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ABSTRACT

Governments often establish economic policy in response to political pressure by interest groups. Since these groups' political activities may alter prices, economies so affected cannot be characterized by perfect competition. We develop a model of a "lobbying economy" in which consumers' choice of political activity simultaneously determines relative prices and income levels. They balance the loss in income due to lobbying payments against the potential gain in wealth from a favorable government price policy. This paper proves the existence of an equilibrium in economies of this sort. We reformulate the economy as a generalized lobbying game and prove the existence of a non-cooperative equilibrium in the game. This equilibrium is then shown to be an equilibrium in the economy.

Keywords: Political economy, rent seeking, generalized game, lobbying equilibrium.

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1. INTRODUCTION

Governments exhibit an enduring willingness to intervene in economic markets. In a pioneering study, Tullock (1967) pointed out that such interventions often create rents which aren't dissipated by the usual competitive forces. He suggested that agents will lobby to influence government policy, and thereby obtain these rents. This line of reasoning lies at the center of the large and growing literature on the theory of rent-seeking and political economic behavior.

Much of the work in this area is built upon models of trade between nations. This is natural for at least two reasons. First, trade is widely distorted by governments, and much is known about modeling the impacts of distortionary trade policy on the economy. Second, when governments intervene in markets, equilibrium prices do not obtain, and domestic markets may not clear. It is sensible to study the affect of such distortions in the context of a model where a market-clearing mechanism, in the form of international trade, is at hand. Hillman and Ursprung (1988), Young and Magee (1986), Anderson and Hayami (1986), Bhagwati and Srinivasan (1980), Bhagwati (1982), and Krueger (1974) combine the results of standard trade theory with some representation of political behavior to discover how rent-seeking behavior might arise and how it affects the economy.

One drawback to much of this literature is that it is essentially macroeconomic in focus. While the models are effective in illustrating the effect of certain policies on the domestic and international economies, they obscure the political choice problem of individuals. In contrast, recent literature on public choice treats this choice problem explicitly in the context of bidding games. These studies offer valuable insights into the relationship between lobbying productivity and the willingness to lobby (Tullock (1980); Applebaum and Katz (1986)). Unfortunately, the relationship of these games to recognizable economies may be difficult to discern, as they proceed in the absence of prices, goods markets, and preferences.

An unanswered question is whether it makes sense, in the context of an equilibrium-based economy, for people to lobby. In this paper we address the problem faced by agents who must choose how to enter the economic market for goods and services, while at the same time determining their willingness to "lobby" in order to influence their economic environment through the political market.

Our model consists of a pair of traders with preferences over two goods, with which they are asymmetrically endowed, and a government which establishes a relative price in the economy in response to lobbying contributions.[1] Each agent in the economy takes the government's pricing rule and the level of his or her opponent's lobbying expenditures as given and chooses a lobbying level and a consumption bundle. He or she must balance the loss in income due to lobbying payments against the potential gain in wealth from an advantageous price movement.

Once the government has set a price in response to lobbying, markets needn't clear. To sustain the mandated price in the face of this

[1]In the interest of brevity, and for lack of a better name, this activity will be called "lobbying." Clearly, real lobbying behavior is hopelessly more complex than this. The term "government" is not entirely satisfying either, but is used for expositional ease in spite of the fact that no real government behaves in exactly this manner.

disequilibrium, we introduce a world market with which the government may trade, at some cost, in order to clear the domestic markets. While there are alternative means of handling disequilibrium situations (e.g., quantity rationing schemes (Benassy, 1982)), the choice here of a trade mechanism is motivated by observed phenomena and by the literature cited above. In order to avoid the free resource problem of unlimited trading in the world market, a feasibility restriction is imposed. The government has "revenue" equal to lobbying donations. Its "costs" are those incurred in its trading operation. Feasibility requires that these costs do not exceed government's revenue.

The purpose of this paper is to show that the lobbying economy possesses an equilibrium. An equilibrium in the lobbying economy is a set of allocations and a pair of lobbying levels where agents optimize in Nash's sense and the government's activity is feasible. First, we respecify the economy as a generalized game in which agents choose optimal responses to each other, but in which the concept of government feasibility is ignored. Restrictions on the lobbying economy are specified which are sufficient for the associated lobbying game to have a non-empty generalized Nash equilibrium set. Then we show that any such equilibrium is also government feasible. This establishes the existence of an equilibrium in the lobbying economy.

Of possible independent interest is a non-convexity which arises in the model. The choice sets of agents, given that they may explicitly influence the relative price, are inherently non-convex. This problem is circumvented by a reformulation of the game as a two-stage optimization problem in which optimal consumption choice is made implicit, and the indirect utility function coincides with the payoff function in the game. While the resulting choice set is convex, this approach introduces the possibility that the payoff function is not quasi-concave. However, a restriction placed upon preferences rules out such non-quasi-concavity.

The model we specify is clearly a drastic simplification of actual lobbying activity. No political markets, voting, or constitutional considerations appear. The "government" simply sets prices according to a function of lobbying donations, and then transacts on outside markets to sustain the price. While this model is simple, something very much like it occurs in U.S. agricultural policy. The government distorts prices (by setting price floors, etc.) and these distortions lead to a notorious oversupply of agricultural commodities. In some markets, the government buys the excess supply at the legal price. Agents are guaranteed the announced price, and are free to make economic decisions based upon this guarantee. Moreover, the prices in these laws are the subject of determined and energetic lobbying efforts each time they are established (see, e.g., Krueger (1988)). While the fixed supply in our model takes us a bit away from the example, it seems that the essential points are illuminated by it.

The remainder of the paper is organized as follows. In section 2 the economic model is specified and developed, and the concept of economic equilibrium is defined. A generalized game is derived from the economic model and a lobbying game equilibrium is defined in section 3. The existence theorem for game equilibria is stated and proved in section 4. In section 5 we prove the existence of a lobbying equilibrium by proving that the game equilibrium of section 4 is feasible for the government. Concluding comments appear in section 6.

2. THE LOBBYING ECONOMY $\mathcal{E}$

The economy under examination is a two-agent exchange economy with two traded goods. Goods are labelled 1 and 2; agents are indexed by $i \in I = \{1,2\}$. Throughout, subscripts denote traders, while superscripts denote commodities. Consumption sets X_i are taken to be $\mathbb{R}^2_+$, the non-negative quadrant in Euclidean space. Agent i's preference relation $\succsim_i$ is a subset of the Cartesian product $X_i \times X_i$, and is assumed to satisfy:

(A1) For each $i \in I$, the preference relation $\succsim_i$ admits a continuous utility function $U_i : X_i \longrightarrow \mathbb{R}$ that is strictly quasi-concave and strictly increasing on $\mathrm{int}(X_i)$ and such that for all x,y in X_i, $U_i(x) \geq U_i(y)$ if and only if $x \succsim_i y$.

At times, we will have use for the following additional restriction on preferences:

(A2) (*r-Differentiability*). The preference relation $\succsim_i$ is said to be r-differentiable if it admits a utility function U_i whose rth-order derivatives all exist.

Assumption (A2) rules out indifference curves with kinks when $r \geq 1$. When $r \geq 2$, demand functions are differentiable, a fact which will be useful below. Unless otherwise specified, r is assumed to equal two.

Agent i is assumed to be endowed only with the ith good. Henceforth, $\omega^i > 0$ will denote the finite scalar value of i's endowment, while ω_i will be used to denote the pair in $\mathbb{R}^2_+$ satisfying $\omega_i^i = \omega^i$ and $\omega_i^{-i} = 0$.[2] A *price vector* is a pair $(p^1,p^2) \in \mathbb{R}^2_{++}$. This exchange economy, in which consumers treat prices parametrically, underlies the lobbying economy. A *competitive equilibrium* is a pair of allocations $(x_i^{1*},x_i^{2*})_{i=1,2}$ and a price vector

[2]This notation is interpreted as $y^{-i} = (y^1,\ldots,y^{i-1},y^{i+1},\ldots,y^n)$ for any n-dimensional vector y.

$p^* = (p^{1*}, p^{2*})$ such that i) agents optimize and ii) markets clear. A standard result from general equilibrium theory guarantees that economies satisfying (A1) and the condition $(p^1, p^2) \in \mathbb{R}^2_{++}$ have non-empty equilibrium sets (Debreu, 1959).

Given a price vector p, let agent i's income be defined as $p \cdot \omega_i$. The *budget set* of agent i is given by $\beta_i(p, p\cdot\omega_i) = \{x \in X_i : p\cdot x \le p\cdot\omega_i\}$. Agent i's *demand*, which maps a price-income pair into a subset of $\beta_i(p, p\cdot\omega_i)$, is given by

$$x_i(p, p\cdot\omega_i) = \{x \in \beta_i(p, p\cdot\omega_i) : \text{for each } k \in \beta_i(p, p\cdot\omega_i),\ x \succsim_i k\}.$$

Under assumption (A1), x_i is a function. When $\succsim_i$ is r-differentiable, $x_i(p, p\cdot\omega_i)$ is (r-1)-differentiable. Agent i's *excess demand* is given by $z_i(p, p\cdot\omega_i) = x_i(p, p\cdot\omega_i) - \omega_i$. *Aggregate excess demand* is simply the sum of individuals' excess demands:

$$z(p) = \Sigma_i\ z_i(p, p\cdot\omega_i).$$

For the purposes of this paper, it is important that the equilibrium price vector p^* be unique. This is assured for exchange economies whenever z(p) is such that for all prices p, all goods are gross substitutes (see, e.g., Arrow and Hahn, 1971, p. 223).

Definition: Two goods, i and j, are *gross substitutes* (GS) at p if $\frac{\partial z^i}{\partial p^j}(p) > 0$ for all $i \neq j$.

It will be assumed that (GS) holds for every price p. Let p^* denote the unique competitive equilibrium price for the undistorted exchange economy.

Let an agent's *characteristic* be given by the pair $a_i = (\succsim_i, \omega_i)$. Let $\mathcal{R}$ be the set of all pairs a_i satisfying (A1), (A2), (GS), and our convention on endowments. The set $\mathcal{R}$ will be called the set of *admissible characteristics*.

Demand functions are easily shown to be homogeneous of degree zero in prices and income. We normalize prices by dividing each p^i by the sum $(p^1 + p^2)$, so that the normalized price vector lies in the one-dimensional simplex $\Delta \subset \mathbb{R}^2_{++}$. In what follows, let $p \in (0,1)$ denote the normalized price of good 1, and let $q = (1 - p)$ denote the price of good 2. A price system for the economy is thus fully specified by the scalar parameter p.

There is, in the background of the economy, a "government" which stands prepared to alter the price in the economy in response to lobbying on the part of consumers. Each consumer may choose to donate a part, η_i, of his or her income to the government to influence the government's price policy. The government is fully specified by the function $p : \mathbb{R}^2 \longrightarrow (0,1)$, given by $p = p(\eta_1,\eta_2)$, by which it sets the price. We will often use the symbol $\mathbf{P}$ to denote the pair $(p,q) \in \Delta$; when it coincides with the government's mandated price, we will write $\mathbf{P}(\eta) = (p(\eta),(1-p(\eta)))$, where $\eta = (\eta_1,\eta_2)$.

The pricing function $p(\eta)$ will be assumed in this paper to satisfy a collection of conditions. The first of these is differentiability.

(A3) The function $p(\eta)$ is $\mathbb{C}^1$.

What's more, if neither agent chooses to lobby, then it is assumed that the government selects the competitive equilibrium price.

(A4) $p(0,0) = p^*$.

Because of the asymmetry of agents' endowments, and under the monotonicity of U_i, Mr. 1 is made better off by an exogenous price increase, while Ms. 2 is made worse off. This divergent interest lends to the model its non-cooperative nature. The following assumption ensures that agents' lobbying has the impact on government policy which they expect, and also that lobbying expenditures don't become more productive at the margin as the level of lobbying increases.

(A5) *(Productive Lobbying)*. $p(\eta_1,\eta_2)$ is strictly increasing and concave (resp. strictly decreasing and convex) in η_1 (resp. η_2).[3]

The final restriction which will be placed on the function $p(\eta_1,\eta_2)$ delivers an upper bound for agents' lobbying activity.

(A6) *(Bounded Lobbying)*. For each agent i, for every η_{-i}, there exists an $\hat{\eta}_i(\eta_{-i}) < +\infty$, depending on η_{-i}, sufficiently large so that $\mathbf{P}(\hat{\eta}_i(\eta_{-i}),\eta_{-i})\cdot\omega_i = \hat{\eta}_i(\eta_{-i})$.

That is, given an η_{-i}, if i chooses to devote $\hat{\eta}_i(\eta_{-i})$ to the government in lobbying expenditures, then none of his or her wealth is left over for purchasing goods. Formally, $\hat{\eta}_i(\eta_{-i}) = \{x \in \mathbb{R}_+ : \mathbf{P}(x,\eta_{-i})\cdot\omega_i = x\}$. By our assumptions on $p(\eta_i,\eta_{-i})$, $\hat{\eta}_i(\eta_{-i})$ is single-valued; that it is a continuous function of η_{-i} follows directly from the continuity of $p(\eta)$ (cf. Debreu (1982), p. 706).

Let $\mathcal{P} = \left\{ p : \mathbb{R}^2 \longrightarrow (0,1) : p(\eta) \text{ satisfies (A3) — (A6)} \right\}$. A generic element $p(\eta)$ of $\mathcal{P}$ is called an *admissible pricing function*. In the remainder of the paper, attention will be restricted to pricing functions defined over $\mathbb{R}^2_+$; let $\mathcal{P}_+$ denote the subset of $\mathcal{P}$ with elements so defined. Allowing $\eta_i < 0$ for some $i \in I$ permits an interesting investigation of tax/transfer schemes; this investigation is undertaken in another paper.

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[3]The concavity restrictions on p prevent the price from "blowing up" in either variable. There are arguments in favor of allowing non-convexities, or more general curvature properties. In this paper, the technical difficulties such non-convexities introduce are avoided in order to focus on more fundamental issues.

Let the set of all lobbying economies be given by the Cartesian product $\mathfrak{E} = \mathcal{R}^2 \times \mathcal{P}$ of admissible characteristics and admissible pricing functions. Let $\mathfrak{E}_+ = \mathcal{R}^2 \times \mathcal{P}_+$ be similarly defined. Henceforth, a lobbying economy, assumed to lie in $\mathfrak{E}_+$, will be denoted $\mathcal{E} = ((\succsim_i, \omega_i)_{i=1,2};\ p(\eta))$.

The optimization program of consumers may now be spelled out. Given an η_{-i}, the set of triples (x_i^1, x_i^2, η_i) in $\mathbb{R}_+^3$ from which agent i may choose is given by

$$\psi_i(\eta_{-i}) = \left\{ (x_i^1, x_i^2, \eta_i) \in \mathbb{R}_+^3 : \mathbf{P}(\eta_i, \eta_{-i}) \cdot (x_i^1, x_i^2) \le \mathbf{P}(\eta_i, \eta_{-i}) \cdot \omega_i - \eta_i \right\}.$$

Given η_{-i}, agent i solves the problem

$$\mathbf{M}_i(\eta_{-i}) \qquad \max_{(x_i^1, x_i^2, \eta_i) \in \psi_i(\eta_{-i})} U_i(x_i^1, x_i^2).$$

Associated with this program is a demand relation different from our $x_i(p, p \cdot \omega_i)$. Given a pair (η_1, η_2), let $\tilde{\omega}^1 = \omega^1 - \eta_1 / p(\eta)$, and let $\tilde{\omega}^2 = \omega^2 - \eta_2 / (1 - p(\eta))$. Let the (after-lobbying) budget set of agent i be given as $\tilde{\beta}_i(p(\eta); \mathbf{P}(\eta) \cdot \tilde{\omega}_i)$. The demand relation of agent i arising from program $\mathbf{M}_i(\eta_{-i})$ may now be defined as $\tilde{x}_i(p(\eta); \mathbf{P}(\eta) \cdot \tilde{\omega}_i)$. After-lobbying excess demand $\tilde{z}_i$ is the difference between $\tilde{x}_i$ and $\tilde{\omega}_i$; $\tilde{z}$ is the sum of $\tilde{z}_i$ over $i \in I$. By our assumptions on preferences and $p(\eta)$, the relations $\tilde{x}_i$, $\tilde{z}_i$, and $\tilde{z}$ are all differentiable functions.

The function $p(\eta)$ is common knowledge; i.e. both agents know $p(\eta)$ with certainty, and they both know that their opponent knows p, etc. Once the rule $p(\eta)$ is announced, the government does nothing further to influence agents' choices. It simply carries through on its promise to enforce the price $p(\eta)$. This is also common knowledge. It does not optimize, and it doesn't choose the function $p(\eta)$ based upon any influence from agents.

Once the price is determined, markets may not clear as a result of trade between agents. An alternative mechanism is employed to deliver a reasonable notion of a feasible equilibrium. It is assumed that the two-agent economy is small relative to a world economy in the two goods. The world price is assumed to equal p^*, and the government carries out trade with the rest of the world without transport cost in order to sustain the prices determined by $p(\eta)$.

We must restrict the model to ensure that the government's market-clearing activity is feasible. The quantity $(\eta_1 + \eta_2)$ is the government's "revenue" in terms of a (non-existent) domestic currency. The "cost" of supporting $p(\eta)$ is $(\mathbf{P}^* - \mathbf{P}(\eta))\cdot\tilde{z}(p(\eta))$. The following definition of feasibility will be employed in our equilibrium definition:

Definition: Given a lobbying economy $\mathcal{E}$, the 6-tuple $(x_i^1, x_i^2, \eta_i)_{i=1,2}$ is *government feasible* if $\pi(\eta) = (\eta_1 + \eta_2) - (\mathbf{P}^* - \mathbf{P}(\eta))\cdot\tilde{z}(p(\eta)) \geq 0$.

Note that if $\eta_1 = \eta_2 = 0$, then $p^* = p(\eta_1, \eta_2)$, so that $\pi(0,0) \equiv 0$.

We are now in a position to define our equilibrium concept for the lobbying economy.

Definition: Given a lobbying economy $\mathcal{E}$, a *lobbying equilibrium*, denoted $LE(\mathcal{E})$, is a 6-tuple $(x_i^{1*}, x_i^{2*}, \eta_i^*)_{i=1,2}$ satisfying:

i) for each i, $(x_i^{1*}, x_i^{2*}, \eta_i^*)$ solves $M_i(\eta_{-i}^*)$; and

ii) $(x_i^{1*}, x_i^{2*}, \eta_i^*)_{i=1,2}$ is government feasible.

We may now proceed to a specification of the game which derives naturally from this economy. In the following section we first formulate the economic model as a generalized game. Then, we study the equilibrium characteristics of the game and relate its equilibria to equilibria in the underlying economy.

3. THE LOBBYING GAME Γ_ε

The central defining characteristic of a game is the dependence of individual players' payoffs on the strategies of all players. A generalized game displays the additional property that players' strategy *sets* are affected by their opponents' strategies. The game which emerges naturally from the lobbying economy is a generalized game.

Let $I = \{1,2,\ldots,I\}$ denote a set of players of a game Γ. Let their strategy sets be given by $H_i \subset \mathbb{R}^m$, with generic element η_i. Given a vector η_{-i} of his or her opponents' strategies, player i's choice is restricted to a subset $\varphi_i(\eta_{-i})$ of H_i. The correspondence[4] $\varphi_i(\eta_{-i})$ is called player i's *constraint correspondence.* The payoff or utility of the ith player resulting from a play $\eta \in H = \times_{i\in I} H_i$ is given by the function $V_i(\eta)$.

Suppose that Nash behavior characterizes interaction between agents. That is, for any vector $\eta \in H$, player i takes η_{-i} as given and chooses an action or strategy t to maximize $V_i(t,\eta_{-i})$ on $\varphi_i(\eta_{-i})$. A *generalized game* is denoted $\Gamma = (H_i,V_i,\varphi_i)_{i=1,\ldots,I}$. An element η^* of H is an *equilibrium* for Γ if for each $i \in I$, η_i^* maximizes $V_i(t,\eta_{-i}^*)$ on $\varphi_i(\eta_{-i}^*)$.

The following theorem, which is a special case of Debreu's (1952) generalization of Nash's (1950) theorem, lists conditions sufficient for the existence of an equilibrium in Γ. This version of the theorem is used in Arrow and Debreu (1954) to prove the existence of an equilibrium for a competitive economy; its statement here follows that of Debreu (1982).

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[4]For $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^n$, a *correspondence* $\varphi : X \twoheadrightarrow Y$ is a rule which associates with every element x of X a non-empty subset $\varphi(x)$ of Y. φ is *convex-valued* if for every x in X, $\varphi(x)$ is a convex subset of Y. Its *graph* is the set $G(\varphi(x)) = \{(x,y) \in X \times Y : y \in \varphi(x)\}$.

Theorem 1 (Debreu). If, for every $i \in I$, the set H_i is a non-empty, compact, convex subset of a Euclidean space, V_i is a continuous real-valued function on $H = \times_{i \in I} H_i$ that is quasi-concave in its ith variable, and φ_i is a continuous, convex-valued correspondence from H to H_i, then the game $\Gamma = (H_i, V_i, \varphi_i)_{i=1,\ldots,I}$ has an equilibrium.

The task at hand is to reformulate the lobbying economy as a generalized game, and to exhibit conditions under which Theorem 1 can be applied to prove the existence of an equilibrium in the game. A natural approach to this problem is to focus on program $\mathbf{M}_i(\eta_{-i})$. Then, the constraint correspondence of player i would coincide with the feasible set $\psi_i(\eta_{-i})$ defined in section 2 above. Unfortunately, this correspondence is not convex-valued in general. In fact, it may be shown that when $p(\eta)$ satisfies (A5) and (A6), for each $i \in I$ the set $\psi_i(\eta_{-i})$ above is not convex for any η_{-i}.[5] Rather than focusing on conditions on $\mathcal{E}$ under which the relevant subset of $\psi_i(\eta_{-i})$ is convex, we take an alternative approach, based on a two-stage maximization formulation of $\mathbf{M}_i(\eta_{-i})$.

Note that for any η_{-i}, once agent i has selected an η_i, p is uniquely determined and i's optimization program over goods is well-defined. We assume

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[5]To see this, consider Mr. 1's problem (the case of Ms. 2 is similar). We know that Mr. 1 may simply eat his endowment, so that $z_1 = (\omega^1,0,0) \in \psi_1$ for any η_2. What's more, he may give all of his income to the government, so that $z_2 = (0,0,\hat{\eta}_1(\eta_2)) \in \psi_1$. The non-convexity of ψ_1 is guaranteed if there is a $t \in (0,1)$ such that $z_t = tz_1 + (1-t)z_2 \notin \psi_1$. But showing the existence of such a t is equivalent to showing that for some $t \in (0,1)$,

$$t\omega^1 > \omega^1 - \frac{(1-t)\cdot\hat{\eta}_1}{p((1-t)\hat{\eta}_1,\eta_2)} .$$

It is readily verified that this last condition is satisfied for *any* $t \in (0,1)$ whenever $p(\eta_1,\eta_2)$ is strictly increasing in η_1 on $[0,\hat{\eta}_1(\eta_2)]$. Thus, ψ_1 is not convex-valued. Similarly, ψ_2 is not convex-valued. This technical curiosity has nothing to do with preferences, and is therefore an inherent feature of the lobbying economy itself.

that agents choose consumption bundles optimally given a price and income vector, and we use the indirect utility functions as payoff functions in the game. With optimal consumption choices assumed, the only strategy open to agent i is a choice of η_i from $[0,\hat{\eta}_i(\eta_{-i})]$, a set which is obviously convex. The problem, given an η_{-i}, is to solve

$$M'_i(\eta_{-i}) \qquad \max_{\eta_i \in [0,\hat{\eta}_i(\eta_{-i})]} V_i(p(\eta),y_i(\eta)),$$

where $y_i(\eta) = P(\eta)\cdot\omega_i - \eta_i$ is i's "after-lobbying income," and $V_i(p(\eta),y_i(\eta)) = \max_{x_i \in \tilde{\beta}_i(p(\eta);P(\eta)\cdot\tilde{\omega}^i)} U_i(x_i)$. If no ambiguity results, the function $V_i(p(\eta),y_i(\eta))$ will be denoted $V_i(\eta_i,\eta_{-i})$, which makes clear the connection to payoff functions in the generalized game. The programs $M_i(\eta_{-i})$ and $M'_i(\eta_{-i})$ are equivalent. We now specify the generalized game which will be used to represent the economy $\mathcal{E}$.

The set of players is the set of agents $I = \{1,2\}$.[6] Players' *strategy sets* are given by $H_i = [0,\overset{*}{\eta}_i]$, where $\overset{*}{\eta}_i = \max_{\eta_{-i}} \hat{\eta}_i(\eta_{-i})$.[7] *Payoff functions* are given by $V_i = V_i(p(\eta),y_i(\eta))$. Player i's *constraint correspondence*, mapping η_{-i} into a subset of H_i, is given by $\varphi_i(\eta_{-i}) = [0,\hat{\eta}_i(\eta_{-i})]$. We may now define a lobbying game for $\mathcal{E}$.

Definition: Given a lobbying economy $\mathcal{E}$, its corresponding *lobbying game*, $\Gamma_{\mathcal{E}}$, is given by the collection $\Gamma_{\mathcal{E}} = (H_i,V_i,\varphi_i)_{i\in I}$.

In this game, player i takes η_{-i} as given and optimizes by choosing a

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[6]We have the government select $p(\eta)$ instead of including the imaginary player whose role is analogous to the Walrasian auctioneer or "market player" in the abstract economy model of Arrow and Debreu.

[7]The existence of this maximum is demonstrated in the proof of Lemma 1 below.

strategy from the set[8]

$$\mu_i(\eta_i,\eta_{-i}) = \left\{ x \in \varphi_i(\eta_{-i}) : V_i(x,\eta_{-i}) = \max_{t\in\varphi_i(\eta_{-i})} V_i(t,\eta_{-i}) \right\}.$$

Let $H = \times_{i\in I} H_i$, with generic element η. An equilibrium in the lobbying game $\Gamma_{\mathcal{E}}$ is defined as follows.

Definition: The vector $\eta^* \in H$ is a *lobbying game equilibrium* of $\Gamma_{\mathcal{E}}$, denoted LGE($\Gamma_{\mathcal{E}}$), if for each $i \in I$, $\eta_i^* \in \mu_i(\eta^*)$.

Equivalently, $\eta^* \in$ LGE($\Gamma_{\mathcal{E}}$) if for each $i \in I$, η_i^* solves $\mathbf{M}'_i(\eta_{-i}^*)$. Defining $\mu(\eta) = \times_{i\in I}\mu_i(\eta_i,\eta_{-i})$, $\eta^* \in$ LGE($\Gamma_{\mathcal{E}}$) if $\eta^* \in \mu(\eta^*)$, or if η^* is a fixed point of the correspondence μ.

Notice that a LGE($\Gamma_{\mathcal{E}}$) differs from a lobbying equilibrium LE($\mathcal{E}$) only by the absence of a feasibility restriction in the game equilibrium. In the following section, it is shown under what conditions on $\mathcal{E}$ the set LGE($\Gamma_{\mathcal{E}}$) is non-empty. Section 5 goes on to state conditions under which at any $\eta^* \in$ LGE($\Gamma_{\mathcal{E}}$), the government feasibility condition is satisfied in $\mathcal{E}$. Attention is now turned to the first main result of the paper — the existence of a lobbying game equilibrium in $\Gamma_{\mathcal{E}}$.

4. EXISTENCE OF A LOBBYING GAME EQUILIBRIUM

The objective in this section is to show that the lobbying game $\Gamma_{\mathcal{E}}$ associated with $\mathcal{E}$ has an equilibrium. This will be accomplished by showing that under certain restrictions on $\mathcal{E}$, $\Gamma_{\mathcal{E}}$ satisfies the conditions of Theorem 1. In applying Theorem 1 to the game $\Gamma_{\mathcal{E}}$, three sets of restrictions must be met: those on H_i, on V_i, and on φ_i. By reformulating agents'

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[8]While μ_i does not depend upon η_i, expressing μ in this manner eases exposition.

optimization programs as $M'_i(\eta_{-i})$, we manage to evade the difficulty related to non-convex-valued constraint correspondences. The reformulation introduces a difficulty in guaranteeing that V_i is quasi-concave in η_i.[9] This difficulty, however, has proven to be more readily surmounted than that concerning φ_i.

The restriction which guarantees that the V_i are quasi-concave requires that agents prefer to consume their own good. We assume that this preference is sufficiently strong. Formally, we have the following definition.

Definition: Consumer i's preference relation $\succsim_i$ is said to satisfy *own good bias* (OGB) if for every $\eta \in H$, $x_i^i(p(\eta),y_i(\eta)) \geq y_i(\eta)$.

The technical content of this definition will become apparent in the proof. Its economic content is that our agents have a proclivity toward consumption of the good they enter the world with. With this definition, the groundwork is now laid for a statement of the game equilibrium existence theorem.

Theorem 2 (Existence of a Lobbying Game Equilibrium). Suppose that in the lobbying economy $\mathcal{E} = ((\succsim_i,\omega_i)_{i=1,2};\ p(\eta))$, for every i, $\succsim_i$ satisfies (A1), (A2), and own good bias; and the function $p(\eta)$ satisfies (A3)—(A6). Then the associated lobbying game $\Gamma_{\mathcal{E}} = (H_i,V_i,\varphi_i)_{i=1,2}$ has an

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[9] Dasgupta and Maskin (1986a,b) study a class of economic games which fail to posses an equilibrium. This failure stems from the failure of payoff functions in these games either to be quasi-concave or to be continuous. Dasgupta and Maskin show that, with non-quasi-concave utility, mixed strategies may correct the non-existence problem. In our model quasi-concavity of the payoff function V_i is shown to follow from more primitive conditions. Absent these, a mixed strategies approach could perhaps be fruitfully employed, an issue needing further research.

equilibrium.[10]

We will prove this theorem via a series of lemmas. The strategy sets H_i, constraint correspondences φ_i, and payoff functions V_i are shown in these lemmas to meet the conditions of Theorem 1; the proof of Theorem 2 follows from them immediately. Theorem 1 requires that for each i in I, H_i be a non-empty, compact, convex subset of a Euclidean space. This is established for the lobbying game $\Gamma_{\mathcal{E}}$ in the first lemma.

Lemma 1. Suppose that $p(\eta)$ satisfies (A5) and (A6). Then for each i, H_i is a non-empty, compact, convex subset of $\mathbb{R}$.

Proof. i.) (Non-emptiness). Clearly, $0 \in H_i$. Thus, $H_i \neq \emptyset$.

ii.) (Compactness). Since $H_i \subset \mathbb{R}$, it is compact precisely when it is closed and bounded. Closedness is immediate from the definition of H_i. To show boundedness, it is enough to show that $\overset{*}{\eta}_i < +\infty$. Note that $\hat{\eta}_i(\eta_{-i})$ is strictly decreasing in η_{-i}. To see this, consider the case of Ms. 2, and suppose not. Then there is a pair $t,t' \in H_1$ such that $\hat{\eta}_2(t) < \hat{\eta}_2(t')$ and $t' > t$. This implies that $\mathbf{P}(\hat{\eta}_2(t),t')\cdot\omega_2 > \mathbf{P}(\hat{\eta}_2(t),t)\cdot\omega_2$, contradicting (A5). We conclude that $\hat{\eta}_2(\eta_1)$ is strictly decreasing in η_1. The argument for Mr. 1 is similar. From this, it follows that $\overset{*}{\eta}_1 = \hat{\eta}_1(0)$, which is finite by assumption (A6). Therefore, H_i is bounded. It follows that it is compact.

iii.) (Convexity). The convexity of H_i follows immediately from the

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[10] In the language of abstract lobbying economies developed above, this theorem may be concisely restated. Let $\mathfrak{E}_+^{\mathrm{OPT}} \subset \mathfrak{E}_+$ denote the set of all lobbying economies for which the corresponding lobbying game $\Gamma_{\mathcal{E}}$ has an equilibrium. Let $\mathcal{R}^{\mathrm{OGB}}$ denote the set of admissible characteristics a_i such that $\succsim_i$ satisfies own good bias. Theorem 2 may be restated as follows: If, for each $i \in I$, $a_i \in \mathcal{R}^{\mathrm{OGB}}$, then $\mathcal{E} \in \mathfrak{E}_+^{\mathrm{OPT}}$. Thus, if preferences are appropriately restricted, the game $\Gamma_{\mathcal{E}}$ will have an equilibrium for any pricing rule in $\mathcal{P}_+$.

definition. This completes the proof of Lemma 1. ∎

Theorem 1 requires that for each i in I, the constraint correspondence $\varphi_i(\eta_{-i})$ be convex-valued and continuous.[11] The next lemma shows that this is indeed the case.

Lemma 2. Under the conditions of Theorem 2, for each i in I, the constraint correspondence $\varphi_i(\eta_{-i})$ is convex-valued and continuous.

Proof. i.) (Convex-valued). That $\varphi_i(\eta_{-i})$ is convex-valued follows from the definition of $\varphi_i(\eta_{-i})$.

ii.) (Continuity). It suffices to show that $\varphi_i(\eta_{-i})$ is upper and lower hemi-continuous. We have noted that $\hat{\eta}_i(\eta_{-i})$ is continuous on $[0,\overset{*}{\eta}_{-i}]$. Thus, the graph of $\varphi_i(\eta_{-i})$ is closed. As $\varphi_i(\eta_{-i})$ is also compact-valued by Lemma 1, it is upper hemi-continuous (see, e.g., Border, 1985, p. 57). To show lower hemi-continuity, consider a sequence $\{\eta^n_{-i}\}$ in H_{-i} converging to η^o_{-i}, and take an arbitrary $\eta^o_i \in \varphi_i(\eta^o_{-i})$. If $\eta^o_i < \hat{\eta}_i(\eta^o_{-i})$, then for N large, we may set $\eta^n_i = \eta^o_i$ for $n \geq N$. Then clearly $\eta^n_i \longrightarrow \eta^o_i$, and the conditions for lower hemi-continuity are satisfied. If $\eta^o_i = \hat{\eta}_i(\eta^o_{-i})$, then let $\eta^n_i = \hat{\eta}_i(\eta^n_{-i})$. As $\hat{\eta}_i(\eta_{-i})$ is continuous, the conditions for lower hemi-continuity are again satisfied. We conclude that $\varphi_i(\eta_{-i})$ is lower hemi-continuous. Thus, it is continuous. This completes the proof of Lemma 2. ∎

Theorem 1 requires that for each i in I, V_i be continuous and quasi-concave in η_i. To demonstrate that V_i is quasi-concave in η_i we will need to prove some intermediate results. As a first step, quasi-concavity is defined.

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[11]See, for example, Hildenbrand and Kirman (1976), Mathematical appendix III, for definitions of upper and lower hemi-continuous and continuous correspondences.

Definition: Let $S \subset \mathbb{R}^m$, $T \subset \mathbb{R}^n$, and $g : S \longrightarrow T$ be a function mapping elements of S into T. g is *quasi-concave* if for every s^1 and s^2 in S, for each $v \in [s^1,s^2]$, $g(v) \geq \min\{g(s^1),g(s^2)\}$.

For differentiable functions from $\mathbb{R}$ to $\mathbb{R}$, an equivalent definition will prove convenient.

Lemma 3. Let $X \subset \mathbb{R}$, $Y \subset \mathbb{R}$, and let $g : X \longrightarrow Y$ be differentiable. Then g is quasi-concave if and only if for every pair of elements x, x′ of X, $[g'(x) < 0$ and $x' > x]$ imply $g'(x') \leq 0$.

Proof. i.) (Necessity). Suppose g is quasi-concave, and that for x,x′ in X, $g'(x) < 0$ and $x' > x$. By way of contradiction, suppose that $g'(x') > 0$. Let $y = \operatorname{argmin}_{z\in\{x,x'\}} g(z)$. Then for any $\varepsilon > 0$, there exists a $y' \in \mathbb{B}(y;\varepsilon)$, the ε-ball around y, with $y' \neq y$, such that $g(y') < g(y)$ and $y' \in (x,x')$. But this contradicts that g is quasi-concave. We conclude $g'(x') \leq 0$.

ii.) (Sufficiency). Suppose that for any x, x′ in X, $[g'(x) < 0$ and $x' > x]$ imply $g'(x') \leq 0$. We must show that for $y \in [x,x']$, $g(y) \geq \min\{g(x),g(x')\}$. Otherwise, suppose not: there exists a $z \in (x,x')$ with $g(z) < \min\{g(x),g(x')\}$. We then have

$$\frac{g(z) - g(x')}{z - x'} > 0.$$

By the Mean Value Theorem, there is a $w \in (z,x')$ such that

$$g'(w) = \frac{g(z) - g(x')}{z - x'} > 0,$$

a contradiction. We conclude that g is quasi-concave. This completes the proof of Lemma 3. ∎

The condition defined in Lemma 3 requires that once g begins declining in x, it may never increase as x increases further. It shall now be demonstrated that for each i, the payoff function V_i satisfies this condition as a function of η_i.

Recall that agent i's indirect utility function is $V_i = V_i(p(\eta), y_i(\eta))$. That V_i is differentiable is immediate from the 2-differentiability of $\succsim_i$ and the differentiability of $p(\eta)$. The derivative of V_i with respect to η_i is given by

$$\partial V_i = \frac{\partial V_i}{\partial \eta_i}(p, y_i) = \frac{\partial V_i}{\partial p}\cdot\frac{\partial p}{\partial \eta_i} + \frac{\partial V_i}{\partial y_i}\cdot\frac{\partial y_i}{\partial \eta_i}.$$

Let $\partial_p V_i = \frac{\partial V_i}{\partial p}\cdot\frac{\partial p}{\partial \eta_i}$ and let $\partial_y V_i = \frac{\partial V_i}{\partial y_i}\cdot\frac{\partial y_i}{\partial \eta_i}$. These expressions will hereafter be referred to as the price and income effect, respectively, of a change in η_i on V_i. They refer to the effect of an incremental change in η_i on indirect utility through the price (with y_i held constant) and through income (with p held constant). In showing that V_i is quasi-concave, we may treat these two terms separately. First, consider $\partial_y V_i$.

Lemma 4. Suppose that $p(\eta)$ satisfies (A5) and (A6), and that preferences are monotone for every i. If $\partial_y V_i(x) < 0$ for some $x \in [0, \hat{\eta}_i(\eta_{-i})]$, then for $x' > x$ with $x' \in [0, \hat{\eta}_i(\eta_{-i})]$, $\partial_y V_i(x') \le 0$.

Proof. Consider $y_i(\eta) = \mathbf{P}(\eta)\cdot\omega_i - \eta_i$. Under assumption (A5), $y_i(\eta)$ is concave in η_i for each i in I. Thus, it is quasi-concave in η_i. By Lemma 3, if $\partial y_i/\partial \eta_i(\eta_i) < 0$ for some $x \in [0, \hat{\eta}_i(\eta_{-i})]$, then for $x' > x$ with $x' \in [0, \hat{\eta}_i(\eta_{-i})]$, $\partial y_i/\partial \eta_i(x') \le 0$.

Under monotonicity of $\succsim_i$, $\partial V_i/\partial y_i > 0$. Thus, $\partial_y V_i$ agrees in sign with $\partial y_i/\partial \eta_i$ at every η_i. We conclude that if $\partial_y V_i(x) < 0$ for some $x \in [0, \hat{\eta}_i(\eta_{-i})]$, then for $x' > x$ with $x' \in [0, \hat{\eta}_i(\eta_{-i})]$, $\partial_y V_i(x') \le 0$. This completes the proof of Lemma 4. ■

Now, it remains only to show that $\partial_p V_i$ doesn't increase in η_i "too much."

By too much is meant that, while $\partial_y V_i$ is always negative in η_i once it becomes negative, the sum ∂V_i goes positive after once having been negative. In Lemma 5 it is shown that as η_i increases, the affect on V_i through the price doesn't offset the eventually negative income effect. In fact, this Lemma shows something stronger: that for a fixed income y_i, under the OGB assumption, $\partial_p V_i$ is non-positive.

Lemma 5. Suppose that for i in I, $\succsim_i$ satisfies own good bias and monotonicity. Then for every $\eta \in H$, $\partial_p V_i(\eta) \leq 0$.

Proof. i.) For an $i \in I$, fix $y_i > 0$. Let $E_i = \{(x^1,x^2) \in \mathbb{R}^2_+ : x^1 \geq y_i\}$. For any $p \in (0,1)$, let $\beta_i(p,y_i) = \{(x^1,x^2) \in \mathbb{R}^2_+ : \mathbf{P}\cdot(x^1,x^2) \leq y_i\}$. By OGB, we have that the demanded bundle $x_i(p,y_i) \in \beta_i(p,y_i) \cap E_i = \beta_i^+(p,y_i)$.

ii.) Now, for $p' > p$, we have that $\beta_1^+(p',y_1) \subset \beta_1^+(p,y_1)$ (resp. $\beta_2^+(p,y_2) \subset \beta_2^+(p',y_2)$). By monotonicity of $\succsim_i$, then, $x_1(p,y_1) \succsim_1 x_1(p',y_1)$ (resp. $x_2(p',y_2) \succsim_2 x_2(p,y_2)$).

Combining i.) and ii.), for any pair p,p' with $p' > p$, $V_1(p,y_1) \geq V_1(p',y_1)$ and $V_2(p,y_2) \leq V_2(p',y_2)$. Since p and p' were arbitrary, and since $\partial p/\partial\eta_1 > 0$ and $\partial p/\partial\eta_2 < 0$, the preceding argument is sufficient to demonstrate that $\partial_p V_i(\eta) \leq 0$, which was to be shown. This completes the proof of Lemma 5.[12] ∎

That V_i is continuous and quasi-concave in η_i is now easily established.

Lemma 6. Under the conditions of Theorem 2, for each $i \in I$, V_i is continuous and quasi-concave in η_i.

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[12] It may be shown that if preferences satisfy OGB, monotonicity, and if they are differentiable, then demands will be such that $x_i^i(p(\eta),y_i(\eta)) > y_i$ on $[0,\hat{\eta}_i(\eta_{-i}))$. From this strict inequality it follows that $\partial_p V_i < 0$, a result which is stronger than is required for the Lemma or for Theorem 2.

Proof. i.) (Continuity). Since $\succsim_i$ is 2-differentiable and $p(\eta)$ is differentiable, V_i is a composition of continuous functions. It is therefore continuous.

ii.) (Quasi-concavity). The quasi-concavity of V_i in η_i is immediate from Lemmas 3, 4, and 5 and the definition of $\partial V_i/\partial\eta_i$. This completes the proof of Lemma 6. ■

Before proceeding to the proof of Theorem 2, some comments upon the results of Lemmas 3 to 6 are in order. First, the requirement of quasi-concavity of payoff functions V_i is more than just a technical restriction. V_i fails to be quasi-concave when it declines in η_i at some point and then rises as η_i continues to increase. There is an intuitive appeal to the idea that such a circumstance may lead to a lobbying game without an equilibrium. Also, the assumption of own good bias is stronger than is needed. Since $\partial_y V_i$ is eventually negative, $\partial_p V_i$ can become positive as η_i increases, as long as the *sum* ∂V_i remains non-positive. There is room, then, for weakening the restriction on preferences required to guarantee the existence of an equilibrium in the lobbying game.

Let us now turn to the proof of Theorem 2.

Proof of Theorem 2.

Combining Lemma 1, Lemma 2, and Lemma 6, it is immediate that $\Gamma_{\mathcal{E}}$ satisfies the conditions of Theorem 1. Thus, $\Gamma_{\mathcal{E}}$ has an equilibrium. That is, there is an $\eta^* \in H$ such that $\eta^* \in \mu(\eta^*)$. This completes the proof of Theorem 2. ■

It has now been established that for economies satisfying the conditions of Theorem 2, there exists a pair η^* of strategies at which each agent is responding optimally to his or her opponent's strategy. In the following section, we specify conditions on the pricing function $p(\eta)$ which guarantee that η^* is also government feasible. Together, Theorem 2 and the feasibility result establish the existence of a lobbying equilibrium for the lobbying economy.

5. EXISTENCE OF A LOBBYING EQUILIBRIUM

The objective of the remainder of the paper is to show that any lobbying game equilibrium η^* for the game $\Gamma_{\mathcal{E}}$, along with the associated consumption bundles $\tilde{x}_i(\eta^*)$, is also an economic equilibrium, i.e. it is a lobbying equilibrium in $\mathcal{E}$. The approach which we will follow involves restricting the pricing function $p(\eta)$ without further restricting preferences. From Theorem 2, we know that if $a_i \in \mathcal{R}^{OGB}$ for each i, then $\mathcal{E} \in \mathfrak{E}_+^{OPT}$ (see footnote 10). To obtain feasibility, a condition on $p(\eta)$, together with OGB, is used to show that $\pi(\eta^*) \geq 0$, and thus that $LE(\mathcal{E}) \neq \varnothing$.

While Theorem 2 ensures that $LGE(\Gamma_{\mathcal{E}}) \neq \varnothing$, it has nothing to say about the location of η^* in H, except that each η_i^* must lie in the set $\varphi_i(\eta_{-i}^*)$. Thus, the feasibility condition $\pi(\eta^*) \geq 0$ must be shown to hold for *every* possible feasible pair η.

Let $\Delta^c = \{\eta \in \mathbb{R}_+^2 : \eta_1 + \eta_2 = c\}$ for an arbitrary $c \geq 0$, and let $\eta_2 = c - \eta_1$. Then the function $p(\eta)$ may be expressed as $p(\eta_1;c)$, where the intervention price depends only on η_1 given c. A characteristic of the pricing function $p(\eta)$ which offers some intuition for the feasibility argument is that for any $c \geq o$, there is an $\bar{\eta}_1 \in [0,c]$ such that $p(\bar{\eta}_1;c) = p^*$,[13] whence

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[13]This follows immediately from (A5), the productive lobbying assumption, which implies $p(c,0) > p^* > p(0,c)$. As $p(\eta)$ is also continuous, there must be an $\bar{\eta}_1$ in [0,c] such that $p(\bar{\eta}_1,c-\bar{\eta}_1) = p^*$.

$\pi(\bar{\eta}) = \bar{\eta}_1 + \bar{\eta}_2 \geq 0$. For a given $c \geq 0$, we may write $\pi(\eta_1;c) = c - (\mathbf{P}^* - \mathbf{P}(\eta_1;c))\cdot\tilde{z}(p(\eta_1;c))$. The government feasibility condition is satisfied either when p^* is "close" to $p(\eta_1;c)$ or when $\tilde{z}(p(\eta_1;c))$ is "small." One possibility, that $p(\eta_1;c) \equiv p^*$, the constant function, is ruled out by the productive lobbying assumption (A5). However, this suggests that if the function $p(\eta_1;c)$ is "flat enough," even if $\tilde{z}(p(\eta_1;c)$ is relatively large, the feasibility condition will be satisfied. Only when $p(\eta)$ is far from p^* with $\eta_1 + \eta_2$ small will the feasibility condition be violated.

To obtain a bound on the degree of flatness required, we exploit the fact that own good bias limits the amount by which demanded bundles (or the tangency points of indifference curves) may separate along the price line.[14] What follows is designed to achieve a steepness restriction on $p(\eta)$ which, together with OGB, ensures $\pi(\eta) \geq 0$.

Proposition 1 (Feasibility of the Lobbying Game Equilibrium)

Suppose that in a lobbying economy $\mathcal{E}$ the conditions of Theorem 2 are satisfied, and take an $\eta^* \in \mathrm{LGE}(\Gamma_{\mathcal{E}})$. Suppose further that for each $i \in I$, $\eta_i^* < \hat{\eta}_i(\eta_{-i}^*)$, so that neither agent devotes his or her entire resource endowment to lobbying activity. If at η^*, $\tilde{\omega}^1 \geq \tilde{\omega}^2$, (resp. $\tilde{\omega}^2 > \tilde{\omega}^1$), then $\pi(\eta^*) \geq 0$ whenever

$$1 - \frac{p^*}{p(\eta^*)} \leq \frac{\eta_1^*}{y_1(p(\eta^*))} \tag{1}$$

[14]For exchange economies, Geller (1986) provides a bound on per capita excess demand which is independent of preferences. This bound is essentially the product of the norm of the average endowment and the square root of the ratio of the number of commodities to the number of traders. Unfortunately, the result ensures only that the bound is satisfied for *some* price vector. Thus, Geller's bound is not helpful here; our interest is in the size of excess demand at the specified price $p(\eta)$.

(resp. whenever $1 - \frac{1-p^*}{1-p(\eta^*)} \le \frac{\eta_2^*}{y_2(p(\eta^*))}$).[15]

Proof: Take a pair $\eta^* \in LGE(\Gamma_g)$. We know that $\eta_1^* + \eta_2^* = c \ge 0$. If $c = 0$, then $\pi(\eta_1^*;c) = 0$ by definition. Now, consider a $c > 0$. Suppose that $\tilde{\omega}^1 \ge \tilde{\omega}^2$. Condition (1) may be written

$$1 - \frac{p^*}{p(\eta_1^*;c)} \le \frac{\eta_1^*}{y_1(p(\eta_1^*;c))}, \tag{2}$$

where $y_1(p(\eta_1^*;c)) > 0$ is guaranteed by our assumption that $\eta_i^* < \hat{\eta}_i(\eta_{-i}^*)$. Rearranging, equation (2) becomes

$$\left(p(\eta_1^*;c) - p^* \right) \cdot \left(p(\eta_1^*;c) - \frac{\eta_1^*}{\omega^1} \right) \le p(\eta_1^*;c) \cdot \left(\frac{\eta_1^*}{\omega^1} \right),$$

or, multiplying by $\omega^1/p(\eta_1^*;c)$,

$$\left(p(\eta_1^*;c) - p^* \right) \cdot \left(\omega^1 - \frac{\eta_1^*}{p(\eta_1^*;c)} \right) \le \eta_1^* \le c. \tag{3}$$

It suffices to show that when (3) is satisfied, $\pi(\eta_1^*;c) \ge 0$. Note that with $\tilde{\omega}^1 \ge \tilde{\omega}^2$ and with the lobbying price determined as $p(\eta^*)$, the condition OGB places a bound on the magnitude of $\pi(\eta_1^*;c)$. Under OGB, $\pi(\eta_1^*;c)$ achieves a maximum if agent 1 consumes on the 45-degree line, where $x_1^1 = x_1^2 = y_1(p(\eta_1^*;c))$, and agent 2 simply consumes her "after-lobbying" endowment $\tilde{\omega}^2$. We have that the aggregate "after-lobbying" excess demand corresponding to this maximum level of π is the pair

$$\tilde{z}^1(p(\eta^*)) = y_1(p(\eta^*)) - \tilde{\omega}^1, \tag{4a}$$

$$\tilde{z}^2(p(\eta^*)) = y_1(p(\eta^*)) + \tilde{\omega}^2 - \tilde{\omega}^2 = y_1(p(\eta^*)). \tag{4b}$$

Now, we claim that (3) holds if and only if $\pi(\eta) \ge 0$. From (3), we have that

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[15]Here, p^* is the competitive equilibrium (lobbying-free) price; $p(\eta^*)$ corresponds to the politically dictated price which results when η^* is an equilibrium outcome in the lobbying game.

$$c - \left(p(\eta_1^*;c) - p^* \right) \cdot \left(\omega^1 - \frac{\eta_1^*}{p(\eta_1^*;c)} \right) \geq 0.$$

Adding and subtracting $y_1(p(\eta_1^*;c))$ in the second bracketed term on the left, and noting that $\left(\omega^1 - \frac{\eta_1}{p(\eta_1^*;c)} \right) = \tilde{\omega}^1$, we may write this as

$$c - \left(p(\eta_1^*;c) - p^* \right) \cdot \left(y_1(p(\eta_1^*;c)) - \tilde{\omega}^1 - y_1(p(\eta_1^*;c)) \right) \geq 0.$$

Upon rearranging, and using the fact that $p^2 = (1 - p^1)$, we obtain

$$c - \left(\mathbf{P}^* - \mathbf{P}(\eta_1^*;c) \right) \cdot \left(y_1(p(\eta_1^*;c)) - \tilde{\omega}^1,\ y_1(p(\eta_1^*;c)) \right) \geq 0, \tag{5}$$

where each of the bracketed terms in (5) is a two-dimensional vector. But (5), together with equations (4), yields the condition

$$\pi(\eta_1^*;c) = \eta_1^* + \eta_2^* - \left(\mathbf{P}^* - \mathbf{P}(\eta_1^*;c) \right) \cdot \left(\tilde{z}(p(\eta_1^*;c)) \right) \geq 0,$$

which was to be obtained. All of the steps in the proof are reversible, so that the claim is established: (3) holds if and only if $\pi(\eta^*) \geq 0$.

For the case with $\tilde{\omega}^2 > \tilde{\omega}^1$, due to the symmetry of our formulation and the price normalization employed, the same argument applies if we let $q(\eta_2^*;c) = 1 - p(\eta_2^*;c)$, and note that $y_2(p(\eta_2^*;c)) = \omega^2 \cdot q(\eta_2^*;c) - \eta_2^*$. This completes the proof of Proposition 1. ■

Before turning to the last theorem, we offer a few remarks on condition (1). With $\eta_1 + \eta_2 = c \geq 0$ fixed, recall that for some $\bar{\eta}_1 \in [0,c]$, $p(\bar{\eta}_1;c) = p^*$. If $\eta_1^* < \bar{\eta}_1$, then $p(\eta_1^*;c) < p^*$, and the left hand side of (1) is negative. As the right hand side is non-negative, the required condition is always satisfied. An intuitive interpretation of the inequality in (1) is that it provides a bound on the degree to which $p(\eta_1^*;c)$ may move away from p^* in response to lobbying donations. The expression $\eta_1^*/y_1(p(\eta_1^*;c))$ is, in some sense, a measure of 1's political involvement. This is the ratio of his lobbying donations to his goods consumption expenditures; it takes values on

$[0,+\infty)$. When it is near zero, the resources of the government are also relatively small. Then, if feasibility is to be satisfied, the lobbying price $p(\eta_1^*;c)$ must be "close to " p^*. When the ratio $\eta_1^*/y(p(\eta_1^*;c))$ is large, $p(\eta_1^*;c)$ is allowed to be larger than *and* to be "far from" p^*. Hence, (1) is precisely the required bound on the maximum steepness of $p(\eta^*)$.

We now combine Theorem 2 with Proposition 1 in a theorem which establishes the existence of a lobbying equilibrium in the lobbying economy $\mathcal{E}$.

Theorem 3 (Existence of a Lobbying Equilibrium)

Consider a lobbying economy $\mathcal{E} \in \mathfrak{E}_+$. If $\mathcal{E}$ meets the conditions of Theorem 2 and of Proposition 1, then $LE(\mathcal{E}) \neq \emptyset$.

Proof: From Theorem 2, we know that $LGE(\Gamma_{\mathcal{E}}) \neq \emptyset$. Take an $\eta^* \in LGE(\Gamma_{\mathcal{E}})$. Proposition 1 guarantees that $\pi(\eta^*) \geq 0$. Therefore, η^* is government feasible. Thus, the set of lobbying equilibria $LE(\mathcal{E})$ is non-empty. This completes the proof of Theorem 3. ∎

6. CONCLUSIONS

Economic behavior is often dependent upon political circumstance. Societies are usually organized so as to permit individuals and interest groups to influence the economic policy of their government. In this event, the neoclassical economic model of agents as price-takers is not very instructive. Recent developments in the theory of rent-seeking and political economic behavior promise to provide new insights into this issue. This study fills a gap in the literature: the demonstration that it is possible to devise a cogent model of equilibrium behavior by economizing agents who lobby to influence prices.

We are still far from a comprehensive, realistic model of the economics of politics. There are no voters, no politicians, no optimizing government agents of any sort in our model; including them will be a challenge. This first step, however, suggests that such an improvement is possible. By insisting on a coherent treatment of agents' choices, we have provided a rigorous foundation for further study.

There is still some unturned ground in this framework, however. The simple two-agent model may also be used to study the efficiency implications of this sort of lobbying behavior. The political economic literature often asserts that when rent-seeking occurs aggregate welfare is reduced. Bhagwati (1982), among others, argues that the theory of the second best might explain when political intervention in the economy may be optimal. Our preliminary results suggest another possibility. Under certain conditions, one agent may be so much better off at the lobbying equilibrium that the other agent cannot arrange a bribe which is improving for him or her, and acceptable for the first. This investigation is undertaken in an upcoming paper.

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