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Abstract

This paper provides a theoretically consistent approach to estimating demand relationships in which kink points occur either in the interior or on the vertices of the budget set. There are important classes of problems in developing countries which demonstrate such kinked budget sets including binding non-negativity constraints. This paper also extends these methods to the estimation of production structures. As an application a translog cost function for three energy inputs is estimated from cross-sections of individual Indonesian firms.

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1. Introduction

Micro-data sets have become increasingly important in applied work in development economics. This new importance reflects both the changing orientation of development economics and the nature of the data available in developing countries. It has become increasingly recognized that formulating development policy requires information that can only be acquired by modeling and estimating the behaviors of individual economic agents. Areas of research that fall under the heading of "the new household economics," such as the fertility, schooling and health behaviors of households, almost always require micro-data from household surveys to estimate the relationships of interest. Micro-data has also been invaluable in estimating the behavior of farmers (and firms) who can choose among discrete technologies (such as high-yielding seed technologies) but who face a variety of market failures. The class of models known as "agricultural household models" - typically estimated with micro-data - have been critical in understanding the complex behaviors governing households which are both producers and consumers.

Even in those areas of empirical investigation in which time-series data are typically relied upon, the absence of sufficiently long time series in the developing countries has necessitated other empirical approaches. For example, there is a large literature which estimates the industrial demand for energy in the developed countries. Almost all of these estimates make use of either a single time series or time series data pooled by subsector or state/country, [Pindyck, 1979]. The absence of similar data sets for developed countries has precluded the same type of analysis of their production structures. This is unfortunate since energy policy issues in the developing countries are as important as in the industrialized countries. Furthermore, most of the existing econometric estimates may be inapplicable to LDCs since it is likely that their structure of production is significantly different.

Cross-section data can be used to surmount the time series constraint in many instances, but only by exploiting a characteristic of cross-section data peculiar to LDCs. That peculiarity is the substantial spatial variation in prices found in single cross-sections, resulting from poor transportation and distributional infrastructure. This cross-sectional price variation has been used to estimate price elasticities for households in large developing countries where spatial price variability is well known - such as island Indonesia, Timmer (1981) and Lee and Pitt (1987), for example - but also in small countries such as Sierra Leone [Strauss (1982) and (1986)], the Dominican Republic, [Yen and Roe (1986] and the Ivory Coast, (Deaton (1986)).In this paper, we make use of spatial cross-section price variation to estimate a cost function for energy inputs used in manufacturing in a developing country (Indonesia). This is the first attempt we know of to estimate a manufacturing cost function from a single price-varying cross-section.

One of the great impediments to using cross-section data from developing countries in econometric research has been the lack of an unrestrictive and theoretically consistent approach to dealing with a common attribute of these data, kink points in the budget sets of consumers or iso-costs sets of firms. These kink points arise quite frequently from binding non-negativity constraints on inputs or outputs in a multiple input/multiple output production technology or from binding non-negativity constraints on the demands of consumers. Ignoring kink points in the data will result in biased estimates. For the case of corner solutions in demand system estimation, the application of standard systems estimators or Tobit estimation will, for systems with more than two goods, result in biased estimates since they fail to consider that consumers response to price depends on the set of goods it consumes at corners. Furthermore, excluding from the sample those observations in which kink points are observed is likely to result in sample selection bias. Recent papers by Wales and Woodland (1983) and Lee and Pitt (1986) have proposed methods for dealing with the estimation of consumer-demand systems with binding non-negativity constraints. Wales and Woodland's approach is based upon the Kuhn-Tucker conditions associated wtih a stochastic direct utility function. Lee and Pitt, taking the dual approach, begin with indirect utility function and show how virtual price relationships can take the place of Kuhn-Tucker conditions.

As of yet, these approaches have not been extended to two areas of importance to applied development economics - binding non-negativity constraints on the inputs and outputs of firms/farms, and kink points that exist in the interior (as opposed to the vertices) of budget or iso-cost sets. The significance of the extension to firms/farms is implied by the importance of agricultural household models in development literature and policy formulation, and by the lack of long time series on the behaviors of firms/farms. In this paper, we extend the earlier work of Wales and Woodland and ourselves on estimating consumer demands with binding non-negativity constraints to the problems of estimating the production structure of firms and farms. As an application of our methods, we estimate a translog energy cost function for two Indonesian manufacturing subsectors with a sample of firms many of whom do not consume one or more fuels.

Generalizing our methods to the problem of estimating demand relationships in which kink points occur in the interior of budget or iso-cost sets is one which is particularly important in the developing countries. The prevalence of such kink points in developing countries is simply a reflection of the continued popularity of market interventions which create "dual" markets for goods and outputs. LDC consumers commonly face dual markets as a result of food rationing systems or "fair price" shops which offer them articles at subsidized prices but in limited quantities. Consumption in excess of these quantities must be purchased in the free (unsubsidized) market. Such systems exist or have existed in almost all of the large developing countries - India, China, Pakistan, Bangladesh, Egypt and Indonesia - and in dozens of smaller ones. Food stamp systems such as found in Sri Lanka, Trinidad and Tobago and Colombia put kinks in consumers budget sets in very much the same manner. Existing econometric work involving dual markets is limited and not altogether satisfactory, partly due to the lack of econometric methods consistent with such kink points.

Kinks occur in the producers optimization problem in a large variety of instances as well. Import licensing and quotas and the rationing of intermediate inputs (including fuels and electricity), and the resulting illegal (black) markets in rationed goods, are still widespread in the developing world. In many LDCs agricultural input, output and credit markets are often targets of government intervention that results in dual markets. In Brazil, quotas on the sale of sugar cane extend down to the level of individual cultivators. The forced sales of agricultural outputs to the state at below free market prices have at one time or another been features of India, Indonesia and many African nations. Modern inputs, such as fertilizer, have often been offered at "subsidized" prices but in limited amount to cultivators who must enter the free market for additional input beyond their ration.

In this article, we present a theoretically consistent approach to dealing with kink points facing both consumers and producers. This paper extends our earlier work on binding non-negativity constraints in the consumer's problem to the study of convex budget sets and to the estimation of production technologies and behaviors. As an application of our methods, we estimate a translog cost function for energy inputs using firm-level data from the Indonesian weaving and metal products sectors. The methods developed are applicable to a wide range of issues in applied development economics and to the cross-section microdata most offen available in LDCs and used in research in applied economic development. The paper is organized as follows.

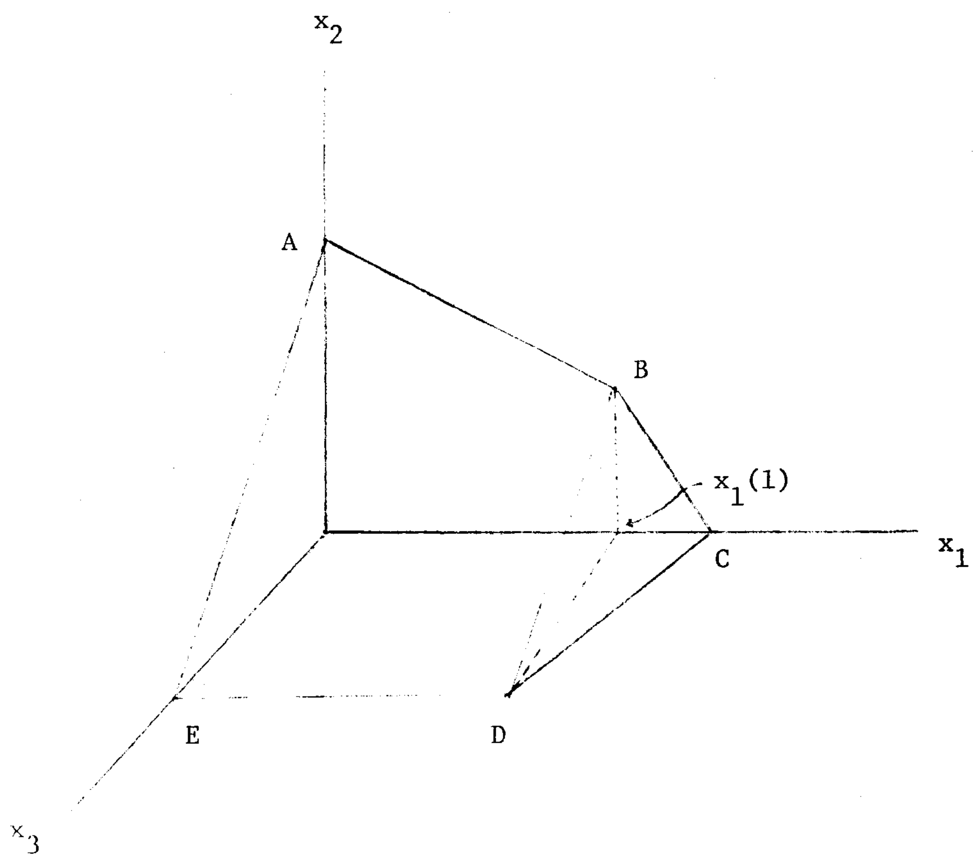
In Section 2 we consider the consumers problem when faced with a convex budget set. In Section 3 we derive econometric specifications of consumer demand systems derived from stochastic formulations of the primal (direct utility function) problem and dual (indirect utility function) problem respectively. Section 4 extends our kink point analysis to the case of production economics. As an application of those methods, a translog energy cost function for Indonesia is estimated and discussed in Section 5. Section 6 summarizes our results.

2. Convex Budget Sets in Consumer Demand

Convex budget sets result naturally from binding non-negativity constraints but also from quantity rationing and increasing block pricing. All of these sources of convexity can be analyzed within a common framework. A simple three goods case with increasing block prices for the commodity x_1 is illustrated in Figure 1. The marginal unit price for quantities of x_1 less than or equal to $x_1(1)$ is p_{11}, and p_{12} (with $p_{12} > p_{11}$) for quantities greater than $x_1(1)$. With income M, the budget plane ABDE is determined by $p_{11}x_1 + p_2x_2 + p_3x_3 = M$ and the budget plane BCD is based on $p_{12}x_1 + p_2x_2 + p_3x_3 = M + (p_{12} - p_{11})x_1(1)$. The point $x_1(1)$ is a kink point for good 1 and so are the non-negativity constraints. Quantity rationing with upper ration limit $x_1(1)$ can be regarded as the special case of $p_{12} = \infty$.

Figure 1

Three-goods case with increasing block price on x_1



In the general multicommodity case, every commodity may be subject to increasing block pricing. For commodity j, assume there are I_j ($I_j > 1$) different block prices $p_{j1} < p_{j2} < \ldots < p_{jI_j}$ corresponding to the kink points $x_j(1), \ldots, x_j(I_j - 1)$ where $x_j(i) < x_j(i + 1)$ for $i=1,\ldots,I_j - 2$. The case $I_j = 1$ is the standard single price situation. If $x_j(I_j - 1)$ is the quantity upper limit for commodity j, $p_{jI_j} = \infty$ for quantity rationing. For notational simplicity, we adopt the conventions $x_j(0) = 0$ and $x_j(I_j) =$ [illegible]

Let $U(x_1,\ldots,x_m)$ be a utility function which is continuously differentiable, increasing and strictly quasi-concave. The utility maximization problem is

$$\max_{x_1,\ldots,x_m} U(x_1,\ldots,x_m)$$

subject to

$$\sum_{j=1}^{m} \sum_{i \in K_j} p_{ji}\, x_{ji} \leq M,$$

$$0 \leq x_{ji} \leq x_j(i) - x_j(i-1) \equiv \bar{x}_j(i) \qquad i \in K_j,\ j=1,\ldots,m \tag{1}$$

$$x_j = \sum_{i \in K_j} x_{ji}$$

where $K_j = \{0,1,\ldots,I_j\}$ is the set of integers describing the kink points for product j and x_{ji} is defined as the purchase of product j in block i.

For econometric analysis it is necessary to determine the conditions under which an optimal solution would occur at each demand regime, given the values of the explanatory variables. For two goods cases, these conditions are readily obtained diagrammatically. Burtless and Hausman (1978) and Hausman (1979) have characterized the optimal solution based on the location of indifference curves for the two goods case. More recently, Hausman and Ruud (1984) describe the case of a three goods model of family labor supply. However, as we will demonstrate below, optimality can be simply characterized by Kuhn-Tucker conditions or with virtual prices, even for the general problem of (1). This analysis generalizes the approach in Wales and Woodland (1983) and Lee and Pitt (1986) for non-negativity constraints with either the direct utility or dual approaches to the convex case.

The Kuhn-Tucker conditions for the problem (1) are

$$\frac{\partial L}{\partial x_{ji}} = \frac{\partial U(x)}{\partial x_j} - \mu p_{ji} - \lambda_{ji} \leq 0 \leq x_{ji}, \qquad \frac{\partial L}{\partial x_{ji}} x_{ji} = 0, \tag{2}$$

$$\frac{\partial L}{\partial \mu} = M - \Sigma_j \Sigma_i \, p_{ji} \, x_{ji} \geq 0 \leq \mu, \qquad \frac{\partial L}{\partial \mu} \mu = 0, \tag{3}$$

$$\frac{\partial L}{\partial \lambda_{ji}} = \bar{x}_j(i) - x_{ji} \geq 0 \leq \lambda_{ji}, \qquad \frac{\partial L}{\partial \lambda_{ji}} \lambda_{ji} = 0 \tag{4}$$

where L is the Lagrange function and μ and λ's are Lagrange multipliers. Because of the block pricing system where $0 < p_{j1} < p_{j2} < \ldots$, purchases will always be made in lower price blocks before higher price blocks. Hence, if $x_{ji} > 0$, $x_{j\ell} = \bar{x}_j(\ell)$ for all $\ell < i$, and that if $x_{ji} = 0$, $x_{j\ell} = 0$ for all $\ell > i$. Thus the demand for good j is

$$x_j = \Sigma_{\ell=1}^{i_j} x_{j\ell}$$

where i_j is the highest integer for which $x_{ji_j} > 0$.

Let x* be a demanded quantity vector such that

$$x_j^* = 0, \qquad j \in J_1$$

$$x_j^* = x_j(i_j), \qquad j \in J_2 \tag{5}$$

$$x_j(i_j - 1) < x_j^* < x_j(i_j), \qquad j \in J_3$$

for some i_j, $j \in J_2 \cup J_3$ where J_1, J_2 and J_3 are some partition of the set $\{1,2,..,m\}$.

Define the virtual prices at x* as

$$\xi_j(x^*) = \frac{\partial U(x^*)}{\partial x_j} / \mu. \tag{6}$$

where $\mu > 0$ follows from assumed strictly increasing property of the utility function. It follows from the Kuhn-Tucker conditions (2) - (4) that

$$\xi_j(x^*) \leq p_{j1} \qquad j \in J_1,$$

$$p_{ji_j} \leq \xi_j(x^*) \leq p_{ji_j+1} \qquad j \in J_2, \tag{7}$$

$$\xi_j(x^*) = p_{ji_j} \qquad j \in J_3.$$

The price $\xi_j(x^*)$ is known as the virtual price for good j at the quentity x*, or also as its shadow price [Rothbarth (1941)]. The kink point $x_j(i_j)$, is the quantity demanded for good j, $j \in J_2$ because the block price p_{ji_j} for good j is less than $\xi_j(x^*)$ and therefore the consumer buys as much of the good as permitted under p_{ji_j}, but the second block price p_{ji_j+1} is sufficiently high so that the consumer does not wish to purchase any more. If $x_j(i_j)$ is purely an upper limit rationed amount, optimality at the rationed limit will be characterized by

$$p_{ji_j} \leq \xi_j(x^*)$$

since $p_{12} = \infty$ for the rationed case. The goods x_j, $j \in J_3$ are purchased at the quantities x_j^* such that their virtual prices equal market prices. The use of the concept of virtual prices is well known in the quantity rationing literature, e.g., Rothbarth (1941), Neary and Roberts (1980) and Deaton (1981) and in the nonlinear tax studies, Burtless and Hausmann (1978).

3. Econometric Model Specification

In their treatment of binding non-negativity constraints, Wales and Woodland (1983) have considered the specification of a direct random utility function and derive its likelihood function through the Kuhn-Tucker conditions. Lee and Pitt (1986) have pointed out that the dual approach, which specifies an indirect utility function or a system of demand equations, is also feasible, because the Kuhn-Tucker conditions can be represented by virtual prices.

For the general convex budget problem (1), the likelihood function can be derived with the aid of the virtual price characterization in (7). Suppose that $D_i(p,M;\varepsilon)$ $i=1,\ldots,m$ are the specified stochastic (notional) demand functions, which are solutions to the utility maximization problem $\max\{U(x) \mid p'x = M\}$. The stochastic utility function $U(x)$ corresponds to the utility function in our problem in (1). Consider the demand vector x^* in (5) where $J_1=\{1,2,\ldots,\ell_1-1\}$, $J_2=\{\ell_1,\ldots,\ell_2-1\}$ and $J_3 = \{\ell_2,\ell_2+1,\ldots,m\}$. The virtual prices and the virtual income c which support x^* are characterized by the inequalities (7) and the demand relations

$$0 = D_j(\xi_1, \ldots, \xi_{\ell_2-1}, p_{\ell_2 i_{\ell_2}}, \ldots, p_{mi_m}, c; \varepsilon) \qquad j=1, \ldots, \ell_1-1 \quad (8)$$

$$x_j(i_j) = D_j(\xi_1, \ldots, \xi_{\ell_2-1}, p_{\ell_2 i_{\ell_2}}, \ldots, p_{mi_m}, c; \varepsilon) \qquad j=\ell_1, \ldots, \ell_2-1 \quad (9)$$

$$x_j^* = D_j(\xi_1, \ldots, \xi_{\ell_2-1}, p_{\ell_2 i_{\ell_2}}, \ldots, p_{mi_m}, c; \varepsilon) \quad j=\ell_2, \ell_2+1, \ldots, m \quad (10)$$

where $c = M + \Sigma_{j=\ell_1}^{m} \Sigma_{\ell=1}^{i_j-1}(p_{j\ell+1}-p_{j\ell})\, x_j(\ell) + \Sigma_{j=\ell_1}^{\ell_2-1} (\xi_j-p_{ji_j})\, x_j(i_j)$. These equations provide an implicit function from the disturbance vector ε to the vector $(\xi_1, \ldots, \xi_{\ell_2-1}, x_{\ell_2}^*, x_{\ell_2+1}^*, \ldots, x_{m-1}^*)$. Since the demand vector x^* lies on a budget plane the equation x_m^* is functionally dependent on the other equations and is redundant. Given a joint density function for ε, the equations (8) - (10) imply a joint density function for $(\xi_1, \ldots, \xi_{\ell_2-1}, x^*_{\ell_2}, x^*_{\ell_2+1}, \ldots, x^*_{m-1})$. Let $g(\xi_1, \ldots, \xi_{\ell_2-1}, x^*_{\ell_2}, x^*_{\ell_2+1}, \ldots, x^*_{m-1})$ denote the implied joint density function. It follows that the likelihood function for this observation is

$$\left(\prod_{j=\ell_1}^{\ell_2-1} \int_{p_j i_j}^{p_j(i_j+1)}\right) \cdot \left(\prod_{j=1}^{\ell_1-1} \int_0^{p_{j1}}\right) g(\xi_1, \ldots, \xi_{\ell_2-1}, x^*_{\ell_2}, \ldots, x^*_{m-1})$$

$$d\xi_1 \ldots d\xi_{\ell_2-1}. \qquad (11)$$

where $(\Pi \int)$ denotes multiple integrals.

There may be various ways to introduce the disturbances ε into a demand system. A possible strategy is to assume that some parameters are stochastic, e.g. Burtless and Hausman (1978). Additive disturbances may not necessarily be compatible with random utility maximization. Given a functional form for the notional demand equations, enough disturbance components need to be introduced such that any possible observed demand vector x^* can be realized by some values of ε; i.e., (8) - (10) have solutions for ε. It is also desirable to introduce enough disturbance components such that the density functions g do not degenerate on lower dimensional spaces. Depending on the specified functional forms and the disturbances, the likelihood function (11) may involve multiple integrals. The evaluation of the likelihood function may be cumbersome and expensive for integrals of more than two dimensions. In Lee and Pitt (1986), we have investigated some stochastic specifications which may result in computationally tractable likelihood functions.

The basic feature of this model is that it assigns a positive probability to observing consumption at a kink. The model is thus well suited to the case of non-negativity constraints where zero consumption is frequently observed in micro-data. In the case of block pricing, observed data may not reveal accumulations of observations at the boundaries of price blocks. This would suggest that another disturbance, such as a measurement error, needs to be included in the model. This additional disturbance will further complicate the likelihood function. This issue is addressed by Burtless and Hausman (1978) and Hausman (1985) for the two goods case.

4. Production Analysis

Our analysis, which has until now focused on consumer demand models, can be extended to the analysis of production technologies. Kink points may occur because of binding non-negativity constraints on inputs or outputs in a multiple input or multiple output technology. Production quotas or the quantity rationing of inputs will also create kink points. Increasing block prices in inputs or decreasing block pricing of outputs are similar to quantity rationing.

Consider the profit maximization problem subject to quantities constraints:

$$\max_{x,q} p'q - r'x \tag{12}$$

subject to $F(q,x) = 0$, $\bar{q} \geqq q \geqq 0$, $\bar{x} \geqq x \geqq 0$

where x and q are $k \times 1$ and $m \times 1$ vectors of inputs and outputs respectively, and $\bar{x}$ and $\bar{q}$ are the upper quantity limits. The production function F is an increasing function of q's and a decreasing function of x's. Other standard regularity conditions on F such as differentiability and strict quasi-concavity are assumed. To illustrate the construction of virtual prices from the production technology F, let us consider a simple regime with $x^* = (0, x_2^*, \ldots, x_k^*)'$ and $q^* = (\bar{q}_1, q_2^*, \ldots, q_m^*)'$ where the first input is not utilized and the first output is produced at the quota level. The Lagrangean function is

$$L = p'q - r'x + \lambda(0 - F(q,x)) + \phi'q + \psi'x + \delta'(\bar{q} - q) + w'(\bar{x} - x)$$

where ϕ, ψ, δ, amd w are vectors of Lagrangean multipliers. The optimality of this x^* is characterized by the following Kuhn-Tucker conditions:

$$-r_1 - \lambda \frac{\partial F(q^*, x^*)}{\partial x_1} + \psi_1 = 0, \qquad \psi_1 > 0;$$

$$-r_i - \lambda \frac{\partial F(q^*, x^*)}{\partial x_i} = 0, \qquad i=2, \ldots, k;$$

$$p_1 - \lambda \frac{\partial F(q^*, x^*)}{\partial q_1} - \delta_1 = 0, \qquad \delta_1 \geqq 0;$$

$$p_j - \lambda \frac{\partial F(q^*, x^*)}{\partial q_j} = 0, \qquad j=2, \ldots, m$$

$$F(q^*, x^*) = 0, \; q^* \geqq 0, \; x^* \geqq 0. \tag{13}$$

Define the virtual price ξ_{d1} for input 1 and virtual price ξ_{s1} for output 1 at x* as

$$\xi_{d1} = -\lambda \frac{\partial F(q^*, x^*)}{\partial x_1}$$

and

$$\xi_{s1} = \lambda \frac{\partial F(q^*, x^*)}{\partial q_1}$$

Since $\frac{\partial F(q^*, x^*)}{\partial x_1} < 0$ and $\frac{\partial F(q^*, x^*)}{\partial q_1} > 0$, ξ_{d1} and ξ_{s1} are strictly positive. It follows that $\psi_1 = r_1 - \xi_{d1}$ and $\delta_1 = p_1 - \xi_{s1}$. Therefore this regime is characterized by

$$r_1 \geqq \xi_{d1}, \; 0 < x_i^* < \bar{x}_i, \quad i=2, \ldots, k$$

and $p_1 \geqq \xi_{s1}, \; 0 < q_j^* < \bar{q}_j, \quad j=2, \ldots, m.$

Input 1 is not used because the market price for this input is too high and output 1 is produced up to the quota limit because the market price for this output is high enough. This technique can be similarly applied to other regimes.

The case of increasing block prices in inputs can be reformulated into the framework (12). Consider the simple case of a single input x with production function $q = f(x)$. Assume the price of input x is r_1

if the purchased amount is less than $x_1(1)$ but a higher price r_2 for amounts in excess of $x_1(1)$. Hence the cost $c(x)$ is

$$c(x) = r_1 x, \qquad \text{if } x \leqq x_1(1);$$

$$= r_1 x_1(1) + r_2(x - x_1(1)), \quad \text{if } x > x_1(1).$$

The problem $\max_x \{pq - c(x) \mid F(q,x) = 0,\ x \geqq 0\}$ can be rewritten into an identical problem with two perfectly substitutable inputs:

$$\max_{x_1, x_2} pq - r_1 x_1 - r_2 x_2$$

$$\text{subject to } q = f(x_1 + x_2), \quad 0 \leqq x_1 \leqq x_1(1), \quad x_2 \geqq 0$$

As the price of x_1 is less than x_2, x_1 will always be purchased first. x_2 will be purchased only if x_1 has been purchased up to its upper limit $x_1(1)$. $x_1(1)$ is a kink point in this model. If the observed sample is $(q^*, x^*) = (q^*, x_1(1))$. The Kuhn-Tucker conditions for $(q^*, x_1(1))$ will be

$$-r_1 + \lambda \frac{\partial f(x_1(1))}{\partial x} - w = 0, \quad w \geqq 0,$$

$$-r_2 + \lambda \frac{\partial f(x_1(1))}{\partial x} + \psi_2 = 0, \quad \psi_2 \geqq 0,$$

$$p - \lambda = 0$$

$$q^* = f(x_1(1))$$

Hence the optimality of this $(q^*, x_1(1))$ is characterized by

$$r_2 \geqq \xi_d(x^*) \geqq r_1$$

where $\xi_d(x^*) = p \frac{\partial f(x^*)}{\partial x}$ is the virtual price of input x at $x_1(1)$. If the sample observation (q^*, x^*) is $x^* > x_1(1)$, then it will be characterized by $\xi_d(x^*) = r_2$.

Similarly, the decreasing block prices in outputs can also be formulated in the framework (12). Consider a single output case where the output quantity q can be sold at price p_1 if the quantity is less than the specified amount q(1); however, quantities in excess of q(1) can only be sold at a lower price p_2. The revenue function will be

$$R(q) = p_1 q, \qquad \text{if } q \leqq q(1);$$

$$= p_1 q(1) + p_2(q - q(1)), \text{ if } q > q(1).$$

The profit maximization problem $\max_{x,q} \{R(q) - rx \mid q = f(x)\}$ can be rewritten identically as a model with two perfectly substitutable outputs:

$$\max_{q_1, q_2, x} \quad p_1 q_1 + p_2 q_2 - rx$$

$$\text{subject to} \quad q_1 + q_2 = f(x), \quad 0 \leqq q_1 \leqq q(1), \; q_2 \geqq 0.$$

The quantity q(1) is a kink point in this model.

For empirical estimation, either the direct or dual approach can be followed. For dual approach, application of Shephard's lemma or the Hotelling-McFadden lemma provides (notional) input demand and output supply functions. Stochastic elements can be introduced into the production function or profit or cost functions. For the direct approach, the stochastic specification $F(q, x; \varepsilon) = G(q, x) + e^{\varepsilon_1'} q + e^{\varepsilon_2'} x$ will be similar to the stochastic specification in section 3. The marginal productivity is the sum of a deterministic part and a stochastic part. Under the assumption that the disturbances are mutually independent, a computationally tractable likelihood function can similarly be derived. In the following section, we apply our methods to the estimation of three input cost functions where non-negativity constraints are binding for a large proportion of firms.

5. An Application: Estimation of an Energy Cost Function

In this section, we will apply the econometric model set out above to the estimation of a translog energy cost function. The production structure used in deriving energy demand relationships parallels that of Fuss (1977) and Pindyck (1979). First, it is assumed that the production function is weakly separable in energy inputs. Thus the cost-minimizing mix of energy inputs is independent of the mix of other factors. Second, the energy aggregate is assumed homothetic in its components so that cost minimization becomes a two-stage procedure: optimize the mix of fuels which make up the energy-aggregate, capital, labor, materials, and other factors. Here we will only estimate the energy aggregator function from which interfuel substitution elasticities can be derived. The data used in the estimation come from the raw data tapes of the annual industrial surveys of Indonesia. Two cost functions for two different sectors will be estimated and compared. In this study, three fuels are identified: (purchased) electricity, fuel oils and other fuels. All three fuels went unconsumed by a substantial number of firms and many firms consumed only one of the three.

The (unobserved) price index for a unit of energy is the linearly homogeneous translog cost function,

$$\ell n P_E = \alpha_0 + \Sigma_{i=1}^{3} \alpha_i \ell n p_i + \frac{1}{2} \Sigma_{i=1}^{3} \Sigma_{j=1}^{3} \beta_{ij} \ell n p_i \ell n p_j + \Sigma_{i=1}^{3} \varepsilon_i \ell n p_i \quad (14)$$

where the disturbance vector $\varepsilon = (\varepsilon_1', \varepsilon_2', \varepsilon_3')'$ is assumed to be distributed $N(0, \Sigma)$. The linearly homogeneous property in input prices yields parameter restrictions $\Sigma_{i=1}^{3}\alpha_i + \Sigma_{i=1}^{3}\varepsilon_i = 1$ and $\Sigma_{j=1}^{3}\beta_{ij} = 0$, for all i. For normalization, $\sum_{i=1}^{3} \alpha_i = 1$ and $\Sigma_{i=1}^{3} \varepsilon_i = 0$. Symmetry on the β's implies that $\beta_{ij} = \beta_{ji}$ for all i, j. The notional cost shares for the inputs from the Shephard's lemma are

$$s_i = \alpha_i + \beta_i' \ell np + \varepsilon_i \qquad i=1,\ 2,\ 3 \tag{15}$$

where $\beta_i' = (\beta_{i1}, \beta_{i2}, \beta_{i3})$ and $\ell np = (\ell np_1, \ell np_2, \ell np_3)'$. To derive the likelihood function for this model, we need to distinguish different regimes. For three goods models, there are seven demand regimes in total. Broadly, there are three types of regimes; namely, all three inputs are used, only two inputs are used, or only one kind of input is used. For the likelihood function to be well defined, the seven regime probabilities need to sum to one - the model coherency requirement. As was earlier noted, if the underlying production structure satisfies the classical properties, the model will be coherent. The translog cost function, however, does not globally satisfy the concavity property and the model may not be coherent. However, as pointed out by van Soest and Kooreman (1986) for the case of a translog indirect utility function, the derived statistical model may still be coherent for some subset of the parameter space. This is also true in our case. Consider the regime that all inputs are used with observed sample s* where $s_i^{*'} > 0$ for all i=1, 2, 3. This regime is characterized by the conditions:

$$\alpha_1 + \beta_1'\ell n\ p + \varepsilon_1 > 0, \qquad \alpha_2 + \beta_2'\ell n\ p + \varepsilon_2 > 0$$

$$\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)'\ \ell n\ p + \varepsilon_1 + \varepsilon_2 < 1. \tag{16}$$

The likelihood function for this interior observation is

$$f(s_1^* - \alpha_1 - \beta_1'\ \ell n\ p, \quad s_2^* - \alpha_2 - \beta_2'\ \ell n\ p)$$

where the f is the bivariate normal density function for $(\varepsilon_1, \varepsilon_2)$. For the second type of regime, $s^* = (0, s_2^*, s_3^*)$ where both inputs $s_2^* > 0$ and $s_3^* > 0$. The logarithmic virtual price for good 1 at s* is

$$\ln\xi_1 = -(\alpha_1 + \beta_{12}\ln p_2 + \beta_{13}\ln p_3 + \varepsilon_1) / \beta_{11}$$

and the observed second share equation becomes[2]

$$s_2^* = \alpha_2 + \beta_2'\ln p + \varepsilon_2 + \beta_{21}(\ln\xi_1 - \ln p_1)$$

$$= \alpha_2 + \beta_2'\ln p + \varepsilon_2 - \frac{\beta_{21}}{\beta_{11}}(\alpha_1 + \beta_1'\ln p + \varepsilon_1)$$

The regime conditions $\xi_1 \leqq p_1$ and $0 < s_2^* < 1$ are equivalent to

$$\frac{1}{\beta_{11}}(\alpha_1 + \beta_1'\ln p + \varepsilon_1) \geqq 0,$$

$$1 > \alpha_2 + \beta_2'\ln p + \varepsilon_2 - \frac{\beta_{21}}{\beta_{11}}(\alpha_1 + \beta_1'\ln p + \varepsilon_1) > 0. \qquad (17)$$

The set of $(\varepsilon_1, \varepsilon_2)$ values which satisfy the regime conditions (17) will not overlap with the $(\varepsilon_1, \varepsilon_2)$ values in (16) only if $\beta_{11} < 0$. With $\beta_{11} < 0$, The likelihood function for $s^* = (0, s_2^*, s_3^*)$ is

$$\int_{-\infty}^{-(\alpha_1 + \beta_1'\ln p)} f(\varepsilon_1, \bar{\varepsilon}_2(s_2^*, \varepsilon_1))\, d\varepsilon_1$$

where $\bar{\varepsilon}_2(s_2^*, \varepsilon_1) \equiv s_2^* - \alpha_2 - \beta_2'\ln p + \frac{\beta_{21}}{\beta_{11}}(\alpha_1 + \beta_1'\ln p + \varepsilon_1)$. This likelihood function can be simplified as a product of some normal density function and normal probability function. Consider now the regime with $s^* = (0, 0, 1)$ where good 1 and good 2 are not used. The virtual prices of good 1 and good 2 satisfy the relations

$$\begin{pmatrix}\ln\xi_1 \\ \ln\xi_2\end{pmatrix} = \begin{pmatrix}\ln p_1 \\ \ln p_2\end{pmatrix} - \begin{pmatrix}\beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22}\end{pmatrix}^{-1} \begin{pmatrix}\alpha_1 + \beta_1'\ln p + \varepsilon_1 \\ \alpha_2 + \beta_2'\ln p + \varepsilon_2\end{pmatrix}$$

and the regime conditions are

$$\frac{1}{\beta_{11}\beta_{22}-\beta_{12}^2}[\beta_{22}(\alpha_1 + \beta_1'\ln p + \varepsilon_1) - \beta_{12}(\alpha_2 + \beta_2'\ln p + \varepsilon_2)] \geqq 0,$$

$$\frac{1}{\beta_{11}\beta_{22}-\beta_{12}^2}[-\beta_{21}(\alpha_1 + \beta_1'\ln p + \varepsilon_1) + \beta_{11}(\alpha_2 + \beta_2'\ln p + \varepsilon_2)] \geqq 0. \qquad (18)$$

The $(\varepsilon_1, \varepsilon_2)$ values which satisfy the inequalities will not overlap with those in (16) or (17) only if $\beta_{11}\beta_{22} - \beta_{12}^2 > 0$. The likelihood function for $s^* = (0, 0, 1)$ is

$$\int_{-\infty}^{\frac{\beta_{12}}{\beta_{22}}(\alpha_2 + \beta_2'\ell np) - (\alpha_1 + \beta_1'\ell np)} \int_{-\infty}^{\frac{\beta_{21}}{\beta_{11}}(\alpha_1 + \beta_1'\ell np) - (\alpha_2 + \beta_2'\ell np)} g(\varepsilon_1^*, \varepsilon_2^*) d\varepsilon_1^* d\varepsilon_2^*$$

where g is the bivariate normal density function of ε_1^* and ε_2^*, where $\varepsilon_1^* = \varepsilon_1 - \frac{\beta_{12}}{\beta_{22}}\varepsilon_2$ and $\varepsilon_2^* = -\frac{\beta_{21}}{\beta_{11}}\varepsilon_1 + \varepsilon_2$. The likelihood functions for the other regimes can similarly be derived. By symmetric arguments for each pair of inputs, the constraints $\beta_{22} < 0$, $\beta_{33} < 0$, $\beta_{11}\beta_{33} - \beta_{13}^2 > 0$ and $\beta_{22}\beta_{33} - \beta_{23}^2 > 0$ are also necessary for model coherency. Denote $s_i = \alpha_i + \beta_i'\ell np + \varepsilon_i$ for i=1, 2, 3. With the above constraints on the β's, the regime conditions for each regime can be summarized:

<u>regime 1</u>: $(s_i^* > 0,\ i=1, 2, 3)$

$$s_1 > 0,\ s_2 > 0,\ s_1 + s_2 < 1$$

<u>regime 2</u>: $(s_1^* = 0,\ s_2^* > 0,\ s_3^* > 0)$

$$s_1 \leqq 0,\ 1 > s_2 - \frac{\beta_{21}}{\beta_{11}} s_1 > 0.$$

<u>regime 3</u>: $(s_1^* > 0,\ s_2^* = 0,\ s_3^* > 0)$

$$s_2 \leqq 0,\ 1 > s_1 - \frac{\beta_{12}}{\beta_{22}} s_2 > 0$$

<u>regime 4</u>: $(s_1^* > 0,\ s_2^* > 0,\ s_3^* = 0)$

$$s_3 \leqq 0,\ 1 > s_1 - \frac{\beta_{13}}{\beta_{33}} s_3 > 0$$

<u>regime 5</u>: $(s_1^* = 0,\ s_2^* = 0,\ s_3^* = 1)$

$$s_1 - \frac{\beta_{12}}{\beta_{22}} s_2 \leqq 0, \quad - \frac{\beta_{21}}{\beta_{11}} s_1 + s_2 \leqq 0$$

<u>regime 6</u>: $(s_1^* = 0,\ s_2^* = 1,\ s_3^* = 0)$

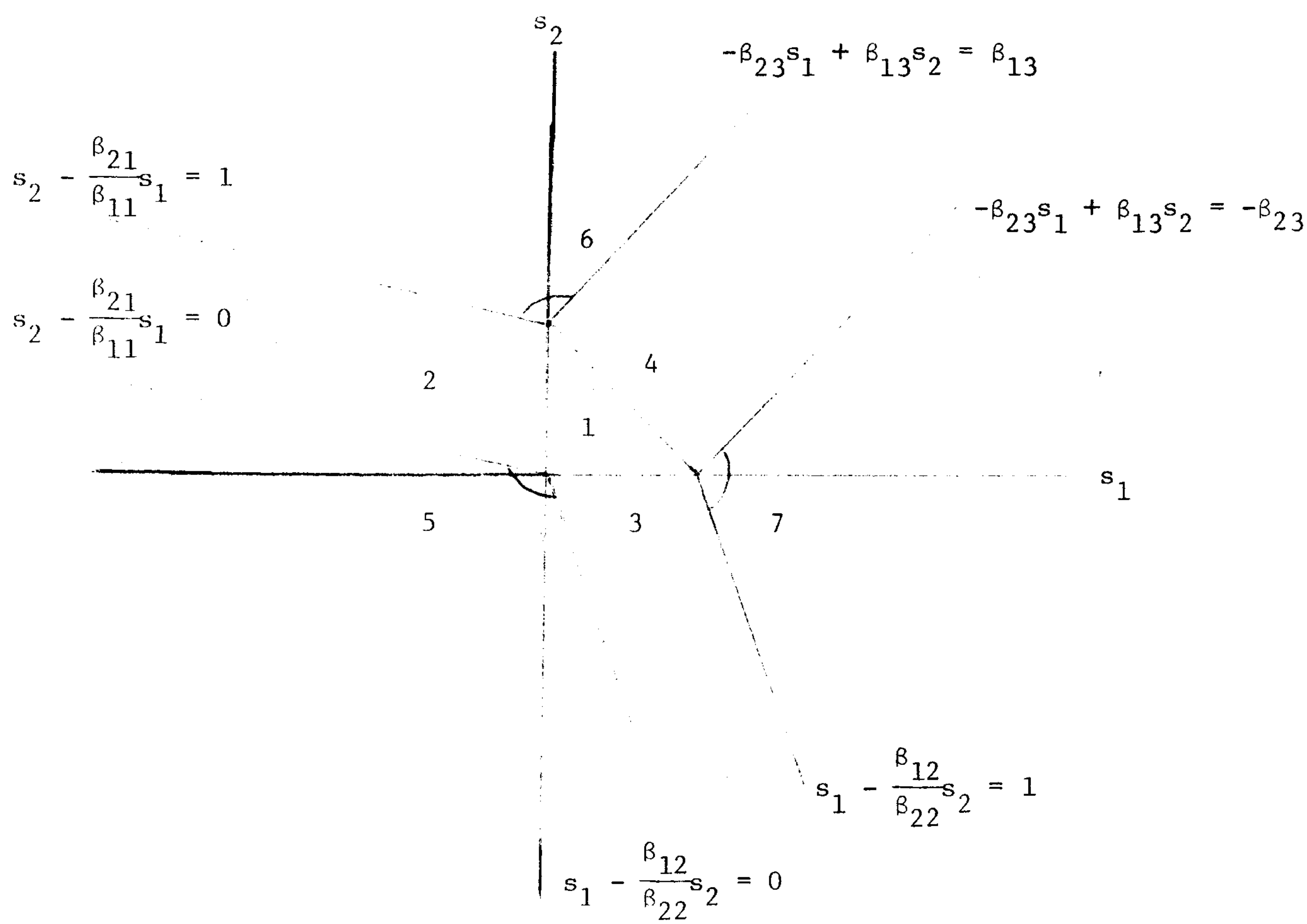
$$s_1 - \frac{\beta_{13}}{\beta_{33}} s_3 \leqq 0, \quad - \frac{\beta_{31}}{\beta_{11}} s_1 + s_3 \leqq 0$$

<u>regime 7</u>: $(s_1^* = 1,\ s_2^* = 0,\ s_3^* = 0)$

$$s_2 - \frac{\beta_{23}}{\beta_{33}} s_3 \leqq 0, \quad - \frac{\beta_{32}}{\beta_{22}} s_2 + s_3 \leqq 0.$$

Since $s_3 = 1 - s_1 - s_2$, all the conditions can also be expressed in terms of s_1 and s_2. The following diagram provides a representation of a coherent model.

Figure 2
Model Coherency



The numbers denote the different regimes.

The data used in the estimation come from the raw data tapes of the 1978 annual industrial surveys of Indonesia (Survey Perusahaan Industri). Two sectors are investigated - fabricated metal products, machinery and equipment (ISIC classification 38) and weaving and spinning (ISIC classification 321). All three fuels were nonconsumed by a substantial number of firms in both sectors.

Not all of Indonesia is electrified and thus firms which are located in areas without electricity may not consume it because of a binding zero ration rather than a negative notional demand. The problem is avoided here by choosing a sample of firms located in large municipalities (kotamadya), all of which are electrified. The nonconsumption of electricity in these cities is treated as the result of firm choice.

A problem in interpreting the results of the energy cost function arises from the transformation of purchased energy inputs into other forms of energy within the firm. For example, a firm which wishes to drive a weaving loom (or most any other piece of mechanical equipment) can do so in any number of ways. It can attach an internal combustion engine to the looms driveshaft, it could heat a boiler which supplies steam to a turbine which in turn drives the loom, or it could drive the loom with an electric motor whose electricity is either purchased or obtained by using fuels to drive an electric generator. All of these methods will provide the force required to drive a mechanical loom but may transform purchased energy inputs into various other forms of energy along the way. In line with other investigators, we treat the within-firm transformation of energy into other energy forms - mechanical, electrical, heat, pressure or otherwise - as part of the production technology itself. Thus, energy input in the cost function is not the force ultimately applied directly to the driveshaft of a weaving loom, but is rather the total quantity of energy used by the firm to achieve the work of the loom. In Indonesia, many firms transform purchased liquid fuels into electrical energy within their plants. Thus, we would expect fuel oils and other petroleum fuels, which are often used to power electric generators as well as prime movers, to be close substitutes for purchased electricity.

Firm specific characteristics, as well as randomness, are allowed to influence energy demands by making the parameters α_i in (15) linear functions of firm characteritics $\alpha_i = \alpha_i' + \Sigma_j \gamma_{ij} z_j$ i=1, 2, 3. The characteristics z_j include the share of the firms equity owed by foreigners, the year the establishment began operation, and the year squared. Foreign ownership is included because foreigners may be less flexible in altering technologies and behaviors in environments that differ from their home country. The year the establishment began operation is included in recognition of the fact that energy use patterns may be somewhat determined by the vintage of capital. Any such effect is unlikely to be linear as older capital equipment is replaced with newer equipment.

Table 1 provides the sample characteristics for the data used in the estimation. Table 2 provides the maximum likelihood estimates of the parameters of the cost function. In estimating the parameters of the cost functions, we impose the restrictions that all the own-price parameters β_{ii} are non-positive. These restrictions are necessary (but not sufficient) for the coherency of the model. Note that for the homothetic translog cost function negative β_{ii} imply elastic own price responses. All the

Table 1

Sample Characteristics

	Weaving and Spinning		Metal Products	
	Mean	S. D.	Mean	S. D.
Foreign share	0.0142	0.1041	0.0554	0.1860
Year started	0.6439	0.0960	0.6423	0.1194
Year squared	0.4238	0.1152	0.4267	0.1327
Electricity share	0.5704	0.3887	0.4168	0.3967
Fuel share	0.2218	0.3297	0.3427	0.3673
Other share	0.2079	0.2736	0.2405	0.2625
Electricity price (ℓn)	3.4026	0.1887	3.4128	0.1661
Fuel price (ℓn)	3.2860	0.0773	3.2663	0.1203
Other price (ℓn)	3.5837	0.0263	2.5061	0.0417
Sample size	362		379	

Table 2

Maximum Likelihood Estimation:

	Weaving and Spinning			Metal Products		
	Electricity	Fuel	Other	Electricity	Fuel	Other
Constant	-0.8946 (3.9558)	0.3096 (1.0973)	1.5850 (2.8970)	1.0754 (0.4514)	0.1988 (0.5194)	-0.2742 (0.2731)
Foreign share	-0.6989 (0.2531)	0.2399 (0.2635)	0.4590 (0.2255)	-0.1492 (0.2000)	0.0717 (0.1600)	0.0775 (0.1100)
Year started	6.1451 (13.0409)	-3.0633 (2.7780)	-3.0819 (10.2987)	2.8072 (1.5507)	-2.1422 (1.7446)	-0.6650 (0.9580)
Year squared	-6.0729 (10.6221)	3.9524 (1.8146)	2.1205 (8.8311)	-4.5769 (1.3365)	3.6135 (1.5070)	0.9634 (0.8734)
β_{i1}	-0.5453 (0.1617)	0.2251 (0.1949)	0.3202 (0.1433)	-1.0767 (0.1928)	0.5688 (0.1745)	0.5079 (0.1158)
β_{i2}	0.2251 (0.1949)	-0.4256 (0.4167)	0.2005 (0.2477)	0.5688 (0.1745)	-0.7000 (0.1929)	0.1313 (0.0970)
β_{i3}	0.3202 (0.1433)	0.2005 (0.2477)	-0.5207 (0.2186)	0.5079 (0.1158)	0.1313 (0.0970)	-0.6392 (0.1345)
σ_i^2	0.3872 (0.0782)	0.4338 (0.0787)	0.1555 (0.0212)	0.3950 (0.0514)	0.2916 (0.0364)	0.1223 (0.0127)
ρ_{12}	-0.8119 (0.0429)			-0.8313 (0.0259)		
ℓn likelihood =	-514.5847			-476.9935		

estimated β_{ii} in both cost functions are indeed negative and not on or near the zero boundary. We confirmed the coherency of both of our estimated cost functions in a manner similar to Figure 2. As these fuels are expected to be close substitutes, the negativity of the β_{ii}'s is not surprising. However, estimation of demands among goods which are not close substitutes may result in the incoherency of the stochastic model, thus limiting the usefulness of this approach. The elasticities of Table 3 suggest that price policies which relatively tax or subsidize one of these fuels relative to the others will have very large consequences on their relative demands.

Our price elasticities are slightly larger than most of those reported in the literature. Almost all the existing estimates of these types of energy price elasticities are for the industrialized countries. We are not aware of any other estimates of partial fuel price elasticities for the manufacturing sector of another LDC. Pindyck (1979) has estimated partial fuel price elasticities using a time series of industrial country cross-sections. The fuels he identified were electricity, oil, gas and coal. Industrial partial fuel own-price elasticities were as large as -.16 for electricity, -1.1 for oil, -2.31 for gas and -2.17 for coal. Mount, Chapman and Tyrrell (1973) have estimated electricity elasticities as high as -1.20 for the U.S. Halvorsen (1976) has reported a partial price elasticity for oil of -2.75 for U.S. industry. Using a panel on LDC total energy demands, Pindyck found fuel oil elasticities as large as -2.89. Thus, our Indonesian estimates are not out of line with the largest of those reported earlier. We would expect our elasticities to be larger than those for industrialized countries. Indonesian firms have chosen technologies that do not rely as heavily on industrial

Table 3

Elasticities and Firm Effects:
Weaving and Spinning

	Electricity	Fuel	Other
Price elasticities	-1.9561	1.0148	1.5406
	0.3946	-2.9188	0.9645
	0.5615	0.9039	-3.5051
Firm effects[a]			
Foreign share	-1.225	1.081	2.208
Year started	10.77	-13.81	-14.83
Year squared	-10.65	17.82	10.20

Metal Products

	Electricity	Fuel	Other
Price Elasticities	-3.5835	1.6596	2.1119
	1.3647	-3.0426	0.5458
	1.2188	0.3830	-3.6576
Firm effects[a]			
Foreign share	-0.3581	0.2091	0.3225
Year started	6.736	-6.251	-2.765
Year squared	-10.98	10.54	4.005

a. $\partial \ell n x_i / \partial z$

machinery driven by purchased electricity. The wide-spread use of machinery driven by prime movers, as opposed to electric motors, and the installed capacity to produce electricity in-plant, make petroleum fuel and purchased electricity closer substitutes than in the industrialized countries.

The asymptotic t-values presented in Table 2 suggest that foreign ownership significantly affects the shares of electricity and other fuels in the weaving and spinning sectors but none of the fuel shares in the metal products sector. Vintage effects captured by the "year started" variable tend to have greater statistical significance in the metal products sector than in weaving and spinning. Table 3 provides derivatives of input quantities with respect to foreign ownership, "year started" and its' square.

6. Summary and Conclusion

This paper extends our earlier work (and that of Wales and Woodland, (1983)) on estimating consumer demand systems with binding non-negativity constraints in two directions. First, we generalize our methods to the problem of estimating demand relationships in which kink points occur in the interior (rather than the vertices) of the budget set. There are important classes of problems in developing countries which demonstrate such kinked budget sets. This generalization differs from the work of Hausman (1985) on convex budget sets in that Kuhn-Tucker conditions are directly utilized which simplifies the analysis in certain situations. These kink points are caused by market failure and incompleteness often resulting from the direct intervention of the state in allocating resources.

This paper also extends our methods on binding non-negativity constraints to the estimation of production structures. As an application of our methods, a translog cost function for three energy inputs is estimated from cross-sections of individual firms. These fuels are thought close substitutes making it more likely that coherency conditions are fulfilled. The results of the estimation confirm both the close substitutability of fuel inputs and the coherency of the model.

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Footnotes

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1. One can, of course, show that the conditions in Hausman (1979) are mathematically equivalent to the Kuhn-Tucker conditions. This can be done, for example, by applying the theorems found in the appendix of Lee (1986).

2. For $\beta_{ii} \neq 0$ it is necessary that cost shares respond to own prices.