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by

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Producer behaviour under strict rationing and quasi-fixed factors

Abstract

The paper examines the effect of rationing outputs and inputs on producer behaviour. Full representation of the modified supply-demand system after rationing, including shadow prices for the constrained netputs, is derived from the unrestricted profit function before rationing, and vice-versa. Attention is focused on cross-price effects which have been less explored than own-price effects which obey to the Le Chatelier principle. The theoretical framework is applied to the EC agricultural sector in order to analyse the effects of output and input rationing on the production structure of the EC agricultural sector, with emphasis on the impact of the milk quota constraint on unrestricted output supplies and input demands.

JEL Classification Numbers : D21, Q11

1. Introduction

Household behaviour under rationing, using duality theory results and the notion of virtual or shadow prices (Rorbarth, 1941) for the constrained goods, has been extensively analysed, e.g., recently by Neary and Roberts (1980), and Deaton and Muellbauer (1980). Both studies have emphasised that "all the properties of the rationed demand and supply functions may be expressed in terms of the properties of the unrationed functions, provided the latter are evaluated at the virtual prices" (Neary and Roberts, p. 26). Several empirical analyses of markets in disequilibrium have made use of this approach (in terms of virtual prices) to specify and estimate demand systems in the case of binding non-negativity constraints (e.g., Lee and Pitt, 1986, 1987). In a recent paper, Madden (1991) analyses the properties of a family of net (i.e., utility constant) substitute-complement classifications and suggests that rationing favours substitution[1].

In producer theory, Sakai (1974) has given analytical relationships between short-run and long-run supply and demand responses. Lau (1976) has characterised the conditions under which supply and demand responses without quantity constraints on inputs or outputs can be derived from the responses estimated under some fixity, and vice-versa. Particularly, he has shown how the Le Chatelier's principle applies. These analytical developments have been the subject of numerous empirical studies applied to the problem of moving form short-run to long-run responses (see, e.g., Brown and Christensen, 1981 ; Kulatilaka, 1985). However, these papers do not use the concept of virtual prices but allow the long-run adjustment of

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[1] "..., as more goods become rationed, substitutes are "likely" ro remain substitutes but complements may "more easily"change and become substitutes" (Madden, p. 1502).

quasi-fixed inputs to their optimal levels.

In this paper, we apply the concept of virtual or shadow prices to the analysis of producer behaviour under output and/or input rationing. We show how the comparative statics of endogenous variables in a regime of effective rationing (i.e., variable netput quantities and shadow prices of fixed netputs) can be characterised from the knowledge of supply and demand responses before the implementation of the constraints. The use of shadow prices makes the analytics simple and allows us to derive the Hessian of the restricted profit function from the Hessian of the unrestricted for any level of the constraints, provided that the latter is calculated at the relevant shadow prices. It makes it possible to characterise the structure of the new supply-demand system and of the shadow prices associated with the rationed outputs or inputs. This approach to producer behaviour under rationing leads to correspondence formulae which are similar to Madden's.

In Section 2, shadow prices and disequilibrium gaps provide a simple manner to analyse various implications of quantity restrictions on supply and demand behaviour. In Section 3, we develop the essentials of the comparative statics of producer behaviour when some netputs are constrained. Emphasis is placed on the comparison of the response parameters of unconstrained variables after some inputs or outputs have been rationed to the parameters without rationing. Attention is focused on cross-price effects which are shown to move towards more substitution and more inferiority[2] when constraints are introduced, in the case of the "normal" technology of Sakai and even under weaker conditions. An empirical example, based on a translog profit function for the European Community (EC) agricultural sector, is presented in Section 4. Section 5 deals with a dynamic approach of the same problem in the case where the adjustment of quasi-fixed inputs is represented by a multivariate partial adjustment process.

2. A framework to derive producer behaviour under rationing from the unconstrained behaviour, and vice-versa

In order to characterise behaviour under constraints, we make use of the knowledge of supply and demand response without or before the implementation of the rationing. This is made easy under fairly general conditions as the comparative statics of unconstrained producer behaviour is better known (Sakai). As there is an evident symmetry between introducing constraints on the one hand and relaxing them on the other, the symmetrical expression

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[2] An input x_h will be said inferior with respect to an output y_i if the output price increase decreases the Marshallian or long-run demand of x_h. By Young's theorem, y_i will be also said inferior or regressive with respect to x_h since the input price increase increases the Marshallian or long-run supply of y_i (see, for example, Hughes, 1981). A netput which is not inferoir will be said superior.

relevant to the problem of moving from constrained to unconstrained response is also derived in our framework.

Notation and preliminary definitions

When all prices are given and all netputs free to adjust, the producer behaviour is described by the familiar problem :

$$\max_{q}[v'q; q \in T] = \pi^{U}(v) \tag{2.1}$$

where q is the vector of $(n+m)$ netput quantities, v' is the (transposed) vector of corresponding prices and $\pi^{U}(v)$ is the unconstrained profit function. The feasible set T is assumed non-empty, closed, bounded from above and convex. Free disposal is also allowed for. Furthermore, we assume that the unconstrained profit function is twice differentiable everywhere so that it is locally strongly convex everywhere in v. The solutions of the optimisation program (2.1) are the unconstrained netput responses :

$$\partial\pi^{U}(v)/\partial v = q^{U}(v) \tag{2.2}$$

When some quantities are pegged, either by policy instruments (output quotas, input use regulations) or by market conditions (constrained outlets, inputs in fixed supply), the remaining variable netputs exhibit constrained response to prices. These responses can be derived from the constrained producer programme, i.e., from the constrained profit function. After partitioning the vector q (respectively v) into a variable netput vector q_1 *(respectively* v_1*)* with dimension n_1+m_1 and a constrained netput vector q_0 *(respectively* v_0*)* with dimension n_0+m_0, we get :

$$\max_{q_1}[v'_1 q_1; (q_1, \bar{q}_0) \in T] = \pi^{R}(v_1, \bar{q}_0) \tag{2.3}$$

and

$$\partial\pi^{R}(v_1, \bar{q}_0)/\partial v_1 = q_1^{R}(v_1, \bar{q}_0) \tag{2.4}$$

The notion of disequilibrium profit function will also be useful in the developments below. It can be defined as the sum of the restricted profit and the value of fixed netputs at market prices, i.e.,

$$\pi^{D}(v_1, \bar{q}_0, v_0) = \pi^{R}(v_1, \bar{q}_0) + v'_0 \bar{q}_0 \tag{2.5a}$$

or, alternatively :

$$\pi^{D}(v_1, \bar{q}_0, v_0) = \max_{q_1, q_0}[v'_1 q_1 + v'_0 q_0; (q_1, q_0) \in T; q_0 = \bar{q}_0] \tag{2.5b}$$

Using the same partition as before, (2.2) can be written as :

$$\partial\pi^U(v_1,v_0)/\partial v_1 = q_1^U(v_1,v_0) \tag{2.6a}$$

$$\partial\pi^U(v_1,v_0)/\partial v_0 = q_0^U(v_1,v_0) \tag{2.6b}$$

From the definitions of $\pi^U(v_1,v_0)$, $\pi^R(v_1,\bar{q}_0)$ and $\pi^D(v_1,\bar{q}_0,v_0)$, it follows that disequilibrium and unconstrained profits are equal when $\bar{q}_0$ just happens to be the optimal vector $q_0^U(v_1,v_0)$, i.e.,

$$\pi^U(v_1,v_0) = \left[\pi^D(v_1,\bar{q}_0,v_0); q_0^U(v_1,v_0) = \bar{q}_0\right] \tag{2.7}$$

Consequently, the unconstrained profit curve is the upper envelope to the family of disequilibrium profit functions and, at the tangency point, q_1^U and q_1^R coincide, i.e.,

$$\begin{aligned} q_1^U(v_1,v_0) &= \partial\pi^U(v_1,v_0)/\partial v_1 \\ &= \partial\pi^D(v_1,q_0^U(v_1,v_0),v_0)/\partial v_1 \\ &= q_1^R(v_1,q_0^U(v_1,v_0)) \end{aligned} \tag{2.8}$$

This relation is strictly valid only when $\bar{q}_0$, given v_1 and v_0, is optimal. In order to characterise the constrained response from the knowledge of $\pi^U(v_1,v_0)$ at any point $(v_1,\bar{q}_0)$ in the constrained regime space, we make use of virtual prices introduced by Rorbarth. The vector of virtual prices η_0 is defined as the system of prices which ensures that the unconstrained quantities q_0^U, as functions of v_1 and η_0, will stay at level $\bar{q}_0$, i.e. :

$$q_0^U(v_1,\eta_0) = \partial\pi^U(v_1,\eta_0)/\partial v_0 = \bar{q}_0 \tag{2.9}$$

Supply response under rationing

When rationed quantities $\bar{q}_0$ are different from optimal levels $q_0^U(v_1,v_0)$, equation (2.9) can be used to determine the virtual price vector η_0 as a function of v_1 and $\bar{q}_0$, i.e., $\eta_0(v_1,\bar{q}_0)$, which will ensure that expression (2.8) holds at the point (v_1,η_0) in the price space, i.e.,

$$q_1^U(v_1,\eta_0(v_1,\bar{q}_0)) = q_1^R(v_1,\bar{q}_0) \tag{2.10}$$

Now, the comparative statics of rationed responses $q_1^R(v_1,\bar{q}_0)$ with respect to v_1 can be derived from $q_1^U(v_1,\eta_0)$ and expression (2.9). A local solution can be obtained by total differentiation of (2.9) and (2.10) and solving for dq_1^R around (v_1,η_0), i.e.,

$$dq_1^R = \pi^U_{v_1v_1}(v_1,\eta_0)dv_1 + \pi^U_{v_1v_0}(v_1,\eta_0)d\eta_0 \tag{2.11a}$$

and

$$d\bar{q}_0 = \pi^U_{v_0 v_1}(v_1, \eta_0)dv_1 + \pi^U_{v_0 v_0}(v_1, \eta_0)d\eta_0 \tag{2.11b}$$

Then (2.11) can be solved for the endogenous variables in rationed regime (i.e., dq_1^R and $d\eta_0$) with respect to the set of exogenous ones which are now dv_1 and $d\bar{q}_0$. Omitting arguments (v_1, η_0), solving (2.11b) for $d\eta_0$ and plugging into (2.11a), we obtain :

$$\begin{bmatrix} dq_1^R \\ d\eta_0 \end{bmatrix} = \begin{bmatrix} \pi^U_{v_1 v_1} - \pi^U_{v_0 v_0}(\pi^U_{v_0 v_0})^{-1}\pi^U_{v_0 v_1} & \pi^U_{v_1 v_0}(\pi^U_{v_0 v_0})^{-1} \\ -(\pi^U_{v_0 v_0})^{-1}\pi^U_{v_0 v_1} & (\pi^U_{v_0 v_0})^{-1} \end{bmatrix} \begin{bmatrix} dv_1 \\ d\bar{q}_0 \end{bmatrix} \tag{2.12}$$

System (2.12) allows us to characterise the comparative statics under rationing constraints on the basis of the unconstrained supply and derived demand responses evaluated at the relevant constrained equilibrium point, i.e., at $((q_1^R, \bar{q}_0),(v_1, \eta_0))$. The first row in (2.12) provides the comparative statics of q_1^R in terms of v_1 and $\bar{q}_0$ from the Hessian of the unconstrained profit function evaluated at the relevant point (v_1, η_0). Similarly, the second row of (2.12) provides the comparative statics of rationed netput shadow prices on the basis of the second partial derivatives of $\pi^U(v_1, \eta_0)$. Note that a unique local solution to (2.11 b) for $d\eta_0$ exists under our assumptions about the feasible production set, as the Hessian of the (non-normalised) profit function is of order n+m-1 (Guesnerie, 1980) and its principal minors of smaller order are therefore of full rank.

Supply response under rationing and de-rationing

There is an obvious correspondence between (2.12) and a similar expression for $dq_1^R(v_1,\bar{q}_0)$ and $d\eta_0(v_1,\bar{q}_0)$ derived from the restricted profit function. At the constrained equilibrium point $(v_1,\bar{q}_0)$, the vector η_0 is also defined by the following system obtained by first differentiation of (2.3) with respect to $\bar{q}_0$ (Lau, 1976) :

$$\partial\pi^R(v_1,\bar{q}_0)/\partial\bar{q}_0 = -\eta_0(v_1,\bar{q}_0) \tag{2.13}$$

If the constrained profit function is known, the comparative statics of endogenous variables (q_1, η_0) in the rationed regime, around the point $(v_1,\bar{q}_0)$, is given by :

$$\begin{bmatrix} dq_1^R \\ -d\eta_0 \end{bmatrix} = \begin{bmatrix} \pi^R_{v_1 v_1} & \pi^R_{v_1 q_0} \\ \pi^R_{q_0 v_1} & \pi^R_{q_0 q_0} \end{bmatrix} \begin{bmatrix} dv_1 \\ d\bar{q}_0 \end{bmatrix} \tag{2.14}$$

Clearly, given our assumptions, (2.12) and (2.14) are two alternative representations of the same behaviour under rationing which can be used alternatively depending on the available information. Note that this analysis is valid only when rationing remains effective throughout. By proper comparison of $\eta_0(v_1,\bar{q}_0)$ to market price v_0, it is possible to describe the change from the unconstrained regime corresponding to (2.6) to the constrained regime defined either by (2.12) or by (2.14), and vice-versa. For example, comparing (2.12) and (2.14) shows that :

$$\pi^R_{v_1v_1} = \pi^U_{v_1v_1} - \pi^U_{v_1v_0}(\pi^U_{v_0v_0})^{-1}\pi^U_{v_0v_1} \qquad (2.15)$$

The correction term in (2.15) is the "contraction effect" due to rationing. The Le Chatelier's principle applies as the diagonal elements of $\pi^R_{v_1v_1}$ differ from those of $\pi^U_{v_1v_1}$ by terms of the contraction effects which are positive-definite quadratic forms. The response of an unconstrained netput to a change of its own price is smaller (in absolute value) with constraints put on some quantities q_0.

Producer response after relaxing constraints can also be derived from the knowledge of the response under rationing, i.e., from the sub-Hessians of the restricted profit function. By total differentiation of (2.10) and (2.13), one can retrieve the Hessian of the unrestricted profit function at the point of rationing $\eta_0(v_1,\bar{q}_0)$. Solution of (2.14) for unrestricted responses $q_1^U(v_1,\eta_0)$ *and* $q_0^U(v_1,\eta_0)$ gives the relation between unconstrained netputs and prices at the disequilibrium point (v_1,η_0) :

$$\begin{bmatrix} dq_1^U \\ dq_0^U \end{bmatrix} = \begin{bmatrix} \pi^U_{v_1v_1}\ \pi^U_{v_1v_0} \\ \pi^U_{v_0v_1}\ \pi^U_{v_0v_0} \end{bmatrix} \begin{bmatrix} dv_1 \\ d\eta_0 \end{bmatrix} = \begin{bmatrix} \pi^R_{v_1v_1} - \pi^R_{v_1q_0}(\pi^R_{q_0q_0})^{-1}\pi^R_{q_0v_1} & -\pi^R_{v_1q_0}(\pi^R_{q_0q_0})^{-1} \\ -(\pi^R_{q_0q_0})^{-1}\pi^R_{q_0v_1} & -(\pi^R_{q_0q_0})^{-1} \end{bmatrix} \begin{bmatrix} dv_1 \\ d\eta_0 \end{bmatrix} \qquad (2.16)$$

It is easy to verify that the expansion effect in (2.16) is the opposite of the contraction effect in (2.12), i.e, $\pi^U_{v_1v_0}(\pi^U_{v_0v_0})^{-1}\pi^U_{v_0v_1} = -\pi^R_{v_1q_0}(\pi^R_{q_0q_0})^{-1}\pi^R_{q_0v_1}$. The Le Chatelier principle applies and the unconstrained response of a netput to a change of its own price is greater (in absolute value) than its constrained counterpart.

While the Le Chatelier effect is easily verified, what happens to cross-price effects is much less obvious (Moschini, 1988), particularly when one uses the restricted profit function where cross effects between quantities of the type $\pi^R_{v_1q_0}$ are not easy to interpret. The situation is much easier when one starts with the unconstrained profit function $\pi^U(v_1,v_0)$, in particular when the unconstrained technology is "normal" (Sakai, p. 272-273).

3. The consequences of rationing on substitutability between inputs and outputs

Own-price effects of rationing follow directly from curvature properties, i.e., the Le Chatelier effect shown by Lau and subsequent authors, the negative effect of reducing the quota on its shadow price (Moschini), and the mirror positive effect of reducing a fixed factor quantity on its shadow price.

As regards cross effects, the comparative statics of supply behaviour under rationing involves at least two broad issues, i) the impact of the level of rationed netputs on output supplies, input demands and on shadow prices of rationed netputs (problem 1), and ii) the consequences of rationing on cross-price effects between the netputs remaining unconstrained (problem 2).

The latter issue has received less attention than the Le Chatelier effect, but it can be important in the assessment of fiscal or trade policies in a multioutput sector subject to rationing. Madden has recently dealt with this problem on Hicksian demand functions in consumer theory. He shows, using a partition of the Hessian of responses under rationing similar to system (2.12), that if two goods are substitutes before, they are also substitutes after addition of one rationing (Madden, theorem 4, p. 1502). He also notes that "substitutes are likely to remain substitutes, but complements may "more easily" change and become substitutes".

In the case of producer theory, a corresponding result can be obtained under a fairly weak assumption that cross effects between two netputs and the price of the netput to be rationed have the same sign before the imposition of rationing.

Rationing under "pairwise similarity"

A pair of netputs q_r and q_s will be said to be similar with respect to a third netput q_0 if the cross-price elasticities of q_r^U and q_s^U with respect to v_0 have the same sign, that is, with usual notation, if :

$$\varepsilon_{r0}^U \varepsilon_{s0}^U = (v_0 / q_r^U . \partial q_r^U / \partial v_0).(v_0 / q_s^U . \partial q_s^U / \partial v_0) \geq 0 \qquad (3.1)$$

For purpose of clarity, unconstrained outputs ($y_i = q_i$; $i = 1,...,n$) and inputs ($x_h = -q_h$; $h = 1,...,m$) are identified. With this notation, quantities of outputs and inputs are positive. Two outputs y_i and y_j will be said to be similar with respect to input x_0 if this factor is superior (or inferior) in the production of both outputs. Two inputs x_h and x_k will be said to be similar with respect to input x_0 if both are substitutes with respect to x_0 or if both are complements with respect to x_0. Output y_i and input x_h will be said to be similar with respect to input x_0 if input x_0 is superior (inferior) in the production of y_i and inputs x_h and x_0 are complements (substitutes)[3].

It is worth noting that the similarity property (3.1) is defined with respect to the

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[3] Inequalities (3.1) may be written in terms of the Hessian of the profit function $\pi^U(v)$. But, in this case, there is not a single definition of similarity and it is necessary to distinguish ouputs and inputs. When q_r and q_s are two outputs or two inputs, (3.1) is equivalent to $\pi_{v_r v_0}^U . \pi_{v_s v_0}^U \geq 0$. When q_r (respectively q_s) is an ouput and q_s (respectively q_r) is an input, (3.1) is equivalent to $\pi_{v_r v_0}^U . \pi_{v_s v_0}^U \leq 0$.

unconstrained Marshallian equilibrium[4] and that two netputs q_r and q_s can be similar with respect to q_0 at point E corresponding to $\pi^U(v)$, but not at point E* corresponding to $\pi^U(v^*)$.

Consider now the case when either one output or one input is rationed. The sub-Hessian $\pi^U_{v_0v_0}$ in (2.15) reduces then to a positive scalar. Under the assumption of similarity of each pair of goods with respect to the rationed netput, the following properties concerning the cross effects at the rationed equilibrium can now be derived :

Property 1. Under the similarity assumption, a binding constraint on an output or an input makes the unrestricted outputs and unrestricted inputs more substitutable.

This follows directly from the definition of similarity and from the convexity in prices of the unconstrained profit function. The contraction effect $\pi^U_{v_rv_o}(\pi^U_{v_ov_o})^{-1}\pi^U_{v_ov_s}$ is non-negative and we have :

$$\frac{\partial y_i^R}{\partial v_j} = \frac{\partial q_i^R}{\partial v_j} = \pi^R_{v_iv_j} = \pi^U_{v_iv_j} - \pi^U_{v_iv_o}(\pi^U_{v_ov_o})^{-1}\pi^U_{v_ov_j} \leq \pi^U_{v_iv_j} = \frac{\partial y_i^U}{\partial v_j} = \frac{\partial q_i^U}{\partial v_j},\ \text{when } i \text{ and } j \text{ are outputs,} \tag{3.3}$$

and,

$$\frac{\partial x_h^R}{\partial v_k} = -\frac{\partial q_h^R}{\partial v_k} = -\pi^R_{v_hv_k} = -\pi^U_{v_hv_k} + \pi^U_{v_hv_o}(\pi^U_{v_ov_o})^{-1}\pi^U_{v_ov_k} \geq -\pi^U_{v_hv_k} = \frac{\partial x_h^U}{\partial v_k} = -\frac{\partial q_h^U}{\partial v_k},\ \text{when } h \text{ and } k \text{ are inputs.} \tag{3.3}$$

Property 2. Under the similarity assumption, a binding constraint on an output or an input makes the unrestricted outputs more regressive and equivalently the unrestricted inputs more inferior.

The proof of property 2 is parallel to the above.

The similarity property defined at the unconstrained Marshallian equilibrium does not allow generally to sign the contraction effect in the case of $n_0 + m_0 \geq 2$ rationed netputs. The contraction effect relating the off-diagonal elements of $\pi^R_{v_lv_l}$ in (2.15) is :

$$(\pi^U_{v_rv_0})(\pi^U_{v_0v_0})^{-1}(\pi^U_{v_0v_s}) \tag{3.4}$$

When $s \neq r$, similarity between netputs r and s with respect to each constrained netput at the unrationed equilibrium is not sufficient to determine the sign of expression (3.4) since

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[4] Obviously, it is also possible to define the similarity property with respect to any constrained equilibrium.

off-diagonal elements of $(\pi^U_{v_0v_0})^{-1}$ are, in general, indeterminate in sign. The diagonal terms of $(\pi^U_{v_0v_0})^{-1}$ are non-negative by convexity. The sign of expression (3.4) will be then defined if off-diagonal elements of $(\pi^U_{v_0v_0})^{-1}$ are non negative too. Technologies such that $(\pi^U_{v_0v_0})^{-1}$ is a non-negative matrix are worth considering.

Let us examine two situations where off-diagonal terms of $(\pi^U_{v_0v_0})^{-1}$ are all non negative, i) $\pi^U_{v_0v_0}$ is a diagonal matrix and, ii) $\pi^U_{v_0v_0}$ is an M-matrix and its off-diagonal elements are all non-positive[5]. In both cases, the inverse of $\pi^U_{v_0v_0}$ includes non-negative elements only and, under the similarity assumption of netputs q_r and q_s with respect to each rationed netput, the contraction effect (3.4) is then non-positive and more substitutability follows from rationing.

i) The matrix $\pi^U_{v_0v_0}$ is diagonal in the case where rationed netputs are outputs only if and only if the technology is non joint in input quantities (Kohli, 1981). Consequently, the unconstrained supply of an output does not depend on other output prices. In this case, binding constraints on outputs makes unrestricted outputs more regressive and unrestricted inputs more substitutable, under the similarity assumption of variable netputs with respect to each rationed output. The constrained supply of a variable output does not depend either on other variable output prices or constrained output quantities. Similarly, if rationed netputs are inputs only, the matrix $\pi^U_{v_0v_0}$ is diagonal if and only if the technology is non-joint in output quantities (Kohli). Consequently, the unconstrained demand of an input does not depend on other input prices. In this case, binding constraints on inputs makes unrestricted inputs more inferior and outputs more substitutable, under the similarity assumption of variable netputs with respect to each rationed input. The constrained demand of a variable input is independent of other variable input prices and of constrained input quantities.

ii) Consider now the case where $\pi^U_{v_0v_0}$ is an M-matrix. This corresponds to the situation where rationed outputs (respectively inputs) are substitutes and outputs are non inferior with respect to inputs in the unconstrained regime. The tendency towards more substitutability and more inferiority is then verified, still under the similarity assumption. In particular, two outputs (respectively two inputs) will be stronger substitutes the more restrictions are implemented.

Rationing under a "normal" technology

It is worth noting that in a procedure by steps, i.e., by implementing constraints one at a time, outputs and inputs tend to become more substitutable and outputs tend to become more inferior with respect to inputs as long as the similarity property is verified at each sub-

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[5] Situation i), which is a special case of ii), is detailed because it corresponds to particular technologies if outputs (respectively inputs) only are rationed.

equilibrium. Since there is apparently no reason why two netputs cannot be similar under one set of constraints but no longer similar under another, technologies which verify the similarity property at each equilibrium are worth considering. Here we consider the case of a normal technology, as defined by Sakai, and show that the tendency toward more substitution and more inferiority follows without ambiguity when, i) outputs only are rationed and, ii) inputs only are rationed.

A multioutput-multiinput technology has been said to be normal by Sakai if the following conditions on the unconstrained cost and revenue functions are satisfied :

$C_{y_i y_j} \leq 0$ for any couple of outputs i and j, $i \neq j$ (cost complementarity), (3.5)

$C_{w_h y_j} \geq 0$ for any input h and output j (non regressivity of outputs), (3.6)

$R_{x_h x_k} \geq 0$ for any couple of inputs h and k, $h \neq k$ (input complementarity), (3.7)

$R_{p_i x_k} \geq 0$ for any output i and input k (non-inferiority of inputs). (3.8)

where $C(y,w)$ is the unconstrained or total cost function, $R(p,x)$ is the unconstrained or total revenue function, y is the vector of outputs (with corresponding vector price p) and x is the vector of inputs (with corresponding vector price w).

Properties (3.5) to (3.8) imply the following restrictions on the technology at the unconstrained Marshallian equilibrium : gross substitution among inputs and outputs is ruled out and regressive (or inferiority) relationships between inputs and outputs are also ruled out. When the arguments of the profit function are p's for outputs and w's for inputs, the following implications hold :

$\pi^U_{p_i p_j} \geq 0$ for any outputs i and j, (3.9)

$\pi^U_{w_h w_k} \geq 0$ for any inputs h and k, and (3.10)

$\pi^U_{p_i w_h} = \pi^U_{w_h p_i} \leq 0$ for any pair of output i and input h. (3.11)

The "normal" technology ensures that all pairs of inputs and outputs are similar at the unconstrained Marshallian equilibrium. Such a technology could be called strongly similar. Properties 1 and 2 therefore extend to the normal technology. Hence, rationing favours substitutability.

In the "normal" case, Moschini has shown how restricting the output of supply-management commodities unambiguously reduces the production of the unconstrained outputs and decreases input use. Moschini's results on the impact of the quota level on unrestricted supplies and demands can be extended to input rationing. We have then the two following properties :

Property 3 a. When the technology is "normal", tightening an output quota constraint reduces variable output supplies and input demands.

In the one rationing case, this property follows directly from the convexity in prices of the profit function which implies a fall in the shadow price associated with the quota and from the normality assumption which implies that the (shadow) price fall of the rationed output leads to a decrease in the supply of complement outputs and in the demand of non-inferior inputs.

When several outputs are constrained, the impact of a constrained output level on the supply of unconstrained commodities may be written as (Moschini, equation 12) :

$$\pi^R_{p_1 y_o} = -(C_{y_1 y_1})^{-1} C_{y_1 y_o} \qquad (3.12)$$

Property (3.1), together with the convexity of the cost function in output quantities, imply that the sub-Hessian $C_{y_1 y_1}$ is an M-Matrix. The elements of its inverse are then non-negative. It follows that the elements of $\pi^R_{p_1 y_o}$ are also non-negative. By a similar argument, one can show that the elements of $-\pi^R_{w y_o}$ are also non-negative since the latter can be written as (Moschini, equation 13):

$$-\pi^R_{w y_o} = C_{w y_o} + C_{w y_1} \pi^R_{p_1 y_o} \qquad (3.13)$$

Property 3 b. Under a "normal" technology, tightening a fixed factor constraint decreases output supplies and variable input demands.

In the one rationing case, complementary between inputs and non regressivity of outputs, together with the rise in the shadow price of the rationed input, imply this result. In the multiple rationing case, this property can be shown in a similar way as above by using the relevant sub-Hessians of the revenue function instead of those of the cost function.

The next property is useful in analysing the impact of unconstrained netput prices on shadow prices of constrained quantities. This property follows directly from symmetry of (2.12).

Property 4. The effect of unconstrained netput prices (v_1) on the shadow price of the rationed netput (η_0) is the negative of the effect of the ration level ($\bar{q}_0$) on unconstrained netputs (q_1).

As regards cross-price effects between unconstrained netputs, the normal technology allows us to establish stronger results than the similarity condition. It turns out that rationing several outputs (respectively several inputs) in the same time strengthens the tendency toward substitution without ambiguity.

Consider the case where all outputs are rationed at once. The restricted profit function then relates directly to the cost function. We have :

$$\pi^R(y,w) = -C(y,w) \tag{3.14}$$

Hence,

$$\pi^R_{ww}(y,w) = -C_{ww}(y,w) \tag{3.15}$$

The unrestricted profit function $\pi^U(p,w)$ may be written as :

$$\pi^U(p,w) = py(w,p) - C(w,y(w,p)) \tag{3.16}$$

Following Sakai's procedure, we have then :

$$\pi^U_{ww} = -C_{ww} + C_{wy}C^{-1}_{yy}C_{yw} \tag{3.17}$$

With consideration of (2.15) and (3.15), it is clear that the contraction effect (from π^U to π^R) and the expansion effect (from C to π^R) are identical, i.e. :

$$\pi^U_{ww} - \pi^R_{ww} = \pi^U_{wp}(\pi^U_{pp})^{-1}\pi^U_{pw} = C_{wy}C^{-1}_{yy}C_{yw} \tag{3.18}$$

The advantage of (3.18) is that the signs of its off-diagonal elements unambiguously follow from the normality assumption. C_{yy} being an M-matrix, its inverse is non-negative. Normality implies the non-negativity of C_{wy}. Hence, all off-diagonal elements in (3.18) are non-negative as well, and rationing will therefore both reduce the magnitude of the own price elasticity (the Le Chatelier principle) and reduce the complementary (increase the substitution) between inputs, i.e., $-\pi^R_{ww} \geq -\pi^U_{ww}$.

A similar argument can be applied to the case where all inputs are rationed at once, using relation (3.7) and (3.8) on the revenue function, to establish the impact of input rationing on output substitutability.

When only a subset y_o of outputs are constrained by quotas, a proper partition of $\pi^U(.), \pi^R(.)$ and $C(.)$, and corresponding Hessian matrices, leads to similar expressions which rely on the equivalence between the expansion matrix based on the cost function (with known signs) and the contraction effect based on the unrestricted profit function[6]. We have :

$$\pi^R(p_1, y_0, w) = p_1 y_1(p_1, w) - C(y_1(p_1,w), y_0, w), \tag{3.19}$$

---

[6] When constrained netputs are inputs only, the proof uses a proper partition of $\pi^U(.), \pi^R(.)$ and $R(.)$.

From (3.19), a similar expression to (3.17) is derived and it is easy to show that expression $\pi^U_{ww} - \pi^R_{ww}$ has non-negative elements everywhere under the normality assumption. Hence, the tendency toward more substitutability (less complementary).

These results do not encompass all cases, however, since, in the examination of the simultaneous imposition of input and output rationing, a similar proof has not been found.

4. Empirical illustration

The previous theoretical framework is applied to the EC agricultural sector in order to analyse the effects of output and input rationing on the production structure of the EC agricultural sector, with emphasis on the impact of the milk quota constraint on unrestricted output supplies and input demands.

The data set spans the period 1960-1984, i.e., before the implementation of the dairy quota system in the EC in end-1984, and are from the SPEL data base (Henrichsmeyer, 1989). The estimated model is based on four outputs (grains, other vegetable products, milk, and other animal products), three variable inputs (animal feed ingredients, fertilisers and other raw materials), and two quasi-fixed inputs (labour and capital). Before 1984, the four output groups distinguished in the model were not subject to supply management policies. Capital includes land and buildings, implements and machinery, and livestock stocks and may be thus considered as fixed in the short run. Effects of (possibly biased) technical change are captured by adding a linear time trend variable. We assume a multioutput-multiinput translog restricted profit function :

$$\begin{aligned} ln\,\pi^R &= a_o + \sum_{r=1}^{7} a_r\, ln\, v_r + \sum_{l=1}^{2} a_l\, ln\, z_l + a_t t \\ &+ 0.5 \sum_r \sum_s b_{rs}\, ln\, v_r\, ln\, v_s + 0.5 \sum_l \sum_k b_{lk}\, ln\, q_l\, ln\, z_k + 0.5 b_{tt} t^2 \\ &+ \sum_r \sum_l c_{rl}\, ln\, v_r\, ln\, z_l + \sum_r c_{rt}\, ln\, v_r t + \sum_l c_{lt}\, log\, z_l t \end{aligned} \tag{4.1}$$

Without loss of generality, we impose symmetry on the coefficients b_{rs} and b_{lk}.

Logarithmic differentiation of the restricted profit function and use of Hotelling's lemma yields profit share equations for each variable netput and profit shadow share equations for the two quasi-fixed input :

$$v_r q_r / \pi^R = S_r = a_r + \sum_{s=1}^{7} b_{rs}\, ln\, v_s + \sum_{l=1}^{2} c_{rl}\, ln\, q_l + c_{rt} t \tag{4.2}$$

$$-\eta_l z_l / \pi^R = a_l + \sum_{k=1}^{2} b_{lk}\, ln z_k + \sum_{s=1}^{7} b_{ls}\, ln v_s + c_{lt} t \qquad (4.3)$$

The estimation is restricted to the set of variable netput share equations (4.2) since our attention is focused on own- and cross-price effects among variable netputs which can be obtained from the sole share system (4.2)[7]. The restricted translog profit function is consistent with theory if and only if it is non-negative, non-decreasing in variable output prices and non-increasing in variable input prices, convex and continuous in prices, and non-decreasing and concave in quasi-fixed input quantities (Diewert, 1974). We impose the theoretical property of homogeneity of degree one in prices using the following linear restrictions :

$$\sum a_r = 1; \sum_s b_{rs} = 0 \forall r; \sum_r c_{rl} = 0 \forall l; \sum_r c_{rt} = 0 \qquad (4.4)$$

One of the variable netput share equations is dropped for estimation because only six of the seven equations (3.2) are linearly independent. Adding-up restrictions, together with the symmetry restrictions, imply the homogeneity restrictions.

The estimating form of the model consists then of six share equations with symmetry and homogeneity in prices imposed. All regressors are assumed to be exogenous and normalised to 1980 = 1. Expected prices of variable netputs and expected quantities of quasi-fixed inputs are measured by one-period lagged values. The estimator employed is Zellner's procedure modified in order to impose convexity in prices of the restricted profit function at the expansion point (i.e., 1980) Convexity in prices is imposed by using the Cholesky decomposition of any definite positive matrix[8] (Lau, 1978). A mathematical programming algorithm, available from the Standford Optimization Laboratory as a Fortran routine called Minos 5.0 (Murtaugh and Saunders, 1983), has been employed. The method, used by Hazilla and Kopp (1986) for example, is discussed only briefly here.

The stochastic version of the share system can be written as :

$$S_t = f(X_t, \theta) + u_t \qquad (4.5)$$

---

[7] Since the profit function is not estimated, the approach does not allow us the computation of shadow price share functions for the two quasi-fixed factors, labour and land.

[8] The Cholesky representation of a real symmetric square matrix A is the factorisation LDL', where L is an unit lower triangular matrix and D is a diagonal matrix whose elements are the Cholesky values. The matrix A is positive semidefinite if and only if all Cholesky values are non negative. In the case of the model estimated in this paper, convexity in prices has been imposed at the approximation point 1980 only. But it turns out that convexity is verified at each point of the data set 1960-84. Finally, it is worth noting that the Cholesky decomposition has been criticised for its use of "brute force" to ensure consistency with economic theory. A second approch, wich uses inequality restrictions as priors, can be applied to impose curvature (Geweke, 1986, 1989).

where t indexes the time observations, S_t is the system of shares at time t, X_t is the vector of regressors at time t and θ is the vector of parameters to be estimated ; u_t is a vector of random errors which are assumed to be independent, normally distributed with mean zero and positive definite covariance matrix. The Zellner's original procedure is then modified in order to include the non-linear constraints of convexity. The estimator reduces to minimising :

$$\underset{\theta}{Min}[S_t - f(X_t - \theta)]'[\Sigma \otimes Id]^{-1}[S_t - f(X_t - \theta)] \qquad (4.6)$$

subject to $h(X_t - \theta) \geq \alpha$

where Σ is the variance-covariance matrix, Id is the identity matrix, $\otimes$ is the Kronecker product, $h(.)$ linear and non-linear equality and inequality constraints, and α are constraint values.

Equation (4.6) is minimised with respect to θ replacing $\Sigma \otimes Id$ with the identity matrix Id. Given $\hat{\theta}$, we derive a new estimate of Σ based on the inner product of estimated residuals and resolve (4.6) with this new estimate employed as the weighting matrix. The estimates are iterated until the coefficient vector and the variance-covariance matrix stabilise.

The parameter estimates with their asymptotic standard errors are shown in Table 4.1. This table contains a total of forty-five parameters, twenty-eight of which are significant at the 1% level. Corresponding price elasticities for the seven variable netput system are presented in Table 4.2 for the year 1984. Table 4.3 illustrates the implications of imposing one rationing on milk on the shadow price behaviour of milk relative to unconstrained netput prices and quota level and on the supply-demand response of variable netputs to market prices and milk quota level.

Table 4.2 suggests that the estimated technology (given pre-existing exogenous levels of some inputs, i.e., capital and labour) is strongly similar. It verifies properties (3.9), (3.10) and (3.11) of a "normal" technology since complementary prevails among outputs and inputs and variable inputs are not inferior in the production of the different outputs. The four outputs have inelastic supply, but supply of animal products is more responsive to own-price movements. The three variable inputs have elastic demand. The demand for feed is the most elastic one.

However, as shown by table 4.3, this structure is seriously modified when an important output such as milk is constrained. The key elasticity of interest is the own-price elasticity of milk which is equal to 0.239. Its inverse is the elasticity of the milk shadow price with respect to the quota level. This estimated elasticity is about 4.18. Property 3.a is verified and tightening the milk quota constraint will reduce both the supply of variable outputs and the demand for variable inputs. By property 4, rising the price of an unconstrained output will increase the milk shadow price whereas rising the price of a variable input will decrease the shadow price. The Le Chatelier effect is verified. The milk quota makes the direct output supply elasticities less positive and the direct input demand elasticities less negative. It is worth noting that own-price elasticities are considerably reduced (in absolute value). Moreover, as expected from the analysis developed above, complementarity between outputs is reduced and substitution between variable inputs now prevails (property 1). Less superiority of inputs is also obtained (property 2) and fertilisers flip to an inferior situation as their demand now depends negatively on variable output prices. The latter result depends on the particular estimated value of unconstrained response of milk supply to its own price and to the price of fertiliser which turn out to be large. It is noticeable that the structure of price response after imposing just one rationing can be altered so deeply.

(Insert Table 4.1)

(Insert Table 4.2)

(Insert Table 4.3)

5. The dynamics of adjustment of fixed quantities and the observable technology

In the previous sections, the nature of cross effects in the behavioural equations was shown to depend on the extent of binding constraints on producer profit-maximising decisions. The underlying dual technology is also clearly dependent on these constraints of fixity. As the producer cannot adjust immediately to price changes, the firm is never observed in an equilibrium either short run or long run, but somewhere in between on a transitory path toward it. Then the speed of adjustment is the key factor in the interpretation of the observed technology in econometric work where for example the adjustment process is not formally included, but implicitly assumed. This section attempts to built on the disequilibrium framework used above to derive a simple dynamic estimable model of supply behaviour in the presence of quasi-fixed factors.

Given the prices (v_{1t}, v_{0t}) of unconstrained and quasi-fixed inputs, two behavioural models are relevant. The first is the long-run equilibrium model which corresponds to costless and immediate adjustment to new prices. Assuming linearity, we obtain :

$$\begin{bmatrix} q_{1t}^U \\ q_{0t}^U \end{bmatrix} = \begin{bmatrix} \pi_{11}^U \pi_{10}^U \\ \pi_{01}^U \pi_{00}^U \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{0t} \end{bmatrix} \tag{5.1}$$

Now, if in fact q_0 cannot adjust immediately to the optimal level, the actually observed quantities q_{1t} and q_{0t} are produced by the same model where virtual prices η_{0t} are substituted for observed prices v_{0t} :

$$\begin{bmatrix} q_{1t} \\ q_{0t} \end{bmatrix} = \begin{bmatrix} \pi^U_{11}\pi^U_{10} \\ \pi^U_{01}\pi^U_{00} \end{bmatrix} \begin{bmatrix} v_{1t} \\ \eta_{0t} \end{bmatrix} \qquad (5.2a)\ (5.2b)$$

Following Norsworthy and Harper (1981), we assume that a multivariate partial adjustment process describes the movement of the firm toward the optimal target :

$$\left[q_{0t} - q_{0t-1}\right] = M\left[q^U_{0t} - q_{0t-1}\right] \qquad (5.3)$$

where M is the adjustment matrix.

By combining (5.1), (5.2) and (5.3), a system of observable equations is obtained. For q_{0t} , we get :

$$q_{0t} = M\left[\pi^U_{01}v_{1t} + \pi^U_{00}v_{0t}\right] + (I - M)q_{0t-1} \qquad (5.4)$$

Norsworthy and Harper review several specifications of such ad-hoc models which differ in the manner in which the adjustment process is incorporated. Obviously, this adjustment scheme is not derived from explicit economic optimising behaviour. Recent developments in dynamic duality theory allow us to specify a multivariate flexible accelerator model whereby behavioural restrictions may be derived for the complete matrix of adjustments coefficients (Epstein, 1981). This model is more elegant but has met with mixed success in applications to aggregate data for the agricultural sector (Tsigas and Hertel, 1989).

The generalised adjustment scheme (5.4) permits disequilibrium in one quasi-fixed factor market to affect the demand for another quasi-fixed input. The actual levels of quasi-fixed inputs are a weighted average of the optimal levels at time t and past observed levels at time t-1, where the adjustment matrices M and $(I-M)$ serve as weights. Through substitution, we can see that equation (5.4) can be rewritten as :

$$q_{0t} = \sum_{i=0}^{\infty}(I - M)^i Mq^U_{0t-i} + (I - M)^{\infty} q_{0t-\infty} \qquad (5.5)$$

$$= \sum_{i=0}^{\infty}(I - M)^i M\left[\pi^U_{01}v_{1t-i} + \pi^U_{00}v_{0t-i}\right] + (I - M)^{\infty} q_{0t-\infty} \qquad (5.6)$$

In equilibrium, $q^U_{0t-i} = q^U_0$. Therefore, in the long run :

$$q_{0t} = \sum_{i=0}^{\infty}(I - M)^i Mq^U_0 + (I - M)^{\infty} q_{0t-\infty}$$

Stability of the adjustment scheme requires that the characteristics roots of the matrix

$(I-M)$ be within the unit circle. Furthermore, the adjustment path is monotonic if the characteristic roots are real positive numbers and oscillates otherwise (Nadiri and Rosen, 1969). From (5.6), short-run, interim and long-run price elasticities of q_0 can be derived.

For observed q_{1t} , there is a virtual price vector η_{0t} which corresponds to the observed vector q_{0t} and which explains the optimal level of unconstrained netputs given the quasi-fixity of factors. First, solving (5.2 b) for η_{0t} and using (5.4), we obtain :

$$\eta_{0t} = (\pi_{00}^{U})^{-1}(M-I)\pi_{01}^{U}v_{1t} + (\pi_{00}^{U})^{-1}M\pi_{00}^{U}v_{0t} + (\pi_{00}^{U})^{-1}(I-M)q_{0t-1} \qquad (5.7)$$

The observed sequence for unconstrained netputs q_1, given the interim level of quasi fixed factors, may now be derived, i.e. :

$$\begin{aligned} q_{1t} &= \pi_{11}^{U}v_{1t} + \pi_{10}^{U}\eta_{0t} \\ &= \pi_{11}^{U}v_{1t} + \pi_{10}^{U}\left[(\pi_{00}^{U})^{-1}(M-I)\pi_{01}^{U}v_{1t} + (\pi_{00}^{U})^{-1}M\pi_{00}^{U}v_{0t} + (\pi_{00}^{U})^{-1}(I-M)q_{0t-1}\right] \end{aligned}$$

Finally, we get :

$$q_{1t} = \left[\pi_{11}^{U} + \pi_{10}^{U}(\pi_{00}^{U})^{-1}(M-I)\pi_{01}^{U}\right]v_{1t} + \pi_{10}^{U}(\pi_{00}^{U})^{-1}M\pi_{00}^{U}v_{0t} - \pi_{10}^{U}(\pi_{00}^{U})^{-1}(M-I)q_{0t-1} \qquad (5.8)$$

In (5.8), it can be checked that if $M = I$, the first line of system (4.1) is retrieved and actual path matches optimal path. Short-run, interim and long-run price elasticities of unconstrained goods q_1 can be derived from (5.8) in a similar way to (5.5).

Equations (5.4) and (5.8) provide analytical forms which are estimable with a proper specification of the error terms. It can be seen that in (5.8) two variables appear in simultaneously, namely the observed quantities of the quasi-fixed inputs and their market rental prices. This is necessary here as it appears clearly that observed quantities are neither in short-run nor long-run equilibrium, but converging to the latter. Clearly, (5.8) shows that by failing to specify the netput interactions created by the quasi-fixities in the estimated model, there is little chance of obtaining consistent estimates of short- or long-run responses. These parameters depend on the speed of adjustment. As for the technology, and particularly for input and output substitution, complementarity and normality relationships, the speed of adjustment and therefore the time frame is also a necessary element of information to be specified clearly. Another advantage of expression (5.8) is that the estimation is made on the long-run parameters directly, that convexity restrictions can be imposed (or tested) on this long-run structure, and that the relation between short-run, interim and long-run responses is less general but more transparent than with the value function approach. The derivation of the long run and short-run responses to various lags is particularly easy.

6. Conclusion

Strict fixities like production quotas or input rationing alter the behaviour of unconstrained supply and derived demand responses to prices. Cross-commodity relationships, i.e., complementarity and substitutability, and inferiority and regressivity, often considered as basic features of the underlying technology, depend highly on the economic or the policy environment. They depend also on the time perspective where they are observed. The observable technology is always in a temporary stage between the short and the long run and cannot be characterised without making a clear reference to existing strict or quasi-fixities or to the time lag after the shock affecting exogenous variables of the firm's environment. Concepts of jointness, technical progress bias, economies of scale and economies of scope must also be looked at with reference to the degree of constraint in the environment and to the time frame.

From a policy point of view, the analysis suggests that the efficiency of public intervention will be more likely to run into problems in the presence of strict or quasi-fixity of quantities. The effect of constraints is to reduce the response of the system to the traditional price incentives. By the Le Chatelier effect and the tendency toward substitutability and inferiority, unconstrained outputs react less to their own prices and less negatively to prices of inputs, but more negatively (or less positively) to the prices of other outputs. The supply system could then be broadly characterised by a smaller reaction to its environment and a higher degree of internal interaction. Policy instruments which apply to, say, only one output or one input are then likely to induce spill-over effects on other goods.

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TABLE 4.1. - COEFFICIENT ESTIMATES

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
a_V	0.5515	$(1.2\ 10^{-2})$	b_{GF}	-0.1227	$(6.5\ 10^{-2})$	c_{GL}	0.2377	(0.11)
a_G	0.2050	$(1.2\ 10^{-3})$	b_{GE}	-0.0254	$(3.8\ 10^{-2})$	c_{MK}	0.8511	(0.29)
a_M	0.3635	$(6.5\ 10^{-3})$	b_{MM}	0.3171	$(9.4\ 10^{-2})$	c_{ML}	0.2483	(0.07)
a_A	0.6964	$(1.2\ 10^{-3})$	b_{MA}	-0.1729	$(7.0\ 10^{-2})$	c_{AK}	1.4990	(0.54)
a_F	-0.3658	$(9.0\ 10^{-3})$	b_{MF}	-0.0126	$(5.8\ 10^{-2})$	c_{AL}	0.1306	(0.12)
a_E	-0.1163	$(4.4\ 10^{-3})$	b_{ME}	-0.0311	$(2.3\ 10^{-2})$	c_{FK}	-1.6117	(0.38)
b_{VV}	0.3800	(0.11)	b_{AA}	0.4259	(0.12)	c_{FL}	-0.3004	(0.08)
b_{VG}	0.0151	$(7.9\ 10^{-2})$	b_{AF}	-0.1222	$(6.9\ 10^{-2})$	c_{EK}	-0.5207	(0.19)
b_{VM}	-0.1151	$(5.3\ 10^{-2})$	b_{AE}	0.0694	$(3.2\ 10^{-2})$	c_{EL}	-0.1113	(0.04)
b_{VA}	-0.2480	$(8.9\ 10^{-2})$	b_{FF}	0.2304	$(6.9\ 10^{-2})$	c_{Vt}	0.0027	$(3.1\ 10^{-3})$
b_{AF}	-0.0073	$(5.5\ 10^{-2})$	b_{FE}	-0.0106	$(2.4\ 10^{-2})$	c_{Gt}	0.0149	$(3.3\ 10^{-3})$
b_{AE}	-0.0001	$(3.0\ 10^{-2})$	b_{EE}	-0.0093	$(1.5\ 10^{-2})$	c_{Mt}	0.0021	$(1.7\ 10^{-3})$
b_{GG}	0.2919	(0.13)	c_{VK}	-0.0429	(0.32)	c_{At}	-0.056	$(3.1\ 10^{-3})$
b_{GM}	-0.0045	$(6.2\ 10^{-2})$	c_{VL}	-0.0540	(0.13)	c_{Ft}	-0.0028	$(2.2\ 10^{-3})$
b_{GA}	-0.0083	$(8.9\ 10^{-2})$	c_{GK}	0.6138	(0.50)	c_{Et}	-0.0005	$(1.2\ 10^{-3})$

Subscript labels are V = other vegetable products, G = grains, M = milk, A = other animal products, F = feed ingredients, E = fertilisers ; K = capital, L = labour, t = time.

TABLE 4.2. PRICE ELASTICITIES OF VARIABLE NETPUT QUANTITIES BEFORE IMPOSING THE DAIRY QUOTA (EVALUATED FOR THE YEAR 1984)

	Elasticities with respect to prices of						
Quantities	Milk	Vegetable products	Grains	Animal products	Feed ingredients	Fertilisers	Other raw materials
Milk	0.239	0.223	0.212	0.201	-0.401	-0.195	-0.280
Vegetable products	0.146	0.239	0.248	0.227	-0.375	-0.110	-0.375
Grains	0.343	0.613	0.547	0.649	-0.922	-0.227	-1.003
Animal products	0.105	0.182	0.210	0.309	-0.546	-0.008	-0.251
Feed ingredients	0.393	0.562	0.559	1.023	-1.998	-0.081	-0.457
Fertilisers	0.636	0.549	0.458	0.052	-0.271	-1.024	-0.401
Other raw materials	0.616	0.667	0.301	0.516	-0.501	-0.132	-1.467

TABLE 4.3. THE COMPLETE SYSTEM OF SUPPLY-DEMAND ELASTICITIES AFTER IMPOSING THE DAIRY QUOTA (EVALUATED FOR THE YEAR 1984)

	Elasticities with respect to						
	quota	prices of					
	milk	Vegetable products	Grains	Animal products	Feed ingredients	Fertilisers	Other raw materials
Milk shadow price	4.184	-0.933	-0.887	-0.841	1.678	0.816	1.172
Quantities							
Vegetable products	0.611	0.103	0.118	0.104	-0.130	0.009	-0.204
Grains	1.435	0.293	0.243	0.361	-0.347	0.053	-0.601
Animal products	0.439	0.084	0.117	0.221	-0.370	0.078	-0.128
Feed ingredients	1.644	0.195	0.210	0.692	-1.339	0.240	0.003
Fertilisers	2.661	-0.044	-0.106	-0.483	0.796	-0.505	0.344
Other raw materials	2.577	0.092	-0.245	-0.002	0.533	0.371	-0.745

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