CRITICAL PATH PLANNING AND SCHEDULING

An Introduction and Example

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1. Summary

2. Introduction
   - *PERT* and *LESS*

3. Basic Concepts, Rules and Procedures
   - Planning
   - Time Scheduling
   - Float Times
   - Critical Path

4. A Practical Example

5. Conclusions

Appendix

1. SUMMARY

Critical Path Planning and Scheduling is a labour management aid considerably in advance of those previously available. First devised for the U.S. Navy in 1958, it divides project programming into two phases—planning, and scheduling. In the planning phase a graphic representation of all activity relations is obtained by constructing an arrow network diagram. In the scheduling phase, duration times are assigned to activities and the three floats (spare time) available to each are computed. This allows the determination of the critical path of the project. Variable activity duration times and associated costs can also be handled.

The techniques of Critical Path Planning and Scheduling are described in Section 3. The discussion, however, is confined to project timing, and cost scheduling is left over to an appendix. This does not detract from the argument but makes it easier to prepare for a practical example of how CPPS may be applied in farming.

The planning and scheduling of some eleven activities required to describe the land preparation and sowing of 250 acres of wheat reveals that, with the assumed precedential ordering, there is spare time available when ordering the seed; and some delays in completing the first scarifying and harrowing do not prevent the entire operation being completed in 42 man-days. With a five and half day working week, it therefore takes between seven and eight weeks to complete the planting. This means that to sow no later than the first week in July, the plough must be put into the ground before the second week in May.

The conclusion is that CPPS could have some use in farm management. Its significance, however, may be less than that of other O-R techniques and its application will be heavily dependent on the gathering of more labour requirements.

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2. INTRODUCTION

On many farms, the introduction of more profitable enterprises is often hampered by labour shortages at certain times of the year. These shortages may be real, in that it is physically impossible for the available labour to handle any further output, or implied by constraints written into a formal re-planning programme.

An approach to solving the "real" problem of labour shortages has been made by introducing work study to farming. Here, activities thought to be time-consuming or unduly exhaustive are studied in detail; timed and charted; and alternative practices evolved. But, while work study is undoubtedly valuable in this context, its application is based on *a priori* assumptions about the relative importance of the activity, and the activity is removed from the complex of all farm operations to be studied *in vacuo*. The particular complaint that can thus be made against its application is that any savings in time or effort might not be profitably re-employable. Further, the whole of the farm operations need not be speeded up or smoothed out as a result of work study since the true bottleneck or limiting activity might not have been identified.

Techniques that have lately been evolved in secondary industry seem likely to obviate such shortcomings in labour management analysis. Known collectively as Critical Path Planning and Scheduling, these techniques supersede the traditional scheduling of job activities on bar charts or Gannt charts. In essence, they permit gathering together all the activities in a project into a composite diagram that shows all relationships and sequential ordering; the fitting to this diagram of activity duration times, and the computation of certain statistics which indicate those activities critical against the prompt completion of the job. Where activity duration times are discreetly variable, and cost data can be assigned to each interval, more advanced techniques permit estimation of the least cost schedule.

PERT AND *LESS*

Formal Critical Path Planning and Scheduling was to some extent anticipated by Kelly[1] in 1956 when examining a cut-and-fill operation in road building but it was not until 1958 that the techniques were fully developed. Early in that year the Special Projects Office, Bureau of Ordinance, U.S. Navy, was given the task of devising a system to evaluate the programme for the Fleet Ballistic Missile weapon system development. The research team rejected on scrutiny the traditional practices of contract scheduling and in one month devised PERT (Programme Evaluation and Review Technique)[2]. Intrinsically, PERT gives "a methodology for providing management with integrated and quantitative evaluation of: (*a*) progress to date and the outlook for accomplishing the objectives of the . . . program, (*b*) validity of established plans and schedules for accomplishing the program objectives, and (*c*) effects of changes proposed in established

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[1] J. E. Kelly, Jr., "Computers and Operations Research in Road Building", *Operations Research, Computers, and Management Decisions*, Symposium Proceedings, Case Institute of Technology (Jan. 31, Feb. 1, 2, 1957), pp. 58-68.

[2] D. G. Malcolm, J. H. Roseboom, C. E. Clark and W. Fazar, "Application of a Technique for Research and Development Program Evaluation", *Operations Research*, Vol. 9, No. 5 (1959), pp. 646-669.

plans"[3]. The main feature of the methodology is the building of a model (termed System Flow Plan in this instance) showing the properly ordered sequence of all inter-related events.

About the same time, Du Pont were perfecting a similar management control system that has come to be known as LESS (**L**east Cost **ES**timating and **S**cheduling)[4]. The basic methodology of LESS is similar to that of PERT but the analysis explicitly assumes the presence and use of at least two sets of times for each activity—the normal time and the shortest ("crash") time in which the activity can be completed—with costs assigned to each—and proceeds to the extent of formulating the project cost curve. It may be noted here that piece-wise linearity of the curve is implicitly assumed in such exercises.

PERT and LESS, and their respectively more commonly termed derivatives Critical Path Method (CPM) and Critical Path Scheduling (CPS)[5] have been widely adopted in commerce and industry. For instance, Bechtel has applied CPM to a grass-roots chemical complex.[6] LESS has been used by Standard Oil of California for turnaround problems in its refineries;[7] and CPS by Dow when expanding a power house.[8] In addition to the firms mentioned above, Martino also cited in 1960 another eight actively using Critical Path Planning and Scheduling, and went on to write "within five years most corporations on this continent (North America) will be actively utilizing the Critical-Path Method in many areas of their organizations. The savings should be in the hundreds of millions of dollars".[9]

3. BASIC CONCEPTS, RULES AND PROCEDURES

It is as well to note at the outset of this section that we shall not be concerned to show the theoretical background and proof of the matter to be presented. Readers interested in this aspect can refer to the contributions of Clark,[10] Frishberg,[11] or Kelly.[12]

In any job or project, technological restrictions force certain activities to be completed before others can be started. For instance, the walls of a building cannot be erected before the foundations are completed, and seed cannot be sown before it is purchased. Thus, initial examination of a job

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[3] *Loc. cit.*, p. 646.

[4] See R. L. Martino, "How Critical-Path Scheduling Works", *Canadian Chemical Processing* (February, 1960), pp. 38-40, and also M. C. Frishberg, "LESS Tells You How Project is Doing", *Hydrocarbon Processing and Petroleum Refiner,* Vol. 41, No. 2 (February, 1962), pp. 130-138.

[5] There is some confusion of terminology in the literature but these four innovations have been recorded, and their titles adequately reflect development up to the present. The comprehensive title "Critical Path Planning and Scheduling" is used to describe the general logic of the techniques involved.

[6] W. G. Kast, "Critical Path Method Ideal Tool for Plant Construction", *Hydrocarbon Processing and Petroleum Refiner,* Vol. 41, No. 2 (February, 1962), pp. 123-130.

[7] M. C. Frishberg, *op. cit.*

[8] J. S. Sayer, J. E. Kelly, Jr., and M. R. Walker, "Critical Path Scheduling", *Factory* (July, 1960), pp. 74-77.

[9] R. L. Martino, *op. cit.*, p. 38.

[10] C. E. Clark, The Optimum Allocation of Resources among the Activities of a Network". *The Journal of Industrial Engineering* (January-February, 1961), pp. 11-17.

[11] *Ibid,* especially pp. 136-138.

[12] J. E. Kelly, Jr., "Critical Path Planning and Scheduling: Mathematical Basis", *Operations Research,* Vol. 9, No. 3 (May-June, 1961), pp. 296-320.

reveals some partial ordering of the activities. Traditional techniques, however, do not allow exploration of *all* order relations, mainly because the planning and scheduling phases must be considered simultaneously.

The basic concept of CPPS is the recognition that planning and scheduling are separate activities. Planning is defined as the act of stating what activities are included in a job and in what order they must occur: scheduling is the act of producing job timetables.

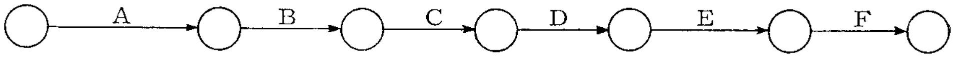
PLANNING

Planning is characterized by the preparation of a network or arrow diagram. This diagram uniquely identifies each activity in the job and presents a complete order relation.

The concepts of arrow diagrams can best be discussed using a hypothetical set of activities. Consider therefore a job containing six activities, conveniently labelled A, B, C, D, E and F. Each activity can be represented by an arrow, whose length is determined only by convenience and whose direction has no vectorial significance but merely shows the passage of time. It is required, however, that the tail of the arrow indicates the start, and the head the finish of the activity.

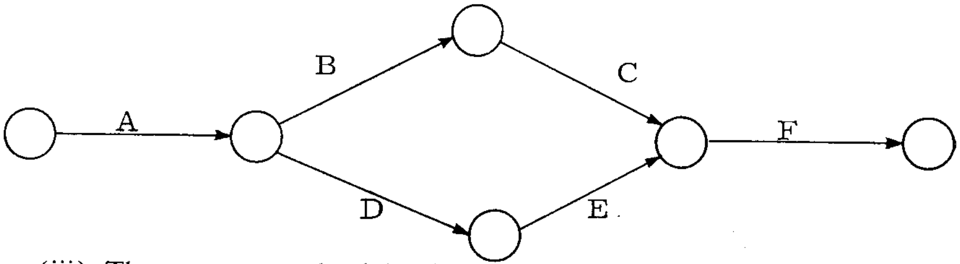
We now proceed to establish some sequential ordering of the activities and construct appropriate arrow network diagrams.

(i) Activities A through E follow in sequence but can only start as the preceding one finishes. The arrow diagram appears as:

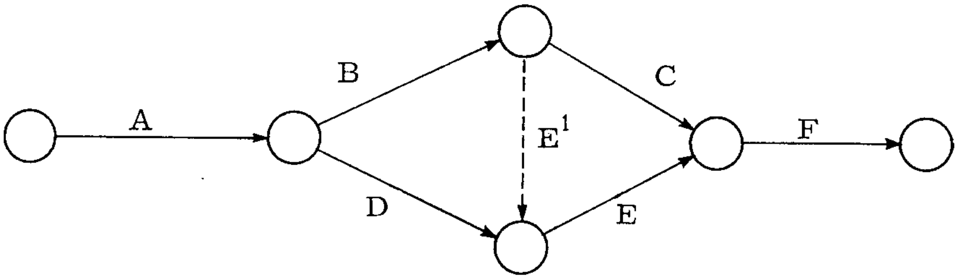


The activity junctions, shown by circles, are termed "events" or "nodes".

(ii) Activity A precedes activities B and D; B precedes C; D precedes E; both C and E precede F. The diagram appears as:



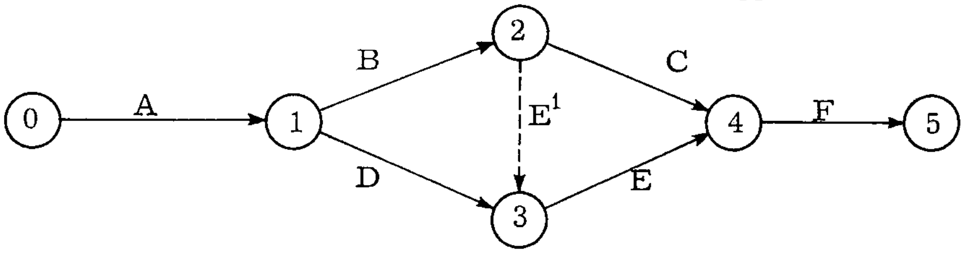
(iii) The same precedential relationships as (ii) are present but, also, the start of activity E is dependent on the finish of activity B. There is no activity relating them so we must insert a dummy one, giving it the labelling E¹. This dummy activity is shown as a dotted arrow and the diagram now appears as:



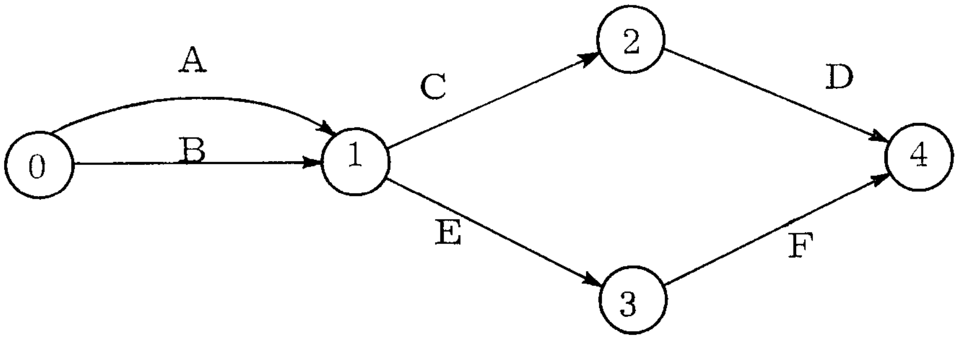
Node numbering

To allow the development of a further refinement, we introduce the concept of node numbering. In general, after an arrow diagram has been constructed the events or nodes that show activity junctions are numbered consecutively—starting at the first node and working through to the last node—in such a way that each activity has assigned to it an unique set of numbers (i,j); and if i designates the tail of the arrow and j the head then $i < j$. Note that by doing this, each activity is uniquely labelled.

Numbering the nodes of the above diagram,[13] it now appears as:

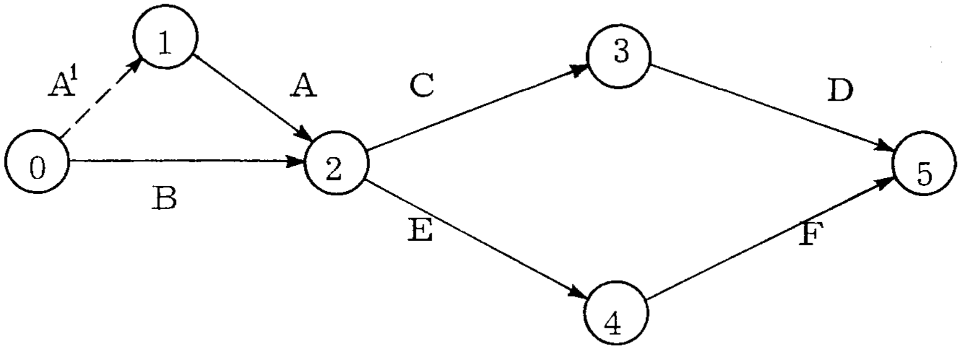


(iv) Activities A and B can take place concurrently and both precede C and E; C precedes D; E precedes F. To diagram it as:



suggests two activities identically labelled (O,1), whereas we have just noted that each activity is to be *uniquely* identified in labelling. To overcome this, a dummy activity must be incorporated. The dummy arrow may either precede or succeed the activity arrow but the preference is for it to precede.

Introducing a dummy activity A^1 to the job allows the correct presentation of the activity ordering:



and though some of the nodes now have different numbers all the activities are uniquely labelled.

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[13] It is customary to begin the numbering from zero, assigning zero to the beginning node.

Arrow Diagramming Rules

It will have become obvious during the above discussion that certain rules must be observed when constructing arrow diagrams. These can be formally stated:

(1) Each activity may have only one arrow. Conversely, one arrow may represent only one activity.

(2) In the diagram all arrows must lead towards the job completion: cycling is not permitted. Further, all activities describing the job must originate from, and terminate at a single node.

(3) The nodes must be numbered consecutively from zero at the beginning node. The manner of numbering must be such that each arrow has an unique (i,j) set of numbers with $i < j$.

(4) A node must describe the complete relationship between all entering and exiting arrows. Where this cannot be immediately satisfied, appropriate dummy arrows must be inserted.

TIME SCHEDULING

When the arrow diagram has been completed the scheduling of the project begins. Scheduling is characterized by the addition of activity duration times to the network, either actually or on an accompanying table. This data is then processed for certain statistics defining the relative importance of an activity to prompt completion of the job.

Consider the activity sequence and second arrow diagram from (iv) above and assume that each of the activities A through F take, respectively, 10; 7; 3; 6; 13; and 4 days for completion. The dummy activity A^1 has zero time duration (and zero cost): it does not form part of the job but is merely added to allow sensible graphic presentation of the precedential relationships. These times are shown under the activity identification letters in Fig. I and in column 2 of Table I.

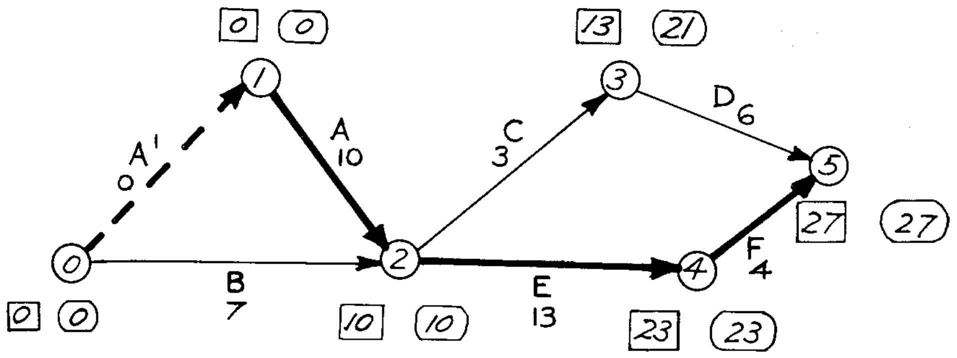

Fig. 1. Arrow Diagram of Project Containing Six Actual and One Dummy Activities, with Activity Duration Times and Earliest (▭) and Latest (⬭) Node Times Inserted.

Earliest, Latest Starting and Finishing Times

The first step in the procedural analysis is the compilation of earliest starting and finishing times. To do this, begin at node zero and sum duration times over all paths until the *highest* last node elapsed time is obtained. This, against the data available,[14] is the *least* time required to complete the project. During the exercise, time sets will have been generated at each node, the number of sets depending on the number of arrows converging to that node. The highest value in any set is the earliest starting time of the succeeding activity, and each value is the earliest finishing time of preceding activities. The earliest starting times at each node are shown in the rectangles on Figure I, and the earliest starting and finishing times for all activities are shown in column 3 of Table I.

TABLE 1

Schedule of Activities and Related Data for Project Containing six Actual and one Dummy Activities

(1) Activity		(2)	(3) Earliest		(4) Latest		(5)	(6)	(7)	(8)
Code	Node Nos.	Duration Times	Start	Finish	Start	Finish	Total Float	Free Float	Independent Float	Critical Path
A^1	0—1	0	0	0	0	0	0	0	0	*
B	0—2	7	0	7	3	10	3	3	0	
A	1—2	10	0	10	0	10	0	0	0	*
C	2—3	3	10	13	18	21	8	0	0†	
E	2—4	13	10	23	10	23	0	0	0	*
D	3—5	6	13	19	21	27	8	8	0	
F	4—5	4	23	27	23	27	0	0	0	*

† The actual value for this activity is negative but is read as zero.

Latest starting and finishing times are obtained by beginning at the last node and successively subtracting activity duration times through all paths. (A partial check to the accuracy of this and the preceding calculation is that times through at least one path must reduce to zero.) Again, time sets are generated at each node. The circles on Figure I contain the latest starting times, and Table I, column 4 shows the latest starting and finishing times for all activities.

FLOAT TIMES

Examining the earliest and latest times associated with each activity (Table I), it appears that some have more time than is absolutely necessary. Activity B (0,2), for instance, only takes seven days but its latest finishing time is given as relative time ten: activity B has three spare days attached to it.

This concept of spare time has been formalised and termed "Float". For each activity there are three identifiable floats.

TOTAL FLOAT. The spare time available when preceding activities finish at the earliest time and succeeding activities do not start until the latest time. It measures the delay that can occur without affecting the project completion time; and inspection will show it to be the *maximum* spare time available.

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[14] We do not here discuss the possibility of variable time durations but the procedures are identical.

FREE FLOAT. The spare time available when preceding activities finish at the earliest time and succeeding ones start at the earliest time. It measures the time an activity can be delayed without upsetting future early start times.

INDEPENDENT FLOAT. The spare time available when preceding activities finish at the latest time but succeeding ones start at the earliest time. Inspection will show this float to be the *minimum* spare time available.

The three floats for our hypothetical project are shown in columns 5, 6. and 7 of Table I. Activity B (0,2), for instance, has a total float of three and a free float of three. In other words, the start of this activity can be delayed for three days, or the duration of it can extend three days beyond that expected without upsetting the early start of succeeding activities or the whole project timing. However, should the activity duration extend beyond relative time ten then the whole project will be delayed. Activity C (2,3) has a total float of eight but no free float. This means that delays up to eight days do not upset the whole project but should any delay occur the succeeding events cannot start at their earliest times.

CRITICAL PATH

Certain activities have no total float. There is no spare time available between the starting and finishing times and if delays occur then succeeding activities must fall behind scheduled timing.

It will be found on examining any arrow diagram and, or, accompanying schedule that there is at least one path through the project made up only of activities with no total float (and also no free or independent float). And, further, that the summed activity duration times through the path(s) point up the least normal time in which the project can be completed.

Such paths are termed the "critical paths". Activities A^1, A, E, and F are shown in Table I to have no total float, and are marked out by asterisks in column 8: the path that these activities trace through the arrow diagram is the thick line in Figure I. This is the critical path for our project.

The whole exercise of CPPS focuses on the determination of the critical path. The activities on this path are critical against the prompt completion time of the whole project. If activity A of our example, for instance, took 11 instead of 10 days then the whole job would last 28 instead of 27 days. On the other hand, activity B could take eight or even up to ten days without affecting the finishing time of the job.

It will be noted that our diagramming, and float computation, have reduced from six to three the number .of activities that determine the project duration time, This is the great value of critical path determination, for it allows management to designate and carefully watch only a few of the many activities that in practice make up a project.[15] Suffice it to say that traditional techniques cannot do the same.

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[15] Sayer *et al, op. cit.* quote the case of a plant team that thought they had 156 critical activities in a job. CPPS reduced the list to seven, of which three were previously overlooked.

4. A PRACTICAL EXAMPLE

To demonstrate the application of CPPS in actual project programming, it is proposed in this section to examine the planting (land preparation and sowing) of 250 acres of wheat. The analysis follows along the same lines as previously: an arrow network diagram depicting the precedential order relations of the various activities is drawn, and then the project is scheduled to determine float times.

Assumptions and Data

Following Tyler,[16] it is assumed that the cultivations required for a wheat crop in the North-west Slope of New South Wales are two ploughings, two scarifyings, two harrowings, and a combine drilling. These activities are partially ordered, to the extent that the second ploughing cannot start until the first is completed and the second scarifying and harrowing cannot start until the first operations are respectively one-quarter and one-third completed. Drilling is dependent on all these previous operations being completed; and also on the procurement of seed.

The eleven activities necessary to adequately reflect all precedential relations are described in column 1, and coded in column 2 of Table 2.

TABLE 2

Description, Coding, and Duration Times of Activities in Planting 250 *Acres of Wheat*

Activity Description	Coding	Activity Duration Times
		Man-days
First ploughing with 7 ft. Disc Plough	A	10·50
Second Ploughing	C	10·50
Scarify one-quarter of area for first time with 8 ft. Scarifier.	D	2·44
Scarify remainder of area for first time	E	7·31
Second scarifying	F	8·75
Harrow one third of area for first time with 24 ft. Diamond Harrows.	G	1·00
Harrow remainder of area for first time	I	2·00
Dummy to ensure second Harrowing does not start until one third harrowed once.	H	0
Second Harrowing	J	2·50
Delivery time for seed wheat	B	12·00
Combine Seed using 10 ft. drill	K	7·75

The "rule of thumb" rates of work[17] determined by Tyler have been adopted without modification as the activity duration times for the two ploughings, the first scarifying and harrowing, and the combining. The activity duration times for the second scarifying and harrowing have been adjusted slightly downward (from 0.039 to 0.035 man-days per acre for the scarifying and from 0.012 to 0.010 for the harrowing) because it was felt that these operations could perhaps be performed slightly quicker. The time required to obtain seed wheat is a reasonable estimate.

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[16] G. J. Tyler, "Labour Requirements on Wheat Farms in the North-western Slope of New South Wales", this *Review,* Vol. 31, No. 2. (June, 1963), pp. 73-100.

[17] *Loc. cit.,* p. 83.

The actual times, in man-days, required for the various activities of planting 250 acres of wheat are shown in column 3 of Table 2.

Arrow Network Diagram

The graphical representation of the activity ordering coded in Table 2 is given in Figure 2. The diagram is straightforward, and the only point to especially note is the dummy activity H (5,6) which restrains the beginning of the second harrowing until one-third of the first harrowing is complete.

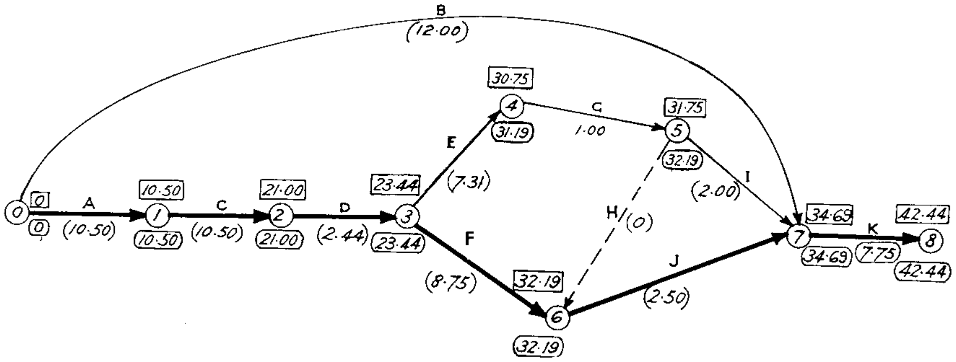

Fig. 2. **Arrow Network Diagram of Activities in Planting 250 Acres of Wheat.**

Activity duration times have been incorporated into the diagram so that, anticipating the schedule, the earliest and latest starting times can be determined and the critical path marked out. Since the critical path is traced by activities which have identical earliest and latest starting times, in this instance it is formed by activities A, C, D, F, J, and K.[18]

Timing Schedule

Even though much of the information has been put on to the arrow network diagram it is still worthwhile to carry out the scheduling phase. For this provides a complete listing of all starting and finishing times and details of float in non-critical activities.

The schedule for planting 250 acres of wheat is given in Table 3. Particular interest centres on the spare time available to activities B, E, G and I. In the case of B, delays of up to 23 man-days can occur in the delivery of seed. Procurement can take 34 instead of 12 man-days without delaying the start of combining. Alternatively, provided that delivery can be made within 12 man-days there are 22 man-days spare from deciding to plant wheat before an order need be placed. This surely gives ample leeway to check on varieties and select the one most suitable. The float of 0.88 in E means that after a quarter of the area has been scarified once nearly a whole day's delay (say 20 acres behind schedule) can occur in getting

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[18] Even if the analysis was to proceed no further than this some vital pointers have been obtained; particularly that the "normal" time (see appendix) required for planting wheat is just over 42 man-days and that some delay is allowed in completing the first scarifying and harrowing.

the remainder done. However, such a delay upsets the early start of G and no further delays must occur or the whole project takes longer because of the restraint imposed by H. On the other hand, if there are no delays then the first harrowing can be a more leisurely affair.

TABLE 3

Schedule of Activities and Related Data for Planning 250 Acres of Wheat

Activity		Duration Times	Earliest		Latest		Total Float	Free Float	Independent Float	Critical Path
Code	Node Nos.		Start	Finish	Start	Finish				
A	0—1	10·50	0	10·50	0	10·50	0	0	0	*
B	0—7	12·00	0	12·00	22·69	34·69	22·69	22·69	0	..
C	1—2	10·50	10·50	21·00	10·50	21·00	0	0	0	*
D	2—3	2·44	21·00	23·44	21·00	23·44	0	0	0	*
E	3—4	7·31	23·44	30·75	23·88	31·19	0·88	0	0†	..
F	3—6	8·75	23·44	32·19	23·44	32·19	0	0	0	*
G	4—5	1·00	30·75	31·75	31·19	32·19	0·44	0	0†	..
H	5—6	0	31·75	31·75	31·19	32·19	0·44	0·44	0	..
I	5—7	2·00	31·75	33·75	32·19	34·19	0·44	0·44	0	..
J	6—7	2·50	32·19	34·69	32·19	34·69	0	0	0	*
K	7—8	7·75	34·69	42·44	34·69	42·44	0	0	0	*

† Actual values negative.

It is interesting to convert the schedule to a calendar day basis. Taking the average working week as five and half man-days, the job of planting 250 acres of wheat takes between seven and eight weeks. Out of this, the two ploughings require approximately half the time, along the critical path the scarifying and harrowing require about three weeks, and sowing just over a week. Should it not be possible to simultaneously carry out the first and second scarifying and harrowing then the whole job would take nearly two weeks longer.

An approximate dead line to ensure a satisfactory crop of wheat in the North-western Slope is the first week in July. Immediately, then, we can see that to meet this deadline at "normal" activity duration times the first ploughing must start not later than the second week in May. It might, of course, be highly desirable for the job to start earlier—for climatic or agronomic reasons that influence activity duration times. These additional considerations can be easily handled in the scheduling phase.[19]

5. CONCLUSIONS

The remarks in the introduction and the principles enunciated in section 3 will serve to show that CPPS has a considerable potential use in many fields. Arrow network diagrams can be constructed with little effort. Manual solutions to time schedules are feasible for all but the largest projects, and computer programmes are becoming available for solutions in the larger projects, as well as for variable time scheduling and cost scheduling in all projects.

The question therefore arises can CPPS be used in farm management? The example used in this article demonstrates that such is the case, though further applications may not be immediately possible for a number of reasons. Chief of these is the lack of data. Few of the labour study reports presently available in Australia can supply the activity duration times necessary for time scheduling and none can supply cost schedule

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[19] See, for example, D. G. Malcolm *et al.*, *op. cit.*

information. A second reason is that, to date, the applications of CPPS have been on projects involving a large number of workers where integration of trades rather than individuals has been a major consideration. It remains to be shown that successful and worthwhile applications can be made to a wide range of small uncomplicated projects.

The first of these problems will, of course, be overcome in the future as more and better organized labour studies are undertaken. The second may equally be overcome, either by test or by further extensions to the techniques.

One thing seems clear; that the criticisms of operations research techniques (that they cannot readily handle the technical restrictions and climatic variability inherent in agriculture) do not apply in the case of CPPS. It has not been demonstrated but it will be obvious on reflection that additional activities in the arrow network diagram can explain technical restrictions, and that variable activity duration times for the schedule will allow for some anticipated delays due to inclement weather. These points do not of themselves indicate an assured future for CPPS as a tool in Farm Management. On the other hand, they suggest that further investigation is warranted.

APPENDIX

The Cost Schedule

In many projects, two discrete sets of activity duration times can generally be established. These are (i) the "normal" or most usual times, and (ii) the "crash" or least possible times. Similarly, two discrete sets of costs can be established—those associated with the normal times and those associated with the crash times.

Given this information, and assuming that costs vary linearly between normal and crash times, it is possible to draw up schedules showing the cost of a project as completion times are progressively compressed.

Normal and crash times and costs, and also the activity cost slopes, for our hypothetical project of Section 3 are contained in Table A1. The activity cost slope shows the extra costs that are incurred as each activity is reduced in length by one day. Activity A, for instance, has a normal time of 10 days with costs of £60, and a crash time of six days with costs of £96. Thus, four days can be saved at an extra cost of £36—a cost slope of £9 per day.

TABLE A1

Normal and Crash Activity Duration Times and Associated Costs

Activity	Normal		Crash		Cost Slope
	Time	Cost	Time	Cost	
	Days	£	Days	£	£/Day
A[1]	0	0	0	0	. .
A	10	60	6	96	9
B	7	85	4	115	10
C	3	20	3	20	. .
D	6	48	3	96	16
E	13	200	5	440	30
F	4	15	2	65	25

The normal time of the project has been established earlier as 27 days, and (from Table A1) the normal costs are £428. This is Schedule I of Table A2.

The cheapest way to shorten the project length is to reduce A. Up to three days can be immediately saved by reducing A by three days, for an extra cost of £27 (Schedule II, Table A2). A could be further reduced (from seven to six days) but this would not cut down the project length since B also takes seven days normally and would under these conditions become the critical activity with A obtaining some float. A further day can, however, be saved by reducing *both* A and B (Schedule III) with additonal costs of £19. This is higher than the cost incurred in reducing D by one day but again it can be noted that there is no point in reducing D at this stage because it is not a critical path activity. The successive steps of arriving at the shortest project length are outlined in Schedules IV through VI.

TABLE A2

Schedules of Some Possible Project Times and Costs

Schedule Number	Number of days Saved on Specified Activities	Extra Cost	Total Cost	Project Length
	Days	£	£	Days
I	......	..	428	27
II	A;3	27	455	24
III	A;1 B;1	19	474	23
IV	F;2	50	524	21
V	E;6	180	704	15
VI	E;2 D;2	92	796	13
VII	D;1	16	812	13

The crash time of the whole project is 13 days, and the crash costs are £796. This is £38 less than the total crash costs in Table A1, which are summations of crash costs over all activities. The difference is explained by the fact that some activities (B and D to be precise), can still take longer than their crash times. Indeed, there is no point in speeding up these activities. Schedule VII bears out this by showing that saving a day on D does not shorten the project time but only adds an unnecessary extra £16 to costs.

The value of cost scheduling can be clearly seen. In this simple example it has been shown that not all activities need be compressed to their crash times, thereby giving a saving of £38. As well, several points on the project cost curve have been obtained. This information could be used with estimates of any additional returns to come from early completion to find the optimum project timing.