# RECENT APPLICATIONS OF THE PRODUCTION FUNCTION IN FARM MANAGEMENT RESEARCH.

by

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Recent years have witnessed a process of refinement in the conceptual basis of farm management analysis. In the most recent text-books in this field economic theory has been applied to farm management problems in a much more rigorous manner than was previously the rule.[1] Verbal analysis has been supplemented to a considerable extent by mathematical exposition. This tendency has been accompanied by an increasing interest in the use of econometric methods in agricultural economics and farm management studies.[2] These methods have supplemented and, to an extent, supplanted accountancy procedures, as used, for example, in the "average farm" or "case study" approach to farm management problems.

This article is concerned with one of these new techniques, viz., the use of the production function concept in the measurement of input-output relationships. It is particularly concerned with its use in measuring the input-output relationships existing within firms, i.e., the relationship between the input of resources (productive services, factors of production) and the output of product.[3] However, passing reference will be made to the measurement of simpler, physical relationships (e.g., that between the input of fertilizer and the output of crop) and of more complex economic relationships, i.e., the input-output relationships of an industry or collection of industries.

[1] The text-books referred to are: Earl O. Heady, *Economics of Agricultural Production and Resource Use* (New York; Prentice Hall, Inc., 1952); Lawrence A. Bradford and Glenn L. Johnson, *Farm Management Analysis* (New York; John Wiley and Sons, 1953).

[2] An excellent survey of the results of the application of econometric methods to agricultural production economics, is given by Glenn L. Johnson, "Results from Production Economic Analysis", *Journal of Farm Economics*, Vol. XXXVII, No. 2 (May, 1955), pp. 206-222.

[3] For a discussion of the use of the production function in farm management analysis, see H. R. Shaw and P. A. Wright, "Alternative Methods of Farm Management Analysis", *Canadian Journal of Agricultural Economics*, Vol. III, No. 1 (1955), esp. pp. 73-76.

The article is in the nature of a review of the literature dealing with the derivation of production functions from farm sample data. It contains a summary of the main problems involved in the use of this technique[4] (Part 2) and illustrations of the sort of finding which have been made (Part 3). In Part 1, the production function approach to the study of economic efficiency is contrasted with some better-known procedures. It is hoped that the article may stimulate research workers to carry out production function studies using Australian data, and that it will provide them with a convenient summary and bibliography of the available literature. The article is also intended to serve as an introduction to an empirical production function study, which it is hoped to publish in the near future.

## 1. INPUT-OUTPUT RELATIONSHIPS AND RESOURCE PRODUCTIVITY.

In any production process, the output depends on the quantities of factors used and the proportions in which they are combined. These input-output relationships provide the starting point for a great deal of economic theory. Although it is not—at least in theory—the task of the economist to elucidate these relationships, they constitute basic "given" data with which he has to work. Furthermore, economists have had to make certain generalizations (e.g., the "law of diminishing returns") and assumptions concerning the production process in order to develop their theory.

In applying their theory, production economists have always been hindered by a lack of empirical input-output data. This is not surprising in view of the complex nature of most production processes whose end products are economic goods. To describe all the physical production relationships involved in the production of wheat, for instance, would be an impossible task—in fact, if not in principle. However, the economist can make use of data relating to single aspects of the wheat production process. For example he can use data showing the relationship between the input of a single resource, fertilizer, and the output of wheat, to calculate the most profitable level of fertilizer application, given the prices of fertilizer and wheat (and making certain other assumptions). He can also study the farm production process, as a whole, by measuring the beginning and end-products of the process and disregarding the complex intermediate stages. This is the sort of study with which this article is concerned. The inputs involved are the productive factors as traditionally defined, i.e., land, labour and capital—of which, however, sub-categories may be distinguished—and the output is the products available for sale. Since output, and at least some of the inputs are invariably measured in value terms, such a relationship may be called a value production function, to distinguish it from the physical or technical type of production function mentioned above.

[4] These problems are reviewed by J. W. Clarke, "The Production Function in Farm Management Research", *Canadian Journal of Agricultural Economics*, Vol. II, No. 2 (Spring, 1954), pp. 36-41. See also Shaw and Wright, *loc. cit.*

The prime object of fitting such a production function is to determine the earning capacity, or productivity, of the different types of resource. The contribution of each category of resource to production is estimated by means of multiple regression analysis, wherein the quantities or values of the various resources used constitute the independent variables, and the value of output, the dependent variable. The regression plane is fitted to data derived from a sample of farms, each farm representing one input-output combination. The resulting function does not therefore apply to any particular farm, but to the sample as a whole.

Knowledge of the productivities of land, labour, and the various types of capital used in agricultural production is constantly being sought by economists and persons associated with rural industries, and its practical importance need not be stressed here. Productivity estimates are basic to the valuation of resources, and to comparisons of economic efficiency, as between individual farms or regions or industries. The value of the production function approach to the estimation of resource productivity can perhaps best be indicated by comparing the type of estimate that it gives with the type of estimate given by other methods.[5]

Resource productivity is usually estimated by means of the method of "residual imputation". If it is desired, for instance, to compute the productivity of the factor land, the quantity or value of all the resources (other than land) that are employed is ascertained, and the cost, at current market prices, of these resources is deducted from the total income of the enterprise or enterprises. (These costs include both cash costs and opportunity costs). The income remaining, i.e., the residual amount, is then regarded as the contribution to income of land. Similarly the return on labour is calculated by deducting from the gross return the cost of land and capital. In efficiency studies, the costs of all three factors—land, labour and capital—is frequently deducted, and the residual quantity is then imputed to the factor, management.

It is obvious that this method of assessing resource productivity is an extremely arbitrary one. It assumes that the factors other than the one being considered just pay for themselves in the contribution they make to production. (True, in a perfectly competitive economy, in equilibrium, resources will be employed to the point where their marginal returns equal their prices. Furthermore, it can be shown that, if constant returns to scale prevail, the sums of the quantities of resources multiplied by their prices will equal the total value product. However, in reality one cannot expect resources to be used in their equilibrium quantities. Nor is there any compelling reason for expecting returns to scale to be constant.) The arbitrariness of the residual imputation method becomes apparent when it is realized that any surplus over the "cost of production" may be attributed to each factor in turn, as the remaining factors are assumed to return only their cost.

[5] The criticisms, which follow, of the residual imputation method of calculating productivities are derived from Earl O. Heady, "Use and Estimation of Input-Output Relationships or Productivity Coefficients", *Journal of Farm Economics,* Vol. XXXIV, No. 5 (December, 1952), pp. 775-7.

When resource productivities are derived by means of fitting a production function to input-output data, the productivities of all resources are assessed simultaneously. No assumption that its marginal productivity equals its price is made regarding any factor. Certain assumptions concerning the mathematical form of the production function, do, indeed, have to be made, but the restrictive assumption of constant returns to scale need not be one of them: in fact, production function parameters may give an indication as to whether production is attended by constant, increasing or diminishing returns to scale.

Residual imputation procedures give average resource productivities, production functions give marginal productivities. Both of these measures are useful in comparing the efficiency with which resources are used from region to region. However, marginal productivities give a better idea of what is possible by changing the quantities of factors used. For example, Tintner and Brownlee calculated the marginal productivities of resources used on Iowa farms and concluded that ". . . these farms are on the average overimproved, and that additional inputs of liquid assets and working assets, and additional cash expenditures on equipment repairs, fuel, oil and feed will probably yield higher returns than additional improvements. The fact that on the average cash operating expenses . . . have a higher marginal productivity than working assets (equipment, horses, breeding stock, etc.) may indicate that on these farms equipment is replaced before it has depreciated to the most economical point. Even though (operating) expenses . . . are increased as the machinery ages, additional expenditures on these items rather than replacement of the machinery would yield higher returns to the operator."[6]

The economic efficiency with which a sample of farms is being operated may also be assessed by comparing the marginal productivities of the factors used with their market prices—it being assumed, subject to certain qualifications, that maximum efficiency is attained when factor inputs are pushed to the point where the last input just pays for itself, i.e., where the marginal return equals the factor price. Some interesting findings have emerged from these comparisons. For example, Heady and du Toit found that in a sample of South African cattle ranches, the marginal labour return was almost double the wage rate, and annual capital expenses yielded a marginal return of nearly 60 per cent., compared with a cost (interest rate) of 6 per cent.[7] (By contrast, most studies using data from United States crop farms have shown the marginal return on labour to be considerably less than the wage rate.) They believe the disequilibrium in the South African sample was due mainly to (i) ranchers' lack of knowledge of the marginal productivities, and (ii) risk and uncertainty, which influence producers to keep factor inputs short of the optimum quantities.

These examples have been quoted in order to give the reader some idea of the sort of findings which may emerge from a production function study. Further examples are given in Part 3 of this article.

[6] Gerhard Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records", *Journal of Farm Economics*, Vol XXVI, No. 3 (August, 1944), p. 571.

[7] See Earl O. Heady and Schalk du Toit, "Marginal Resource Productivity for Agriculture in Selected Areas of South Africa and the United States", *Journal of Political Economy*, Vol. LXII, No. 6 (December, 1954), pp. 494-505.

## 2. PROBLEMS IN DERIVING PRODUCTION FUNCTIONS FROM FARM SAMPLE DATA.

### Choosing an Appropriate Function.

The relationship between the inputs of resources and the output of product may be written, in algebraic terminology, as

$$Y = f\ (X_1, X_2, \ldots, X_n)$$

where Y is the quantity of output and $X_1$, $X_2$, etc., are the quantities of the various inputs. The relationship is called the production function.

Despite the disarming simplicity of this mathematical formulation of the production function, it is almost invariably a highly complex relationship, to which any manageable mathematical equation is likely to be, at best, only a reasonable approximation. In our choice of an appropriate function we are guided by our conception of the production process involved. For example, in the case of the relationship between fertilizer applications and yields, a simple linear relationship (i.e., that equal increments of fertilizer result in equal increments of yield) may be assumed. Such a relationship has the mathematical form

$$Y = a + bX$$

where Y is yield, X is fertilizer input and $a$ and $b$ are constants. However, an hypothesis more in keeping with our knowledge of biological science is that as fertilizer is added in units of uniform size successive increments in yield diminish at a constant rate. This hypothesis can be formulated in mathematical terms as

$$Y = M - ar^{x}$$

where Y and X have the same meanings as before, M is the theoretical maximum yield and $a$ and $r$ are constants.[8]

The Cobb-Douglas type of production function has been used most frequently in attempts to express mathematically the input-output relationship of firms or industries.[9] This function is of the form,

$$Y = aX_1^{b_1}\ X_2^{b_2} \ldots .\ X_n^{b_n}$$

which, if transformed into logarithms, reduces to the simple linear equation,

$$\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \ldots + b_n \log X_n$$

which can be solved by the familiar means of least squares. This property simplifies the computational work involved in the use of the function. The function has a number of other properties, too, which, since they correspond to well-known economic concepts and axioms, enhance its value in an economic context.

[8] This is known as the "Spillman" function. See W. J. Spillman, *Use of the Exponential Yield Curve in Fertilizer Experiments,* U.S. Department of Agriculture Technical Bulletin 348, 1933. This function has also proved useful in describing the relations between feeding and milk and meat production (Einar Jensen et al., *Input-Output Relationships in Milk Production,* U.S. Department of Agriculture Technical Bulletin 815, 1942, pp. 24-26; Aaron G. Nelson, *Relation of Feed Consumed to Food Products Produced by Fattening Cattle,* U.S. Department of Agriculture Technical Bulletin 900, 1945, p. 27).

[9] So called after Charles W. Cobb and Paul H. Douglas, who used the function in investigating the relation (for American manufacturing industry) between production and labour and capital, in "A Theory of Production", *American Economic Review,* Vol. XVIII, Supplement (March, 1928), pp. 139-165.

1. The coefficient ($b_1$, $b_2$ ...) associated with each factor input ($X_1$, $X_2$ ...) corresponds to what is known in economic terminology as the *elasticity of production* of that factor. That is, it expresses the percentage change in output which results from a one per cent. change in the input of the factor. Elasticities are independent of the units of measurement of input and output and hence are directly comparable one with another.

2. By adding together the elasticities associated with each factor, it is possible to determine whether the production process as a whole yields constant, diminishing or increasing returns to scale. A sum of coefficients equal to unity indicates constant returns to scale, less than unity, diminishing returns to scale, and greater than unity, increasing returns to scale.

3. The marginal productivity of a factor (say $X_1$) can be obtained directly from any production function by partial differentiation with respect to the factor concerned. In the case of the Cobb-Douglas function, this partial derivative is of the form

$$\frac{\partial Y}{\partial X_1} = a\ b_1\ X_1^{b_1-1}\ X_2^{b_2} \ldots X_n^{b_n}$$

This relationship has two important properties[10]:—

(*a*) It allows for diminishing marginal productivity of any factor, i.e., it allows successive equal increments of a factor to give rise to successively smaller increments of output. This occurs when the elasticity of production ($b_1$) of a factor ($X_1$) is less that unity: then the coefficient ($b_1$—1) associated with $X_1$ in the equation above becomes negative, and hence the larger the value assumed by $X_1$, the smaller the increment in Y associated with a given increment in $X_1$.

(*b*) The marginal productivity of any single factor is influenced by the level of input of the other factors. This is indicated by the presence in the partial derivative of terms involving $X_2$, $X_3$, etc.

Both these properties are in accord with the assumptions of economic theory.

The Cobb-Douglas function also has a number of restrictive features. For instance, it does not allow for changing elasticities of production or of substitution to accompany changes in the size and ratio of factor inputs. These restrictions can be overcome by the use of general equations of the second degree, but these involve much more computational

[10] This equation may be written

$$\frac{\partial Y}{\partial X_1} = b_1 \frac{a\ X_1^{b_1}\ X_2^{b_2} \ldots X_n^{b_n}}{X_1}$$

which reduces to

$$\frac{\partial Y}{\partial X_1} = \frac{b_1\ Y}{X_1}$$

work than the Cobb-Douglas type.[11] As Heady has pointed out, "choice [of an appropriate function] must rest on the relevant economic logic in relation to the funds available for computation and the purposes and extremes over which the predictions will apply".[12] In practice, the Cobb-Douglas function has given quite encouraging results.

### Statistical Problems Associated with the Use of Economic Data.

In describing the nature of a production function, we have used the example of the relationships between the "input" of fertilizer and the "output" (yield) in an agricultural experiment. However, the unqualified acceptance by the economist of this experimental analogy can lead to the use of inappropriate statistical methods and erroneous interpretation of results. The data available to the economist differs, in a very important respect, from that available to the agricultural experimenter. This difference has been very clearly explained by Marschak and Andrews:—

> "To describe and measure causation, the economist cannot perform experiments. That is, he cannot choose one variable as 'dependent', and, while keeping the other 'independent' ones under control (i.e., while making them assume deliberately chosen sets of values), watch the values taken by the dependent, i.e., uncontrolled variable. The economist has no independent variables at his disposal because he has to take the values of all variables as they come, produced by a mechanism outside his control. This mechanism is expressed by a system of simultaneous equations, as many of them as there are variables. The experimenter can isolate one such equation, substituting his own action for all the other equations. The economist cannot.
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> For example, in agricultural experimentation preassigned quantities of fertilizers are added to the soil of various plots, and the yields compared. Substitute 'firms' for 'plots', and 'labor, capital' for 'nitrate, phosphate'. Can the economist measure the effect of changing amounts of labour and capital on the firm's output—the 'production function'—in the same way in which the agricultural research worker measures the effect of changing amounts of fertilizers on the plot's yield? He cannot because the manpower and capital used by each firm is determined by the firm, not by the economist. This determination is expressed by a system of functional relationships; the production function, in which the economist happens to be interested, is but one of them."[13]

A production function is supposed to represent the input-output situation confronting the individual "average" entrepreneur: it purports to answer the question "how much will a firm produce, if a certain amount of labour and capital is forced upon it?" But a production function derived from data in which each input-output combination relates to a separate firm "will tell us what likely production we shall expect from a

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[11] For example, with six factors, six regression coefficients must be estimated for a Cobb-Douglas; 27 must be estimated for a function involving linear, squared and cross-product terms for each factor.

For an example of the use of a second degree function, see R. B. Hughes, Jr., "Marginal Returns on Agricultural Resources in a Southern Mountain Valley", *Journal of Farm Economics,* Vol. XXXVI, No. 2 (May, 1954), pp. 334-339. In this study, both a Cobb-Douglas type and a function involving squared and cross-product terms were fitted to the same data. The value of $R^2$ was only 0.24 for the former, as against 0.62 for the latter.

[12] Heady, "Use and Estimation of Input-Output Relationships or Productivity Coefficients", *op. cit.,* p. 781.

[13] Jacob Marschak and William H. Andrews, Jr., "Random Simultaneous Equations and the Theory of Production", *Econometrica,* Vol. 12, Nos. 3 and 4 (July-October, 1944, pp. 143-144.

firm whose technical and economic efficiency and other characteristics are such as to make it hire a given amount of manpower and capital".[14] These considerations led Marschak and Andrews to recommend the use of the simultaneous equations approach to the derivation of production functions, rather than the traditional single equation method of fitting a regression plane to the data by means of least squares. This recommendation was made in the course of a controversy stimulated by the work of Douglas and his associates, who, by ordinary regression analysis, fitted production functions to aggregative data relating to the whole, or to broad sectors of manufacturing industry, where differences in "technical and economic efficiency and other characteristics" are obviously very great.[15] However, the single equation approach has continued to be used in investigations involving farm sample data since, in comparison with the extremely varied collection of firms included in Douglas's analyses, the farms composing the sample constitute a very homogeneous group. At the same time, the homogeneity of the samples of farms used has varied greatly from study to study: great care has been taken in some recent studies in order to ensure an appropriate degree of homogeneity.[16] Nevertheless it remains a fact that,

> "Strictly, the function which has been calculated (by means of least squares) justifies only an estimate of the form 'the farms which have inputs $A_1$, $L_1$, $F_1$, have an average gross output $P_1$, whilst the (other) farms which have inputs $A_2$, $L_2$, $F_2$ have output $P_2$'.
>
> In estimating marginal productivities and output elasticities it is necessary to go somewhat beyond this and to infer that if the inputs of a given farm were varied from $A_1$, $L_1$, $F_1$ to $A_2$, $L_2$, $F_2$ then its expectation of output would change in consequence from $P_1$ to $P_2$. This is indeed an important

[14] *Ibid.*, p. 150.

[15] Cobb and Douglas, *op. cit.;* Paul H. Douglas and Marjorie L. Handsaker, "The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria", *Quarterly Journal of Economics,* Vol LII, No. 4 (November, 1937), pp. 1-36; Vol. LIII, No. 1 (February, 1938), pp. 215-254; Paul H. Douglas and M. Bronfenbrenner, "Cross-Section Studies in the Cobb-Douglas Function", *Journal of Political Economy,* Vol. XLVII, No. 6 (December, 1939), pp. 761-785; Paul H. Douglas and Grace T. Gunn, "Further Measurement of Marginal Productivity", *Quarterly Journal of Economics,* Vol. LIV, No. 2 (May, 1940), pp. 399-428; Douglas and Gunn, "The Production Function for American Manufacturing in 1919", *American Economic Review,* Vol. XXXI, No. 1 (March, 1941), pp. 67-80; Douglas and Gunn, "The Production Function for Australian Manufacturing", *Quarterly Journal of Economics,* Vol. LVI, No. 4 (November, 1941), pp. 108-129; Douglas and Gunn, "The Production Function for American Manufacturing for 1914", *Journal of Political Economy,* Vol. L, No. 4 (August, 1942), pp. 595-602; Douglas, Daly and Olsen, "The Production Function for Manufacturing in the United States, 1904", *Journal of Political Economy,* Vol. LI, No. 1 (February, 1943), pp. 61-65; Douglas and Daley, "The Production Function for Canadian Manufactures", *Journal of American Statistical Association,* Vol. 39 (June, 1943), pp. 178-186. David Durand, "Some Thoughts on Marginal Productivities, with Special Reference to Professor Douglas' Analysis", *Journal of Political Economy,* Vol. XLV, No. 6 (December, 1937), pp. 740-758; M. W. Reder, "An Alternative Interpretation of the Cobb-Douglas Function", *Econometrica,* Vol. 11 (July-October, 1943), pp. 259-264; M. Bronfenbrenner, "Production Functions: Cobb-Douglas, Interfirm, Intrafirm", *Econometrica,* Vol. 12, No. 1 (January, 1944), pp. 35-44.

[16] For example, Heady has used a sample of Iowa farms, homogeneous both with respect to area and soil type. See Earl O. Heady, *Resource Productivity and Returns on 160-acre Farms in North Central Iowa,* Iowa Agricultural Experiment Station Research Bulletin 412 (Ames, Iowa; July, 1954).

assumption which might be gravely wrong if the data from which the estimates were made related to a number of production functions with very different characteristics."[17]

The unfortunate consequences of making this assumption when some of the unmeasured "other characteristics" of farms happen to be systematically related to those that *are* measured and included in the production relationship, will be discussed later in this article (see page 229).

In addition to errors which arise due to the omission of variables from the analysis, errors of measurement, etc., are associated with the variables that are included. In the presence of such errors—or "disturbances" as they are often called—the ordinary multiple regression method is an inappropriate means of measuring the structural relationships involved, since it assumes that only the dependent variable is subject to disturbance. To deal with this problem, a method of "weighted regression" has been developed.[18] The problem of multicollinearity also owes its origin to the existence of errors in variables. Multicollinearity refers to the presence of relationships between the independent variables themselves, as well as between them and the dependent variable. It may be a serious problem in the derivation of production functions since, although the input of any factor may vary considerably from firm to firm, there is a tendency for most firms to use similar proportionate amounts of the various factors. If the correlation between the inputs is high, regression analysis may give indeterminate results. How this comes about may be illustrated by an extreme case. Suppose that we have a series of observations of labour, capital and output, such that each variable is perfectly correlated with each other variable. If these observations are plotted in three-dimensional space, they will fall along a straight line. A regression analysis using this data will give indeterminate results, as an infinite number of regression planes will satisfy the condition of passing through one straight line. However, since the observations are subject to errors of measurement the relations between the variables will not be perfect and an apparently determinate result will ensue. The resulting coefficients will be meaningless, however, as they are merely the consequences of the presence of random errors in the data. Thus when the independent variables are highly correlated with each other, the errors associated with their measurement assume very great significance.

A test for multicollinearity has been developed within the framework of the "weighted regression" method mentioned above. A more familiar means of dealing with the problem is that of "confluence analysis".[19] Antill used both these methods in connection with the derivation of a production function from farm sample data.[20] In general, however, these advanced statistical procedures—which were developed in response to the problems of deriving supply, demand, production and consumption

[17] A. G. Antill, "Towards a Production Function for Dairy Farms", *The Farm Economist,* Vol. VIII, No. 1 (1955), p. 11.

[18] See Gerhard Tintner, *Econometrics* (New York; John Wiley and Sons, 1952), p. 121 *et seq.*

[19] There is an extensive literature on these problems, references to which may be found in Tintner, *op. cit.* A short and relatively non-technical account of the problems and techniques mentioned in this section, and further references to the literature, are given by D. Cochrane, "Measurement of Economic Relationships", *Economic Record,* Vol. XXV, No. 49 (December, 1949), pp. 7-23.

[20] Antill, *op. cit.*

functions from aggregative data—have not been used in studies using sample data. In such studies, a simpler approach to the problem of multicollinearity is offered by the use of appropriate sampling methods. Correlation among input categories is apt to be high in random samples of farms. Hence the possibility arises of deliberately selecting farms so as to reduce this correlation. Such purposive samples are more efficient for estimating regression coefficients, but they are less efficient for estimating means, variances and correlation coefficients.[21]

## Classifying and Measuring Inputs.

The significance and content of any production function derived from empirical data depends on the way in which the factors entering into the production relationship are defined and measured.

In production functions derived from farm records, land and labour are frequently treated as homogeneous factors, and measured in physical terms (e.g., acres and man-months respectively), but the various capital goods and services used in farm production are very heterogenerous and have no common physical measure. They are therefore measured in value terms. Output, too, almost invariably consists of various grades of more than one product and hence its value is measured rather than its quantity. Thus the clear theoretical distinction between a physical production function and a value production function cannot, in practice, be retained when firm or industry data are being analysed. The generality of the resulting function is thus greatly reduced, as, strictly, it applies only to the particular factor price situation that prevailed when the data were collected.[22]

### Problems of Classification.

The valuation of capital inputs avoids, but, from the point of view of interpretation, does not solve the problem posed by their heterogeneity. To say that capital inputs yielded, on the average, a return of so many pounds per pound invested, says nothing concerning the productivity of particular types of capital inputs, so the above statement has to be qualified to the effect that the composition of the capital invested corresponds to the average composition of capital employed on the sample properties.[23] This problem can be minimized, however, by classifying capital inputs into a number of value categories. The particular categories chosen will depend on the investigator's assessment of the strategic inputs in the relationship, the bookkeeping habits of farmers, the purpose of the investigation, etc. Factors, the input of which are highly correlated with each other, i.e., factors forming complementary pairs of groups, should be combined in the one input category, as it is meaningless to derive separate productivities for complementary factors.

[21] Johnson, *op. cit.*, p. 212.

[22] A "technique for adjusting value productivity extimates derived from 'Cobb-Douglas functions' for changes in input and output prices" has recently been presented by Gerald I. Trant, "Adjusting for Changes in Price Levels in Value Productivity Studies", *Journal of Farm Economics,* Vol. XXXVII, No. 3 (August, 1955), pp. 563-566.

[23] Also to the effect that as the quantity involved increases, its composition changes in accordance with the way the average composition of capital changes in the sample.

Two general principles affecting the choice of input categories are (1) that the inputs within a category be as nearly perfect substitutes or perfect complements as possible, and (2) that the categories of inputs be neither perfect complements nor perfect substitutes relative to each other.[24]

The classification of inputs used in a number of published investigations is given below[25]:

*Tintner* (1944):[26] Land; labour; farm improvements (buildings, fences, etc.); liquid assets (livestock, feed, seed, fertilizers); working assets (machinery, breeding stock, equipment); cash operating expenses (repairs, fuel, oil, purchased feed).

*Heady* (1946)[27] Real estate (land and improvements); labour; machinery and equipment (inventory value plus value of repairs, fuel and lubricants); livestock and feed (stock on hand and purchased, livestock expense and feed fed); miscellaneous operating expenses.

*Johnson* (1952):[28] Land; labour; machinery investment (inventory value); livestock (inventory value) and forage production investment (replacement value of hay and pasture stands plus investment in structures or land clearing necessary to establish such crops); other expenses (cash operating expenses).

*Heady* (1954): [29] Land fixed.

I. Crop function: Labour; machinery expenses (depreciation, repairs, fuel, oil, etc.); annual crop expenses (seed, fertilizer, lime, seed treatment, etc.).

II. Livestock function: Labour; annual livestock expenses.

*Heady and Shaw* (1954):[30]

I. Crop function: Cropland; crop labour; capital services used on crops (in addition to annual cash expenses, includes depreciation on all items used directly or indirectly in crop production).

[24]Bradford and Johnson, *op. cit.*, p. 144. See also James S. Plaxico, "Problems of Factor-Product Aggregation in Cobb-Douglas Value Productivity Analysis", *Journal of Farm Economics*, Vol. XXXVII, No. 4 (November, 1955), pp. 664-675. Unfortunately, this interesting study did not come to hand until after the present article was prepared for publication. Plaxico's purpose is: "(1) to show how the aggregation of products into a dependent variable and the aggregation of inputs into different independent variables may affect the value and the reliability of the estimated parameters, (2) to indicate the conditions associated with an optimum aggregation, and (3) to question the usefulness of Cobb-Douglas estimates as guides for intra-farm and policy decisions".

[25] Land in all cases is measured in acres; real estate in dollars or pounds; labour in man-months or man-years; all capital inputs in dollars or pounds.

[26] Gerhard Tintner, "A Note on the Derivation of Production Functions from Farm Records", *Econometrica*, Vol. 12, No. 1 (January, 1944), pp. 26-34. Also Tintner and Brownlee, *op. cit.*

[27] Earl O. Heady, "Production Functions from a Random Sample of Farms", *Journal of Farm Economics*, Vol. XXVIII, No. 4 (November, 1946), pp. 989-1004.

[28] Glenn L. Johnson, *Sources of Income on Upland McCracken County Farms*, 1951, Progress Report No. 2, Kentucky Agricultural Experiment Station, 1952. Also reported in Bradford and Johnson, *op. cit.*, p. 145 *et. seq.*

[29] "Resource Productivity and Returns on 160-acre Farms in North-Central Iowa", *op. cit.*

[30] Earl O. Heady and Russel Shaw, "Resource Returns and Productivity Coefficients in Selected Farming Areas", *Journal of Farm Economics*, Vol. XXXVI, No. 2 (May, 1954), pp. 243-257.

II. Livestock function: Labour; capital services used on livestock (including depreciation on buildings, etc., and depreciation on breeding stock, and purchase value of feeding stock).

*Heady and du Toit* (1954):[31] Land; labour; annual capital services (including depreciation, depreciation on breeding stock, purchase value of feeding stock).

*Antill* (1955):[32] Land; labour; purchased feeds; other capital inputs (including interest on crops, livestock, machinery and equipment inventories, depreciation on machinery and equipment, rent or rental value of real estate, cost of salaried management).

This listing illustrates the diversity of possible approaches to the classification of capital inputs. Tintner's classification is probably the most appropriate in an economic context, but unfortunately the elasticities obtained for some of his categories were, for the most part, not significant in the statistical sense; "farm improvements" and "working assets" in particular usually appeared to have little influence on production. On the other hand, cash operating expenses appeared to have a substantial effect. It is presumably because they have encountered the same difficulties that later investigators have tended to (1) reduce the number of capital input categories and (2) measure the input of durable assets by the actual and imputed costs associated with their use, rather than by their capital values.[33]

Antill's classification is an example of the singling out for separate treatment of one category of capital input which is considered to be of key importance in the production process—in this case purchased feeds in milk production. Heady's 1946 study illustrates the appropriate treatment of complementary factors: a preliminary analysis showed high correlations between inventory value of machinery and equipment on the farm, and the cost of machinery repairs, fuel and lubricants: hence these items were combined to form a single input category. The value of livestock on hand and livestock and feed expenses were lumped together for the same reason.

Included in cash operating expenses as usually defined are some items the input of which is largely determined by the volume of production. Selling charges, where they constitute a fixed commission on the amount of the sale, are an example of such an expense. If all producers sold their produce through the same channels and were charged a fixed percentage of the revenue as selling commission, there would be a perfect correlation between selling charges and output; and if selling charges were included in a production function as a separate input, they would have an elasticity of unity, and a very high marginal productivity. But such results would be meaningless, as it is clearly inappropriate to include these expenses in a production function: they are completely determined *by* the value of the output and do not determine it to any

[31] Heady and du Toit, *op. cit.*

[32] Antill, *op. cit.*

[33] Heady's 1946 study represents a transitional case where the *capital value* of particular types of assets is combined with the cash costs associated with their use to form a single input category.

extent, since, in the example above, it is assumed that the producer cannot influence the price or the revenue that he receives by varying his selling policy. His only choice is between selling for a fixed commission or not selling at all.

Expenses of this nature should, then, be excluded from the inputs in the derivation of a production function. Otherwise they may substantially contribute to the degree of correlation found between output and the class of capital inputs in which they are included and thus give rise to spuriously high elasticities of production. However, there are very few examples of this type of expense so clear-cut as the case mentioned above. In most lines of agricultural production there is *some* opportunity for the entrepreneur to exercise greater or less care (spend more or less money) in marketing his product, and hence to influence the return that he receives. The cost of packing and handling the product is another example of an input which is closely related to the volume of output: for instance, the number of bags used in grain harvesting and marketing is largely determined by the quantity of grain produced. But again there are opportunities for varying the intput of bags: a smaller number of bags may be filled and emptied more frequently (i.e., labour substituted for bags) or bulk handling equipment installed (capital substituted for bags). The extent to which this type of input is directly geared to output will vary from industry to industry and from region to region, and clearly some judgment is required as to whether significant opportunities exist for varying these inputs independently of the output. There is no record of this having been done in published investigations, but Heady has made reference to the problem: he pointed out that the high marginal productivity shown by the item "cash operating expenses" may be partly explained on the grounds that "this miscellaneous category includes twine and some similar items where the quantity used is dependent on total product".[34] One cannot help suspecting that such items have made a substantial contribution to the high marginal productivity that cash operating expenses have shown in other published investigations. The authors have had an opportunity to study the influence of these factors in an investigation that they have carried out using Australian data. In a New South Wales woolgrowing area annual capital expenditure, including the cost of shearing and wool selling expenses, was found to have an elasticity of 0.55 and a marginal productivity of £1.73 per £1 spent. When shearing and wool selling expenses were excluded, however, the elasticity of capital was reduced to 0.43 and its marginal productivity to £1.59.[35]

**Problems of Measurement and Interpretation.**

The main problems associated with the derivation of production functions from farm data do not, however, arise from the inclusion of too *many* input items, but from the inclusion of too *few*. It was stated

[34] "Production Functions from a Random Sample of Farms", *op. cit.*, p. 998.

[35] Wool is sold at auction by firms which adopt standardized procedures and charge a standardized commission. There is scope for devoting more or less care in shearing and classing the clip, so that the amount spent on shearing can influence the return. However, these practices are also more or less standardized, and it is felt that the exclusion of shearing expenses from the capital input is probably desirable.

above that land and labour are frequently treated as homogeneous factors and measured in physical terms. This procedure can only be regarded as an expedient adopted in the absence of information concerning differences in the quality of these resources. The analysis would certainly benefit if these differences could be measured and incorporated in it. The virtual impossibility of obtaining any measure of the input of management in a production process is another major lacuna in the analysis. These problems will now be discussed in some detail.

*Labour.*—Where large numbers of men are employed under supervision, as in factory production, the wages bill or the number of man-hours worked represents a reasonable measure of the labour input.[36] But in most types of farm production, the available measures of labour input are much less satisfactory. Most farm work is usually performed by self-employed family labour. Studies have shown that the work performed by farmers varies considerably according to their age, and it is commonly observed that there are great differences in the labour performance of self-employed individuals of the same age. The measurement of labour in man-months takes no account of these differences, nor does it allow for the very great seasonal fluctuations in labour input which are characteristic of farming. Even if it is agreed to ignore these differences, some sort of convention has to be adopted in the valuation of the work of junior and female family members. The small number of persons employed per farm increases the importance of individual differences in work capacity. It also introduces important discontinuities in the labour supply function, which are at variance with the marginalist assumptions of production function analysis.

The measurement of the input of hired agricultural labour is not subject to these difficulties—hired employees work under supervision and receive wages which presumably correspond, more or less, with their work capacities and skills—but the aggregation of hired and family labour input presents some problems. In most published studies hired labour has been measured, along with family labour, in man-months, but it seems a pity not to use wage payments as its measure when these are usually part of the available data. Contract (custom) work has usually been included with capital inputs.

In other contexts some success has been achieved in estimating differences in the labour performance of individual farmers.[37] Such estimates are subjective and have a considerable margin of error, but even so should be preferable to the crude unweighted data so far used in production function studies, and could be obtained relatively easily in a properly designed survey.

*Land.*—In production function analysis, differences in land quality can be a more serious, but at the same time a less intractable problem than differences in labour quality. They can be more serious because they are more likely to be systematically associated with differences in the

[36] Both these measures were used in a study of the productivity of labour used in meat-packing operations. See William H. Nicholls, *Labour Productivity Functions in Meat Packing* (Chicago, Illinois; The University of Chicago Press, 1948).

[37] See, in particular, Ewen J. Long and Kenneth H. Parsons, *How Family Labor Affects Wisconsin Farming,* University of Wisconsin Research Bulletin 167 (May, 1950).

input of other factors, so that their influence, rather than having a random effect, tends to be attributed to these other factors. They are less intractable because records of land values, or staff to carry out land valuation can usually be obtained. Alternatively, samples can be selected from areas that are relatively homogeneous as regards land quality.

Differences in land quality are likely to be associated with differences in the input of other factors, since the better quality land tends to be farmed more intensively. In the analysis, the higher output of this land is associated only with the higher inputs of capital and labour, and hence is attributed to them alone. The resulting marginal productivities would suggest that labour and capital, if employed in larger quantities on the poorer land, would yield returns as high as, or higher than, those achieved on the better land. The situation can best be visualized in terms of a simple two dimensional input-output diagram.[38] The input-output curves (production functions) associated with different qualities of land would differ from each other in both position and shape; the better the land, the higher will the curve be on the graph (the better the land, the greater the output, inputs remaining constant) and the further to the right will be its point of inflexion. In a sample of farms drawn from differing land types, the observations will not be randomly located along these curves, but, for the higher curves (better quality land) will be concentrated to the right, and for the lower curves, to the left. The fitted "prediction" curve will therefore rise from left to right more steeply than any of the "structural" curves, which it will intersect. In these circumstances the fitted curve can in no sense be regarded as a mean estimate of the various structural curves. It is a "hybrid" curve which exaggerates the productivity of labour and capital and which may show increasing returns to scale, when, in fact, a movement along any of the structural curves may soon produce diminishing returns.

*Management.*—If a production function derived from sample data could be interpreted as expressing the underlying physical production possibilities confronting the entrepreneur or manager, the omission of the factor, *management,* from the analysis would be, of course, entirely appropriate. Unfortunately it cannot be so regarded, since each of the separate combinations of inputs has been made, not by the investigator, but by a different farm manager, and the managerial skills of farmers are likely to vary widely. The outputs therefore reflect not only the combinations of inputs, but the skill and care with which the inputs have been combined. Hence management should be included in the analysis as a productive factor.

As in the case of differences in land quality, the main danger here lies in the possibility that differing "inputs of management" are systematically related to differences in the input of other factors. Heady has pointed out that ". . . . the true outcome of our study (which indicated diminishing returns to scale) would differ depending on whether the actual input of management for the farm studied increased, (1) at an

[38] See Earl O. Heady, "Elementary Models in Farm Production Economics Research", *Journal of Farm Economics,* Vol. XXX, No. 2 (May, 1948), pp. 213-215.

increasing or (2) at a decreasing rate as the input of other resources or output of production increased by a given percentage. Were it possible to include management inputs, then the statistical results in case (1) would tend to show a greater degree of diminishing returns than those presented while case (2) would turn the results in the direction of constant or increasing returns".[39] Since it is hard to conceive of an input of management (is it an amount of cerebration expended on managerial problems?) the problem is probably more usefully formulated in terms of operator's differing managerial abilities. The situation is then analogous to that just discussed in connection with differences in land quality. If, as is commonly believed, the better managers are found on the farms using larger quantities of inputs, the fitted production function will overestimate the elasticities of the various resources, and will imply a greater degree of increasing returns than, in fact, exists.

Although the importance of achieving sample homogenity in respect of land quality (or alternatively of valuing land) has been stressed in the literature, differences in farmers' managerial abilities do not appear to have received the attention they deserve. "It (management) has been omitted as a factor affecting output due to the difficulty of measurement or because it was thought that management was not a sufficiently important factor of production or both. It is difficult, however, to accept past suppositions that management is not directly related to output." [40] We are of the opinion that in Australia, at any rate, differences in farmers' managerial abilities are very marked—so marked, indeed, as to be of even greater importance, in many instances, than differences in land quality.[41] In Australia, at the present time, the productivity of land in many areas is capable of great improvement by the implementation of pasture improvement techniques and the use of trace elements. These innovations tend to diminish the relative importance of unimproved land, and management is the key factor in their (rapid) adoption and exploitation.

There is one context, however, in which the contribution of management to the production process may be legitimately ignored. This is when the findings of a production function analysis are used as an exhortatory or educational device in extension work. It seems as if extension officers have to assume, at any rate overtly, that members of their farmer audience are of equal managerial ability. To infer marginal productivities on the assumption that farmer A, with a low level of inputs, output and competence can attain the same output as highly efficient farmer B, if only he adopts the same level of inputs as the latter, is merely another way of holding up the more efficiently conducted farms as examples to be emulated.

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[39] "Production Functions from a Random Sample of Farms", *op. cit.*, p. 995.

[40] Clarke, *op. cit.*, p. 38.

[41] For some evidence and argument on this point, particularly in relation to farmers' adoption of innovations, see Ross Parish, "Innovation and Enterprise in Wheat Farming", *Review of Marketing and Agricultural Economics*, Vol. 22, No. 3 (September, 1954), pp. 189-218 .

## 3. SOME EMPIRICAL FINDINGS.

Despite its theoretical limitations and the many practical difficulties associated with its use, the Cobb-Douglas production function has been widely employed in recent years in investigations based on farm sample data. The explanation of this interest is not merely—although it is partly—that empirical results are so meagre in economics that any such results are of interest, no matter how serious the qualifications surrounding them. The practical consequences of some of the imperfections in the technique may be small in relation to the usually accepted standards of accuracy in economic investigations. By appropriate techniques a number of the difficulties may be surmounted, or their consequences minimized. And finally, many of the findings so far made have been sufficiently in accord with what would be expected on other grounds, as to give encouragement to the investigators and to potential investigators. Of course, not all results have been consistent with expectations, but such unexpected results have frequently been sufficiently plausible to stimulate a re-examination of accepted views.

Some of the empirical findings in this field will now be discussed.

### Returns to Scale.

Douglas's and his associates' assumptions and findings regarding returns to scale in manufacturing industries gave rise to some controversy, so that it is not surprising that Tintner and later investigators have paid attention to this aspect of their own findings. The sum of the elasticities in both of Tintner's studies was less than, but close to unity. He came to the conclusion that:

> "It is not unlikely that the production function of Iowa farms is a homogeneous function of degree 1 in the factors of production and shows no economies or diseconomies of large-scale production. This would to a certain extent confirm the previous results of Paul Douglas and his collaborators.
>
> These results, if actually true, or approximately true, would of course be of great importance for economic theory and policy. The fact that we have constant returns to scale is to a certain extent corroborated by the fact that we observe in many industries the co-existence of and survival of firms of various size. This condition prevails also with farms in Iowa.
>
> A theoretical explanation of this phenomenon can perhaps be given along the lines of Kaldor's theory of the firm. There is one factor of production, which (by necessity) has not been included in the empirical production functions: this is entrepreneurship. It is likely, as Kaldor suggests, that it is this scarce factor of production which eventually causes decreasing returns to scale and actually may determine the optimum size of the enterprise."[42]

Tintner's studies related to samples composed, on the whole, of larger and better-managed Iowa farms. Heady fitted production functions to a *random* sample of Iowa farms, but obtained similar results, which indicated diminishing or constant returns to scale.[43] He also tested the hypothesis that "although the function used indicated diminishing returns for all farms, increasing returns might hold for small farms, while decreasing returns holds for large farms". However, diminishing returns were found to hold for both the large and the small farms in his sample. In fact, returns to small farms appeared to diminish at a more rapid rate than did returns to large farms (the sums of the elasticities for small and large farms were 0.8051 and 0.9605, respectively).

[42] *Econometrics, op. cit.*, p. 55.

[43] "Production Functions from a Random Sample of Farms", *op. cit.*

Heady and Shaw fitted separate crop and livestock functions to sample data derived from four contrasted agricultural regions of the United States.[44] For crops, the sums of elasticities were significantly greater than 1.0 at a 5 per cent. probability level in three areas, and at approximately an 8 per cent. level in the fourth. These results seem reasonable, as larger farms might be expected to achieve greater economies of scale through the use of modern crop machinery. (However, Heady and Shaw believe that "ranges of increasing and decreasing returns to scale appear logical, especially in crop production", and point out that since the Cobb-Douglas function assumes constant elasticity over all ranges of input its use in this connection is not entirely appropriate. All their equations except one—which indicated increasing returns—suggested constant returns to scale in livestock production.

In a study of 160 acre farms in North Central Iowa Heady found "diminishing returns as more and more of the various resources are used for crops on a given land area.[45] (All the crop land was in cultivation on all the farms in the sample; the sum of elasticities was 0.3076.) On the other hand, resources used in livestock production apparently yielded constant returns. "While diminishing returns might be expected on the 160-acre farms if they carried an extreme amount of livestock, few farms approached programmes of this intensity."

Evidence has been presented which suggests that cattle ranches in a South African region operate under constant returns to scale[46] and that increasing returns obtain for British dairy farms.[47]

These results have a number of encouraging features:—

(i) There is a strong tendency for the sum of elasticities to approximate to unity. This is in accord with the fact that in most rural industries studied there are no compelling reasons for expecting either very large economics or very large diseconomics of scale.

(ii) In most cases where increasing or constant returns to scale have been indicated, the finding appears to be consistent with our knowledge of the technological conditions of production in the industry or area. For instance, the fact that American studies have several times indicated increasing returns in crop production, but almost invariably constant returns in livestock industries, is consistent with the greater importance of mechanization in the former type of production.

## Marginal Productivities.

It should be noted that a Cobb-Douglas type function does not give, for any region or industry, a *single* estimate of a resource's marginal productivity, for this varies according to the quantities of inputs used. How-

[44] Heady and Shaw, *op. cit.*

[45] "Resource Productivity and Returns on 160-Acre Farms in North-Central Iowa", *op. cit.*

[46] Heady and du Toit, *op. cit.*

[47] Antill, *op. cit.*

ever an estimate of the "mean" marginal productivity of each input category can be obtained by calculating the marginal productivities for an enterprise which is "representative" of the industry or region, in that it employs per firm mean quantities of all resources.[48]

In investigating the efficiency with which resources are used, their calculated mean marginal productivities may be compared with either their market prices or the productivities of similar resources employed in other areas or industries. A third type of comparison can also be made: mean marginal productivities may be calculated for different strata within the one sample and these may be compared. (For example, the larger farms may be compared with the smaller.)[49]

For these comparisons a statistical test is available which shows whether the two quantities differ significantly at some selected probability level. The test involves calculating an elasticity coefficient which, given the mean input of all resources and the mean product in each area, would have resulted in a marginal productivity equal to the market price, or the mean marginal productivity in the contrasted area or stratum. The *t*-test is then applied to determine whether the actual and hypothetical elasticities differ significantly from each other.

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[48] Mean resource productivities are usually calculated for *geometric* mean quantities of all inputs and of output. The use of geometric means has two advantages. (i) In a function linear in the logarithms, the actual and predicted geometric mean outputs are the same, so that the actual mean output ($\overline{Y}$) can be substituted in the marginal productivity formula.

$$\frac{\partial \overline{Y}}{\partial \overline{X}_1} = \frac{b_1 \overline{Y}}{\overline{X}_1}$$

(ii) In agricultural data the distribution of inputs and outputs is often positively skewed (i.e., the longer tail of the curve lies to the right). In this circumstance the geometric mean is a more appropriate measure of central tendency than the arithmetic mean, since the former is closer to the mode.

[49] These comparisons are all based, of course, on the well known equilibrium conditions for a perfectly competitive economy. In making them one is measuring the performance of the actual economy, as regards the allocation of resources, in comparison with this theoretical norm. Some measure of the importance of market imperfections is thus obtained. However, there is one factor making for a divergence between actual and theoretical marginal productivity which has nothing to do with market imperfections, as normally understood. This is uncertainty, which, in many agricultural industries is high in respect of both output and price. There is thus a danger of inferring the existence of market imperfections from a situation which really only reflects the existence of uncertainty.

Fortunately, the possibility of such ambiguous situations arising is lessened by the fact that uncertainty consistently makes for marginal productivities in excess of the market prices of resources (since it causes entrepreneurs to demand risk premiums in their expected returns, and lending institutions to resort to capital rationing) whereas market imperfections may keep marginal returns either above or below market prices, as they inhibit both the inflow and outflow of resources. Thus the situation where the marginal return of a resource is greater than its market price is ambiguous: it may mean that greater quantities of the resource could be employed profitably; on the other hand, the difference between the two may simply represent the risk premium appropriate to the degree of uncertainty accompanying production. But the converse situation is not ambiguous: if a factor yields a marginal return less than its market price, we can be confident that it would be *economically* advantageous to employ less of it.

Examples of all three types of comparison, and of the inferences which it is possible to draw from them, are contained in the work of Heady and Shaw.[50] Some of their findings are set out below.

### Inter-Area Comparisons.

Heady and Shaw made use of data relating to four "distant and contrasting areas of the United States", viz., Montana, North Iowa, South Iowa and Alabama. They found that differences between the areas in respect of the marginal productivities of land, labour, and capital were in accordance with "the experience of agricultural extension workers and others closely acquainted with the nation's agriculture".[51] However, only in the case of land were these differences found to be consistently significant, in the statistical sense. None of the differences in labour productivity were significant, and only one significant difference in capital productivities was found. Heady and Shaw point out that, "from these data, we cannot even say that at 'the margin of mean labor inputs', marginal productivity of labor in Alabama differs significantly from that of other areas, although we know that it differs in average productivity terms. It is entirely possible that the return to 'the last month of labor' use 'on the average farm' in the four areas is entirely similar."[52] This possibility, they state, "is in contrast to certain previous inferences on productivity based on extremely gross, residual data for parts of the same region."[53] While thus arguing for the superiority of the production function approach in investigating resource productivities (as compared with their derivation by means of "residual" analysis) they do, however, admit that, "significant differences in mean marginal labor productivity might be obtained in such contrasting areas as North Iowa and Alabama by obtaining the labor records and by lessening variance through samples stratified more closely by soil type and techniques of production".[54]

### Inter-Strata Comparisons.

Farms in all areas were stratified into nine groups, by capital and labour thirds, and marginal labour products predicted for each strata separately. From the results it is inferred that "relative differentials in labour productivity are just as great or greater in Montana and Iowa as in Alabama; absolute differences are even greater."[55] This result is also put forward by Heady and Shaw as demonstrating the weaknesses

[50] Heady and Shaw, *op. cit.*

[51] *Ibid.*, p. 250.

[52] *Ibid.*, p. 252.

[53] *Loc. cit.*; here reference is made to Colin Clark, *The Economics of 1960*, Macmillan, London, 1940, Chs. 3-11; T. W. Schultz, *Production and Welfare of Agriculture*; McGraw Hill, New York, 1949, Ch. 7; and L. J. Ducoff and M. J. Hagood, *Differentials in Productivity and Income of Agricultural Workers by Regions*, Bureau of Agricultural Economics Mimeograph, 1944.

[54] Heady and Shaw, *loc. cit.*

[55] *Ibid.*, p. 256.

in the approach adopted by Clark and Schultz. The fact that, "while the level of resource returns is different, the relative degree of inefficiency can be just as great in areas conventionally classified as 'highly efficient', as those classified as 'inefficient' ", is, they believe, "a point which, has too often been overlooked in analyses of agricultural resource productivity".[56]

**Comparisons between Resource Returns and Resource Prices.**

The mean marginal productivity of labour used in crop production was found to be significantly lower than the wage rate in three of the four areas. Marginal productivity of land was significantly greater than rental rates in all areas. Capital returns were significantly greater than the cost of capital in two areas, but were lower in the highly mechanised North Iowa region. There was no significant difference between the cost and marginal product of capital in Alabama. Heady and Shaw point out, however, that capital in Alabama is of a form representing "poor techniques"; "capital in the form of other techniques and in an amount to allow gains to scale economies might allow much higher returns".[57] This comment illustrates the point that the production function approach offers a guide to policy only in so far as marginal adjustments are concerned. In some "problem" areas or industries, adjustments to be effective may have to be of a much more far-reaching character.

**International Comparisons.**

Further resource productivity comparisons, utilizing South African data as well as the American data referred to above, have been made by Heady and du Toit.[58] The South African findings relate to a sample of cattle ranches, and are remarkable for revealing a very wide discrepancy between factor prices and factor returns. The marginal productivity of (native) labour, though low, in absolute terms, at 21 dollars per month, was almost double the average monthly wage rate of 11 dollars. Land yielded a marginal return approximately 80 per cent. greater than its average rent, and capital (annual capital expenses) had a marginal productivity of 1.57 dollars, whereas it cost only 1.06 dollars (one dollar principal, plus interest of ·6 per cent). Heady and du Toit suggest that: (I) lack of knowledge of the marginal productivities and (II) risk and uncertainty are the main reasons why the ranchers have kept factor inputs so far short of the theoretical optimum amounts.

The found that "the marginal productivity of labor computed for the African sample does not differ significantly from the figure computed for the Alabama sample. . . . . Hence this inference can be drawn: differentials in labor productivities are just as great as between selected segments of United States agriculture as they are between the American and African sample areas".[59] This finding, they believe, will "help to

[56] *Loc. cit.*

[57] *Ibid.*, p. 255.

[58] Heady and Du Toit, *op.* cit.

[59] *Ibid.*, p. 505.

clear up some widespread notions that only American farmers use labor efficiently". They also point out that African labour "has a relatively high productivity considering the low ratio of capital to labor inputs that characterizes its use".

Capital used on the African properties was found to differ significantly in productivity from that employed in the livestock enterprises in all the American areas. These differences indicated a less efficient use of this resource in the African sample—efficiency being indicated, not by the high capital productivity in Africa "but by the near equality of capital productivity and the price of capital in the American samples".[60]

Interesting as these comparisons are, those relating to land and labour are of little practical importance, since land is a fixed resource, and population transfers from Africa to the United States are not possible. There are, however, opportunities for international movements of capital. Heady and du Toit conclude that, "given the productivity estimates of this study, it appears that the world product could be augmented by transfers of capital from the United States areas to the African areas".

[60] *Loc. cit.*