ESTIMATION OF STOCKING RATES BY MULTIPLE
REGRESSION ANALYSIS

by

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In an article published in a previous issue of this Review Gruen demonstrated the possibility of using a multiple correlation analysis to estimate average stocking rates for different types of pasture grazed in common. We have applied the same technique to data obtained from a sample of South-west Slope grazing properties in an attempt to estimate average stocking rates for natural and improved pastures. The attempt has not been notably successful. However, a discussion of the statistical findings may be of some interest insofar as it throws light on the theoretical and practical difficulties involved in using this technique.

The data on which the estimates are based (and on the adequacy of which more will be said later) were obtained by means of a field survey carried out in late 1954 and early 1955. The survey sample consisted of 150 randomly-selected grazing properties located in the shires of Gundagai, Tumut, Tumbarumba, Holbrook and Kyeamba. The following data were used in the correlation analysis:

\[ y: \text{the number of livestock, expressed in terms of dry sheep equivalents, on hand at shearing, 1954.} \]
\[ x_1: \text{the acreage of unimproved grazing land (not forest) on the property.} \]
\[ x_2: \text{the acreage of improved pasture, sown in 1953 or in earlier years.} \]

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1 F. H. Gruen, "Stocking Rates in the Berriquin and Wakool Irrigation Districts", this Review, Vol. 21, No. 2 (June, 1953) pp. 113-140.

2 Descriptive information concerning these properties has already been published in the Review. See Vol. 23, No. 2 (June, 1955), pp. 59-82 and Vol. 24, No. 2 (June, 1956) pp. 74-100.

3 The following conversion ratios, which are intended to express the relative grazing requirements of different types of stock, were used:
   1 Merino, Polwarth or comeback wether = 1 dry sheep.
   1 Merino, Polwarth or comeback ewe and lamb = 1½ dry sheep.
   1 Corriedale, crossbred or British breed hogget, ewe or wether = 1½ dry sheep.
   1 Corriedale, crossbred or British breed lamb = 2 dry sheep.
   1 ram (any breed) = 2 dry sheep.
   1 mature beef animal = 8 dry sheep.
   1 steer or heifer = 5 dry sheep.
   1 weaner calf = 3 dry sheep.
   1 milking cow = 9 dry sheep.
   1 bull = 8 dry sheep.
(Thus the grazing contribution made by forest land has been ignored as being insignificant, and areas newly-sown to improved pasture have been regarded as being unimproved.) The mean values, for each shire, of $y$, $x_1$ and $x_2$ are shown in Table I.

**Table I**

*Characteristics of Survey Properties*

*(Average Quantities per Property)*

<table>
<thead>
<tr>
<th>Shire</th>
<th>Number of Stock Carried ($y$)</th>
<th>Area of Natural Pasture ($x_1$)</th>
<th>Area of Improved Pasture ($x_2$)</th>
<th>Proportion of Grazing Land Pasture Improved</th>
<th>Overall Rate of Stocking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tumut</td>
<td>2,758</td>
<td>1,092</td>
<td>384</td>
<td>26</td>
<td>1.9</td>
</tr>
<tr>
<td>Gundagai</td>
<td>4,734</td>
<td>1,709</td>
<td>647</td>
<td>27</td>
<td>2.0</td>
</tr>
<tr>
<td>Holbrook</td>
<td>2,130</td>
<td>804</td>
<td>538</td>
<td>38</td>
<td>1.5</td>
</tr>
<tr>
<td>Tumbarumba</td>
<td>3,402</td>
<td>483</td>
<td>754</td>
<td>61</td>
<td>2.3</td>
</tr>
<tr>
<td>Kysamba</td>
<td>2,559</td>
<td>1,202</td>
<td>540</td>
<td>31</td>
<td>1.3</td>
</tr>
<tr>
<td>Tumut and Gundagai</td>
<td>3,902</td>
<td>1,449</td>
<td>533</td>
<td>27</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Regression equations of the form—

$$ y = b_0 + b_1 x_1 + b_2 x_2 $$

were fitted separately to the data obtained from each shire. This formulation assumes that there is a linear relationship between area of pasture and stock numbers: that 5,000 acres of pasture will usually be stocked with ten times as many sheep as 500 acres. However, it is often observed that larger properties tend to carry fewer stock per acre than the smaller properties. This may be because the larger properties tend to be found on the poorer land and/or because graziers with smaller holdings have a greater incentive to use their land more intensively. Thus it might be expected that the relationship between stock numbers and farm size is of a curvilinear nature, with stock numbers increasing at a diminishing rate as farm size increases. A straight line fitted to a set of observations which is best described by means of a curve, will be to a greater or less extent misleading but may be an acceptable approximation if the degree of curvature is not great. Such a line will not pass through the origin, but will intersect the vertical axis at some positive value of $y$ (stock numbers). Also, its slope will best approximate to the slope of the true curve over the middle range of observations.

The regression and correlation coefficients obtained for each shire are listed in Table II. The correlation coefficients obtained for Tumut, Gundagai and Holbrook shires indicate that the fitted regression plane
fits the observations quite well—that, in fact, 92, 91 and 86 per cent of the variation in stock numbers in each shire, respectively, can be explained by variations in the acreage of natural and improved pastures available. The values of $b_0$ (which indicates that point where the regression plane intersects the y axis) are positive—as would be expected on the assumption that the larger properties tend to stock less heavily than the smaller—but the standard errors associated with their estimation are such that it cannot be asserted that they differ significantly from zero.

### Table II

**Regression and Correlation Coefficients for each Shire.**

<table>
<thead>
<tr>
<th>Shire</th>
<th>$n^*$</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$R^2$†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tumut</td>
<td>24</td>
<td>316.9 ± 205.4</td>
<td>1.19 ± 0.12</td>
<td>2.97 ± 0.40</td>
<td>.915</td>
</tr>
<tr>
<td>Gundagai</td>
<td>33</td>
<td>412.7 ± 387.7</td>
<td>1.21 ± 0.15</td>
<td>3.42 ± 0.29</td>
<td>.900</td>
</tr>
<tr>
<td>Holbrook</td>
<td>36</td>
<td>354.6 ± 177.6</td>
<td>0.75 ± 0.16</td>
<td>2.09 ± 0.20</td>
<td>.846</td>
</tr>
<tr>
<td>Tumbarumba</td>
<td>25</td>
<td>1,028 ± 358.6</td>
<td>1.37 ± 0.32</td>
<td>2.27 ± 0.38</td>
<td>.771</td>
</tr>
<tr>
<td>Kyeamba</td>
<td>25</td>
<td>-1,424 ± 964.7</td>
<td>1.54 ± 0.30</td>
<td>3.95 ± 1.30</td>
<td>.505</td>
</tr>
<tr>
<td>Tumut and Gundagai</td>
<td>57</td>
<td>327.7 ± 233.6</td>
<td>1.21 ± 0.12</td>
<td>3.42 ± 0.22</td>
<td>.914</td>
</tr>
</tbody>
</table>

* $n = \text{Number of properties.}$
† $R^2 = \text{Square of the multiple correlation coefficient adjusted according to the size of the sample.}$
‡ Errors quoted are standard errors of regression coefficients.

Although the degree of correlation between stock numbers and pasture acreages is reasonably good in Tumbarumba shire, the high value of $b_0$ suggests that the stocking rates on large and small properties differ markedly, so that it would be misleading to quote average stocking rates for all farms. Accordingly, the analysis has been rejected as unsatisfactory. The results shown for Kyeamba shire are also unsatisfactory both on account of the low coefficient of correlation obtained, and the anomalous negative $b_1$ term.

Since the regression coefficients for Tumut Shire did not differ significantly from those obtained for Gundagai Shire, both sets of data were combined, and a regression plane fitted to the pooled data. The coefficients obtained are very similar to those resulting from the earlier analyses, but their standard errors are smaller. (See Table I.)

The high degree of correlation obtained from Tumut, Gundagai and Holbrook shires, together with the fact that the values of $b_0$ are not unduly large, suggests that the assumption of linear relationships is not likely to be grossly misleading, and that the regression coefficients $b_1$ and $b_2$ may be interpreted, with fair accuracy, as average stocking rates. Furthermore, these apparent rates—1.2 and 3.4 dry sheep equivalents per acre of natural and improved pasture respectively, for Tumut.

* In this article, all assertions regarding significant differences are based upon five per cent level tests.
and Gundagai shires, and 0.75 and 2.1 dry sheep for Holbrook shire—
do not conflict with the opinions of agriculturalists acquainted with the
grazing industry in these shires. Thus the results appear to be both
statistically acceptable and reasonable from a common-sense point of
view. However, further analysis of the data has shown that the statisti-
cal evidence is not so straightforward as the previous analyses have
suggested.

<table>
<thead>
<tr>
<th>Shire and Type of Farm</th>
<th>Number of Stock Carried ((\bar{x}))</th>
<th>Area of Natural Pasture ((\bar{x}_1))</th>
<th>Area of Improved Pasture ((\bar{x}_2))</th>
<th>Proportion of Grazing Land Pasture Improved</th>
<th>Overall Rate of Stocking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tumut-Gundagai</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Farms</td>
<td>3,902</td>
<td>1,449</td>
<td>533</td>
<td>27</td>
<td>2.0</td>
</tr>
<tr>
<td>Large Farms</td>
<td>7,594</td>
<td>2,834</td>
<td>1,065</td>
<td>27</td>
<td>1.9</td>
</tr>
<tr>
<td>Small Farms</td>
<td>1,936</td>
<td>702</td>
<td>245</td>
<td>26</td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Holbrook</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Farms</td>
<td>2,130</td>
<td>864</td>
<td>538</td>
<td>38</td>
<td>1.5</td>
</tr>
<tr>
<td>Large Farms</td>
<td>3,353</td>
<td>1,473</td>
<td>910</td>
<td>38</td>
<td>1.4</td>
</tr>
<tr>
<td>Small Farms</td>
<td>1,591</td>
<td>596</td>
<td>374</td>
<td>39</td>
<td>1.6</td>
</tr>
</tbody>
</table>

In view of the suggestion that average stocking rates per acre may
be related to farm size, it was decided to subject the large and the small
farms to separate analyses. In the pooled Tumut-Gundagai sample
there were 20 farms greater than 2,000 acres and 37 farms of 2,000
acres or less. The Holbrook sample contained 11 properties greater
than 1,500 acres and 25 of lesser area. The mean values of the data
relating to these four groups of farms are given in Table III and the
regression and correlation coefficients obtained are set out in Table IV.

In each district, large farms showed almost as high a degree of
correlation between stock numbers and pasture areas as was found
in the sample as a whole, despite the fact that the number of observa-
tions were substantially fewer. Also, the regression coefficients for
the large farms did not differ substantially nor significantly from those
obtained for all farms. On the other hand, in both groups of small
farms, only a tenuous relationship between area and stock numbers was
evident; in each case little more than 40 per cent of the variation in
the latter was explicable in terms of variations in the former. And as would be expected, the regression coefficients for the small farms showed some wide differences from those obtained using the data from the whole sample.

\[\text{Table IV}\]

Regression and Correlation Coefficients for All, Large and Small Farms

<table>
<thead>
<tr>
<th>Shire and Type of Farm</th>
<th>(n)</th>
<th>(b_0)</th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{Tumut-Gundagai:}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Farms ...</td>
<td>57</td>
<td>327.7 ± 233.6</td>
<td>1.21 ± 0.12</td>
<td>3.42 ± 0.22</td>
<td>0.914</td>
</tr>
<tr>
<td>Large Farms</td>
<td>20</td>
<td>315.2 ± 277.1</td>
<td>1.13 ± 0.20</td>
<td>3.50 ± 0.39</td>
<td>0.886</td>
</tr>
<tr>
<td>Small Farms...</td>
<td>37</td>
<td>917.7 ± 241.6</td>
<td>1.01 ± 0.20</td>
<td>1.33 ± 0.34</td>
<td>0.422</td>
</tr>
<tr>
<td>\text{Holbrook:}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Farms ...</td>
<td>36</td>
<td>354.6 ± 177.6</td>
<td>0.75 ± 0.16</td>
<td>2.09 ± 0.20</td>
<td>0.856</td>
</tr>
<tr>
<td>Large Farms</td>
<td>11</td>
<td>333.2 ± 595.8</td>
<td>0.90 ± 0.34</td>
<td>1.98 ± 0.37</td>
<td>0.815</td>
</tr>
<tr>
<td>Small Farms...</td>
<td>25</td>
<td>437.9 ± 412.9</td>
<td>0.54 ± 0.42</td>
<td>2.22 ± 0.52</td>
<td>0.486</td>
</tr>
</tbody>
</table>

These findings lead us to conclude that the analyses made using the whole sample for each shire are dominated by the large farms. This is easily understood in terms of a three-dimensional scatter diagram: the small farms are represented by a fairly amorphous group of coordinates clustered relatively close to the origin, while the coordinates representing the large farms tend to fall about a plane extending far from the origin in each dimension. Quite drastic changes in the data relating to the small farms would have relatively little effect on a regression plane fitted to the large farm data.

The poor correlations found in some shires and for small farms are partly the result of deficiencies in the available data. These deficiencies arise from the fact that the survey sample and questionnaire were not specifically designed for the purpose of estimating stocking rates, and become apparent when the data are compared with those used by Gruen in his study of the Berriquin and Wakool Irrigation Districts. Gruen’s farms were selected on the basis of soil type, whereas ours were chosen at random without reference to this factor. His data took account of

*Thus nothing conclusive emerges from the regression analyses regarding differences in stocking rates as between large and small properties. However, it is evident from Table III that the larger farms do tend to adopt lower stocking rates per acre than the small farms. In each area, both the small and the large properties have been pasture improved, on the average, to practically the same degree (26 and 27 per cent of improved pasture, respectively, in Tumut-Gundagai, and 30 and 38 per cent respectively, in Holbrook) so that direct comparisons of the overall stocking rates may be made. In Tumut-Gundagai, the average overall rate of stocking on large farms is approximately 10 per cent lower than the rate on small farms, while in Holbrook, the large farms carry on the average 12.5 per cent fewer stock per acre than the small farms.

*Note particularly the differing values of \(b_3\) in Tumut-Gundagai.
seasonal variations in stock numbers but ours did not. Furthermore, the distinction between irrigated and dry-land pasture in the Berriquin-Wakool area is more clear cut than the distinction between improved and natural pasture in the South-West Slope: the differences in carrying capacities are greater, and variations in the quality of irrigated pasture are likely to be less.¹

However, even if “perfect” data had been available, far from perfect correlations would have been obtained. Stocking rates and carrying capacities of farms reflect the influence of many factors in addition to area and type of pasture. The degree of subdivision, the availability of water, of stock handling facilities and of conserved fodder, the amount and skill of labour employed and the managerial competence of the grazer, all contribute substantially to the stock carrying capacity of a property. In view of the wide variation from farm to farm in these factors, and of the crudeness of the available data, it is not surprising that on the small farms little correlation was found between stock carried and pasture areas.² Among the large farms, however, the size range was sufficiently wide for differences in area to overshadow differences in the other factors affecting rate of stocking.

Although dominated by the large properties, the analyses using data from all farms still provide the best estimates of stocking rates in the shires concerned. These estimates are attended by a fair degree of error. All that can be said with a high degree of confidence is that stocking rates lie with the ranges quoted below.³

**Tumut and Gundagai Shires:**

- Natural pasture 0.97—1.45 dry sheep per acre.
- Improved pasture 2.98—3.86 dry sheep per acre.

**Holbrook Shire:**

- Natural pasture 0.43—1.07 dry sheep per acre.
- Improved pasture 1.60—2.49 dry sheep per acre.

¹ Establishment of improved pastures under irrigation tends to be an “all-or-nothing” type of proposition: the cost of irrigation provides a strong incentive to strive for a highly-improved pasture. On the South-West Slope, on the other hand, such an incentive does not exist.

² To the extent to which the poor correlation found among the small farms was due to factors other than the inadequacy of the data, the usefulness of the concept of inherent carrying capacity or average stocking rate may be questioned. And, from the economist's point of view, the concept can be regarded as misleading since it focuses attention on only one factor—land—of the many which contribute to farm production. However, there can be no doubt that among farmers the concept is of operative significance, i.e., farmers' stocking policies are based on notions of the carrying capacities of various types of country. For this reason, rate of stocking is likely to vary less from farm to farm than either input of productive factors or output of product. The differences between well-improved and little-improved, well-managed and poorly-managed properties are often in the care with which stock are treated—and consequently show up in such matters as lambing percentages, wool cut per head, drought losses, wool and meat quality—rather than in the number of stock carried.

³ These ranges represent 95 per cent confidence intervals about the estimated regression coefficients.
There are some theoretical grounds for believing that these figures exaggerate the carrying capacity of improved pasture and under-estimate that of natural pasture. A regression analysis would give such a result if either the better land tended to be pasture improved first, or the more competent graziers tended to sow more improved pastures than the less competent. It is impossible to say how important these two tendencies might be, but it seems likely that to some extent they would tend to nullify each other.

Despite their deficiencies the estimates given may be of some interest, on account of both the importance of pasture improvement in the area, and the difficulty of securing estimates of stocking rates by other means. Experimental evidence and case studies, although useful in showing how much it is possible to increase carrying capacity by pasture improvement, give little clue as to the average increases which have occurred. The study of time-series data is complicated by the fact that the increase in the area of improved pasture over the years has been accompanied by many other changes affecting carrying capacities.