TOWARDS A PRODUCTION FUNCTION FOR SUPPLEMENTARY IRRIGATION ON FAR NORTH COAST DAIRY FARMS

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Introduction

A number of methods have been employed to estimate the production increases obtained from supplementary irrigation. Details of these methods and some of the disadvantages from which they suffer will be discussed later. Suffice it to say at this stage that, in the opinion of this author the methods fail to establish the fundamental relationships involved, namely, to define a production function for water, or, more basically, for soil moisture. The present paper is intended as a progress report of an attempt to relate butter production to soil moisture on three separate dairy farms. The assumption is that a functional relationship exists between soil moisture and pasture production, and, carrying this one stage further, between soil moisture and animal production. If this relationship can be defined mathematically it will be possible to estimate production responses to soil moisture changes within known limits of accuracy, an advantage which the methods referred to above do not possess.

Theoretical Basis for the Production Function

Some of the basic soil moisture plant growth concepts used to define the production functions described in this paper have been reviewed by Beringer[1] and have already been employed to produce useful economic assessments of irrigation. Two studies in the United States, one by Moore[2], the other by Reutlinger and Seagraves[3] are significant contributions in this field. Moore has postulated a particular functional relationship between soil moisture depletion and relative rate of growth for crops generally. He applies this relationship to each irrigation cycle, several of which may occur during a season, according to irrigation requirements. On this basis he is able to predict, for a series of irrigations at specified levels of soil moisture depletion, a corresponding percentage attainment of the potential crop yield which is possible under physically optimum moisture conditions.

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Although this paper arose out of preliminary work done in connection with a study of the economics of sprinkler irrigation on the Far North Coast, a project being conducted jointly with Mr. J. G. Bird, from whom the author has received a great deal of help in preparing the present paper, the author accepts full responsibility for any deficiencies in this paper.

[1] C. Beringer: An Economic Model for Determining the Production Function for Water in Agriculture *Giannini Foundation Research Report* No. 240, February, 1961.

[2] C. V. Moore: "A General Framework for Estimating the Production Function for Crops Using Irrigation Water". *Journal of Farm Economics*, Vol. 43, Part 1, No. 4, November, 1961, pp. 876-888.

[3] S. Reutlinger and J. A. Seagraves: "A Method of Appraising Irrigation Returns". *Journal of Farm Economics*, August, 1962, pp. 837-850.

Reutlinger and Seagraves have adopted a more direct approach in establishing a relationship between an annual index of soil moisture, estimated from weather data, and yield of tobacco leaf on experimental plots in North Carolina. Regression analysis was used to quantify this relationship and a coefficient of determination (r^2) of .83 was obtained after trying various methods of estimating the daily soil moisture conditions and various ways of weighting these estimates to calculate the annual soil moisture index. They also demonstrate how long term estimates of soil moisture levels can be used to obtain a probability distribution of moisture deficits, and consequently an estimate of the expected amount of irrigation required annually. Finally, the standard error of the average soil moisture index for the life of an irrigation system is used to evaluate the likelihood that the weather will be too wet for the system to pay for itself.

The Concept of a Production Function for Soil Moisture

It might be just as well at the outset to make quite clear what it is we are trying to measure. The production function being defined relates only to a particular soil—pasture situation together with a particular fertilizer regime, and attempts to measure the changes in production due to soil moisture alone. As the production function is estimated from time series data it must be reasonably certain that the data refers to a period when the management policy has been constant. This situation may be difficult to find in some farming areas, but the author has observed that it is not uncommon on dairy farms on the Far North Coast, typically where the farm operator has passed middle age and becomes content to settle down to a constant pattern of management. This kind of situation is usually associated with a rather low level of farm inputs such as fertilizers, but this may not always be the case. It is farms which have been operating at a constant level of management for some years which have been chosen as sources of data for estimating the production functions described in this paper. It should be added also that the farms chosen are ones which do not practice irrigation, the changes in soil moisture[4] from year to year being due entirely to climatic factors.

There is another point worth making at this stage. The introduction of irrigation is quite often accompanied by a number of other innovations, for example, paddock subdivision may be modified and the area to be irrigated may be sown to improved pasture species which give a greater response to irrigation than the original pastures. Because of this simultaneous introduction of a number of innovations, the question commonly arises in studies of the economics of supplementary irrigation—how much of the production increase is due to irrigation alone and how much to the other changes which have taken place? If we define exactly what the production function refers to it will be seen that it is possible, by the method proposed below, to make a distinction between these two sources of production increases. The advantage of being able to do this will be questioned later, but for the sake of demonstrating the point the application of the estimated production function is now considered.

[4] Details of the way in which soil moisture is estimated and the method of deriving the production function are dealt with later.

The Application of the Production Function

1. *The use of the production function to estimate the likely response from irrigating the existing pastures of a farm.*

This may be illustrated by reference to Fig. 1[5] where PF1 represents the production function for the pastures before irrigation. If x_1 represents the level of soil moisture of the non-irrigated area (a) for the year under consideration, and x_2 is the soil moisture level of the irrigated area (ai), then the increased production from irrigation is ai (y_2-y_1).

2. *If irrigation and other innovations are adopted together, the production function may be used to assess (a) what the production would have been that year without any of the innovations, and (b) what the production would have been with only irrigation and none of the other innovations. Finally, because the actual total increase in production will be known, it is possible to assess, (c) the contributions which innovations other than irrigation have made.*

Referring again to Fig. 1, x_1 and x_2 represent the soil moisture level without, and with, irrigation (in the same year). With irrigation, only y_2 production is achieved, but with irrigation plus the other innovations the production level is y_3. The addition of other innovations means that a new production function PF2 now operates. If we simplify the situation and say that the innovations introduced with irrigation also apply to the area irrigated only, then, the increase due to irrigation alone, as before is ai (y_2—y_1). The difference between this and the total increase in production (known) is the production due to the associated improvements.[6]

3. *Another possible use of the production function would be to assess the contribution made by certain innovations adopted without irrigation.*

Say, for example, pasture improvement was carried out, the production function would provide an estimate of what the existing pasture would have produced given the particular soil moisture conditions prevailing during the period under consideration. In Fig. 1 if x_1 is the level of soil moisture which is the result of the climatic conditions for period under consideration

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[5] For convenience the production function has been expressed on a per acre basis. In the empirical part of this study "whole farm" production functions are first estimated and then related to production per unit area of land. The production function for unit area may be derived from the "whole farm" production function on the assumption that each unit area has identical productivity. The validity of this method of deriving the unit area production function will depend entirely on the uniformity of the property being considered.

[6] Although a possible dissection of production increases has been suggested, the author believes that this serves little real purpose because it is usually expected that the introduction of irrigation will involve other innovations if the full benefit of irrigation is to be realized. This being the case, it is the increased production from a given group of associated improvements that is important, not the individual contributions. There is another point here also. The assessment of the individual contribution is based on performances of each innovation (in this case irrigation only) carried out as an entirely separate innovation. The introduction of a group of innovations may result in interaction giving a resultant increase in production greater than the sum of the individual innovations had they been introduced separately. This sort of situation would invalidate the attempt above to separate the effects of irrigation from those of the other innovations.

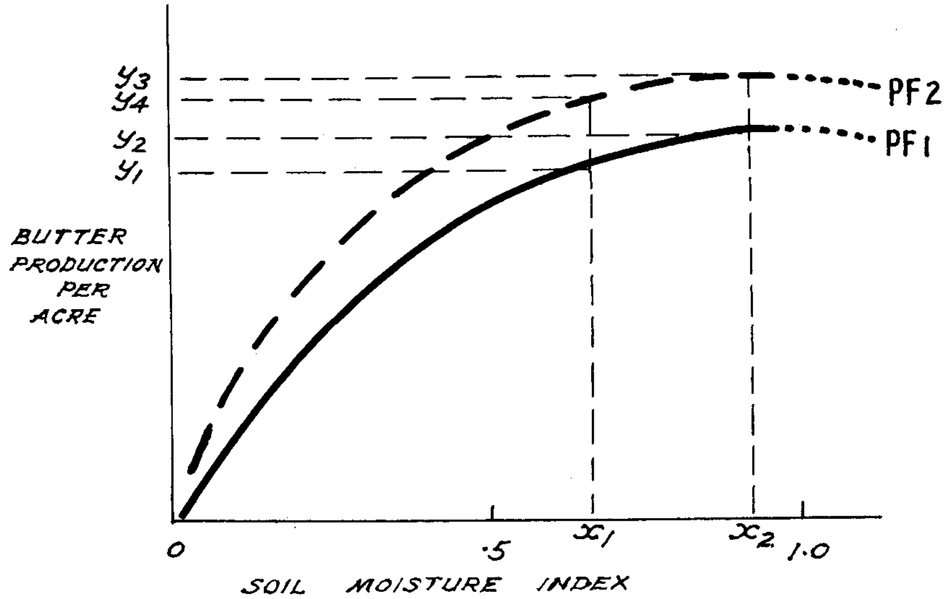

Fig. 1. Showing Theoretical Production Functions Relating Butter Production per Acre to a Soil Moisture index (where 1.0 = Field Capacity)

and y_1 is the level of production associated with it, y_4 might represent the production achieved from the introduction of pasture improvement and the movement on to a new production function PF2.

As the total farm production after the introduction of pasture improvement is known, the increased production gained from the introduction of pasture improvement is $T - Ay_1$ where T = total production after pasture improvement, A = total area of property.

Estimation of Production Functions for Three North Coast Farms

The author has adopted a similar procedure to that used by Reutlinger and Seagraves and related butter production to estimates of soil moisture level[7] for the three properties on the North Coast of New South Wales, one near Murwillumbah on the Tweed River and two near Kyogle in the Richmond River Valley. The results offer encouragement for further studies in this field. Thornthwaite's[8] water balance concept has been used to estimate monthly soil moisture levels which have been averaged on an

[7] Based on rainfall and temperature recordings adjacent to the properties.

[8] C. W. Thornthwaite and J. R. Mather: Instructions and Tables for Computing Potential Evapotranspiration and the Water Balance. *Drexel Institute of Technology, Publications in Climatology*, vol. 10, No. 3, 1957. For purposes of these estimates the soils of the three properties have been classed as clay loams, the Murwillumbah property having meadow soils derived from alluvial material while the Kyogle properties have a combination of meadow and chocolate soils derived from basalt, according to the classification of J. W. McGarity in "The Soils of the Richmond-Tweed Region. A Study of their Distribution and Genesis". Unpublished M.Sc. Agr. thesis 1956, University of Sydney. Effective pasture root depth for soil moisture calculations has been taken as one foot, giving an estimated soil moisture capacity of 3 inches as suggested by Thornthwaite and Mather, *op. cit.*, p. 244.

annual basis to obtain the soil moisture index (X) expressed as a decimal fraction of soil moisture capacity (field capacity).[9] The production variable (Y) is the annual commercial butter production in lb.

The following regression equations have been estimated from the data (See Fig. 2 for scatter diagrams):

(1) Murwillumbah property (N = 10, 1951-52 to 1960-61)
$Y = 1594 + 5307\,X$, $r = .89$ (significant at the .1 per cent level)

(2) Kyogle property (N = 10, 1952-53 to 1961-62)
$Y = 7594 + 8242\,X$, $r = .71$ (significant at the 5 per cent level)

(3) Kyogle property (N = 10, 1952-53 to 1961-62)
$Y = 10965 + 17407\,X$, $r = .59$ (significant at the 10 per cent level)

From this the relationships shown in Table 1 can be calculated.

TABLE 1

Results of the "Whole Farm" Regressions Expressed per Acre-inch of Soil Moisture and of Water Applied

Total Farm Area	Commercial Butter per 3-inch Soil Moisture		Commercial Butter per Acre-inch of Soil Moisture	Equivalent Butterfat per Acre-inch Soil Moisture	Butterfat per Acre-inch of Water Applied (80 per cent Efficiency)
	Whole Farm	Per Acre			
Acres	lb.	lb.	lb.	lb.	lb.
(1) 100	5,307	53.1	17.7	14.7	11.8
(2) 190	8,242	43.4	14.5	12.1	9.7
(3) 280	17,407	62.2	20.7	17.2	13.8

It will be noted that butter production has been related to soil moisture expressed in acre-inches. Soil moisture is customarily expressed in "inches per unit depth of soil" so that it may be related directly to irrigation water supplied, measured in acre-inches.

These results may be compared with other methods used for assessing returns from irrigation on the North Coast of New South Wales. It will be seen incidentally that the estimates obtained are of a similar order of magnitude.

(1) The "before and after" comparison has been used by Yabsley[10] to estimate the returns to irrigation combined with pasture improvement, on demonstration projects financed by Dairy Industry Extension Grants.

[9] It is expected that the correlation would be improved by either weighting each of the monthly values of soil moisture to correspond with optimum pasture growth possibility curves, or else by grouping certain months into separate variables on the same basis. The reason being that soil moisture will have a greater effect during the period of maximum growth of pasture.

[10] Gordon Yabsley, Department of Agriculture, Murwillumbah. Information by personal communication.

A typical example is a property on the Tweed River on which irrigation and associated pasture improvement was introduced for the 1957-58 season. After adjusting the difference in returns before and after these improvements by a "district" variation in production (which was taken as reflecting the variation due to climate) the increased butterfat per irrigation acre was 147 lb. Twelve inches of water had been applied so the increase was approximately 12.2 lb. per acre inch.[11]

One rather unsatisfactory aspect of this method is the use of the variation in production from the local dairy factory as an indication of the variation due to climate. This is unsatisfactory (*a*) because supplies to the factory will be drawn from an area sufficient in extent to allow considerable variation in rainfall pattern and will therefore not necessarily reflect the climate being experienced by the farm for which the estimates are being made, and (*b*) although climate may be the main cause of variation in factory production, part of the variation is likely to be due to managerial changes on some of the farms, as the property under consideration is not likely to be the only one in the district which is being developed.

It may be mentioned here that the "soil moisture" method of assessing production changes is most likely to be used in the form of a "before and after" comparison, with the production function providing a more accurate means for assessing what the production would have been without irrigation.

(2) Waring [12] has used feeding standards to calculate the likely butterfat production from the results of irrigated pasture trials. In the reference given a New Zealand irrigation trial is cited in which 1 ton of clover-ryegrass hay was produced per 12 acre-inches of water applied. By applying conversion ratios obtained from stall feeding experiments Waring calculated that the increased hay production was equivalent to 140-160 lb. of butterfat per acre-foot of irrigation water used. This corresponds to 11.7-13.3 lb. per acre-inch.

The main criticism of this approach is that results comparable to experimental yields are rarely attained in commercial practice.[13] This is recognized by Waring because he regards the estimate cited above as a kind of "maximum attainable" increase from irrigation.

(3) Another method which has also been used by Waring[14] is the estimation of the effects of rainfall on butter production. Waring[15] has recently further developed these estimates by including various combinations of monthly rainfall as independent variables in regression analyses with butter production as dependent variable. The question here is just what is being measured by rainfall. It is suggested that the effect of rainfall on plant growth and subsequent dairy production is almost entirely due to the changes in soil moisture which it brings about. However, as the

[11] The total increase in butterfat, after adjustment, has been ascribed to the irrigation area.

[12] E. J. Waring, "Supplementary Irrigation of Pastures in Humid Areas", this *Review*, Vol. 27, No. 4 (December, 1959), p. 244.

[13] B. R. Davidson, "Crop Yields in Experiments and on Farms", *Nature*, Vol. 194, No. 4827 (May 5, 1962).

[14] *Op. cit.* p. 242.

[15] Personal communication.

correlation between rainfall and soil moisture must be less than one because of the occurrence of runoff one would expect to obtain better estimates of variation in production by using soil moisture as the independent variable. The "rainfall" method then is related to the method propounded in this paper and the justification for using soil moisture as the independent variable relies on the ability to estimate soil moisture with a reasonable degree of accuracy.

Significance of the Soil Moisture—Production Correlations

It is suggested that the high coefficient of determination ($r^2 = .80$) obtained for property (1) is fairly convincing evidence of the applicability, in this environment, of Thornthwaite's method of estimating soil moisture[16]. The basis for this assertion is that soil moisture is unlikely to be merely an index of other factors which are directly responsible for the variations in production. Temperature and rainfall are two factors which could be looked at in case they should be contributing to a "spurious" correlation in this way between soil moisture and production, both being used in the Thornthwaite estimation of soil moisture. Temperature has a definite direct link with the physiology of plant growth, and because of its effect on evapotranspiration, is also linked with soil moisture. However, although temperature is a predisposing factor as far as potential evapotranspiration is concerned this does not necessarily mean that the level of soil moisture attained is highly correlated with temperature because although temperature sets the rate at which evapotranspiration can take place the level of soil moisture achieved depends on the incidence of rainfall. Also on an annual basis, there is not much variation in average temperature, certainly not as much variation as there is in the level of soil moisture.

With regard to rainfall, the effect of this on production is mainly indirect, soil moisture being the operative factor.

Regarding the "shape" of the functional relationship between butter production and soil moisture, it might be expected that zero soil moisture would be associated with zero production and that maximum production might be reached somewhat before soil moisture level reached a value of 1. The range of observations does not provide any information about these portions of the production function. For the range of values observed (between .4 and .95) the high coefficient of determination (r^2) and the distribution of the observations as shown in Fig. 2 suggest that the functional relationship is very close to a linear one.

The properties used for these correlations were especially chosen because no significant changes in management had taken place over the period of study and because no irrigation was practised[17]. Properties (1) and (3)

[16] It could be noted here that the lower r^2 (.34) for property (3) is due, in part to the degree of under-stocking which is practiced. High pasture production is not fully reflected in milk production.

[17] The estimation of a production function for soil moisture only has significance if the soil moisture measurement, or index, is for an area which has a uniform soil moisture value. Where portion of the property is irrigated, different soil moisture values prevail, and the composite soil moisture index which might be estimated for the farm as a whole has little value for deriving a production function.

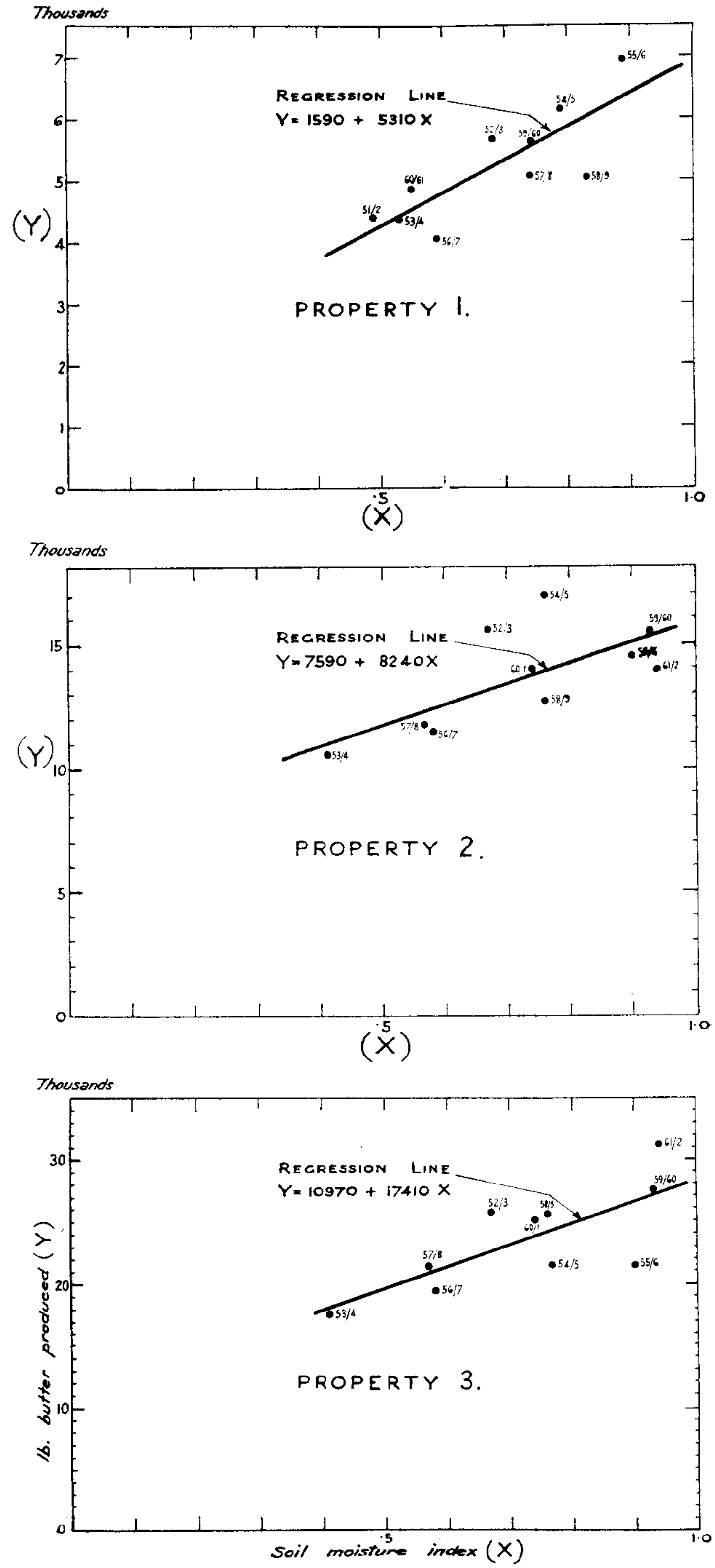

Fig. 2. Relation of Annual Butter Production to an Index of Estimated Soil Moisture for 3 North Coast Dairy Farms.

made little use of fertilizers and pastures were paspalum-dominant and contained little clover. Property (2) used superphosphate regularly and renewed a constant area of pasture annually. Rye grass and white clover were well established in the pastures of this property.

The properties were not chosen for any similarity of production factors such as soils, climate and management, so that no claim is made that they have comparable production possibilities or that they constitute a "sample" of a larger "population" of comparable farms. In fact, because property (2) shows a lower increment in production per unit of soil moisture,[18] despite the improved pastures, this indicates that the properties are not comparable.

For farms which are comparable in this respect, a substantially larger response to changes in soil moisture can be expected from the irrigation of improved pastures as has been demonstrated at Badgery's Creek by Crofts and his colleagues.[19]

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[18] See Table 1.

[19] F. C. Crofts, H. J. Geddes, and O. G. Carter, "Water Harvesting and Planned Pasture Production at Badgery's Creek", *University of Sydney, School of Agriculture Report No.* 6, January, 1963, p. 15.