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FARM PLANNING, LINEAR PROGRAMMING AND SOIL HETEROGENEITY

P. A. RICKARDS* AND W. F. MUSGRAVE†

The accumulation of experience in the application of linear programming and its extensions by farm management workers, together with the growth of the farm advisory profession, should ultimately result in a fruitful interaction. However, this attractive prospect should not be allowed to mask the fact that to date most Australian uses of programming in agriculture have been basically research oriented. To do this it has often been necessary to abstract from those unique characteristics of a particular farm situation which would severely detract from study of general principles and policy.

While there is no denying the usefulness of such generalized studies it should be recognized that farm investigations for advisory purposes may require more of a "warts and all" approach. A problem which could typically be involved in such an approach arises when the optimum location of various enterprises is not determinable by orthodox programming procedures because of the lack of coincidence of soil-type and paddock boundaries. A procedure which should, in general, enable determination of a near optimum solution in such cases is presented.

A DESCRIPTION OF THE CASE PROPERTY

An analysis of farm planning on an 8,333 acre holding near Goondiwindi was commenced early in 1964 with the intention of generating a plan for the following twelve months.[1] The property considered is located in a part of the "brigalow belt" which is characterized by the heterogeneous nature of its soils and the wide range of enterprises which are successful on these soils.

At the commencement of this study, most of the property had been cleared and 2,200 acres were available for immediate cultivation. The accompanying map, Figure 1, shows the range of soil types, their distribution

* Research Fellow, University of New England.

† Senior Lecturer in Agricultural Economics, University of New England.

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[1] A longer planning period was not considered because advances in technology in this district have been so rapid as to make a detailed approach to long-term planning unrewarding. Hence, rotational constraints were not built into the matrices but the immediate past history of various paddocks was considered when deciding upon the yield coefficients to be used.

and the paddock layout in 1964. This map was constructed after identifying soil production types and cultivation land on a scaled aerial photograph. Shadelines, wasteground, and subdivision fences were also located. A planimeter was then used to measure the acreages of soil types and paddocks.

The lack of coincidence of soil type and paddock boundaries would clearly prevent the simple allocation of crops according to soil distribution and other restraints. In order to demonstrate the significance of this problem, as well as to introduce the property production possibilities in more detail, an initial linear programming matrix (Matrix A) was drawn up and the optimum plan derived. The details of this matrix are contained in Appendixes 1 and 2. Apart from the acreages of the various soil types available for cultivation the restraints were determined in consultation with the operator. The remaining coefficients were selected by choosing the appropriate information from that provided by the operator, members of a district survey,[2] local technical experts and market reports. It is worth repeating, and emphasizing, that paddock size and distribution are not included in the restraints of this matrix. Accordingly, it would be unreasonable to expect the computed optimum plan to be practicable.

The programmed solution to Matrix A, plan 1, is included in Table 1. This plan represents an optimum farm plan for the case study property, given that the location of crop production activities need only to be restrained by soil-type distribution. If plan 1 were fitted to the property, according to the soil type boundaries marked on Figure 1, insurmountable difficulties would arise in attempting to put it into operation.

This can be well illustrated by considering activity X_1 which represents wheat production on the 519 acres of soil type 1 under cultivation. This soil type occurs in parts of paddocks B, C, F, G, H, I, K and L. Should wheat be grown on these areas it would be impractical to use these paddocks for grazing purposes. In other words land under wheat should be fenced separately from that used for grazing. To erect subdivision fences on each of the eight paddocks on which soil type 1 occurs would scarcely be practicable from a managerial viewpoint due to the small size and irregular shape of the areas concerned.

[2] A survey of production enterprises on eight neighbouring properties was conducted in order to provide a more comprehensive pool of information from which the appropriate coefficients could be selected without fear of excessive bias. In the absence of this type of precaution the farmer estimates of crop yields on different soil types may include undetected significant errors. Should this be so there is probably little advantage in using the vector weighting technique, discussed later, for assessing likely paddock yields for all feasible paddock-crop combinations rather than using the available farm records of yields of particular crops in individual paddocks, augmented by the operator's subjective estimates of yields for paddock-crop combinations which have not been tried.

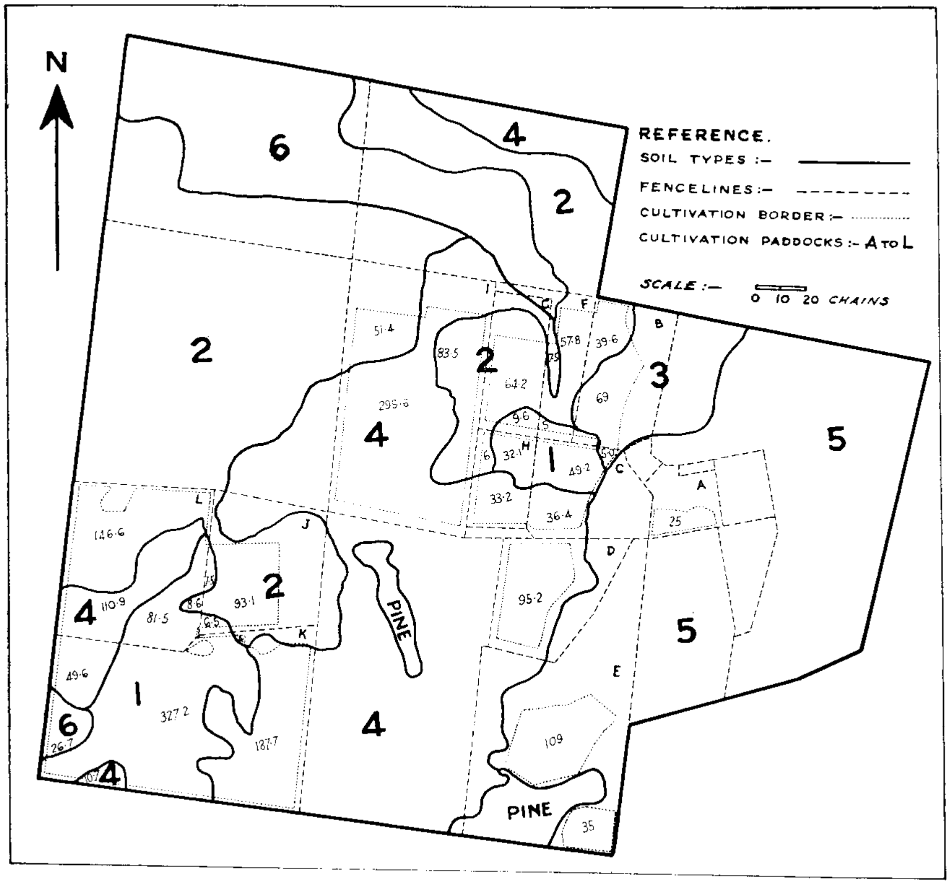

Figure 1. Soil production types, paddock code, fencelines and cultivation acreages—May, 1964.

LEGEND

Number	Mapping Unit	Cultivated Acreage	Major Characteristics of Dominant Soil
1	Production Type 1 ..	519	Heavy dark grey self mulching clays.
2	Production Type 2 ..	555	Weakly gilgaied brown clays with strongly acid subsoils.
3	Production Type 3 ..	74	Deep heavy grey clays with little profile differentiation.
4	Production Type 4 ..	730	Weakly solonized red brown earths.
5	Production Type 5 ..	169	Weakly solonized soils related to solodized-solonetz, red brown earths and grey and brown soils of heavy texture.
6	Production Type 6 ..	27	Very strongly gilgaied grey clays with strongly acid sub-soils.

Similar problems arise if an attempt is made to fit any of the other crop activities into the property organization in the manner specified in plan 1.[3] It will be recalled that this outcome was anticipated and it is suggested that such shortcomings will be inevitable in any similar analysis of properties with heterogeneous soil resources and a range of crop alternatives.

TABLE 1

PLAN 1—The Programmed Solution from Matrix A

	Activity	Unit	Activity Level
X_1	Wheat Type 1	Acre	519·0
X_2	Wheat Type 2	Acre	101·5
X_6	Early Oats Type 3	Acre	74·0
X_7	Early Oats Type 4	Acre	149·9
X_{12}	Late Oats Type 4	Acre	68·6
X_{13}	Late Oats Type 5	Acre	169·0
X_{16}	Early Grazing Sorghum Type 4	Acre	216·6
X_{19}	Late Grazing Sorghum Type 2	Acre	480·5
X_{25}	Wheat Sown Lucerne Type 4	Acre	294·9
X_{27}	Native Pasture X	Acre	1,875·0
X_{28}	Native Pasture Y	Acre	762·0
X_{29}	Native Pasture Z	Acre	2,857·0
X_{30}	May-June Oat Transfer	D.M.E.*	191·3
X_{33}	June-March Feed Transfer	D.M.E.	13,268·4
X_{34}	April-June Feed Transfer	D.M.E.	13,491·6
X_{36}	October-December Consumption Transfer	D.M.E.	10,768·7
X_{37}	January-March Consumption Transfer	D.M.E.	15,641·8
X_{38}	April-June Consumption Transfer	D.M.E.	7,277·8
X_{39}	July-September Consumption Transfer	D.M.E.	14,731·2
X_{40}	Flock Sheep	Breeding Ewe.	321·0
X_{41}	Stud Sheep	Breeding Ewe.	500·0
X_{42}	Wethers	Wether	155·5
X_{43}	Crop Wethers	Wether	3,504·5
X_{46}	Partially Competitive Cattle	Breeding Cow.	101·4
X_{48}	Crop Fatteners	Steer	8·7
Surplus Resources—			
R_{12}	Wheat Maximum	Bags	829·2
R_{13}	January Tractor	Hour	40·7
R_{17}	Arable Type 2 Wheat Supply	Acre	101·5
Objective Function			£28,355·1

* One Dry Merino Ewe equivalent (D.M.E.) is the energy requirement of an adult merino ewe, neither pregnant nor lactating for normal maintenance and wool growth over a one-month period. One D.M.E. is taken as being equivalent to 36 lb T.D.N.

[3] On the other hand the information included in this solution suggests that a systematic subdivision of the larger paddocks according to soil type distribution constittues an attractive consideration for longer term planning.

For crop management reasons the size and location of cultivation paddocks, as well as soil types, is an effective restraint on the spatial distribution of crop production activities. Clearly the production situation must be specified more rigorously. In order to do this a new matrix, Matrix B, was constructed.[4] In this matrix acreage restraints and crop production activities were specified for *each* cultivation paddock. The way in which this was done will now be described in some detail.

CONSTRUCTION OF MATRIX B

Restraints

Twelve cultivation paddocks, distinguished by letters A to L in Figure 1, were available for cropping in the 1964 planning period. Restraints R_1 to R_{12} represent the acreages of arable land in these paddocks.

Restraints R_{13} to R_{23} in Matrix B are defined identically with restraints R_6 to R_{16} of Matrix A (see p. 204). In addition the feed retraints R_{24} to R_{34} of Matrix B are identical with the corresponding feed restraints, R_{18} to R_{27} of Matrix A (see p. 205).

Activities

Six alternative crops were considered for each cultivation paddock. These crops were wheat, early grazing oats, late grazing oats, early grazing sorghum, late grazing sorghum and lucerne.

The vector for a particular crop activity on a particular paddock represents the production process that would be operative if the whole of the paddock were committed to that crop. This vector is derived by weighting the relevant crop production process for each soil type in the paddock, according to the proportion of the paddock which is made up of that soil type, and then summing over all soil types in the paddock. In other words the new paddock crop production processes are weighted linear combinations of the old soil type processes of Matrix A. The procedure used to determine these vectors is elaborated in Appendix 3.

While a full complement of alternative crops was initially considered for each cultivation paddock, discussions with the farm operator revealed that 11 of these alternatives were agronomically infeasible. After deleting these, 61 crop-paddock alternatives remained and these were specified in activities P_1 to P_{61}.

[4] For reasons of space this second matrix is omitted as it is felt that its construction could be understood from study of Matrix A and reading the following pages. However, copies of Matrix B are available from the authors.

Matrix B was completed by specifying three native pasture activities P_{62} to P_{64}, ten feed transfer activities P_{65} to P_{74} and nine livestock activities P_{75} to P_{83}. These 22 activities, P_{62} to P_{83}, are defined identically with activities X_{27} to X_{48} in Matrix A (see p. 206).

Even though Matrix B constitutes a more comprehensive specification of the original farm planning problem than does Matrix A, it still fails to be sufficiently rigorous to ensure that the programmed solution will provide a practicable farm plan. A necessary condition for an acceptable farm plan would be that (with the exception of paddock G, I and L for which temporary subdivision fences are available) the whole of each cultivation paddock should be placed either under a single crop or under a combination of agronomically compatible crops.

Conceptually this problem appears ideal for the application of integer programming.[5] This could be done by specifying integer restraints for alternative crop activities on each cultivation paddock, thereby ensuring that the programmed solution contained no more than one crop on each cultivation paddock. Such a solution would normally be acceptable to the farm operator. Unfortunately current experience with the integer programming algorithm indicates problems of slow convergence to an optimum with matrices containing more than 40 activities.[6] The possibility of this occurring in the present study was sufficient to deter the use of integer programming. This stimulated an investigation into ways in which the orthodox linear programming model could be used to obtain acceptable farm plans in such a situation.

The Results from Matrix B

The programmed solution to Matrix B is included in Table 2.

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[5] R. E. Gomory, *"An Algorithm for the Mixed Integer Problem"*, RAND, R. M.—2597, July 7, 1960.

[6] See J. M. Bennett and R. J. Dakin, *"Experience with Mixed Integer Linear Programming Problems"*, Technical Report No. 18, Basser Computing Department, University of Sydney, October, 1961. However, there is hope that this problem may be transitory. See R. J. Dakin, *A Mixed Integer Programming Algorithm*, Technical Report No. 31, Basser Computing Department, University of Sydney. The presence of the more profound problem of the interpretation of the shadow prices provided by a completed integer programme could also be involved. For a more detailed discussion of this problem see H. M. Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems*, Prentice Hall, Englewood Cliffs, 1963, Ch. 5.

TABLE 2

*PLAN 2—The Programmed Solution from Matrix B**

	Activity	Unit	Activity Level
P_1	Early Oats Paddock A	Acres	25·0
P_4	Early Oats Paddock B	Acres	49·8
P_6	Lucerne Paddock B	Acres	58·8
P_7	Early Oats Paddock C	Acres	30·2
P_{10}	Late Grazing Sorghum Paddock C	Acres	60·4
P_{15}	Early Grazing Sorghum Paddock D	Acres	95·2
P_{21}	Lucerne Paddock E	Acres	144·0
P_{25}	Late Grazing Paddock F	Acres	70·3
P_{31}	Late Grazing Sorghum Paddock G	Acres	73·8
P_{34}	Wheat Paddock H	Acres	71·3
P_{41}	Early Oats Paddock I	Acres	190·0
P_{43}	Early Grazing Sorghum Paddock I	Acres	244·5
P_{49}	Late Grazing Sorghum Paddock J	Acres	107·1
P_{51}	Wheat Paddock K	Acres	505·9
P_{57}	Wheat Paddock L	Acres	178·7
P_{60}	Late Grazing Sorghum Paddock L	Acres	168·9
P_{62}	Native Pasture X	Acres	1,875·0
P_{63}	Native Pasture Y	Acres	762·0
P_{64}	Native Pasture Z	Acres	2,857·0
P_{65}	May-June Oat Transfer	D.M.E.	1,835·2
P_{66}	July-September Oat Transfer	D.M.E.	32·8
P_{68}	January-March Feed Transfer	D.M.E.	13,194·6
P_{69}	April-June Feed Transfer	D.M.E.	15,663·4
P_{71}	October-December Consumption Transfer	D.M.E.	10,807·0
P_{72}	January-March Consumption Transfer	D.M.E.	15,069·3
P_{73}	April-June Consumption Transfer	D.M.E.	4,781·8
P_{74}	July-September Consumption Transfer	D.M.E.	15,687·5
P_{76}	Stud Sheep	Breeding Ewes.	500·0
P_{77}	Wethers	Wether	764·1
P_{78}	Crop Wethers	Wether	3,655·1
P_{81}	Partially Competitive Cattle	Breeding Cow.	97·9
P_{83}	Crop Fatteners	Steer	11·7
	Surplus Resources—		
R_{19}	Wheat Maximum	Bags	1,075·3
R_{20}	January Tractor	Hours	83·1
	Objective Function		£27,465·1

* The tape linear programme for the G.E.225 electronic computer at the University of Queensland was used to solve Matrix B. Solution was reached in approximately 10 minutes computing time and the final matrix was saved on magnetic tapes.

A comparison of plans 1 and 2 shows that the grossly impracticable placement of crops in the former plan has largely been corrected in plan 2. Improved acceptability of the programmed solution has only been achieved with a decline of £890 in the objective function. This revenue decline is to be expected as a result of the added restrictions included in the second matrix. In restraining the placement of crops by paddock boundaries, rather than by soil type boundaries (as in Matrix A), several paddocks with a range of soil types have been forced to accept one crop alone in plan 2.

In such a case it is inevitable that some soil types, within particular paddocks, support crops which would be suboptimally placed according to the crop location criterion used in deriving plan 1, and a decline in revenue must result.

Analysis of plan 2 shows that four of the twelve paddocks, namely paddocks B, C, I and L, contain more than one crop. However, the crop combinations in paddocks I and L, are quite acceptable as they may be separated by temporary subdivision fences.[7]

Paddock B is divided in its use between early oats and lucerne in the ratio of 4:5. These two crops would be compatible, because they would both be required to provide winter grazing for the 500 stud ewes of plan 2. The proportional acreage of each crop implies a rotation with four years of oats followed by five years of lucerne. Such a rotation would be acceptable on agronomic grounds.

The recommendation for paddock C is for its division between early oats (P_7) and late grazing sorghum (P_{10}). In practice, these two crops would not be grown in the same paddock since they provide forage for different classes of livestock. This is just the type of unacceptable situation which could be expected with this type of specification in the absence of integer restraints. It is apparent that either P_7 or P_{10} must be excluded from plan 2 in order to reach an acceptable farm plan.

The "true" or practicable optimum solution to this problem would belong to a set of production possibilities which must be a subset[8] of the set defined by Matrix B. Should it be possible to define this subset, the "true" optimum, on being determined, would result in a lower value of the objective function than does plan 2. Unfortunately this is as far as conventional static programming analysis can take us. That is, we know that the "true" optimum must be similar to plan 2 but that it must produce a somewhat lower revenue. Because of our inability to use integer programming we are forced to move beyond conventional programming analysis.

Though a more rigorous specification of the problem than that represented by Matrix B does not seem possible, this does not mean that we cannot move *toward* the "true" optimum within a programming framework. For example, by successively reprogramming the problem using a series of unreal activity price[9] combinations in the objective function of Matrix B, it would be possible to inspect a multiplicity of alternative plans. This technique amounts to forcing successive solutions into the basis which do not represent optimum resource use under the actual price regime. By using sufficient price combinations it would be possible to inspect all plans

[7] The authors realize that the weighted production processes estimated for crop activities on the whole of paddocks I and L need not necessarily hold for lesser portions of these paddocks as is implied in plan 2. This problem was investigated and it was found that the crop boundaries could be located so that discrepancies between the assumed and the likely real production processes for the paddock portions were not material.

[8] A set of points is called a subset of another set of points when all the points of the former are also points of the latter.

[9] The use of arbitrarily selected objective coefficients to ensure the inclusion or exclusion of certain activities from the final solution is called by Heady and Candler the *m technique*. For a description see E. O. Heady and W. V. Candler, *Linear Programming Methods,* Ames, Iowa State College Press, 1958, Ch. 4.

TABLE 3

Range Calculations and Border-plan Information for P_7 and P_{10} in Plan 2

Original Basis Variable	Level in Plan 2	Objective Coefficient	Objective Coefficient Range	Border Price	New Basis Variable in Border-plan	New Basis Variable Level	Change in Level of Original Variable
P_7 Early Oats Paddock C.	30·2	—1·86	—0·013	—1·873	P_9 Early Grazing Sorghum Paddock C.	30·2	—30·2
			+0·556	—1·304	P_{44} Late Grazing Sorghum Paddock I.	60·4	+60·4
P_{10} Late Grazing Sorghum Paddock C.	60·4	—0·54	—0·556	—1·096	P_{44} Late Grazing Sorghum Paddock I.	60·4	—60·4
			+0·134	—0·406	P_{59} Early Grazing Sorghum Paddock L.	30·2	+30·2

at corner points adjacent to the present solution. The programmer could then select that plan which satisfies all "real" restraints for the property, but for which the revenue decrement from plan 2 is minimized, as the true optimum plan. Of course, the computing time required for such an investigation in a problem of the present dimensions would be so great as to preclude its use as a practicable technique.

Fortunately, the computing routine used is able to calculate the minimum change in the objective coefficients of each current basis activity, which would be necessary to induce a change in basis variables. For purposes of brevity, these objective-coefficient changes are referred to as "range values"; the value of an objective coefficient after adding the "range value" to its original value is called the "border price" while the new plan that becomes optimum at a "border price" of any activity is known as the "border plan". These concepts are basic to variable-price programming[10] which broadly consists of an analysis of a series of plans at successively increasing border prices of one or more activities.

The border-price information associated with plan 2 can be used to examine the border plans nominated by shifts in the revenue of the "problem" activities P_7 and P_{10}. It is felt that such a procedure is justified by the fact that any movement towards a practicable near-optimum solution must involve changes in the level of these two activities and hence it is not unreasonable to think that a plan nominated by shifts in the objective function in the P_7, P_{10} plane is highly likely to provide as good an approximation as could be hoped for.

A summary of border price and plan information obtained for activities P_7 and P_{10} is included in Table 3. All rows in this table have similar interpretation but for purposes of illustration the specific meaning of the first row is given as follows: Activity P_7 is included in plan 2 at the 30.2 acre level. The objective coefficient of P_7 for which plan 2 is optimum is—£1.86 but, *ceteris paribus,* a change of —£0.013 in this value would be sufficient to induce a change in basis variables. Hence the lower border price of P_7 is —£1.873. The new basis variable at this border price is P_9 which is introduced at the 30.2 acre level and totally displaces P_7.

It should be noted that four border plans are located in all because there is an upper and lower border price for each of the two activities. At each border price of P_7 and P_{10} the new plan which becomes optimum contains either P_7 or P_{10} but not both. Hence it is not unlikely that one or more of these border plans may constitute a practicable farm plan. The next step is to determine a criterion which enables a selection of the most attractive of these border plans. The selected border plan should satisfy all "real" restraints for the property and should minimize the revenue decrement from that of plan 2. In the present instance the relevant choice indicator becomes the minimum revenue decrement that would result when a border plan is adopted but *under the price regime which made plan 2 optimum.*

Table 4 includes information on changes in the basis variables of plan 2 at both upper and lower border prices of P_7 and P_{10} as well as indicating the revenue decrements to be expected if these border plans were adopted

[10] *Op. cit.,* Ch. 8.

TABLE 4

Changes in Plan 2 at both upper and lower " Border Prices " of Activities P_7 and P_{10}

(1) Border plan	(2) Plan 2 Basis Variable Identity		(3) Border price	(4) Activities on Paddock C in border plan			(5) Other crop activities changed in level in border plan		(6) Real Objective function decrement with border plan under original revenue regime
						Level			
A	P_7	Early Oats Paddock C.	—1·873	P_9 *	Early Grazing Sorghum.	30·2	Nil	.. .. ..	—30·2 × 0·013 = £0·39
				P_{10}	Late Grazing Sorghum.	60·4			
B	P_7	Early Oats Paddock C.	—1·304	P_7	Early Oats	90·6	P_{41}	Early Oats Paddock I.	—60·4 × 0·556 = —£33·58
							P_{43}	Early Grazing Sorghum Paddock I.	
							P_{44}*	Late Grazing Sorghum Paddock I.	
C	P_{10}	Late Grazing Sorghum Paddock C.	—1·096	P_7	Early Oats	90·6	P_{41}	Early Oats Paddock I.	
							P_{43}	Early Grazing Sorghum Paddock I.	—60·4 × 0·556 = —£33·58
							P_{44}*	Late Grazing Sorghum Paddock I.	
D	P_{10}	Late Grazing Sorghum Paddock C.	—0·406	P_{10}	Late Grazing Sorghum.	90·6	P_{57}	Wheat Paddock L	—30·2 × 0·134 = —£3·99
							P_{59}*	Early Grazing Sorghum Paddock L.	
							P_{60}	Late Grazing Sorghum Paddock L.	

* New Basis Activity.

under the original price regime.[11] The technique used to calculate these revenue decrements will now be discussed.

A border price, as the name implies, marks a boundary in the value of an objective coefficient at which both the current plan and another plan, the so-called border plan, are optimum. Hence the border price marks the unique value of activity revenue at which it is possible to pass from the current plan to the border plan without a loss of income.

As indicated in Table 4, the lower border price for activiity P_7 is $-£1.83$. At this price both plan 2 and border plan A yield the same revenue. This revenue, however, will be $£(30.2 \times 0.013) = £0.39$ lower than that of plan 2 due to the increase of £0.013/acre in the cost of P_7 of which there is 30.2 acres in plan 2. Since border plan A does not contain P_7 the revenue from this plan is insensitive to changes in the price of this activity. Therefore if border plan A were adopted under the original price regime, instead of that containing the border price of P_7, there would be no change in the objective function, i.e. border plan A could be adopted under the original price regime at a "cost" of £0.39. This is the procedure generally followed in determining the level of revenue decrement for each border plan: the relevant objective-coefficient range value is multiplied by the change in level of the activity to which the objective coefficient applies.

In border plan A, corresponding to the lower border price of P_7, early oats have been replaced by early grazing sorghum at a cost of only £0.39. This leads to a situation in which Paddock C is divided in its use between early and late planted grazing sorghum crops. This combination is agronomically feasible provided a small adjustment is made in the forage production assumed for the former crop. If both crops are grown on the same paddock, as is implied in border plan A, then the first three months' grazing of P_9 would have to be sacrificed in order to allow the late crop, P_{10}, to reach the grazing stage. This would involve a loss of 2.9 D.M.E. per acre of January-March "consumption" feed over 30.2 acres of early grazing sorghum. Since January-March "consumption" feed on plan 2 was marginally valued at £0.1123 per D.M.E. unit, one could expect that an operationally feasible plan including P_9 and P_{10} at the levels specified in "border plan" A could be achieved with a further revenue decrement of £9.83. This would make the total decrement only £10.22.

Border plan B for the upper border price of P_7 and border plan C for the lower border price of P_{10} are identical. An investigation of these plans is unnecessary in that the associated real objective function decrement of £33.58 exceeds the corresponding decrement for the feasible version of border plan A.

[11] Different computer routines provide differing amounts of information on border plans and prices and this determines the length of time involved in performing the operations entailed in constructing the above table. For example, routine CD.225 D7.005 of the G.E.225 automatically provides all the information in Table 4 except that for column 6 so that appraisal of border plans is only a matter of minutes. On the other hand, if no such information is provided by the computer it would be necessary to have the final matrix printed and the calculations required to derive border plan information could take several hours for each plan.

TABLE 5

PLAN 3—A Practicable Solution to Farm Planning (The levels of activities included in Plan 2 are included for purposes of comparison)

Activity		Unit	Activity Level	Activity Levels in Plan 2
P_1	Early Oats Paddock A	Acres	25·0	25·0
P_4	Early Oats Paddock B	Acres	49·8	49·8
P_6	Lucerne Paddock B	Acres	58·8	58·8
P_7	Early Oats Paddock C	Acres	....	30·2
P_9 *	Early Grazing Sorghum Paddock C	Acres	30·2	....
P_{10}	Late Grazing Sorghum Paddock C	Acres	60·4	60·4
P_{15}	Early Grazing Sorghum Paddock D	Acres	95·2	95·2
P_{21}	Lucerne Paddock E	Acres	144·0	144·0
P_{25}	Late Grazing Sorghum Paddock F	Acres	70·3	70·3
P_{31}	Late Grazing Sorghum Paddock G	Acres	73·8	73·8
P_{34}	Wheat Paddock H	Acres	71·3	71·3
P_{41}	Early Oats Paddock I	Acres	220·2	190·0
P_{43}	Early Grazing Sorghum Paddock I	Acres	214·3	244·5
P_{49}	Late Grazing Sorghum Paddock J	Acres	107·1	107·1
P_{51}	Wheat Paddock K	Acres	505·9	505·9
P_{57}	Wheat Paddock L	Acres	178·7	178·7
P_{60}	Late Grazing Paddock L	Acres	168·9	168·9
P_{62}	Native Pasture X	Acres	1,875·0	1,875·0
P_{63}	Native Pasture Y	Acres	762·0	762·0
P_{64}	Native Pasture Z	Acres	2,857·0	2,857·0
P_{65}	May-June Oat Transfer	D.M.E.	1,822·3	1,835·2
P_{66}	July-September Oat Transfer	D.M.E.	70·8	32·8
P_{68}	January-March Feed Transfer	D.M.E.	13,107·1	13,194·6
P_{69}	April-June Feed Transfer	D.M.E.	15,645·3	15,663·4
P_{70}	October-December Consumption Transfer.	D.M.E.	10,807·0	10,807·0
P_{72}	January-March Consumption Transfer.	D.M.E.	15,156·9	15,069·3
P_{73}	April-June Consumption Transfer	D.M.E.	4,742·9	4,781·8
P_{74}	July-September Consumption Transfer.	D.M.E.	15,678·4	15,687·5
P_{76}	Stud Sheep	Breeding Ewes.	500·0	500·0
P_{77}	Wethers	Wether	753·8	764·1
P_{78}	Crop Wethers	Wether	3,673·6	3,655·1
P_{81}	Partially Competitive Cattle	Breeding Cow.	98·0	97·9
P_{83}	Crop Fatteners	Steer	7·7	11·7
Surplus Resources—				
R_{19}	Wheat Maximum	Bags	1,075·3	1,075·3
R_{20}	January Tractor	Hours	83·1	83·1
Objective Function			£27,454·9	£27,465·1

P_9* represents a modification of the original P_9 vector such that P_9* and P_{10} are compatible activities.

In border plan D which would become optimum at the upper border price of P_{10}, the real objective function decrement is £3.99. In this plan the whole of Paddock C is under late grazing sorghum. Operational feasibility in Paddock C, however, has only been achieved at the expense of Paddock L, which is divided in its use between three crop activities, namely,

P_{57}, P_{59} and P_{60}. The assumed production coefficients of all of these activities could not be realized if it became necessary to produce them in the proportions specified in border plan D. If the only subdivision fence available in Paddock L were used to fence in P_{57} (wheat on Paddock L) then adjustments could be made to P_{59} (early grazing sorghum Paddock L) in order to make it agronomically compatible with P_{60} (late grazing sorghum Paddock L). This adjustment would involve the sacrifice of 92.11 D.M.E.s of January-March consumption feed valued at £10.34. In this case the total real objective function decrement for border plan D would be £14.33, as compared with a real decrement of £10.22 for border plan A. Hence the latter plan is selected as the most attractive of the limited range of practicable farm plans investigated, and no further computation is necessary.

While it is not essential to the exercise, it is of interest to check the validity of the revenue fall predicted above. To do this the problem was reprogrammed after making changes in Matrix B. Two changes were necessary. Firstly, the supply of January-March "consumption" D.M.E. was reduced by 87.58 units to allow agronomic compatibility between P_9 and P_{10} and secondly the objective coefficient of P_7 was set below the border price of −£1.873 to ensure exclusion of P_7 from the programmed solution. One advantage of this complete enumeration of the selected border plan is the definition of precise levels of activities in the fodder-livestock complex.

Since the original solution to Matrix B was saved on magnetic tape the new solution incorporating these changes was reached in a very short time. In fact, only one additional matrix inversion was required. For purposes of comparison both solutions are included in Table 5.

Plan 3 represents an operationally feasible solution to farm planning for the case study property. As predicted, the revenue from this plan lies £10.2 below the objective function of plan 2.

It does not necessarily follow that plan 3 represents the true optimum farm plan, since it was selected from a limited range of the complete set of border plans surrounding plan 2. The revenue from the true optimum plan, however, must be less than plan 2 in which production restraints were inadequately specified. Hence we can say that the upper limit of likely revenue difference betwen plan 3 and the true optimum plan is less than the difference between the objective functions of plans 2 and 3, e.g., less than £10.2. Apart from this test we have no way of checking on the likely extent of "sub-optimality" introduced. On the other hand, it is clear that the extent of the deviation from the true value of optimum revenue is hardly significant.

Conclusion

It is suggested that if linear programming is to find widespread application in individual farm planning, then advisers will need to grapple with the problem posed by the failure of paddock and soil-type boundaries to coincide. In this article it is shown how the presence of this problem prevents derivation of an optimum by conventional programming procedures. In addition, it is shown how programming methods can be used to explore a production situation involving this problem in such a way as to obtain a satisfactory approximation to the optimum.

It should be emphasized that the problem posed by soil heterogeneity and paddock location should be ideal for the application of integer programming and that the present suggested procedure would be but an inferior substitute for successful application of that technique. However, as long as the convergence problem associated with integer programming continues, some "cut and try" procedure such as has been employed herein will be required.

Of course, there is no guarantee that widespread use of the suggested procedure will not produce yet another convergence problem, for example, in cases where several paddocks are divided in their use among incompatible crops. Certainly, it does seem possible that convergence could be less rapid than was the case in this study. However, it is believed that intelligent use of border information in a systematic fashion, as indicated, should lead to an acceptable approximation within a reasonable time period.

APPENDIX 1

A DESCRIPTION OF MATRIX A

1. The Programmed Restraints

Five discrete soil production types were under cultivation on the case study property in 1964. (The cultivation portion of soil production type 6 has been amalgamated with that of soil production type 2 since these two soils have identical crop productivity once the gilgais have been removed from the former soil type.) The acreage of these soil types comprised the restraints R_1 to R_5.

In addition to cultivation land it was estimated that 5,494 acres of the property were under cleared native pasture in 1964. This land was subdivided into three types on the basis of pasture productivity and the acreage of each type constituted the restaints R_6 to R_8.

R_9 *The Maximum Stud Ewe Restraint*

The number of stud ewes was restricted to 500 as this was the maximum number that the operator could manage in the 1964 period.

R_{10} *The Lucerne Restraint*

This restraint ensures that there will be an adequate supply of lucerne feed available for those stock requiring it for strategic grazing purposes.

R_{11} *The Maximum Late Grazing Sorghum Restraint*

The operator has expressed a preference for a restricted acreage of this crop and has indicated that 160 acres would be the maximum he would be prepared to plant in any one year.

R_{12} *The Maximum Wheat Restraint*

Wheat is harvested with an auto-header capable of handling 600 bags daily.

The safe harvesting period is considered to be 12 days so that an upper limit of 7,200 bags was placed on the annual wheat harvest.

R_{13}, R_{14} and R_{15} *Tractor Hour Restraints*

The plant capacity only becomes limiting from December to March. Restraints R_{13} to R_{15} constitute the level of available tractor hours in the January, February-March and December periods respectively.

R_{16} *Supplementary Sheep Units*

It is the general concensus of opinion of graziers in the Goondiwindi district that during the summer flush, from October to April, approximately 5 breeding cows can be run to every 100 breeding ewes (or equivalent dry sheep) without active competition for feed. During the winter months, when less tall feed is available, this ratio falls to 3 per cent. This relationship was expressed in the R_{16} restraint.

R_{17} *Arable Type 2 Wheat Supply*

This restraint was specified so that adequate wheat would be available to act as a zero cost cover crop for lucerne on this land type.

R_{18} to R_{27} *Feed Restraints*

The supply of forage by crops and pastures and the demands for forage by livestock were all expressed in Dry Merino Ewe equivalents (D.M.E.s). The feed year was divided into four equal periods which roughly coincided with the four seasons of the pasture year. In addition three feed pools were established in order to differentiate crops and pastures according to the characteristics of forage supplied for livestock production.

The first or so-called "transferable" feed pool collects all forage from perennial crops and pastures. A characteristic of forage supplied to this pool is that it is freely transferable at the cost of some loss in nutritional value from the period in which it is produced to a future period. Livestock do not consume directly from this pool but all feed, after inter-period transfers, is supplied to a separate pool called the "consumption" pool.

As far as this study is concerned, the consumption requirements of livestock have been broken up into two components:

(i) a requirement of oats forage for special purpose grazing;

(ii) a requirement of forage of at least maintenance quality for general purpose grazing.

Separate consumption pools have been established for these two feed components. The "oats" feed pool collects all forage from grazing oats crops and supplies the need of those livestock activities requiring special purpose grazing.

The general purpose consumption requirements of livestock are met from the so-called "consumption pool". This pool collects forage supplies from annual crops (excepting grazing oats during the May-September period) as well as transfers from the "transferable" feed pool. No transfer activities operate within this pool as forage from annual crops usually has zero substitutability with respect to time.

The restraints R_{18} to R_{21} relate to the levels of supply of forage in the "transferable" feed pool in the four periods of the feed year: R_{22} and R_{23} relate to the levels of supply of special purpose forage in the "oats" feed pool in the May-June and July-September periods respectively and R_{24} to R_{27} relate to the levels of supply of forage in the "consumption" pool in the feed year periods.

R_{28} *The Grazing Sorghum Supply Restraint*

This restraint ensures an adequate supply of sorghum forage for the "crop wether" activity which requires four months of crop forage during the April to September period.

2. THE ACTIVITIES CONSIDERED

Five discrete soil types were under cultivation on the property in 1964. A maximum of six alternative crops was allowed on each soil type, these crops being wheat, early grazing oats, late grazing oats, early grazing sorghum, late grazing sorghum and lucerne.

The first three activities considered, X_1 to X_3, were grain wheat activities on soil types 1, 2 and 3 respectively.

Activities X_4 to X_8 represent early season grazing oats on soil types 1 to 5 respectively while activities X_9 to X_{13} represent late season grazing oats on the same soil types.

October planted forage sorghum or so-called "early grazing sorghum" on soil types 1, 2, 4 and 5 respectively is represented by activities X_{14} to X_{17}. This crop has a three-year cycle on soil types 1, 2 and 4, on which it readily produces ratoon growth, but only a two-year cycle on soil type 5 where the ratoon stand is not successful.

The next four activities X_{18} to X_{21} represent January planted or "late grazing sorghum" on soil types 1, 2, 4 and 5 respectively. This crop is normally planted with the auxiliary tractor so that activities X_{18} to X_{21} are not competitive for tractor hours in January.

Activities X_{22} to X_{26} represent lucerne growing activities on soil types 1 to 5 respectively. On soil types 1 and 2, wheat which is optimally produced in the programmed solution provides a zero cost cover crop for lucerne. On soil types 3 and 4 where wheat alone would not normally be optimally produced, the lucerne activities include wheat as an initial cover crop and the revenue from wheat sales is spread evenly over the life of the lucerne stand. On soil type 5 no cover crop is used due to the unsuitability of this soil for wheat production.

Land under cleared native pasture was subdivided into three types, namely X, Y and Z, on the basis of pasture productivity. Activities X_{27} to X_{29} respectively refer to pasture activities on these three land types. The production coefficients used for each pasture type represent the seasonal feed productivity of the pastures and are not a measure of annual production under some specific form of pasture management.

The next ten activities X_{30} to X_{39} are feed transfer activities and relate to transfer of forage within and among the three feed pools previously defined. The first of these activities X_{30} and X_{31} allow transfer of oats forage from the "oats" feed pool to the "consumption" pool. The remaining feed transfer activities were specified so that the programme would be enabled to select the optimum form of pasture management for the property.

Activities X_{32} to X_{35} allow transfer of feed within the "transferable" feed pool from each of the four periods of the feed year to the following period at a loss of nutrient value. A nutrient decline of 35 per cent was assumed for transfers into a frost-free period and a decline of 50 per cent assumed for transfers into a period of frost incidence.[12]

In order to relate the "transferable" feed pool, with its associated inter-period transfer activities, to the consumption requirements of livestock, four additional activities, X_{36} to X_{39}, were specified to allow the transfer of feed from any period in the "transferable" feed pool to the corresponding period in the "consumption" pool.

Matrix A was completed by specifying nine livestock activities. The nutrient requirements of all livestock were expressed in Dry Merino Ewe equivalents, one D.M.E. being set at 36 lb T.D.N.[13]

[12] For a discussion on the seasonal decline in nutritive value of tropical pastures see R. Milford "Nutritive Evaluation of Tropical Pastures" in *The Tropical Grassland Society of Australia Field Meeting*, April, 1964, Proceedings No. 2, Mimeo.

[13] The nutrient requirements for cattle activities were taken from *Nutrient Requirements of Domestic Animals*, No. 4., a report of the Committee of Animal Nutrition, Pub. 579, National Academy of Sciences.

The requirements for sheep activities were taken from, *Nutrient Requirements of Domestic* Animals, No. 5, a report of the Committee of Animal Nutrition, Pub. 504, National Academy of Sciences.

The first livestock activity X_{40} represents flock breeding sheep and one unit of this activity is taken to be a Poll merino breeding ewe and her normal supporting stock; approximately 3 per cent rams, all lambs and 2 tooths. It is implicit in this vector that ewes and lambs are grazed on oats for six weeks after lambing, that all weaner ewes are carried on oats from May until October, that ewes and weaners cut 12 lb of wool and that lambs cut four pounds.

X_{41} represents the stud Poll merino ewe activity which is defined similarly to the flock ewe activity.

The next two activities X_{42} and X_{43} are Poll merino wether activities. In X_{42} it is assumed that the wethers are grazed on natural pastures throughout the year and cut an average of 12.5 lb of wool per head. In contrast X_{43} represents an activity in which it is assumed that wethers are given access to forage sorghum crops for four months during the winter and consequently cut an average of 15 lb. of wool per head.

X_{44} represents breeding cattle which are fully competitive with sheep as regards feed requirements. One unit of this activity is taken as a Hereford breeding cow and her normal supporting stock; approximately 4 per cent bulls, all calves, weaners, all steers and heifers up to 24 months. It is assumed that all steers and 70 per cent of heifers are fattened on oats and sold at 24 months.

X_{45} represents vealer production which is also fully competitive with sheep as regards feed requirements. One unit of this activity is assumed to be a Hereford breeding cow plus 4 per cent bulls, all calves and weaners and carry over stock up to 24 months of age. It is assumed that all weaners are cent carry over stock up to 24 months of age. It is assumed that all weaners are fattened on oats but only 75 per cent reach sale condition in 12 months. The carry over stock, with the exception of replacement heifers, are sold fat at 24 months.

The majority of graziers interviewed in the Goondiwindi district estimated that the border between the supplementary and competitive range of cattle grazing in association with sheep is 5 per cent during October to March and 3 per cent during April to September. Consequently it would be feasible to run a limited number of either breeding cows or vealer mothers in a supplementary relationship with sheep in addition to X_{44} and X_{45} which were assumed to be fully competitive activities. X_{46} and X_{47} representing "partially competitive" breeding cows and "partially competitive" vealers respectively were specified such that the upper limit of these activities was set at 5 per cent of the breeding ewe numbers (or 1.67 per cent of the wether numbers). At this level X_{46} and X_{47} only become competitive for pasture feed in the April-September period and even then 60 per cent of their pasture feed requirements can be supplied without diminishing the amount of feed available for sheep activities.

The final activity X_{48} represents crop fattening. This activity entails the purchase of 30 month old store cattle during May, June and July, followed by intensive grazing on oats and sale of fat cattle in September and early October.

An arithmetic description of Matrix A is included in Appendix 2. Only non zero elements are shown in the body of the matrix.

APPENDIX 2

MATRIX A

Resources	"B" Col.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Revenue	..	12·74	9·68	8·17	—1·86	—1·86	—1·86	—1·86	—1·86	—2·1	—2·1	—2·1	—2·1
Arable Type 1	519	1	..	..	1	..	..	..	..	1	..	..	..
Arable Type 2	582	..	1	..	..	1	..	..	..	..	1	..	..
Arable Type 3	74	..	..	1	..	..	1	..	..	..	..	1	..
Arable Type 4	730	..	..	..	..	..	..	1	..	..	..	..	1
Arable Type 5	169	..	..	..	..	..	..	..	1	..	..	..	..
Pasture X	1,875	..	..	..	..	..	..	..	..	..	..	..	..
Pasture Y	762	..	..	..	..	..	..	..	..	..	..	..	..
Pasture Z	2,857	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Stud Ewes	500	..	..	..	..	..	..	..	..	..	..	..	..
Lucerne Restraint	0	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Late Grazing Sorghum	160	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Wheat	7,200	10	8	7	..	..	..	..	..	..	..	..	..
January Tractor	240	·143	·143	·143	·143	·143	·143	·143	·143	·286	·286	·286	·286
February-March Tractor	480	·528	·528	·528	·235	·235	·235	·235	·235	·256	·256	·256	·256
December Tractor	240	·16	·16	·16	·303	·303	·303	·303	·303	·16	·16	·16	·16
Supplementary Sheep Units	0	..	..	..	..	..	..	..	..	..	..	..	..
Arable 2 Wheat Supply	0	..	—1	..	..	..	..	..	..	..	..	..	..
Transferable Feed—													
January-March D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
April-June D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
July-September D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
October-December D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
Oats Feed—													
March-June D.M.E.	0	..	..	..	—17	—17	—15·7	—11·4	—8·5	..	..	..	..
July-September D.M.E.	0	..	..	..	—25·5	—25·5	—23	—17·1	—12·8	—27	—27	—24·3	—18·1
Consumption Feed—													
January-March D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
April-June D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
July-September D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
October-December D.M.E.	0	—6	—4·8	—4·2	..	..	..	..	..	—9·3	—9·3	—8·4	—6·2
Grazing Sorghum Supply	0	..	..	..	..	..	..	..	..	..	..	..	..

APPENDIX 2—*continued*

MATRIX A—*continued*

Resources	"B" Col.	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}
Revenue	..	—2·1	—·42	—·42	—·42	—·5	—·54	—·54	—·54	—·56	—·14	—·14	1·87
Arable Type 1	519	..	1	..	..	..	1	..	..	..	1	..	..
Arable Type 2	582	..	..	1	..	..	..	1	..	..	..	1	..
Arable Type 3	74	..	..	..	..	..	..	..	..	..	..	..	1
Arable Type 4	730	..	..	..	1	..	..	..	1	..	..	..	..
Arable Type 5	169	1	..	..	..	1	..	..	..	1	..	..	..
Pasture X	1,875	..	..	..	..	..	..	..	..	..	..	..	..
Pasture Y	762	..	..	..	..	..	..	..	..	..	..	..	..
Pasture Z	2,857	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Stud Ewes	500	..	..	..	..	..	..	..	..	..	—24·1	—24·1	—14·3
Lucerne Restraint	0	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Late Grazing Sorghum	160	..	..	..	..	..	·33	·33	·33	·5	..	..	..
Maximum Wheat	7,200	..	..	..	..	..	..	..	..	..	..	..	1·75
January Tractor	240	·286	..	..	..	..	..	..	..	..	..	..	·036
February-March Tractor	480	·256	..	..	..	..	..	..	..	..	..	..	·132
December Tractor	240	·16	..	..	..	..	·048	·048	·048	·072	..	..	·04
Supplementary Sheep Units	0	..	..	..	..	..	..	..	..	..	..	..	..
Arable 2 Wheat Supply	0	..	..	..	..	..	..	..	..	..	..	·2	..
Transferable Feed—													
January-March D.M.E.	0	..	..	..	..	..	..	..	..	..	—12·7	—12·7	—6·3
April-June D.M.E.	0	..	..	..	..	..	..	..	..	..	—6·4	—6·4	—3·2
July-September D.M.E.	0	..	..	..	..	..	..	..	..	..	—5·1	—5·1	—2·5
October-December D.M.E.	0	..	..	..	..	..	..	..	..	..	—5·5	—5·5	—2·3
Oats Feed—													
March-June D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
July-September D.M.E.	0	—13·5	..	..	..	..	..	..	..	..	..	..	..
Consumption Feed—													
January-March D.M.E.	0	..	—7·3	—7·3	—3·7	—1·8	—3·7	—3·7	—1·8	..	..	..	..
April-June D.M.E.	0	..	—18·6	—18·6	—9·3	—4·7	—21·7	—21·7	—10·9	—6·2	..	..	..
July-September D.M.E.	0	..	..	..	..	..	—6·2	—6·2	—3·1	—3·1	..	..	..
October-December D.M.E.	0	—4·7	—7·3	—7·3	—3·7	—1·8	—7·3	—7·3	—3·7	—1·8	..	..	..
Grazing Sorghum Supply	0	..	—18·6	—18·6	—9·3	—4·7	—27·9	—27·9	—14	—9·3	..	..	..

APPENDIX 2—*continued*

MATRIX A—*continued*

Resources	"B" Col.	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	X_{30}	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}
Revenue	..	1·1	—·57	—·11	—·11	—·11	0	0	0	0	0	0	0
Arable Type 1	519	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 2	582	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 3	74	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 4	730	1	..	..	..	..	..	..	..	..	..	..	..
Arable Type 5	169	..	1	..	..	..	..	..	..	..	..	..	..
Pasture X	1,875	..	..	1	..	..	..	..	..	..	..	..	..
Pasture Y	762	..	..	..	1	..	..	..	..	..	..	..	..
Pasture Z	2,857	..	..	..	..	1	..	..	..	..	..	..	..
Maximum Stud Ewes	500	..	..	..	..	..	..	..	..	..	..	..	..
Lucerne Restraint	0	—14·3	—9·5	..	..	..	..	..	..	..	..	..	..
Maximum Late Grazing Sorghum ..	160	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Wheat	7,200	1·25	..	..	..	..	..	..	..	..	..	..	..
January Tractor	240	·036	·036	..	..	..	..	..	..	..	..	..	..
February-March Tractor	480	·132	·063	..	..	..	..	..	..	..	..	..	..
December Tractor	240	·04	·036	..	..	..	..	..	..	..	..	..	..
Supplementary Sheep Units	0	..	..	..	..	..	..	..	..	..	..	..	..
Arable 2 Wheat Supply	0	..	..	..	..	..	..	..	..	..	..	..	..
Transferable Feed—													
January-March D.M.E.	0	—6·3	—5·0	—4·9	—4·9	—4·9	..	..	—·65	1	..	..	..
April-June D.M.E.	0	—3·2	—2·5	—3·3	—1·7	—1·3	..	..	..	—·65	1	..	..
July-September D.M.E.	0	—2·5	—2·0	—1·7	—1·3	—1·1	..	..	..	..	—·5	1	..
October-December D.M.E.	0	—2·3	—1·8	—2·2	—1·7	—1·7	..	..	1	..	..	—·65	1
Oats Feed—													
March-June D.M.E.	0	..	..	..	..	..	1	..	..	..	..	..	..
July-September D.M.E.	0	..	..	..	..	..	..	1	..	..	..	..	..
Consumption Feed—													
January-March D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
April-June D.M.E.	0	..	..	..	..	..	—1	..	..	..	..	..	..
July-September D.M.E.	0	..	..	..	..	..	..	—1	..	..	..	..	..
October-December D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	—1
Grazing Sorghum Supply	0	..	..	..	..	..	..	..	..	..	..	..	..

APPENDIX 2—*continued*

MATRIX A—*continued*

Resources	"B" Col.	X_{37}	X_{38}	X_{39}	X_{40}	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	X_{48}
Revenue	..	0	0	0	7·23	12·88	2·19	2·87	29·7	26·38	30·05	26·73	11·25
Arable Type 1	519	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 2	582	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 3	74	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 4	730	..	..	..	..	..	..	..	..	..	..	..	..
Arable Type 5	169	..	..	..	..	..	..	..	..	..	..	..	..
Pasture X	1,875	..	..	..	..	..	..	..	..	..	..	..	..
Pasture Y	762	..	..	..	..	..	..	..	..	..	..	..	..
Pasture Z	2,857	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Stud Ewes	500	..	..	..	..	1	..	..	..	..	..	..	..
Lucerne Restraint	0	..	..	..	5·1	5·1	..	..	..	..	..	..	..
Maximum Late Grazing Sorghum	160	..	..	..	..	..	..	..	..	..	..	..	..
Maximum Wheat	7,200	..	..	..	..	..	..	..	..	..	..	..	..
January Tractor	240	..	..	..	..	..	..	..	..	..	..	..	..
February-March Tractor	480	..	..	..	..	..	..	..	..	..	..	..	..
December Tractor	240	..	..	..	..	..	..	..	..	..	..	..	..
Supplementary Sheep Units	0	..	..	..	—1	—1	—·33	—·33	..	..	20	20	..
Arable 2 Wheat Supply	0	..	..	..	..	..	..	..	..	..	..	..	..
Transferable Feed—													
January-March D.M.E.	0	1	..	..	..	..	..	..	..	..	..	..	..
April-June D.M.E.	0	..	1	..	..	..	..	..	..	..	..	..	..
July-September D.M.E.	0	..	..	1	..	..	..	..	..	..	..	..	..
October-December D.M.E.	0	..	..	..	..	..	..	..	..	..	..	..	..
Oats Feed—													
March-June D.M.E.	0	..	..	..	1·7	1·5	..	..	12·8	13·6	12·8	13·6	9·3
July-September D.M.E.	0	..	..	..	5·9	5·3	..	..	29·6	25·6	29·6	25·6	27·9
Consumption Feed—													
January-March D.M.E.	0	—1	..	..	7·7	7·3	3·3	3·3	60	46·6	..	..	..
April-June D.M.E.	0	..	—1	..	7·4	7·0	3·3	3·3	48·4	36·9	19·4	14·8	..
July-September D.M.E.	0	..	..	—1	5·4	5·2	3·3	3·3	32·1	32·0	12·8	12·8	..
October-December D.M.E.	0	..	..	..	9·9	9·3	3·3	3·3	58·3	46·4	..	..	..
Grazing Sorghum Supply	0	..	..	..	..	..	..	4·4	..	..	..	..	..

APPENDIX 3

AN EXAMPLE OF VECTOR WEIGHTING

Paddock H is made up of three soil production types, namely, types 1, 2 and 4. Hence if this paddock is planted to wheat, for example, differential yield rates would be experienced in different parts of the paddock. The average yield and average revenue per acre of paddock H (when wheat is grown) can be found by weighting the yields and revenues per acre of the constituent soil types according to the proportional distribution of these soil types, as indicated in Tables 6 and 7.

TABLE 6

Calculations for Average Yield

Soil Type	Acreage	Wheat Bags/ Acre of Soil Type	Total Yield/ Soil Type
Type 1	32·1	10	321
Type 2	6·0	8	48
Type 4	33·2	5	166
Total Acreage	71·3	Total Yield ..	535

∴ Weighted Average Yield = 535 ÷ 71·3

= 7·5 bags per acre of Paddock H.

TABLE 7

Calculations for Average Revenue

Soil Type	Acreage	Revenue/ Acre of Soil Type	Total Revenue/ Soil Type
Type 1	32·1	£12·74	£409·0
Type 2	6·0	9·68	58·1
Type 4	33·2	5·09	169·0
Total Acreage	71·3	Total Revenue	£636·1

∴ Weighted Average Revenue = 636·1 ÷ 71·3

= £8·9 per acre Paddock H.

The vector P_{34} representing wheat on paddock H is completed by weighting all 34 input-output coefficients in by the same method. The non zero coefficients of P_{34} and the corresponding coefficients of the three crop vectors from which P_{34} was derived are shown as in Table 8.

TABLE 8

Calculations of Vector Coefficients

Restraint	Non-zero Coefficients of Soil Type Vectors			Non-zero Coefficients of Composite Vector, P_{34}
	Wheat Type 1	Wheat Type 2	Wheat Type 4	
Wheat Maximum	10	8	5	7·5
January Tractor	·143	·143	·143	·143
February-March Tractor ..	·528	·528	·528	·528
December Tractor	·16	·16	·16	·16
October-December Consumption D.M.E.	—6	—4·8	—3	—4·5
Revenue..	12·74	9·68	5·09	£8·9

The derivation of weighted vectors is not a very time consuming process. In the present case the construction of Matrix B from the original matrix was completed within a day.