The Term Structure of Interest Rates and Net Present Value

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Abstract: *Net present value, as traditionally taught and applied to agricultural problems, assumes a discount rate that is constant over the life of the investment. This article draws on the theory of the term structure of interest rates to illustrate a means of estimating future non-constant discount rates. An agricultural capital investment example highlights the consequences of using this method of calculating the discount rate on net present value.*

Key Words and Phrases: *Discount rate, Term structure of interest rates, Net present value.*

One mainstay of financial economics as it applies to agriculture has been net present value analysis. Traditionally, the economic consequences of an action have been analyzed by discounting the future cash flows generated by that action. The basic theory behind this technique involves the intertemporal trade-off between consumption now and later (Hirshleifer; Fama and Miller). Given the existence of a capital market, this trade-off can be simplified to a discounting method using the appropriate interest rate or cost of capital.

In the typical application of net present value, certain assumptions must be made in order to investigate practical problems. For example, it is typically assumed future cash flows resulting from a particular action are fixed and predictable at the time the action is taken. The potential shortcomings of this assumption are obvious, thus, some potential corrections or methods of compensating for this simplicity have been suggested (Levy and Sarnat; Bussey). Another potential shortcoming of the traditional approach involves the typical assumption of a constant discount rate. This shortcoming can also be damaging. Given the stochastic nature of interest rates over time it is possible the error caused by the deviation in the discount rate is as damaging as the error caused by assuming deterministic returns.

The objective of this study is to examine the term structure theories that explain interest rates in the future and relate these future interest rates to net present value calculations. Specifically, this study will pedagogically present

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a procedure for incorporating information from the term structure of the interest rate into the net present value capital budgeting technique. In a manner analogous to a farmer's using commodity futures to "lock in" a future price, it is possible, using observable security prices, to estimate implied forward discount rates.

The Term Structure of Interest Rates

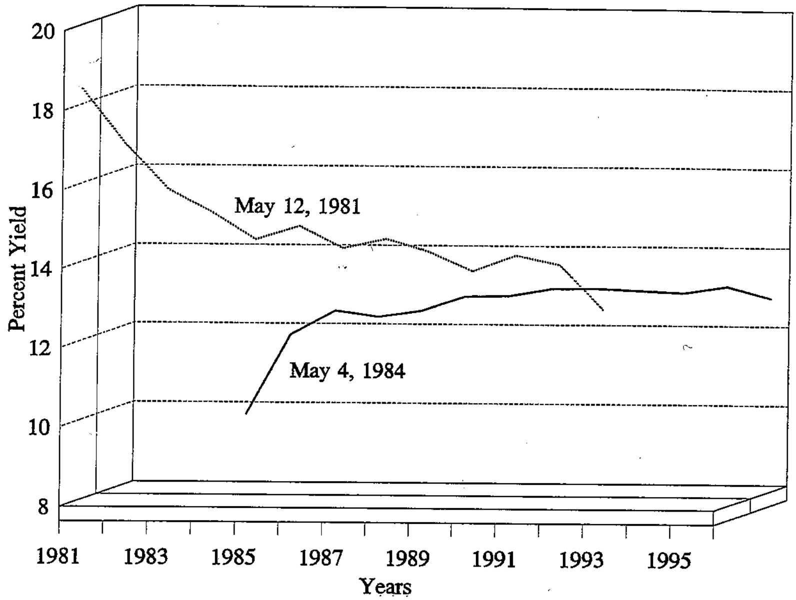
Changes in interest rates over time reflect changes in basic macroeconomic conditions. Differences in interest rates across financial instruments or financial securities are a reflection of the different characteristics of the instruments. If all instruments had homogeneous characteristics, such as risk, taxability, and fixed or variable income, then there would be only one interest rate. However, as the characteristics differ and their magnitude changes across instruments then the interest rates will change accordingly.

The term structure of interest rates refers to the difference in interest rates associated with one very important characteristic, time-to-maturity variations. For the term structure to be meaningful, all other characteristics of the instruments, except time to maturity, must be the same. Government securities are one type of financial instrument that vary only in terms of time to maturity. In addition they are generally regarded as having minimal risk. Therefore, government securities, such as notes and bonds, are typically used to examine term structure. For example, the yield on a thirty-year government bond with five years to maturity will probably be much different than the yield on a thirty-year government bond recently issued.

The term structure relationship is typically represented graphically by the yield curve. The solid yield curve in Figure 1 depicts the term structure of interest rates for government securities on May 4, 1984. Each point on the curve represents the geometric yield on a particular government instrument on May 4, 1984, where the year is the date of maturity for each particular note or bond. The year of maturity, prices of the securities, annual coupon rates, and yields[1] assuming annual payouts are given in Table 1. The yield curve is then a comparison of the yields on different terms to maturity for financial instruments that differ only in time to maturity.

Though presently relatively flat, the yield curve since the early 1980s has taken on dramatically different shapes.[2] The yield curve for May 4, 1984 in Figure 1 displays an upward slope, while the yield curve for May 12, 1981 displays a downward slope. At any one particular point in time only one yield curve will exist. These two examples show the range that has been experienced since 1980.

Figure 1.
Upward and Downward Term Structure



Though subject to debate, financial economists feel that market expectations about future interest rates affect the term structure (Cox, Ingersoll and Ross). If the shape of the yield curve does provide information about future interest rates, the term structure may prove useful in projecting future interest rates. Specifically, the term structure may be used to predict how the interest rate, hence the discount rate used in net present value analysis, may change over time.

In order to utilize the information encompassed in the term structure, the three major hypotheses regarding the term structure of interest rates must be explored. The unbiased expectations hypothesis (UEH), first suggested by Fisher and later refined by Lutz, states that the long-term spot rate (the observed rate on an instrument that matures in the future) is an unbiased average of the one-period observed spot rate and the expected future one-period short-term rates. Let ${}_0R_j$ be the spot interest rate observed now for an instrument that matures in period j. Thus, ${}_0R_1$ is the observed spot interest rate on an instrument that matures at the end of period 1, and ${}_0R_N$ is the observed spot interest rate on an instrument that matures at the end of

Table 1.
Maturity, Instrument Price, Coupon Rates and Yields

Year[a]	May 4, 1984			May 12, 1981		
	Price[b]	Coupon[c]	Yield[d]	Price[b]	Coupon[c]	Yield[d]
		----- *percent* -----			----- *percent* -----	
1981				$99.15	9.750	18.14
1982				92.30	8.000	17.01
1983				87.14	7.875	15.87
1984	$99.26	8.875	9.87	86.18	9.250	15.30
1985	98.19	9.875	11.90	87.80	10.375	14.62
1986	92.23	7.875	12.50	76.21	7.875	14.97
1987	99.16	12.000	12.35	79.20	9.000	14.41
1988	87.20	8.250	12.51	73.10	8.250	14.65
1989	87.24	9.250	12.87	76.70	9.250	14.33
1990	90.40	10.500	12.89	72.19	8.250	13.84
1991	106.23	14.500	13.09	93.50	13.000	14.26
1992	103.10	13.750	13.10	63.10	7.250	14.02
1993	85.00	10.125	13.05	63.30	7.500	12.89
1994	78.30	9.000	13.00			
1995	84.24	10.375	13.17			
1996	91.80	11.500	12.88			

[a]Year that particular issue will mature.
[b]Dealer selling price in secondary market on that day (asked price).
[c]Original coupon interest rate.
[d]Yield to maturity, assuming annual coupon payment.
Source: The Wall Street Journal, May 5, 1984 and May 13, 1981.

period N. The expected (but unobserved) interest rate between period t and period $t+1$ at period 0 is expressed $E_0\,[{}_tR_{t+1}] = {}_t\hat{r}_{t+1}$, for $t = 1, \ldots, N\text{-}1$. The UEH then implies (Van Horne)

$$(1 + {}_0R_N)^N = (1 + {}_0R_1)(1 + {}_1\hat{r}_2)(1 + {}_2\hat{r}_3) \ldots (1 + {}_{N\text{-}1}\hat{r}_N) \qquad (1)$$

with both ${}_0R_N$ and ${}_0R_1$ being observed and ${}_t\hat{r}_{t+1}$ being implied unobservables.

There are two ways of looking at this representation of the term structure. First, on the left-hand side of (1) investing \$1.00 today yields $(1 + {}_0R_N)^N$ dollars in period N. Alternatively, the right-hand side of (1) implies that \$1.00 can be invested today at ${}_0R_1$ and then reinvested in subsequent periods at ${}_t\hat{r}_{t+1}$ for $t = 1, \ldots, N$-1. Equation (1) implies that in equilibrium the two scenarios should yield the same return. The justification for this equilibrium is rooted in arbitrage. If ${}_t\hat{r}_{t+1} \neq E_0\,[{}_tR_{t+1}]$, then arbitrage profit could be obtained by buying and selling securities until equilibrium is restored. This equilibrium is based on three assumptions: 1) securities of different maturities are perfect substitutes;[3] 2) transaction costs are small; and 3) capital markets are efficient.

By comparing the observed rates ${}_0R_N$ and ${}_0R_{N-1}$, it is possible to compute the projected interest rate ${}_{N-1}\hat{r}_N$. Specifically, dividing (1) by $(1 + {}_0R_{N-1})^{N-1}$ yields:

$$
\begin{aligned}
{}_{N-1}\hat{r}_N &= \frac{(1 + {}_0R_N)^N}{(1 + {}_0R_{N-1})^{N-1}} - 1 \\
&= \frac{(1 + {}_0R_1)(1 + {}_1\hat{r}_2)(1 + {}_2\hat{r}_3)\ldots(1 + {}_{N-1}\hat{r}_N)}{(1 + {}_0R_1)(1 + {}_1\hat{r}_2)(1 + {}_2\hat{r}_3)\ldots(1 + {}_{N-2}\hat{r}_{N-1})} - 1.
\end{aligned} \tag{2}
$$

Therefore, by using ${}_0R_K$ and ${}_0R_{K-1}$ it is possible to estimate ${}_{K-1}\hat{r}_K$ for $K = 2, \ldots, N$. Thus, the UEH implies that expected future short-term interest rates can be computed from present spot rates.

In complete certainty the computed rates in (2) would be the certain forward rates. However, uncertainty exists in making long-term financial commitments. A second hypothesis to explain term structure by Hicks contends that this uncertainty leads to a liquidity premium. Specifically, Hicks states:

> A person engaging in a long-term contract (i.e., making a long-term loan) puts himself into a more risky position than he would be in if he refrained from making it; however there are some people for whom this will not be true, because they are already committed to needing loan capital over extensive future periods.

Thus, lenders have shorter time horizons due to risk and are motivated to make short-term loans while borrowers for opposite reasons are motivated to borrow long. As a result, the borrower must provide a premium in excess of the expected future short-term rate to induce the lender to make long-term loans. Further, as the length of the loan grows, the liquidity premium increases. This would result in a forward rate, ${}_t\hat{r}_{t+1}$, derived in (2) as an overestimate of the expected future single period rate at time t since it also includes a liquidity premium. Specifically,

$$_t\hat{r}_{t+1} = E_t\,[_tR_{t+1}] + {}_tL_{t+1} \tag{3}$$

where $_tL_{t+1}$ is the necessary liquidity premium such that

$$0 < {}_tL_{t+1} < {}_{t+1}L_{t+2} < {}_{t+2}L_{t+3} < \ldots,$$

which suggests an upward sloping yield curve.

The third hypothesis is known as market segmentation (Walker; Culbertson). Market segmentation suggests that there is little substitution between securities and instruments of different maturity, as opposed to the assumption in the first two hypotheses, and that institutions tend to match maturities with the lives of purchased assets. As a result, separate markets develop for instruments of different maturities such that participants in one market are unlikely to cross over into another market. As a result, the interest rate in a market composed of instruments of a similar maturity is determined only by the supply and demand in that market. Therefore, the market segmentation hypothesis suggests that calculated forward rates are not the expected future short rates and there is actually no way of estimating expected future short-term rates.

Though far from conclusive, the empirical evidence supports the role of interest rate expectations in the term structure of interest rates (Van Horne). In addition, the evidence suggests that the liquidity premium in (3) may influence forward rates. Thus, the observed term structure contains the market expectations of future one-period interest rates. This market expectation information has implications for capital budgeting (Copeland and Weston, Chapter 3). However, as Figure 1 implies *ex ante* market expectations and *ex post* market realizations can differ. For example, early 1980s market expectations about interest rates in the 1990s were significantly higher than present interest rate levels.

Net Present Value Capital Budgeting

Net present value analysis has gained widespread acceptance as the preferred method for analyzing the economic potential of a capital investment. Net present value directly incorporates the time value of money and is not sensitive to mixed investment cash flows unlike the internal rate of return (Bussey). Typically, in the absence of capital restrictions, the net present value (*NPV*) is defined as:[4]

$$NPV = \sum_{t=1}^{N} \frac{NCF_t}{(1 + r)^t} - I_0, \tag{4}$$

where NCF_t is the expected net cash flow in period t, I_0 is the initial investment, and r is a constant discount rate applied to each period.

However, implicit in (4) is the assumption that the term structure of interest rates is flat, or that the economic forces generating interest rates are such that the interest rate is constant over time. Nothing in economic theory suggests this will be the case. Instead, such things as changing tax rates, technology, savings rates and wealth will generally result in changing interest rates over time.

The assumption of a constant interest rate may actually distort the true profitability of the investment. Specifically, an investment with a given series of cash flows using the standard procedure of assuming a constant discount rate might generate a positive (negative) net present value. However, incorporating the information on future spot interest rates embodied in the yield curve may lead to a different and possibly even negative (positive) net present value. Such a situation will be presented later in the paper. Thus, a modification of (4) allows for a more complete depiction of the investment decision over time:

$$NPV = \sum_{t=1}^{N} \frac{NCF_t}{\prod_{j=1}^{t} (1 + {}_{j-1}r_j)^t} - I_0. \tag{5}$$

With this formulation of net present value, each period's net cash flow is discounted by its own discount rate. The missing piece of the puzzle given (5) is an algorithm for estimating the future period discount rates.

The Term Structure and Net Present Value

The vehicle for calculating ${}_{j-1}\hat{r}_j$, $j = 1, 2, \ldots, t$ will be an algorithm of selling short and buying long the government securities (Table 1) in a manner identical to hedging that locks in a sequence of one-period loans for time $t_1, t_2, t_3, \ldots, t_{T-1}$. The interest rate on these loans will then serve as a prediction of the forward interest rates. This will establish that it is technically possible for an individual to guarantee a future short-term interest rate through the appropriate sequence of security transactions.

These transactions will enable the producer to fix the opportunity cost of capital for any given period in the future.

The strategy for establishing ${}_1\hat{r}_2$ will be to use the one-year, 1985 government bond, and the two-year, 1986 bond to set up cash flows at times t_1 and t_2 that replicated a one-period loan with no net cash flows occurring in any other period. First, at t_0 the producer will sell short a one-year bond and buy X number of two-year bonds such that the net cash flow is zero. Using the prices and coupons rates for the government security in Table 1 the price of the bond maturing in one year (1985) is \$98.19 while the price of a bond maturing in 1986 with two \$7.875 coupons is \$92.23. Therefore, if the producer sells short the one-year bond with no additional money, 1.065 bonds maturing in the second period can be purchased

$$98.19 - X(92.23) = 0, \text{ or } X = 1.065.$$

Then, at t_1 the producer will pay out a coupon of \$9.875 and the \$100.00 face amount on the one-year bond that was sold short and receive a coupon for \$7.875 per unit of the two-year bond. The total cash flow from this transaction at t_1 is

$$-\$109.875 + 1.065(\$7.875) = -\$101.49.$$

In the second period, the 1.065 period two bonds mature yielding

$$1.065(\$107.875) = \$114.85.$$

The net cash flows for these transactions are

$$t_0 = \$0.00,\ t_1 = -\$101.49, \text{ and } t_2 = \$114.85.$$

This is equivalent to a loan over period one for \$101.49 with repayment of \$114.85 in principal and interest expense. The result is an interest rate

$${}_1\hat{r}_2 = \frac{\$114.85}{\$101.49} - 1 = 0.1316.$$

This is repeated to derive ${}_2\hat{r}_3$, except now the actions involve short selling the two-year 1986 bond and 0.0335 units of the 1985 bond while purchasing 0.9633 units of the three-year 1987 bond. The condition for a zero cash flow at t_0 is

$$\$92.23 + Y(\$98.19) - X(\$99.16) = 0.00 \tag{6}$$

The second condition for a zero cash flow at t_1 is

$$-\$7.875 - Y(\$109.875) + X(\$12.00) = 0.00. \tag{7}$$

Solving (6) and (7) for Y and X, the amounts of the 1985 bond and the 1987 three-year bonds that lead to zero cash flows in the first two periods are

$$X = 0.9633, \text{ and } Y = 0.0335.$$

The resulting net cash flows are \$0.00 at t_0, \$0.00 at t_1, -\$96.31 at t_2, and \$107.89 at t_3.[5] This is identical to making a loan of \$96.31 at t_2 that repays \$107.89 at t_3, with a resulting interest rate of 12.02 percent.

To derive ${}_3\hat{r}_4$, the actions now involve selling a three-year bond, while purchasing 0.0289 units of the two-year bond, and 0.0263 units of a one-year bond, and 1.077 units of a four-year bond. The net cash flows will be \$0.00 at t_0, \$0.00 at t_1, \$0.00 at t_2, -\$103.11 at t_3 and \$116.59 at t_4. This is equivalent to making a loan of \$103.11 at t_3 that repays \$116.59 at t_4, with a resulting interest rate of 13.07 percent. (See appendix for computational details).

Table 2 summarizes the imputed interest rates for both the upward sloped yield curve (May 1984) and the downward sloped yield curve (May 1981). These forward rates have significant implications for the capital budgeting decision.

An Empirical Application to a Capital Budgeting Decision

To show the effect of the term structure on capital budgeting, this study examines the net present value of a Florida citrus grove assuming initially a constant discount rate. Then the *NPV* is recalculated assuming first an upward sloping yield curve and, finally, assuming the yield curve is downward sloping.

Table 3 gives the expected gross revenues, grove care expenses, and net cash flow for twelve years for a Florida citrus producer. Year one represents the first year in which a positive cash flow is experienced. Subsequent years show a gradual increase in production as the grove reaches maturity with full production obtained in year eleven. Data used to construct these annual cash flows are based on yields from Florida Agricultural Statistics' *Citrus Summary* and cost of producing from extension budgets (Muraro, Hurner and Oswalt). It is assumed the oranges are sold on tree for \$5.00 per box throughout the twelve years. The investment in

Table 2.
Estimated Forward Spot Interest Rates, for Upward Sloped Yield Curve, May 1984; and Downward Sloped Yield Curve, May 1981

Forward Spot Interest Rate ($_t\hat{r}_{t+1}$)	May 1984	May 1981
	------------ *percent* ------------	
$_0\hat{r}_1$	11.90	17.04
$_1\hat{r}_2$	13.16	14.65
$_2\hat{r}_3$	12.02	14.08
$_3\hat{r}_4$	13.07	11.94
$_4\hat{r}_5$	14.72	16.88
$_5\hat{r}_6$	13.17	10.69
$_6\hat{r}_7$	15.25	16.68
$_7\hat{r}_8$	12.97	10.90
$_8\hat{r}_9$	12.56	8.21
$_9\hat{r}_{10}$	12.09	19.31
$_{10}\hat{r}_{11}$	16.52	11.33
$_{11}\hat{r}_{12}$	6.61	12.52

the grove for land, land preparation, irrigation and other similar items is assumed to be $7,500.00 per acre.

Table 4 gives the present value of the net cash flows assuming a constant discount rate of 13 percent (which is approximately the yield on the government security that matures in 12 years for both examples of term structure) is $7,865.49, resulting in a net present value of $365.49. Assuming a 12-year planning horizon and no risk means this would be a profitable investment. Recalculation of the present values of the cash flows, assuming the decision is being made in an economy characterized by the upward sloping yield curve, and these future spot rates result in a $7,823.74 present value ($323.14 net present value). This is only a minor difference, less than 1 percent, from assuming a constant rate. However, if we examine the capital budgeting decision under the downward sloping yield curve, the resulting present value of the cash flow is $7,358.14 and the net present value is negative (-$141.86). The $507.35 reduction in the present value of the net cash flows now results in a rejection of the investment.

Table 3.
Estimated Per Acre Annual Net Cash Flows and Net Present Values for South Florida Citrus Production

Revenue/Expense	Production Year												
	0	1	2	3	4	5	6	7	8	9	10	11	12
	dollars												
Adjusted Gross Revenue	0	745	1,155	1,630	2,035	2,300	2,415	2,525	2,635	2,735	2,835	2,930	2,930
Operating Expenses:													
Cost to remove trees or brush	0	13	17	17	25	25	25	25	25	25	25	25	25
Grove care costs	0	427	427	460	482	482	507	507	507	532	532	532	532
Young tree care - resets	0	83	83	83	83	83	83	83	83	83	83	83	83
Plant reset/solidset trees	0	33	33	33	33	33	33	33	33	33	33	33	33
Property taxes	0	15	15	15	15	15	15	15	15	15	15	15	15
Total operating expense	0	570	575	608	638	638	663	663	663	688	688	688	688
Grove Investment	-7,500												
Annual Net Cash Flow	-7,500	175	580	1,022	1,397	1,662	1,752	1,862	1,972	2,047	2,147	2,242	2,242

Table 4.
Present Value and Net Present Value of Per Acre Annual Net Cash Flows

Forward Spot Interest Rate $({}_t\hat{r}_{t+1})$	Present Value of Annual Net Cash Flows	Net Present Value (I_0 = \$7,500)
	---------------------- *dollars*	----------------------
Constant Rate (13%)	7,865.49	365.49
Upward Yield Curve	7,823.74	323.14
Downward Yield Curve	7,358.14	-141.86

Conclusions

The capital budgeting decision is generally the most important decision made by the firm. These decisions typically represent significant dollar expenditures and ultimately determine the long-term profitability of the firm. The importance of finding alternative investments and forecasting their influence on the firm cannot be ignored. When the investment alternative will produce a very large certain return with minimal cost or investment it is a simple decision and a detailed formal analysis is not required. However, few investment decisions have such nice characteristics. As the uncertainty of the forecasts and the impact of financial complexity of the investment on the firm increase so does the need for an economically sound decision-making method.

The net present value method is the preferred method for handling the capital budgeting decision. This basic formulation is first complicated by the inclusion of various tax related factors including depreciation, capital gains and losses, and income taxes. The common approach for handling these factors is to forecast NCF_t on an after-tax basis. As a result, attention must be paid to depreciation methods, capital gains or losses, and the expected terminal value of the investment at the end of the planning period (Casler, Anderson and Aplin).

Next the importance of crucial items such as capital structure or financing, inflation, term structure and risk must be considered. It is general recommended that these be handled using adjustments in the cost-of-capital or the discount rate (Levy and Sarnat; Bussey). The cost of capital will need to accurately reflect the influence of these internal and external conditions.

An initial cost-of-capital consideration is the debt structure of the business. Typically a weighted average cost of capital (WACC) is used to reflect the level of debt and equity financing for the firm. The WACC separates the investment decision from the financing decision by assuming a future optimal capital structure. This basic cost of capital is then adjusted consistent with *NPV* method of (5) to reflect changes in external variables that will modify the opportunity cost of debt and equity (WACC) in the future.

Obviously one external factor that can affect the investment decision is inflation. If the forecast NCF_t include inflation that changes over time then the WACC should also include an allowance for inflation. Empirical evidence suggests that the term structure provides an accurate forecast of market inflation expectation (Fama). The WACC can be adjusted to reflect these differences. However, if the NCF_t are in real terms then the WACC will not include the inflation adjustment.

As illustrated by this study the term structure of interest rates contains information about expected future single-period interest rates. Failure to adjust the cost of capital when the term structure is downward sloping will result in biased *NPV* estimates. If, as in this study, the deterministic rate used is the lower, more distant future rate, the bias is upward; however, if the deterministic rate is the higher near-term rate, the bias will be downward. The existence of an upward sloping term structure will cause an opposite bias.

The treatment of risk is arguably the most important step in *NPV* calculation. Methods for handling risk range from the simple to the theoretical. A simple, but practical approach, is a sensitivity analysis wherein the WACC is increased to reflect a 'pessimistic' or risky outcome (greater opportunity cost) and decreased to reflect an 'optimistic' outcome. The use of this approach captures the combined sources of risk—production, market price, term structure, etc.—in an unsystematic manner.

Modern finance theory provides several different, but theoretically consistent, models for incorporating risk into the *NPV* calculation. One method based on expected utility theory uses the CAPM or single index model to adjust the cost of capital for capital budgeting (Collins and Barry). Other methods for handling risk, particularly stochastic interest rates, model the risk as a stochastic process, i.e., a random walk or a Wiener process (Dothan and Williams). These later models do not depend on restrictive assumptions concerning individual risk preferences.

It is evident term structure is one of several potentially important items that need to be considered when making the capital budgeting decision. Given the complexity of making the decision all these items must be considered. The process of evaluating all these variables will help provide the important information needed to make a sound economic decision.

Notes

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1. The yield on each bond is computed using the internal rate of return on the sequence of annual cash flows arising from the purchase of the bond. For example, the internal rates of return for the 1985 bond maturing one year, and the 1986 bond maturing two years, in the future are computed as:

$$0 = -98.19 + \frac{(9.875 + 100.00)}{(1 + \tau)^1}, \quad or \quad \tau = 0.119,$$

and

$$0 = -92.23 + \frac{7.875}{(1 + \tau)^1} + \frac{(7.875 + 100.00)}{(1 + \tau)^2}, \quad or \quad \tau = 0.125,$$

where τ is the internal rate of return.

2. See Wood for an historic perspective on yield curves and term structure.

3. Typically, financial assets are perfect substitutes except for risk. Thus, if we restrict our analysis to securities differing only by maturity, the financial assets are sufficiently close to perfect substitutes for a sufficient segment of investors.

4. We recognize that the notion of scarcity is fundamental to economic literature typically defined as the allocation of scarce resources among unlimited and competing human wants and desires. However, by borrowing at the appropriate interest rate, the entrepreneur can typically expand the capital frontier for any sufficiently profitable investment. Alternative formulations under constrained capital are also consistent with the net present value framework.

5. The net cash flows are found as:

$$t_0 = 92.23 + Y(98.19) - X(99.16) = 0,$$

$$t_1 = -7.875 - Y(109.875) + X(12.00) = 0,$$
$$t_2 = -107.875 + X(12.00) = -96.31, \text{ and}$$
$$t_3 = X(112.00) = 107.89.$$

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Appendix

General Formulation of Shorts and Longs

Obviously as the lock-in period moves forward in time, the procedure for determining the buys and sells becomes more complex. It is useful to follow this construction from a linear algebraic perspective. The first case is simple enough that linear algebra may appear a step backward. However, it is informative so we will discuss it briefly. One way to visualize the problem is that we want to determine X_1 and X_2 such that:

$$98.19X_1 - 92.23X_2 = 0,$$

where X_1 is the number of 1985 bonds sold short, i.e., a cash inflow, and X_2 is the number of 1986 bonds bought, i.e., a cash outflow. If we normalize this expression to $X_1 = 1.0$, for selling short one 1985 bond we have:

$$98.19 - 92.23X_2 = 0, \text{ or } 92.23X_2 = 98.19,$$

or

$$X_2 = 1.065.$$

Again to duplicate the text results and fix the interest rate in the third year, the net cash flow in the first and second period is constrained to zero:

$$98.19X_1 + 92.23X_2 - 99.16X_3 = 0,$$

and

$$-(100.00 + 9.875)X_1 - 7.875X_2 + 12.00X_3 = 0,$$

where X_1 is the number of 1985 bonds sold, X_2 is the number of 1986 bonds sold and X_3 is the number of 1987 bonds bought. Setting $X_2 = 1.0$, which is equivalent to selling short 1.0 1986 bond, the linear system becomes:

$$98.19X_1 \ - \ 99.16X_3 = -92.23,$$

and

$$-109.875X_1 + 12.00X_3 = 7.875,$$

or

$$\begin{bmatrix} 98.19 & -99.16 \\ -109.875 & 12.00 \end{bmatrix} \begin{bmatrix} X_1 \\ X_3 \end{bmatrix} = \begin{bmatrix} -92.23 \\ 7.875 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0.0335 \\ 0.9633 \end{bmatrix}$$

Thus to lock-in the third period interest rate the investor would short 0.0335 1985 bonds and 1.0 1986 bonds while purchasing 0.9633 1987 bonds.

Finally, the fourth period lock-in procedure would involve 1985 through 1988 bonds where the 1988 bonds purchased are denoted X_4. In this case, we wish to fix the cash flows in years 1 to 3 to zero, such that

$$98.19X_1 + 92.23X_2 + 99.16X_3 - 87.20X_4 = 0,$$

$$-(100.00 + 9.875)X_1 - 7.875X_2 - 12.00X_3 + 8.25X_4 = 0,$$

and

$$-(100.00 + 7.875)X_2 - 12.00X_3 + 8.25X_4 = 0.$$

By normalizing X_3 (i.e. $X_3 = 1.0$), the system becomes

$$\begin{bmatrix} 98.19 & 92.23 & -87.20 \\ -109.875 & -7.875 & 8.25 \\ 0 & -107.875 & 8.25 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix} = \begin{bmatrix} -99.16 \\ 12.00 \\ 12.00 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.0263 \\ -0.0289 \\ 1.077 \end{bmatrix}$$

Therefore, the investor purchases 0.0263 of 1985 bonds, and 0.0289 of 1986 bonds, sells one 1987 bond short and buys 1.077 1988 bonds. The remaining transactions are then simple extensions of this paradigm.