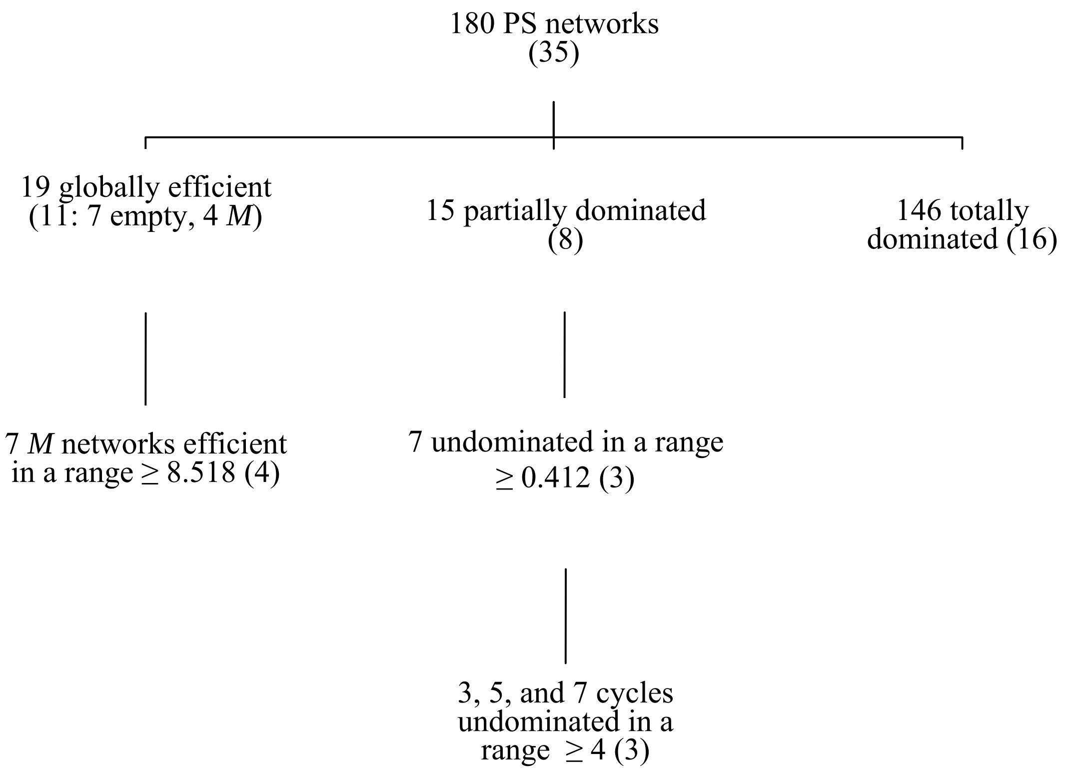

Figure 4: Efficiency of PS networks.

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