PROBLEMS IN AGGREGATE AGRICULTURAL SUPPLY ANALYSIS: II—PRELIMINARY RESULTS FOR CEREALS AND WOOL

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The Australian wool/cereals complex provides an almost larger-than-life example of the textbook situation of multi-product enterprise. In the Agricultural and Pastoral Census of 1959-60, 85 per cent of all wheat farms were also engaged in sheep enterprises.[26] A traditional tool for the analysis of product-mix decisions on the multiple enterprise farm is the iso-resource, or production possibility, curve. Whilst the mechanism of the shift around such a frontier in response to changing relative prices has received excellent textbook treatment[27], an empirically very useful implication of this analysis seems to have been neglected. We refer to the symmetry of the derivatives which measure price responsiveness around a production frontier.

The use of symmetry in *demand* analysis has become classical since the publication in 1939 of Hicks' *Value and Capital.* Indeed, the fundamental proposition in this area was published originally in 1915 by Eugen Slutsky in a previously little known article in an Italian journal. Since its rediscovery by Hicks and Allen in the 1930's, the symmetry property has gradually been incorporated into textbook treatises on demand, and even more recently has been used with considerable success in empirical demand analysis.

The emergence of the symmetry postulate in demand analysis was inevitable once indifference curves had been discovered. Equally, and for mathematically the same reasons, symmetry of supply responses follows inexorably once the production possibility frontier has been accepted as a legitimate tool of analysis. For suppose we measure the responsiveness (around a production frontier) of the product-mix ratio, to changes in the marginal rate of transformation, by

$$\tau_{12} = -\frac{d\left[\frac{y_1}{y_2}\right]}{d\left[\frac{\partial y_1}{\partial y_2}\right]} \cdot \frac{\frac{\partial y_1}{\partial y_2}}{\frac{y_1}{y_2}} \qquad (8)$$

where the y's are outputs, and the subscripts are used to distinguish product 1 from product 2. Formula (8) defines the elasticity of transformation between products 1 and 2. Then, as Allen has shown, $\tau_{12} \equiv \tau_{21}$.[28] That is to say, the transformation elasticity between product 1 and product 2 is identically the transformation elasticity between product 2 and product 1 (as indeed common sense would suggest).

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[26] Commonwealth Bureau of Census and Statistics, *Classification of Rural Holdings by Size and Type of Activity* 1959-60 (Bulletin No. 7, Canberra), Table 3, p. 5.

[27] See, *e.g.*, Earl O. Heady, *Economics of Agricultural Production and Resource Use* (New York: Prentice-Hall, 1952), Ch. 8; C. E. Bishop and W. D. Toussaint, *Introduction to Agricultural Economic Analysis* (New York: John Wiley and Sons, 1958), Ch. 11.

[28] R. G. D. Allen, *Mathematical Analysis for Economists* (London: Macmillan, 1938), pp. 341-342.

Whilst the situation requires more careful interpretation in the case of a firm producing more than two products, it seems plausible to postulate that pair-wise symmetry between partial transformation elasticities might be preserved. This we have done, exploiting the consequent reduction in the number of parameters to be estimated within the context of the conventional linear supply systems. A detailed account of what this involves may be found elsewhere:[29] however, below we give a heuristic interpretation of the *CET* (constant elasticity of transformation) transform of a linear supply system.

Linear Supply System and Its *CET* Transform

Suppose we are concerned to fit a system of N linear supply equations,

$$(9)\quad y_{it} = \Gamma_{it} + \sum_{j=1}^{N} a_{ij}\,\pi_{jt} + \varepsilon_{it} \qquad (i = 1, \ldots, N)$$

in which planned output (y_{it}) of the i^{th} product during t is composed of a price-inelastic part (Γ_{it}) plus a linear combination of the prices expected to prevail during t, $\{\pi_{jt}\}$; plus a random shock (ε_{it}). The price-unresponsive part of supply is free to move through time as investment and technology push the supply curves rightward;[30] however, the slopes $\{a_{ii}^{-1}\}$ of the supply curves, as well the parameters $\{a_{ij}\}$ $(i \neq j)$ determining the movement of these curves in response to changes in the prices of other products, are fixed. This constitutes the simplest plausible model.

We have shown that a *CET* transform of this model is [31]

$$(10)\qquad y_{it} = \Gamma_{it} + \sum_{j \neq i} \tau_{ij}\, x_{ijt} + \varepsilon_{it}$$

where

$$(11)\qquad x_{ijt} = \bar{y}_i \left[\frac{\bar{\pi}_j \bar{y}_j}{\bar{\pi}_j \bar{y}_j + \bar{\pi}_i \bar{y}_i}\right] \left\{\frac{\pi_{jt}}{\bar{\pi}_j} - \frac{\pi_{it}}{\bar{\pi}_i}\right\}$$

In equation (11), superscript bars indicate sample means over the observed period. Equation (9) had N^2 price parameters $\{a_{ij}\}$; the exploitation of symmetry has reduced the number of price parameters to $\frac{1}{2}N(N-1)$ partial transformation elasticities $\{\tau_{ij}\}$.[32]

How does one interpret the variables x_{ijt} in equation (11)? Clearly these have replaced raw price expectations $\{\pi_{it}\}$ with entities which reflect pair-specific relative prices. In equation (11), the expression in "curly" brackets measures the price of product j relative to product i by taking the difference of *price relatives* for these products.[33]

The term within square brackets on the right-hand side of (11) gives an average measure of the share of product j in the total value of output of

[29] Powell and Gruen, "The Constant Elasticity of Transformation Production Frontier and Linear Supply Systems", *International Economic Review* (in press).

[30] Thus the Γ_{it} themselves are envisaged as functions of further variables, the identities of which are made explicit below.

[31] Powell and Gruen, *op. cit.*

[32] Ancillary relationships not here reported enable one to transform a set of τ_{ij}'s into the equivalent a_{ij}'s.

[33] The price relatives each have the mean over the sample period of expected prices as base.

products i and j. Such a correction for value share results in this model from the fact that transformation elasticities are, in themselves, *scale free*. By this we mean that the relative scales on which two enterprises are conducted does not, in itself, affect their basic degree of substitutability as measured by a partial transformation elasticity. But of course, the relative scales on which the products are produced is relevant for assessing the impact of a change in the price of one product upon the output of another. Our formulation ensures that if the price of a product whose relative share in income is large should rise by a given amount, this will not require an unduly high τ-value in order to "explain" the response in the output of another product; for example, whilst we would expect a change in the price of wheat to have quite an impact on the output of coarse grains, part of this expected response must be attributed to the sheer size itself of the wheat industry (relative to course grains), rather than to the basic technological possibilities for transformation of potential wheat output into production of coarse grains.[34] The final element of the right-hand side of (11), —namely, $\bar{y}_i$ —merely ensures that what would otherwise have been a pure index number is converted back into the units in which the output of product i is measured, thus preserving dimensionality. It is clear that the expected signs of all τ_{ij}'s are negative. From equations (10) and (11) we see that negative transformation elasticities are necessary if a rise in the price of a competing product j is to reduce output of product i.

We cannot here go into details of the statistical methodology by which we have fitted the model whose economic properties are described above. Suffice it to remark that a fairly minor adjustment to the least squares technique was necessary[35].

Distributing Lags and the Shifting Frontier[36]

We have deferred until now the problem of identifying the variables which shift the production frontier over time.[37] For an analysis of the long run it is clear that we would need indexes of the flow of capital services, for these essentially will determine the schedule of production possibilities. (Whether it is possible or not to construct such indicators independently of output measurements in a moot point. In any event, their construction could not be attempted within this study: Australian data on the amount of agricultural capital available—let alone the flow of services from such capital—is very inadequate.) For the analysis of annual data, however, we need to pin-point, with a reasonable degree of accuracy, the capacity of the system to expand various lines of production over a very short period. We are thus concerned with capacity in the very short run. An appropriate indicator of this short run capacity may be lagged production. On this view our linear supply system would

[34]The 'scale' and 'pure transformation' effects we have distinguished here are both components of a substitution effect; *i.e.*, of a movement *around* a production frontier—they should not be confused with substitution and expansion effects of textbook fame.

[35]For further details see Powell and Gruen, *op.cit.*

[36]In this and in the following section, we draw freely upon material contained in Powell and Gruen, *ibid.*

[37]*I.e.*, the arguments of the functions Γ_{it} occurring in equation (9).

be written

(12) $y_{it} = b_i + c_i y_{it-1} + \sum_j a_{ij} \pi_{jt} + \varepsilon_{it}$

where y_{it} is output during t; π_{jt} is price expected to prevail for product j during t from the viewpoint of $(t - 1)$; ε_{it} is a random term; and the a_{ij}, b_i and c_i are parameters. However we wish to mention explicitly the possibility that lagged output y_{it-1} may be in some cases a less than adequate indicator of the position of the frontier. In such cases we propose to introduce additional shift variables into the system.

The format (12) of the linear supply system has a number of advantages. First, it only attempts to explain the current level of output conditional upon information regarding last year's output. This is admittedly less ambitious than attempting to explain the level of output as a function of prices only; but, in view of the cumulative impact of droughts and other extraneous influences upon the level of output, this less ambitious approach at least has some prospect of success. Second, inclusion of last year's output establishes a formal equivalence between the linear supply system used here and the work of Nerlove.[38]

For if

(13) $y_{it} - y_{it-1} = \gamma_i (y^*_{it} - y_{it-1})$,

where y^*_{it} is the desired long-run equilibrium output from the viewpoint of time t, and γ_i is the "coefficient of adjustment" of product i (reflecting technological 'stickiness' in the adjustment of output towards its desired long-run level), then (13) is equivalent to

(14) $\gamma_i y^*_{it} = b_i + \sum_j a_{ij} \pi_{jt} + \varepsilon_{it}$,

in which $\gamma_i \equiv (1 - c_i)$. The long-run supply equation is hence determined, γ_i giving the proportion of the eventual total adjustment occurring in the first year. Long run elasticities hence may be estimated by dividing short-run estimates by an estimate of the appropriate γ_i.[39]

Distributed Lags and Price Expectations

For two of the three products considered here (wheat, coarse grains, wool), we have adhered to an orthodox treatment of price expectations. (The exception: wheat.) Using the Koyck/Nerlove model of adaptive expectations we have written

(15) $$\pi_{it} = \sum_{l=1}^{\infty} \beta_i (1-\beta_i)^{l-1} p_{i,t-l}$$

in which π_{it} is the price expected to prevail for product i in time t from the viewpoint of time $(t - 1)$; β_i is the coefficient of expectations for product i, and $p_{i,t-l}$ is the actual price pertaining l years ago from the viewpoint of year t. Thus price expectations are assumed to be a weighted average of prices prevailing in the past.[40] However, on pragmatic grounds we have truncated the distributed lag series after seven years, so that prices occurring eight or more years previously have no influence on expected prices in this model.[41]

[38]Marc Nerlove, *The Dynamics of Supply: Estimation of Farmers' Response to Price* (Baltimore: The John Hopkins Press, 1958).

[39]Implicit in the above treatment is the postulate that the only prices to which suppliers are responsive are their estimates of long-run or "normal" price. (On this point see Nerlove, *op.cit.*, pp. 45-59.)

[40]Nerlove, *ibid.*, p. 55.

[41]The weights of lagged prices have been adjusted upwards to sum to unity.

All results in this study have been obtained as conditional estimates for given, arbitrary, sets of values on the coefficients of expectations β_i, the latter being varied parametrically. In order to limit the range of possibilities, we have assumed that the coefficient of expectations for any given product is inversely proportional to the coefficient of variation of its actual price series. The rationale here is that the reliability of last year's price as an estimate of the longer run price level will be high for stable series, but less so for erratic ones. The coefficients of variation for the two products concerned are shown below in Table 16.

TABLE 16

Coefficients of Variation of Price Series for Wool and Coarse Grains, 1940-41 to 1961-62

	Wool	Coarse Grains
Coefficient of Variation	0.63	0.44
Ratio (*relative to wool* = 1)	1	0.70
Inverse Ratio	1	1.43

Sources: Based on Table 16, Part I of this paper, this *Review*, Vol. 34, No. 3 (September, 1966), pp. 112–135.

With few exceptions, for each set of assumptions about price expectations we have maintained among the assumed β-values the proportions given in line three of Table 16.

A Pilot Study of Two Key Sectors—Wool/Wheat

Before attempting a simultaneous analysis of the three products included in this study, we explored the partial response surface for the two products which enter most directly into competition for the resource base; namely, wool and wheat. The purpose of this pilot study was to delineate the most promising lines of attack to follow in our larger studies. In particular, we wished to gain some feeling for the sensitivity of results to variations in the structure of distributed lags and to clarify our ideas on the shift mechanism for the production frontier. Whilst the results of this pilot study are of considerable interest in their own right, our principal motive for recording them in some depth here is to document a variety of arbitrary decisions relating to our research methodology in the broader context.

Whilst rather full allowance for the impact of drought has been made in our adjusted acreage series for wheat, further corrections to our output indicator for wool (number of adult sheep shorn) seemed potentially fruitful. Thus we included an indicator of drought mortality among sheep as an additional shift variable in the supply equation for wool. This index is recorded with other material in Table 17. It is simply the deviation of the crude mortality rate about 6·3 per cent, the latter figure being the mean of such annual rates over the period 1947-48 to 1963-64.

Wheat is one of the two commodities where we deemed the distributed lag model of price expectations to be inappropriate.[42] Our justification is twofold: First, the government stabilization machinery ought to create an atmosphere of greater confidence about minimum prices for wheat than could be expected to prevail for other products. Moreover, on the basis of either Series I or Series II (Table 7), wheat prices possess much greater stability than prices of other products considered here. Thus the β-value for wheat should be in the neighbourhood of unity. Second, many lag effects have already been built into Series I, and the interpretation of further lags is obscure. As a final pragmatic justification, we add that extension of a wheat price series back to 1940-41 was not feasible, so that in order to adopt a model of distributed lags for wheat we would have been forced to shorten our already scant time series of 18 years by seven observations.[43]

TABLE 17

Additional Shift Variables for Supply Equations

Fiscal Year	Drought Index for Sheep 1	Current Input Series 2
		($m. in constant prices of 1953–54
1946–47	1·4	311·4*
1947–48	0·2	323·6*
1948–49	0·4	348·0
1949–50	−1·5	360·0
1950–51	3·3	364·0
1951–52	2·0	360·0
1952–53	−1·9	380·0
1953–54	−0·3	390·0
1954–55	−1·3	410·0
1955–56	−0·7	422·0
1956–57	0·6	426·0
1957–58	2·2	444·0
1958–59	−0·8	460·0
1959–60	−0·2	472·0
1960–61	−0·1	482·0
1961–62	−1·7	496·0†
1962–63	0·1	526·0†
1963–64	−1·2	532·0†
1964–65	−1·4	

* Data not available; figure shown is backward extrapolation from linear trend (R^2 for regression of current inputs on time : 98 per cent).
† Our estimates.

Sources and Notes:
1 Based on estimated numbers of sheep and lambs at beginning of year and on mortality estimates from Commonwealth Bureau of Census and Statistics, *Statistical Bulletin: Livestock Numbers Australia*, No. 23 (31 March 1965, Canberra); and earlier annual issues. Number tabulated is deviation of crude annual mortality rate about 6·3 per cent (see text).
2 Series compiled by F. Juhasz and B. Hillsdon, "Farm Income Elasticities in Relation to Economic Changes", paper presented to Section G, Australia and New Zealand Association for the Advancement of Science, 37th Congress, 1964.

[42]The other — beef — is not discussed in this paper.
[43]At the time of this pilot analysis only 16 years' data were available, covering the period 1947-48 to 1962-63. All 2-sector results are based on output data for this 16-year period.

The initial choice confronting us was between price Series I and II for wheat. We have resolved this issue on the basis of relative performance of the series in two-sector models. Using a model of the type (12), but in which we introduced a drought mortality index as an additional shifter of the wool equation, we fitted the two supply equations simultaneously for values of the coefficient of expectations for wool ranging from 0·4 to 1·0. This was done in duplicate, once using each series. Over the entire range of β-values (0·4 to 1·0) Series I (liquidity sensitive) consistently gave higher R^2s for both equations. Moreover, Series I gave uniformly higher Student's t values for the elasticity of transformation. On these grounds we have chosen to work with Series I throughout the remainder of this study. Series I also gave higher estimates of the elasticity of transformation, and thus of price responsiveness.

Both versions of the analysis gave low Durbin-Watson statistics for wheat, and the residuals from the wheat equation in both instances were suggestive of a neglected trend. The faulty specification was diagnosed to be due probably to our failure to take account of the importance of current inputs in wheat production, and the analysis was rerun using Series I but this time including a lagged current input series as an argument of the supply function for wheat.[44] This had the effect of raising the Durbin-Watson statistic for the wheat equation above critical levels for all of the β-values for wool considered. The cost was a slight lowering throughout the β-range for the Durbin-Watson statistic (d) for the wool equation, but these decreases were very slight, and in no case were resultant d-values brought below critical upper levels for positive autocorrelation. At the same time, however, the apparent significance[45] of the elasticity of transformation suffered a marginal decline throughout the β-range.[46]

Three-Sector Model: Enter Coarse Grains

Whilst coarse grains[47] account for only a small share in the value of output of the wool/cereals complex, the relative ease with which they can be substituted for wheat (and *vice-versa*) suggests that they may, nevertheless, contribute quite importantly to the flexibility of wheat supply. For this to hold, the partial transformation elasticity between wheat and coarse grains would need to be "high" in absolute value. As will be seen presently, this hypothesis is supported by our model's interpretation of the data.

[44]Some ambiguity existed as to whether a current or lagged series would be more appropriate. The high correlation of the current input series with time ($R^2 = \cdot 98$) robbed the issue of operational significance.

[45]It is not proper to speak of the statistical "significance" in this study. For this to be so would need an independent set of data for each hypothesis we test. In fact, we have attempted to test a multiplicity of hypotheses using the same data. Moreover, we have based our estimates of standard errors of regression coefficients on classical least-squares formulae, which are not strictly applicable for our iteratively fitted generalised least-squares method. However, we conform to the common usage by using the word "significance" synonymously with the ratio of parameter estimate to its apparent sampling standard deviation.

[46]Tabular results for the three versions of the analysis discussed in preceding paragraphs are given in Table 18.

[47]We are concerned here mainly with oats and barley. Our series (Tables 7 and 8) also contains a small component due to maize.

TABLE 18

*Summary of Principal Results for Two Key-Sector Model (Wool/Wheat)**

β-Value Assumed for Wool		Coefficient of Multiple Determination		Coefficient of Adjustment γ		Regression Constant		Regression Coefficient for Drought Variable §	Regression Coefficient of Lagged Current Inputs ‖	Elacticity of Trans-formation	|t| Value of Previous Column	Durbin-Watson Statistic for		Own Price Elacticity‡			
														Wool		Wheat	
		Wool	Wheat	Wool	Wheat	Wool	Wheat					Wool	Wheat	Short Run	Long Run	Short Run	Long Run
1·0	a	·971	·781	·0371	·0585	7·812	0·883	−0·8615		−0·1159	2·071	1·854	1·113	0·035	0·935	0·084	1·390
	b	·971	·817	·0243	·0752	6·350	1·001	−0·7282		−0·1510	2·555	2·069	1·106	0·042	1·728	0·109	1·449
	c	·973	·887	·0309	−·0053	7·102	−4·156	−0·6758	·01050	−0·1177	2·142	1·999	1·903	0·033	1·061	0·085	†
·8	a	·971	·783	·0383	·0842	7·943	1·177	−0·7609		−0·1341	2·179	1·818	1·094	0·040	1·055	0·094	1·113
	b	·972	·821	·0230	·1059	6·201	1·356	−0·5745		−0·1815	2·739	2·073	1·102	0·051	2·213	0·131	1·233
	c	·974	·887	·0298	·0205	6·970	−3·771	−0·5563	·01025	−0·1423	2·272	2·024	1·896	0·040	1·339	0·102	†
·6	a	·972	·775	·0452	·1123	8·720	1·499	−0·6397		−0·1462	2·142	1·817	1·050	0·045	0·985	0·102	0·832
	b	·974	·811	·0300	·1421	6·989	1·773	−0·3932		−0·2022	2·695	2·086	1·026	0·057	1·913	0·145	1·020
	c	·975	·885	·0345	·0503	7·50	−3·540	−0·4110	·01055	−0·1650	2·334	2·065	1·878	0·047	1·357	0·118	†
·4	a	·974	·757	·0600	·1411	10·407	1·829	−0·5321		−0·1541	2·028	1·882	0·974	0·048	0·796	0·106	0·753
	b	·976	·781	·0485	·1710	9·094	2·106	−0·2469		−0·2014	2·428	2·155	0·884	0·058	1·202	0·143	0·837
	c	·976	·881	·049	·080	9·150	−3·649	−0·2673	·01165	−0·1845	2·411	2·149	1·863	0·053	1·090	0·131	†

Notes:

a Wheat Series II, wool equation shifted by drought index.

b Wheat Series I, wool equation shifted by drought index.

c Wheat Series I, wool equation shifted by drought index; wheat shifted by lagged current input series.

* Based on 16 years' data 1947-48 through 1962-63. Because of minor inaccuracies in the early version of the time series used, results may not be 100 per cent repeatable.

† Indeterminate: long-run estimates cannot be computed with current inputs held constant.

‡ Evaluated at mean outputs and price expectations.

§ In no case differs significantly from zero under classical tests (5 per cent significance level).

‖ In all cases differs significantly from zero under classical tests (5 per cent significance level).

What of the partial production frontier between wool and coarse grains? Our prior hunches in this case favoured a low elasticity of transformation: in fact, the empirical value obtained in the earliest version of our 3-sector analysis[48] had a perverse sign, low absolute value, and high sampling standard deviation. This led us to conjecture that, since coarse grains can to some extent enter into wool production as an input—witness the winter grazing of oats which are subsequently harvested—the output response between wool and these grains cannot be formulated in terms of a simple shift around a production frontier. Hence it seemed better to exclude the τ-coefficient between wool and coarse grains altogether; *i.e.*, to constrain this value arbitrarily to zero.[49]

For the same reasons as prompted our inclusion of a series on lagged current inputs (Table 17) in the 2-sector model reported above, this variable was retained as a shifter of the wheat equation, and added in a similar role to the coarse-grains supply equation. The consequent 3-sector results given in Table 19 are not directly comparable with those of Table 18—in the first place, eighteen (rather than sixteen) years' data have been used for estimation; in the second, the assumed ranges of values for coefficients of expectations differ.[50] However, these differences are fairly minor, and some illuminating comparisons can be made.

In a model (such as ours) in which short-run flexibility of output is attributed largely to the existence of alternative uses of resources, the exclusion of any important alternative product may lead to a serious underestimation of the responsiveness of supply. The wheat supply elasticities reported in Table 18 assume that wool is the only alternative; those given in Table 19 allow for the possibility of switching into coarse grains. Since Table 19 is based on cereals equations which include a current input series, it must be compared with the results labelled "*c*" in Table 18; *i.e.*, with parameter-estimates based on equations also containing current inputs in the case of the supply equation for wheat. Using linear interpolation between wool β-values of 0·6 and 0·8 to obtain from Table 18 approximate values of the short-run wheat supply elasticity which would correspond to a wool β-value of 0·7, one finds this figure to be 0·11. That is to say, taking into account only the wool alternative, the estimated short-run supply elasticity would be about 0·1. The equivalent figure from Table 19 is 0·189: a rather dramatic increase in excess of 70 per cent.[51] This illustrates that the "importance" of an alternative in contributing to the responsiveness of the supply of another product bears no necessary relation to the relative scales on which the two

[48]See, *e.g.*, the brief report of 3-sector results in Powell and Gruen, *op.cit.*

[49]This constraint has two possible interpretations: (i) that the production frontier is right-angled; (ii) that the relative prices prevailing between the two products in question does not enter into the farmers' decision process. In-so-far as an interpretation is forced on us we favour the latter.

[50]Under our method of tieing together the β-values for wool and coarse grains, the maximal value of wool's β-value is 0·7 (since otherwise our assumed β-value for coarse grains would exceed unity). Results conditional on an assumed β-value of 0·4 for wool are common to both Tables 18 and 19; because of the (somewhat surprising) lack of sensitivity of results to these sorts of assumptions, however, results conditional on wool β-values of 0·6 or 0·8 in Table 18 may be compared, at least roughly, with results conditional on a wool β-value of 0·7 in Table 19. Alternatively, linear interpolation may be used.

[51]The increase is rather less dramatic in the case of an assumed β-value for wool of 0·4—44 per cent—but is still quite substantial.

are produced. If partial transformation elasticities are high enough—in Table 19 the estimated τ-values for wheat/coarse grains are uniformly about three times as great as the wheat/wool values—then even relatively "minor" products can be "important" in contributing to the flexibility of supply.

Judging the Most Likely Elasticity of Supply

Economists have long realised that there is no unique value for the elasticity of supply—*i.e.*, that the responsiveness of supply to price depends on the length of run envisaged. Nerlove and other writers have therefore distinguished between a short run (one season) elasticity and elasticities for different periods of adjustments.

A further problem arises when we are confronted with a multi-equation system. If the conventional "goodness of fit" criterion is applied to more than one equation, it may not be possible to arrive at a unique value for the elasticity of supply (for a given length of time). Thus we obtain somewhat different supply elasticities for different wool β-values, but there is no unique value of β which "best fits" the data. For instance in Table 18, the R^2 for the wool equation rises slightly as its β is reduced from 1·0 to 0·4, whilst the R^2 for the wheat equations tends to rise when wool's β-value is increased from 0·4 to 0·8. Fortunately, this problem has not occurred in an extreme form in either our 2-, or our 3-sector models.

The Short Run Elasticity of Supply

Table 20 provides a summary of own and cross short-run elasticities of supply. Additional flexibility would come from the responsiveness of the supply of cereals to (lagged) current inputs. Thus these estimates assume that there is no instantaneous feed-back from prices into investment, and are appropriate for measuring short-run responsiveness.

(a) *Wool.* The short run elasticity for wool ranges from 0·039 to 0·041 between our extreme assumptions about its β. This is in line with the *a priori* reasoning of other writers[52] and with the limited amount of other empirical work on this subject.[53] It is of interest however that our method is able to obtain statistically significant or near-significant results (when measured by conventional tests), of such low positive elasticities.

(b) *Wheat.* Changes in assumptions about price expectations leave our short run elasticity of wheat supply stable at 0·19. This, perhaps, is somewhat lower than some of us would have expected. Duloy and Watson, using single equation estimates, obtained short run elasticities ranging from 0·34 to 0·51 for four "old established wheat areas" though their short run elasticities for two "new" wheat growing areas in New

[52]K. O. Campbell, "Australian Wool Supply Prospects", *Wool in the Australian Economy*, Economic Papers No. 10, Economic Society of Australia and New Zealand (New South Wales Branch), 1955, p. 57; and (by the same author) "The Inelasticity of Supply of Wool", *Economic Record*, Vol. 31, No. 61 (November, 1955) pp. 311-317.

[53]D. L. Dahlberg, "Supply Responses for Wool in South Australia", *Australian Journal of Agricultural Economics*, Vol. 8, No. 4 (June, 1964) pp. 62-63.

TABLE 19

*Summary of Results for Three Sector Model (Wool—Wheat—Coarse Grains)**

Equation for	β-value for wool†	Regression Constant	Variables Shifting Supply Curve		Estimated Transformation Elasticities		Estimated Coefficient of Adjustment	Own Price Elasticity (Short Run)	R^2
			Name of Variable	Coefficient	Other Product	Elasticity			
Wool‡	·4	7·1512 [1·548]	Drought Mortality Index ‖ Lagged Output ‡	−0·4794 [0·843] 0·9701 [24·809]	Wheat	−0·1376 [1·948]	0·0299	0·0414	·979
	·7	5·7370 [1·199]	Drought Mortality Index ‖ Lagged Output ‡	−0·5675 [0·678] 0·9821 [24·240]	Wheat	−0·1319 [2·145]	0·0179	0·0386	·979
Wheat§	·4	−2·3214 [1·609]	Lagged Current Inputs ¶ Lagged Output §	0·01035 [3·581] 0·8517 [9·123]	Wool Coarse Grains	−0·1376 [1·948] −0·3716 [1·824]	0·1483	0·1890	·928
	·7	−2·0166 [1·420]	Lagged Current Inputs ¶ Lagged Output §	0·00850 [2·937] 0·8915 [10·471]	Wool Coarse Grains	−0·1319 [2·145] −0·3744 [2·154]	0·1085	0·1888	·932

Coarse Grains§	·4	−0·4508 [0·525]	Lagged Current Inputs ¶ Lagged Output §	0·00725 [1·814] 0·4740 [2·140]	Wheat	−0·3716 [1·824]	0·5260	0·2788	·867
	·7	−0·5675 [·678]	Lagged Current Inputs ¶ Lagged Output §	0·00670 [1·823] 0·5477 [2·766]	Wheat	−0·3744 [2·154]	0·4523	0·2789	·875

* All figures based on Price Series I for wheat. Student's | t | 1—values for estimated coefficients are shown in square brackets.

† Coefficient of expectations used for coarse grains is based on Table 16.

‡ Units are millions of adult sheep shorn.

§ Units are millions of acres.

|| Deviation of annual gross percentage mortality rate about 'normal' value of 6·3 per cent.

¶ Units are $m. in constant prices of 1953-54.

TABLE 20

*Estimated Short-Run Own and Cross Price Elasticities of Supply**

Product	β-value for Wool†	Elasticity‡ with Respect to the Expected Price of Wool	Wheat	Coarse Grains	Sum
Wool	0·4	+0·0414	−0·0414	0	0
	0·7	+0·0386	−0·0386	0	0
Wheat	0·4	−0·0962	+0·1890	−0·0928	0
	0·7	−0·0933	+0·1888	−0·0955	0
Coarse Grains	0·4	0	−0·2788	+0·2788	0
	0·7	0	−0·2790	+0·2790	0

* Elasticities evaluated at sample mean outputs and expected prices. Computed with (lagged) current inputs held constant in equations for wheat and coarse grains. The figure in the table gives the percentage increment in the output indicator for each product estimated to result one year after a one per cent rise in the *expected* price of the product named at the head of each column.

† Coefficient of expectations assumed in case of coarse grains is 1·41 times value shown for wool.

‡ Zero entries indicate elasticity constrained as such.

South Wales (0·13 and 0·16) are fairly close to those obtained by us.[54] We have also run one single equation estimate for wheat (with $\beta = 0{\cdot}8$) and obtained a short run elasticity 2·3 times higher than our short run estimate with the 2-sector model. This raises some interesting problems of interpretation which we have not as yet fully explored. However it seems to us likely that the two main reasons for this difference in the value of the short run elasticities obtained by the two techniques are:

(1) In the 2-sector model we are attempting to estimate movements *around* a production frontier, whereas in the single equation estimate, other supply responses (*e.g.* increased investment in the short run and even some technical change) may add to the supply response recorded.

(2) In our multi-sectoral analysis, the inherent statistical reliability of an equation (as measured by the inverse of its estimated residual variance), as well as the apparent price responsiveness of the product in question, will determine estimated τ-values and hence estimates of supply elasticities.

Thus the higher degree of success in "explaining" the wool supply ($R^2 = {\cdot}98$) relative to wheat ($R^2 = {\cdot}93$), coupled with a lower apparent price responsiveness in the wool supply, has been partly responsible for bringing our estimated wheat supply elasticities below single-equation values.

[54] J. H. Duloy and A. S. Watson, "Supply Relationships in the Australian Wheat Industry: New South Wales" *Australian Journal of Agricultural Economics*, Vol. 8, No. 4 (June, 1964) pp. 28-45.

(c) *Coarse Grains.* This somewhat heterogeneous conglomerate yielded the highest of the three short run own price elasticities estimated. The value in question, 0·28, was insensitive to changing assumptions about the structure of price expectations. One can speculate with reasonable safety that the individual aggregands—oats, barley, maize—would possess even higher own price elasticities in the short run.

Long Run Elasticity Estimates

Our approach to longer-run estimates has necessarily been somewhat different. Even though, in a formal sense, estimates of Nerlove's coefficients of adjustment [the γ_i of equations (13) and (14)] are automatically obtained, it would be inappropriate to use these coefficients to estimate long run elasticities from short run values, when the latter themselves were computed from versions of the analysis in which the level of (lagged) current inputs was included as an explanator. The reason, of course, is that the missing link—namely, the feed-back from prices into inputs—has proved intractable within the strictures imposed on this study by time and by its other objectives. This problem, basically one of determining the investment function for the agricultural sector, certainly deserves further attention, and accurate estimates of long-run responsiveness of agricultural supply will not be feasible until we have come to grips with it. In the meantime, however, we have abided by the established Nerlovian practice; *i.e.*, we have estimated long run elasticities simply by inflating our short run estimates by factors ($1/\gamma_i$), using for this purpose a system whose cereals' equations excluded current inputs. Whilst this involved ignoring some embarrassingly low Durbin-Watson statistics (1·15 for wheat with wool's β-value = 0·7),[55] the only statistically satisfactory way out involved using higher powered versions of generalised least-squares than were available with our existing computational resources.[56] Thus our estimates of structural parameters remain unbiased, though of less than optimal efficiency. In the interests of brevity, we have not reported results similar to those of Table 19 for an equivalent system excluding current inputs; instead we have restricted tabulations to estimated long-run own price elasticities based on the method outlined above. These are given in Table 21, where intermediate run (*i.e.*, 5-year) elasticities may also be found.

Our long run elasticity estimates range from 3·1 to 4·4 for wheat (for different values of β) and from 1·4 to 3·6 for wool. The γ-values found for wool were lower than those estimated for wheat (*i.e.*, the rate of adjustment of desired wool output to its equilibrium level was estimated to take longer than in the case of wheat). There is, of course, considerable plausibility, *a priori*, in expecting wool production to adjust more slowly. Since the "long run" may be very long indeed in the case of wool we

[55]This value lies in the range for which the test is inconclusive; *i.e.*, the evidence for the existence of positive autocorrelation in the residuals is not "conclusive". Things were much worse, however, when the β-value for wool was assumed to be 0·4. In this case the *d*-value for wheat was 0·9, indicating significant positive autocorrelation at the 5 per cent level.

[56]Our technique of fitting throughout has been a version of generalised least-squares which takes into account the equivalent of heteroskedasticity between equations, but ignores the additional possibility of autocorrelated disturbances within equations as well as the possibility of non-zero contemporaneous and lagged correlations of disturbances between equations.

have also tabulated an "intermediate run" elasticity in Table 21 which gives the supply response which may be expected over a period of five years. In terms of this five-year concept, wheat, surprisingly, shows greater flexibility when a β-value of 0·4 for wool is assumed, and fares scarcely worse than coarse grains (0·88 *vs.* 0·89) when wool's coefficient of expectations is taken to be 0·7. Whilst the (operationally obscure) infinite period—*i.e.*, long run—elasticities are very sensitive to the structure of price expectations, such sensitivity is mercifully lacking in the case of short- and intermediate run constructs. A possible exception to this happy dictum is the case of the 5-year coarse grains elasticity, which swings from 0·6 through 0·9 as the assumed β-value for wool ranges from 0·4 to 0·7.

As pointed out above, the long run elasticities are very sensitive to the values of γ obtained. The γ-values in turn depend mainly on the extent of the auto-correlation present in the original output data. Since this in only one (likely) interpretation of the possible cause of auto-correlation it seems desirable to record our reservations regarding the validity (and accuracy) of the long run estimates.

On the one hand, in spite of our suppression of current inputs as shifters in the underlying equations, these long run estimates may be too low, being only magnifications of responsiveness around a given production frontier, whereas long run response must depend to a considerable degree on outward shifts of the frontier itself. (We have already noted our [impotent] desire to quantify this outward shift by constructing an investment function which allowed for the feed-back of prices upon the

TABLE 21

Nerlovian Estimates of Long and Intermediate Run Own Supply Elasticities*

β-Values Assumed for Wool	Wool		Wheat		Coarse Grains	
	Intermediate	Long†	Intermediate	Long†	Intermediate	Long†
0·4	0·20	1·43 (·0296)	0·70	3·14 (·0490)	0·61	1·04 (·1612)
0·7	0·23	3·58 (·0131)	0·88	4·40 (·0437)	0·89	1·81 (·1269)

* 5-year response period.

† Estimated coefficient of adjustment (γ) shown in parentheses.

position of the frontier.) On the other hand, our γ-estimates may be biased downwards because there are other possible explanations of auto-correlation in the output data. In particular, secular technical change escaping our initial filters (Part I of this paper) may account for some of the autocorrelation, in which case a further problem of interpretation arises. Theoretically, it would be desirable to catch in our long run price elasticity estimates, that part of technical change which is price-induced.[57] However it seems likely that the observed autocorrelation is to some extent due to "autonomous" technological progress. This would have the effect of lowering our estimated γ's below their appropriate values, thus causing a rise in the scale factor $(1/\gamma)$ by which our short run estimates were inflated. We are unable to judge in which direction, on balance, our long run elasticity estimates may be biased.

Perspectives

In this article we have reported on a new technique of analysing supply responses which, it is hoped, will prove useful in quantitatively estimating that part of the supply response which comes from the movements around partial production frontiers in response to changes in expected price relationships. Much remains to be done. It is our hope that this study will encourage others to experiment with this technique, to estimate supply responses for less grossly aggregated commodities, to study more uniform regional districts in greater depth than we have been able to within the scope of our aggregate Australian study. In addition we have only briefly experimented with estimating supply responses between wool, lamb, mutton and beef—commodities where production lags are less definite than for annually planted crops and where very substantial problems of estimation and interpretation arise. Lastly, another problem which requires consideration consists of the relation between supply responses estimated by means of this technique and those obtained from other statistical methods. It may be that a combination of techniques will enable us to arrive at better and more explicit estimates of the outward shift of the production frontier resulting from increased investment and better technology—a process which has so far largely defied empirical analysis. However this is purely speculative at this stage. What is certain is that, with the growth in administrative pricing of agricultural products, there is an urgent need for more reliable techniques of estimating and analysing the supply relationships prevailing in Australian agriculture.

[57]If the pessimism of Dale W. Jorgensen is justified, this is something we may never be able to do (even in principle). See his "The Embodiment Hypothesis", *Journal of Political Economy*, Vol. LXXIV, No. 1 (February, 1966) pp. 1-17.