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Government Turnover in Parliamentary Democracies

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Abstract

In this paper we consider a dynamic model of government formation and termination in parliamentary democracies. Our analysis accounts for the following observed phenomena: (1) Cabinet reshuffles; (2) Cabinet replacements; (3) Early elections; (4) Surplus governments; (5) Minority governments; (6) The relative instability of minority governments.

JEL classification: D72, H19, C73.

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1 Introduction

The distinctive characteristic of parliamentary democracies is the fact that the executive derives its mandate from and is politically responsible to the legislature. This has two consequences. First, unless one party wins a majority of seats, the choice of executive is not determined by an election alone, but is the result of an elaborate bargaining process among the parties represented in the parliament. Second, parliamentary governments may lose the confidence of the parliament at any time, which leads to their immediate termination.

The following is a list of prominent empirical regularities about the formation and termination of parliamentary governments.⁴

1. Governments frequently terminate before the end of the legislative period. While most governments are immediately replaced by a new cabinet, 45% of all governments terminate in an early election (Diermeier and Stevenson 1998).
2. Cabinets frequently reshuffle the allocation of cabinet posts and other government positions during their lifetime (Laver and Shepsle 1996).
3. Minority governments are common (Strom 1990; Laver and Schofield 1990). They occur in about 37% of all cases where no party controls a majority of seats (33% overall). In some countries they are the norm. Of the 20 Danish governments between 1945 and 1987, 18 were minority governments.
4. Minority governments are, on average, less stable than other governments (Strom 1990), even though some minority governments survive until the next regular election. Moreover, if a minority government terminates, it is frequently replaced by another minority government even after an early election.
5. Minimal winning coalitions are not the norm (Laver and Schofield 1990). They occur in only 39% of the cases where no party has a majority of seats (36% overall).

⁴For recent overviews of the large empirical literature on government formation and termination see Laver and Schofield 1990, Strom 1990, and Warwick 1994.

6. Surplus majority governments are not rare (Laver and Schofield 1990). 23% of all governments where no party controls a majority of seats are of this type (21% overall). In Italy the percentages are 45% and 40%, respectively.

In spite of the fact that these regularities are well documented empirically, no theoretical model exists that can simultaneously explain all of them. Recently, a series of non-cooperative models have been proposed to account for the first regularity. These models interpret cabinets as equilibria in a legislative bargaining process.

Lupia and Strom (1995) consider a one round bargaining model with outside options or “events”. They focus on a particular type of events related to electoral prospects. That is, events are interpreted as common knowledge information about what would happen if parliament were dissolved and an election held immediately. This may be interpreted, for example, as the current state of public opinion. A party with favorable electoral prospects, however, does not need to realize its favorable prospects by going to the polls. Rather, it can alternatively extract benefits through bargaining with parties that would be disadvantaged by an early election. The (credible) threat of early elections thus can be used to exploit less fortunate parties without having to dissolve parliament and call an election. Thus, events are the parameters that make or break governments. Whether gains are realized in a new government coalition or in the calling of early elections depends, in the Lupia and Strom model, on the relative magnitude of election and negotiation related transaction costs.

Baron (1998) proposes a dynamic model of government stability. He uses an infinite horizon version of the legislative bargaining model proposed by Diermeier and Feddersen (1998). In this framework, legislators vote to redistribute a fixed unit of distributive benefits in each period. A randomly drawn reservation value is associated with each legislator. Governments are assumed to control the legislative agenda which allows the members of the governing coalition to extract rents from other legislators. Diermeier and Feddersen demonstrate that the vote of confidence procedure—a constitutional mechanism typical of parliamentary democracies that links the survival of the cabinet with the acceptance of the government’s bill—permits legislative majorities to capture almost all the distributive benefits leaving the opposition with almost nothing. This result holds if sufficiently many bargaining periods are left and future payoffs are not discounted too heavily.

Baron shows that if this condition is not satisfied, governments may fall if the opposition can table a vote of no confidence. This outcome may occur if a governing party's current reservation value is too high compared to the future benefits of maintaining the current government. Baron interprets the draws of reservation values as events in the sense of Lupia and Strom, but does not distinguish between different types of events. In his model, in contrast to the Lupia and Strom model, any government termination is a replacement.

In line with the previous literature we interpret cabinets as the outcome of a legislative bargaining process. Like Baron, but in contrast to Lupia and Strom, we resort to a dynamic bargaining model with random events. In contrast to Baron we also allow for the dissolution of parliament and cabinet reshuffles.

As recently pointed out by Laver and Shepsle (1997), changes in public opinion are not the only type of critical events governments face. Norway's constitution, for instance, does not allow for an early dissolution of parliament under any circumstances, yet cabinets terminate regularly before the end of a legislative term. We focus on shocks to the parties' reservation payoffs. We show that in a model where parties care about both policy and office related benefits, each coalition is associated with a "cake" of varying size that can be distributed among the parties in the legislature. The size of the cake depends both on the composition of the coalition and on the current state of the political and economic environment. Changes in this environment may make the current coalition no longer viable.

The assumption of shocks to reservation payoffs is critical. As we show below, in the absence of such shocks parties can always reshuffle government positions in response to public opinion shocks. This suggests that Lupia and Strom's results are not robust once one allows for reshuffles in the current government.

Our model also accounts for the occurrence of minority and surplus coalitions and their associated stylized facts mentioned above. Most of the traditional literature views minority governments as pathologies.⁵ Strom (1990), however, suggests that minority governments may be the result of rational calculations by parties. His account of minority governments focuses on non-formateur parties that are asked to participate in a government, but decline to do so. Strom identifies two relevant factors that may make it disadvantageous to be part of a proposed coalition. First, in many countries parties

⁵For an overview see chapter 1 of Strom (1990).

can influence policy outcomes without taking part in the government (e.g., through corporatist bargaining mechanisms). This may lower the benefit of being in the government. Second, parties that participate in the government are frequently punished at the polls. This "anti-incumbency effect" may increase the cost of participating in a government.⁶

While there is some empirical support for this view, to date there is no satisfactory theoretical account for minority governments. Laver and Shepsle (1990) propose a structure-induced equilibrium model of minority governments, but, as demonstrated by Austen-Smith and Banks (1990), their existence results do not generalize. Baron (1998) considers a model where minority governments could be sustained, but they are never chosen in equilibrium.

To build a model that allows for minority governments we first need to clarify what we mean by a "government". Following Laver and Shepsle (1990, 1996) we identify a government with an allocation of cabinet portfolios. Thus a party that supports a minority government on critical votes but does not hold any portfolios is not part of the government, but is only part of the supporting coalition. In our model this assumption has two consequences. First, holding a ministry implies political control of the bureaucracy. This is important, since in parliamentary democracies the effective power to draft and implement public policy rests with the civil service.⁷ Since members of the government alone can control and verify the implementation of policies, bargains on policy with parties outside of the cabinet are not credible. It follows that only the members of a government decide on policies.

Second, membership in the cabinet allows control over government posts. While some of these posts have direct influence over policy, others are better interpreted as perks that are valued by all actors and thus can be freely distributed. Examples are well-paid positions on boards of state-owned businesses or the national television. These government posts can be interpreted as transferable benefits or "money" that can be allocated by the cabinet.⁸ Since money can be exchanged for policy concessions, governing coalitions

⁶This view has recently been challenged by Stevenson (1996), who argues that the prime minister's party usually has an incumbency advantage while the other members of the governing coalition suffer at the polls.

⁷See Laver and Shepsle (1994) for supporting empirical evidence.

⁸See, e.g., Snyder (1990).

can bargain efficiently on policies.⁹ These benefits may also be allocated to parties in the supporting coalition in exchange for their votes. Given that appointments to government jobs are easily verifiable, cabinets may thus buy parliamentary support by allocating money to opposition parties. However, since only the cabinet controls the allocation of perks, transfers can only be made from the government to outside parties.

To summarize, a government is a set of parties that can efficiently bargain over policy and the distribution of perks. Perks may also be allocated to parties that are not members of the government. In particular, they can be used to sustain minority governments.

In the remainder of the paper we investigate the consequences of this notion of parliamentary government with respect to the formation and termination of governments. In the next section we introduce our model. The third section contains a characterization of the equilibrium. This is followed by a discussion of the results and a conclusion.

2 The Model

We consider a two-period spatial model of government formation and termination in a parliamentary democracy which builds on the framework developed by Baron and Diermeier (1998). Let $N = \{1, \dots, n\}$ denote the set of parties in a parliamentary democracy and assume that $n = 3$. Each party $i \in N$ has time-separable quasi-linear preferences over policy outcomes $x \in \mathbb{R}^2$ and distributive benefits $y_i \in \mathbb{R}$. We assume that the per-period utility function of party i , $i = 1, 2, 3$, is given by

$$U_i(x, y_i) = u_i(x) + y_i \quad (1)$$

where

$$u_i(x) = -(x_1 - z_1^i)^2 - (x_2 - z_2^i)^2 \quad (2)$$

and the parties ideal points $z^i = (z_1^i, z_2^i) \in \mathbb{R}^2$, $i = 1, 2, 3$, are located symmetrically. Without further loss of generality, we normalize the parties ideal points so that $z^1 = (0, 0)$, $z^2 = (1, 0)$, and $z^3 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$. This specification captures the intuition that parties care both about policy outcomes and the benefits from holding office. We normalize aggregate transfers to be zero in

⁹Laver and Shepsle (1990) and Austen-Smith and Banks (1990) consider models where each minister is a dictator on his policy dimension. In our model policies are the result of efficient cabinet bargaining.

each period (i.e., $\sum_{i \in N} y_i = 0$), and assume that utility in the second period is discounted at a common discount factor $\beta \in [0, 1]$.

Period 1 begins after a general election (not modeled here), which determines the parties relative shares in the parliament $\pi = (\pi_1, \pi_2, \pi_3)$. We assume that $\pi \in \Pi = \{(\pi_1, \pi_2, \pi_3) : \pi_i \in (0, \frac{1}{2}) \text{ and } \sum_{i \in N} \pi_i = 1\}$. This assumption implies that no single party has a majority of seats, but any two-party coalition is winning under majority rule. Also given in period 1 is a *default policy* $q \in Q = \{z^1, z^2, z^3\}$. This is the policy that is implemented if no government forms in that period. It determines each party's payoff in period 1 if such an event occurs. Our assumption about Q implies that the default policy may be particularly favorable to one of the parties.¹⁰ If $q = z^j$, we refer to party j as the party *avored* by the default policy.

Let $s \equiv (q, \pi) \in S = Q \times \Pi$ denote the *state* of the political system in period 1, which is summarized by the default policy and the distribution of parliamentary seat shares among the parties.

At the beginning of period 1, the head of state chooses one of the parties to try to form a government. We refer to the selected party $k \in N$ as the *formateur*. We assume that the head of state is non-strategic and each party $i \in N$ is selected to be a formateur with probability equal to its seat share π_i .¹¹ The formateur then chooses a *proto-coalition* $D \in \Delta_k$, where Δ_k denotes the set of subsets of N which contain k . (For example, if party 1 is the formateur, then $\Delta_1 = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$.) Intuitively, a proto-coalition is a set of parties that agree to talk to each other about forming a government together. After the proto-coalition is chosen, D selects a set of non-negative transfers to parties outside the proto-coalition, $t(D, s) = (t_j(D, s))_{j \in N \setminus D} \in \mathbb{R}_+^{|N \setminus D|}$. These transfers can be interpreted as payments to non-coalition parties to sustain the proposed government coalition.

Given D and t , the parliament votes to approve the formateur's proposal under majority rule. If the proposal is defeated, the default policy is implemented and each party $i \in N$ receives a period 1 payoff of $U_i(q, 0)$. If the formateur's proposal is accepted, the members of D bargain over a policy $x(D, s) \in \mathbb{R}^2$ and transfers within the coalition $r(D, s) = (r_j(D, s))_{j \in D} \in \mathbb{R}^{|D|}$. The bargaining procedure is such that for as long as no agreement

¹⁰The default policy can be interpreted, for example, as the current state of the economy, or the policy that would be implemented by a caretaker government.

¹¹For an empirical justification of this assumption see Diermeier and Merlo (1998).

is reached, each party in D is independently selected to make a proposal with probability $\frac{1}{|D|}$ and agreement entails unanimous approval of the proto-coalition members.¹² If the members of D do not reach an agreement on a common policy and vector of transfers, then the government formation attempt fails and each party $i \in N$ receives a period 1 payoff of $U_i(q, 0)$. If instead an agreement is reached, then D forms the government and each party $i \in D$ receives a period 1 payoff of $U_i(x(D, s), r_i(D, s))$ while each party $j \notin D$ receives a period 1 payoff of $U_j(x(D, s), t_j(D, s))$.

At the beginning of period 2 a new default policy $q' \in Q = \{z^1, z^2, z^3\}$ is realized. We assume that the default policy follows a Markov process with transition probabilities $\Pr[q' = z^i | q = z^i] = \lambda$ and $\Pr[q' = z^j | q = z^i] = \frac{1-\lambda}{2}$, $i \neq j = 1, 2, 3$. Also at the beginning of period 2, the parties receive a common signal about the seat share each party would receive if the current parliament were dissolved and an early election called. Let $\pi' = (\pi'_1, \pi'_2, \pi'_3)$ denote the vector of the new shares. We assume that $\pi' = \pi + \varepsilon$, where $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is a random vector that takes the value $(0, 0, 0)$ with probability ρ and takes each of the values $(-2e, e, e)$, $(e, -2e, e)$, or $(e, e, -2e)$ with probability $\frac{1-\rho}{3}$, and e is small.¹³ In particular, we assume that it is still the case that $\pi' \in \Pi$, and that $E[\pi'] = \pi$. This assumption captures the fact that in multiparty parliamentary democracies it is very unlikely that one party could gain (or lose) significant shares in a short period of time.

Let $s' \equiv (q', \pi', \pi) \in S' = Q \times \Pi \times \Pi$ denote the state of the political system in period 2, which is summarized by the default policy in that period, the distribution of seat shares parties would receive if an early election were called, and the current seat distribution.

If a government formed in period 1, then after observing s' , the incumbent government can renegotiate its agreement. Renegotiation is similar to government formation, with the incumbent government being the chosen proto-coalition. Hence, first the government may choose a set of period 2 transfers to the parties outside the government coalition, $t(D, s') = (t_j(D, s'))_{j \in N \setminus D} \in \mathbb{R}_+^{|N \setminus D|}$. Given $t(D, s')$, a vote is then taken to determine whether the incumbent government has the confidence of a parliamentary majority to continue its ruling. If the government retains the confidence of the parliament, it then bargains over a policy $x(D, s')$ and transfers within the government coalition

¹²We assume that bargaining takes no time and hence there is no within period discounting.

¹³We may think of e as one parliamentary seat.

$r(D, s') = (r_j(D, s'))_{j \in D} \in \mathbb{R}^{|D|}$ for period 2. If an agreement is reached, then D continues as a government and period 2 payoffs to the parties are determined as a function of $x(D, s')$, $r(D, s')$ and $t(D, s')$. If D fails to reach an agreement or loses the confidence of the parliament, then D terminates.

If the incumbent government terminates or no government formed in period 1, then the parliament decides under majority rule whether to dissolve and call early election or to continue. In the event an early election is called, a new government formation process begins with the head of state selecting a formateur with probabilities π' . If no early election is called, a new government formation process begins with the head of state selecting a formateur with probabilities π . Like in period 1, the outcome of the government formation process determines the period 2 payoffs to the parties. In particular, if a government D' forms, then each party $i \in D'$ receives a period 2 payoff of $U_i(x(D', s'), r_i(D', s'))$ while each party $j \notin D'$ receives a period 2 payoff of $U_j(x(D', s'), t_j(D', s'))$. If instead no government forms, then each party $i \in N$ receives a period 2 payoff of $U_i(q', 0)$.

At the end of period 2 a regularly scheduled election takes place and any incumbent government has to resign.

3 Results

Since the model we consider is a game with complete information and a finite horizon, we focus on the characterization of its subgame perfect equilibrium using backwards induction. Our characterization is presented in a series of lemmata which illustrate the main properties of the equilibrium of each subgame. A proposition containing the main result of the paper concludes the analysis.

The first lemma pertains to the outcome of the government formation process in period 2 for a given proto-coalition, if government formation is necessary.

Lemma 1: *Suppose a government formation process begins in period 2 and D' is chosen as the proto-coalition. Then for any $s' \in S'$ and for any $D' \subseteq N$, D' forms the government. Furthermore, the chosen policy is*

$$x(D', s') = \frac{1}{|D'|} \sum_{i \in D'} z^i \quad (3)$$

and transfers are equal to

$$r_i(D', s') = -\frac{1}{|D'|} \sum_{\substack{j \in D' \\ j \neq i}} u_j(q') + \frac{|D'| - 1}{|D'|} u_i(q'), \quad i \in D' \quad (4)$$

and

$$t_j(D', s') = 0, \quad j \in N \setminus D'. \quad (5)$$

Proof: Suppose first that D' obtains the confidence of the parliament given transfers $t_j(D', s')$, $j \in N \setminus D'$. For any policy $x \in \mathfrak{R}^2$ coalition D' may choose to implement, the requirement that all coalition members have to agree defines a “cake” to be allocated among the coalition partners if they agree on that policy

$$c(x; D', s') = \sum_{i \in D'} [u_i(x) - u_i(q')] - \sum_{j \in N \setminus D'} t_j(D', s').$$

Given the bargaining procedure specified, the unique stationary subgame perfect equilibrium outcome of the bargaining game which determines how the cake is allocated (and hence the transfers within the coalition), is that parties immediately agree on a split of the cake such that each party $i \in D'$ receives an equal share

$$\frac{1}{|D'|} c(x; D', s')$$

(see, Binmore (1987) for the two-player case and Merlo and Wilson (1998) for the general case).

Since each party wants to maximize its share of the cake, it immediately follows that all parties in D' unanimously agree to select the policy that maximizes the size of the cake:

$$x(D', s') = \frac{1}{|D'|} \sum_{i \in D'} z^i.$$

Hence, if $c(x(D', s'); D', s') \geq 0$, D' forms the government and each party $i \in D'$ receives a payoff equal to

$$u_i(q') + \frac{1}{|D'|} c(x(D', s'); D', s')$$

or equivalently,

$$u_i(x(D', s')) + r_i(D', s'),$$

where

$$r_i(D', s') = -\frac{1}{|D'|} \sum_{\substack{j \in D' \\ j \neq i}} u_j(q') + \frac{|D'| - 1}{|D'|} u_i(q') - \frac{1}{|D'|} \sum_{j \in N \setminus D'} t_j(D', s').$$

If instead $c(x(D', s'); D', s') < 0$, then the government formation attempt fails and each party $i \in N$ receives a payoff of $U_i(q', 0)$. These results hold for minority coalitions as well as minimal winning and surplus coalitions for any $s' \in S'$.

Consider now how transfers to parties outside the proto-coalition are determined. If D' is a majority coalition, then it does not need the support of any other party outside the proto-coalition to obtain the confidence of the parliament. Hence, if D' is a majority,

$$t_j(D', s') = 0, j \in N \setminus D'.$$

This implies that $c(x(D', s'); D', s') > 0$, and hence D' forms the government.

If instead D' is a minority coalition (i.e., $D' = \{i\}$, $i \in N$), it needs the external support of at least another party to obtain the confidence of a parliamentary majority. Let $D' = \{\mathbb{k}\}$, $\mathbb{k} \in N$, and note that $x'(\{\mathbb{k}\}, s') = z^{\mathbb{k}}$. To obtain the support of party $j \neq \mathbb{k}$, $\{\mathbb{k}\}$ needs to pay that party at least

$$\max\{0, u_j(q') - u_j(z^{\mathbb{k}})\}.$$

Since the external support of one party is enough to obtain the confidence of the parliament, $\{\mathbb{k}\}$ pays the least amount possible to at most one party. Note that for any $q' \in Q$, there always exists some party $j \in N$, $j \neq \mathbb{k}$, for which $u_j(q') - u_j(z^{\mathbb{k}}) = 0$. Hence, if D' is a minority,

$$t_j(D', s') = 0, j \in N \setminus D'.$$

This implies that $c(x(D', s'); D', s') > 0$, and hence D' forms the government. Q.E.D.

Two observations are particularly noteworthy. First, like in Baron and Diermeier (1998) the policy choice of any government coalition is independent of the default policy. Second, in the last period before a regularly scheduled

election, no government coalition makes transfers to parties outside the coalition.

The second lemma characterizes which government coalitions can form in period 2, if government formation is necessary. To answer this question, for any state in period 2 and for any formateur, we have to solve the following maximization problem the formateur faces:

$$\max_{D' \in \Delta_{\mathbf{k}}} U_{\mathbf{k}}(x(D', s'), r_{\mathbf{k}}(D', s')). \quad (6)$$

Let $D'_{\mathbf{k}}(s')$ denote the solution to this optimization problem.

Lemma 2: *Suppose a government formation process begins in period 2.*

- (i) *If $q' = z^{\mathbf{k}}$, then for any $\mathbf{k} \in N$, and for any $\pi, \pi' \in \Pi$, $D'_{\mathbf{k}}(z^{\mathbf{k}}, \pi', \pi) = \{1, 2, 3\}$ forms the government.*
- (ii) *If $q' = z^i$, then for any $\mathbf{k} \in N$, and for any $\pi, \pi' \in \Pi$, $D'_{\mathbf{k}}(z^i, \pi', \pi) = \{\mathbf{k}\}$ forms the government with the external support of party j , $i, j, \mathbf{k} \in N$, $i \neq j \neq \mathbf{k}$.*

Proof: By Lemma 1, for any $\mathbf{k} \in N$, any $q' \in Q$, and any $\pi, \pi' \in \Pi$,

$$\begin{aligned} U_{\mathbf{k}}(x(\{1, 2, 3\}, s'), r_{\mathbf{k}}(x(\{1, 2, 3\}, s'))) &= u_{\mathbf{k}}\left(\frac{z^1 + z^2 + z^3}{3}\right) - \\ &\quad \frac{1}{3} \sum_{\substack{i \in N \\ i \neq \mathbf{k}}} u_i(q') + \frac{2}{3} u_{\mathbf{k}}(q'), \\ U_{\mathbf{k}}(x(\{i, \mathbf{k}\}, s'), r_{\mathbf{k}}(x(\{i, \mathbf{k}\}, s'))) &= u_{\mathbf{k}}\left(\frac{z^i + z^{\mathbf{k}}}{2}\right) - \frac{1}{2} u_i(q') + \frac{1}{2} u_{\mathbf{k}}(q'), \\ U_{\mathbf{k}}(x(\{j, \mathbf{k}\}, s'), r_{\mathbf{k}}(x(\{j, \mathbf{k}\}, s'))) &= u_{\mathbf{k}}\left(\frac{z^j + z^{\mathbf{k}}}{2}\right) - \frac{1}{2} u_j(q') + \frac{1}{2} u_{\mathbf{k}}(q'), \end{aligned}$$

and

$$U_{\mathbf{k}}(x(\{\mathbf{k}\}, q'), r_{\mathbf{k}}(x(\{\mathbf{k}\}, q'))) = u_{\mathbf{k}}(z^{\mathbf{k}}),$$

$i, j \in N$, $i \neq j \neq \mathbf{k}$.

Next, note that for any $i, j, \mathbf{k} \in N$, $i \neq j \neq \mathbf{k}$, $u_{\mathbf{k}}\left(\frac{z^1 + z^2 + z^3}{3}\right) = -\frac{1}{3}$, $u_{\mathbf{k}}\left(\frac{z^i + z^{\mathbf{k}}}{2}\right) = u_{\mathbf{k}}\left(\frac{z^j + z^{\mathbf{k}}}{2}\right) = -\frac{1}{4}$, $u_i(z^{\mathbf{k}}) = u_j(z^{\mathbf{k}}) = -1$, and $u_{\mathbf{k}}(z^{\mathbf{k}}) = 0$.

(i) If $q' = z^{\mathbf{k}}$, then

$$U_{\mathbf{k}}(x(\{1, 2, 3\}, z^{\mathbf{k}}), r_{\mathbf{k}}(x(\{1, 2, 3\}, z^{\mathbf{k}}))) = \frac{1}{3},$$

$$\begin{aligned}
U_{\mathbf{k}}(x(\{i, \mathbf{k}\}, z^{\mathbf{k}}), r_{\mathbf{k}}(x(\{i, \mathbf{k}\}, z^{\mathbf{k}}))) &= \frac{1}{4}, \\
U_{\mathbf{k}}(x(\{j, \mathbf{k}\}, z^{\mathbf{k}}), r_{\mathbf{k}}(x(\{j, \mathbf{k}\}, z^{\mathbf{k}}))) &= \frac{1}{4},
\end{aligned}$$

and

$$U_{\mathbf{k}}(x(\{\mathbf{k}\}, z^{\mathbf{k}}), r_{\mathbf{k}}(x(\{\mathbf{k}\}, z^{\mathbf{k}}))) = 0,$$

which establishes the first part of the lemma.

(ii) If $q' = z^i$, $i \neq \mathbf{k}$, then

$$\begin{aligned}
U_{\mathbf{k}}(x(\{1, 2, 3\}, z^i), r_{\mathbf{k}}(x(\{1, 2, 3\}, z^i))) &= -\frac{2}{3}, \\
U_{\mathbf{k}}(x(\{i, \mathbf{k}\}, z^i), r_{\mathbf{k}}(x(\{i, \mathbf{k}\}, z^i))) &= -\frac{3}{4}, \\
U_{\mathbf{k}}(x(\{j, \mathbf{k}\}, z^i), r_{\mathbf{k}}(x(\{j, \mathbf{k}\}, z^i))) &= -\frac{1}{4},
\end{aligned}$$

and

$$U_{\mathbf{k}}(x(\{\mathbf{k}\}, z^i), r_{\mathbf{k}}(x(\{\mathbf{k}\}, z^i))) = 0.$$

Furthermore, party j is willing to support government $\{\mathbf{k}\}$ since

$$U_j(z^i, 0) = U_j(z^{\mathbf{k}}, 0),$$

which proves the second part of the lemma.

Q.E.D.

Note that Lemma 2 implies that if government formation occurs in the last period before a regularly scheduled election, a minimal winning coalition government never forms. Only minority or surplus governments form.

Another interesting observation is that if the party favored by the default policy is selected as formateur, it chooses to form a surplus government rather than forming a minority government and implementing its most preferred policy. The reason this happens, is that by forming a surplus government and compromising on the policy choice, the formateur can elicit transfers from the other two parties when their willingness to pay is the highest.¹⁴

The next lemma establishes the conditions under which early elections occur in period 2, if the parliament is called to decide upon the matter.

¹⁴For another framework that generates surplus governments, see Baron and Diermeier (1998).

Lemma 3: *Suppose the parliament has to decide whether to dissolve and call early elections in period 2. For any $q' \in Q$, whenever $\pi' \neq \pi$, an early election is called.*

Proof: Suppose at the beginning of period 2 there is no incumbent government or the incumbent government terminates. Using Lemma 2, given q' , π , and π' , for each party $i \in N$ we can compute its expected continuation payoff under the two alternative scenarios where an early election is called or no early election is called. Let $W_i(q', \tilde{\pi})$ denote party i 's expected continuation payoff, where $\tilde{\pi} \in \{\pi, \pi'\}$.

Without loss of generality, let $q' = z^\ell$. Then, with probability $\tilde{\pi}_\ell$ party ℓ is chosen as formateur yielding the party the payoff

$$U_\ell(x(\{1, 2, 3\}, z^\ell), r_\ell(x(\{1, 2, 3\}, z^\ell))) = \frac{1}{3}$$

while with probability $1 - \tilde{\pi}_\ell$ some other party $j \neq \ell$ is chosen as formateur yielding party ℓ the payoff $U_\ell(x(\{j\}), 0) = -1$. Hence,

$$W_\ell(z^\ell, \tilde{\pi}) = \frac{4}{3}\tilde{\pi}_\ell - 1.$$

Now consider the expected payoff to party $j \neq \ell$. With probability $\tilde{\pi}_j$ party j is chosen as formateur yielding the party the payoff

$$U_j(x(\{j\}, z^\ell), r_j(x(\{j\}, z^\ell))) = 0$$

with probability $\tilde{\pi}_\ell$ party ℓ is chosen as formateur yielding party j the payoff

$$U_j(x(\{1, 2, 3\}, z^\ell), r_j(x(\{1, 2, 3\}, z^\ell))) = -\frac{2}{3}$$

Finally, with probability $1 - \tilde{\pi}_i - \tilde{\pi}_\ell$ party h is chosen as formateur yielding party j the payoff $U_j(x(\{h\}), 0) = -1$. Hence,

$$W_j(z^\ell, \tilde{\pi}) = \frac{1}{3}\tilde{\pi}_\ell + \tilde{\pi}_j - 1$$

Similarly, the expected continuation payoff to party h is equal to

$$W_h(z^\ell, \tilde{\pi}) = \frac{1}{3}\tilde{\pi}_\ell + \tilde{\pi}_h - 1,$$

$h, j, \ell \in N, h \neq j \neq \ell$.

It is easy to see that for any party $i \in N$, for any realization of ε such that $\pi'_i > \pi_i$, $W_i(z^\ell, \pi') > W_i(z^\ell, \pi)$ and hence party i votes in favor of an early election. Obviously, if $\pi = \pi'$, then $W_i(z^\ell, \pi') = W_i(z^\ell, \pi)$ for all $i \in N$, and hence no party votes in favor of an early election. Therefore, for any realization of ε such that $\pi' \neq \pi$, since it is always the case that two parties gain at the expenses of the third party, a majority strictly prefers to dissolve the parliament and call early elections.

Q.E.D.

The previous lemma implies that in so far as governments can terminate before the expiration of a parliamentary term, early elections are likely to occur. How likely they are to occur, however, depends on the probability they would alter the current distribution of seat shares which is equal to $1 - \rho$.

Lemma 3 also implies that to include π in the description of the state of the political system in period 2 is redundant. Hence, in the remainder of the section we refer to $s' \equiv (q', \pi') \in S = Q \times \Pi$.

By combining the results of the previous lemmata, given s' we can compute the parties expected payoffs in period 2 if there is no incumbent government or if the incumbent government terminates. For any party $i \in N$, let

$$W_i(s') = \begin{cases} \frac{4}{3}\pi'_i - 1 & \text{if } q' = z^i \\ \frac{1}{3}\pi'_j + \pi'_i - 1 & \text{if } q' = z^j \\ \frac{1}{3}\pi'_\ell + \pi'_i - 1 & \text{if } q' = z^\ell \end{cases} \quad (7)$$

$j, \ell \in N, i \neq j \neq \ell$, denote party i 's expected payoff before the beginning of the government formation process in period 2.

The next lemma pertains to the renegotiation stage of an incumbent government and characterizes the conditions under which a government would prematurely terminate.

Lemma 4: *Let $D \subseteq N$ denote the incumbent government at the beginning of period 2.*

(i) Suppose D is a majority government. Then for any $s' \in S$, D remains in power throughout period 2. Furthermore, the chosen policy is

$$x'(D, s') = \frac{1}{|D|} \sum_{i \in D} z^i \quad (8)$$

and transfers are equal to

$$r'_i(D, s') = -\frac{1}{|D|} \sum_{\substack{j \in D \\ j \neq i}} W_j(s') + \frac{|D| - 1}{|D|} W_i(s'), \quad i \in D \quad (9)$$

and

$$t'_j(D, s') = 0, \quad j \in N \setminus D. \quad (10)$$

(ii) Suppose D is a minority government and let $D = \{i\}$. Then for $q' = z^j$, $i, j \in N$, $i \neq j$, and for any $\pi' \in \Pi$, D remains in power throughout period 2. Furthermore, the chosen policy is

$$x'(\{i\}, s') = z^i \quad (11)$$

and if $\pi'_j < \pi'_\ell$ transfers are equal to

$$r'_i(\{i\}, s') = -(W_j(s') - u_j(z^i)) \quad (12)$$

and

$$t'_j(\{i\}, s') = W_j(s') - u_j(z^i) \text{ and } t'_\ell(\{i\}, s') = 0, \quad (13)$$

whereas if $\pi'_j > \pi'_\ell$ transfers are equal to

$$r'_i(\{i\}, s') = -(W_\ell(s') - u_\ell(z^i)) \quad (14)$$

and

$$t'_\ell(\{i\}, s') = W_\ell(s') - u_\ell(z^i) \text{ and } t'_j(\{i\}, s') = 0, \quad (15)$$

$i, j, \ell \in N$, $i \neq j \neq \ell$.

(iii) Suppose D is a minority government and let $D = \{i\}$. Then for $q' = z^i$, if $\pi'_j < \frac{2}{5}(1 - \pi'_\ell)$ and $\pi'_\ell < \frac{2}{5}(1 - \pi'_j)$, $\{i\}$ terminates and the continuation payoff of each party $i \in N$ is given by $W_i(s')$. If instead $\pi'_j > \frac{2}{5}(1 - \pi'_\ell)$ or $\pi'_\ell > \frac{2}{5}(1 - \pi'_j)$, $\{1\}$ remains in power throughout period 2. In this case, the chosen policy is

$$x'(\{i\}, s') = z^i \quad (16)$$

and if $\pi'_j < \pi'_\ell$ transfers are equal to

$$r'_i(\{i\}, s') = -(W_j(s') - u_j(z^i)) \quad (17)$$

and

$$t'_j(\{i\}, s') = W_j(s') - u_j(z^i) \text{ and } t'_\ell(\{i\}, s') = 0, \quad (18)$$

whereas if $\pi'_j > \pi'_\ell$ transfers are equal to

$$r'_i(\{i\}, s') = -(W_\ell(s') - u_\ell(z^i)) \quad (19)$$

and

$$t'_j(\{i\}, s') = W_\ell(s') - u_\ell(z^i) \text{ and } t'_j(\{i\}, s') = 0, \quad (20)$$

$i, j, \ell \in N, i \neq j \neq \ell$.

Proof: The argument is analogous to the one presented in the proof of Lemma 1. The main difference is the fact that the failure of a renegotiation leads to a government formation attempt, and hence to a payoff equal to $W_i(s')$ for each party $i \in N$.

Suppose first that D has maintained the confidence of the parliament given transfers $t_j(D, s')$, $j \in N \setminus D$. Then the cake to be allocated among the coalition partners if they remain together and agree on a policy $x \in \mathfrak{R}^2$ is

$$c(x; D, s') = \sum_{i \in D} [u_i(x) - W_i(s')] - \sum_{j \in N \setminus D} t_j(D, s').$$

and each party $i \in D$ receives an equal share

$$\frac{1}{|D|} c(x; D, s').$$

Hence, by the same argument given in the proof of Lemma 1,

$$x(D, s') = \frac{1}{|D|} \sum_{i \in D} z^i$$

and if $c(x(D, s'); D, s') \geq 0$, D stays in power and each party $i \in D$ receives a period 2 payoff equal to

$$U_i(x(D, s'), r_i(D, s')),$$

where

$$r_i(D, s') = -\frac{1}{|D|} \sum_{\substack{j \in D \\ j \neq i}} W_j(s') + \frac{|D| - 1}{|D|} W_i(s') - \frac{1}{|D|} \sum_{j \in N \setminus D} t_j(D, s').$$

If instead $c(x(D, s'); D, s') < 0$, then the renegotiation attempt fails and each party $i \in N$ receives its expected continuation payoff $W_i(s')$.

If D is a majority coalition, then it does not need the support of any other party outside the government coalition to obtain the confidence of the parliament. Hence, we have that

$$t_j(D, s') = 0, j \in N \setminus D'.$$

This implies that $c(x(D, s'); D, s') > 0$, and hence D remains the government. This proves the first part of the lemma.

If D is a minority coalition, it needs the external support of at least another party to maintain the confidence of a parliamentary majority. Without loss of generality, suppose that $q' = z^\ell$.

First consider case (ii). That is, let $D = \{i\}$ with $i \neq \ell$. Then $x(\{i\}, z^\ell) = z^i$. To obtain the support of some party $h \neq i$, $\{i\}$ needs to pay that party at least

$$\max\{0, W_h(z^\ell, \pi') - u_h(z^i)\}.$$

Note that for any $s' \in S$ and any $h \in N$, $W_h(s') - u_h(z^i) > 0$. Hence, to stay in power $\{i\}$ needs to make a positive transfer either to party j or to party ℓ , $i \neq j \neq \ell$. Since the external support of one party is enough to obtain the confidence of the parliament, $\{i\}$ seeks the support of the cheapest outside party. Now

$$W_\ell(z^\ell, \pi') - u_\ell(z^i) \geq W_j(z^\ell, \pi') - u_j(z^i)$$

if and only if

$$\frac{4}{3}\pi'_\ell \geq \frac{1}{3}\pi'_\ell + \pi'_j$$

if and only if

$$\pi'_\ell \geq \pi'_j.$$

Suppose that $\pi'_\ell \geq \pi'_j$. Then, to stay in power $\{i\}$ needs to make the transfer

$$t_j(\{i\}, z^\ell, \pi') = W_j(z^\ell, \pi') - u_j(z^i)$$

in which case party i 's period 2 payoff is equal to

$$U_i(z^i, -(W_j(z^\ell, \pi') - u_j(z^i))).$$

Alternatively, $\{i\}$ could terminate, in which case party i would receive a period 2 expected payoff equal to

$$W_i(z^\ell, \pi').$$

Obviously, $\{i\}$ chooses to make the transfer and stay in power if and only if

$$U_i(z^i, -(W_j(z^\ell, \pi') - u_j(z^i))) \geq W_i(z^\ell, \pi')$$

if and only if

$$u_i(z^i) + u_j(z^i) \geq W_i(z^\ell, \pi') + W_j(z^\ell, \pi')$$

if and only if

$$-1 \geq \frac{1}{3}\pi'_\ell + \pi'_i - 1 + \frac{1}{3}\pi'_\ell + \pi'_j - 1$$

if and only if

$$0 \geq -\frac{1}{3}\pi'_\ell$$

which is always true.

Now suppose that $\pi'_\ell \leq \pi'_j$. Then $\{i\}$ chooses to make the necessary transfer to party ℓ and stay in power if and only if

$$U_i(z^i, -(W_\ell(z^\ell, \pi') - u_\ell(z^i))) \geq W_i(z^\ell, \pi')$$

if and only if

$$u_i(z^i) + u_\ell(z^i) \geq W_i(z^\ell, \pi') + W_\ell(z^\ell, \pi')$$

if and only if

$$-1 \geq \frac{1}{3}\pi'_\ell + \pi'_i - 1 + \frac{4}{3}\pi'_\ell - 1$$

if and only if

$$1 \geq \frac{5}{3}\pi'_\ell + \pi'_i$$

if and only if

$$\pi'_j \geq \frac{2}{3}\pi'_\ell$$

which is always true since we are assuming that $\pi'_\ell \leq \pi'_j$. This proves the second part of the lemma.

Finally, consider case (iii). That is, let $D = \{\ell\}$. Then $x(\{\ell\}, z^\ell) = z^\ell$. As in the previous case $\{\ell\}$ seeks the support of the cheapest outside party. Now

$$W_i(z^\ell, \pi') - u_i(z^\ell) \geq W_j(z^\ell, \pi') - u_j(z^\ell)$$

if and only if

$$\frac{1}{3}\pi'_\ell + \pi'_i \geq \frac{1}{3}\pi'_\ell + \pi'_j$$

if and only if

$$\pi'_i \geq \pi'_j.$$

Suppose that $\pi'_i \geq \pi'_j$. Then $\{\ell\}$ chooses to make the necessary transfer to party j and remain in office if and only if

$$U_\ell(z^\ell, -(W_j(z^\ell, \pi') - u_j(z^\ell))) \geq W_\ell(z^\ell, \pi')$$

if and only if

$$u_\ell(z^\ell) + u_j(z^\ell) \geq W_\ell(z^\ell, \pi') + W_j(z^\ell, \pi')$$

if and only if

$$-1 \geq \frac{4}{3}\pi'_\ell - 1 + \frac{1}{3}\pi'_\ell + \pi'_j - 1$$

if and only if

$$\pi'_i \geq \frac{2}{5}(1 - \pi'_j).$$

Now suppose that $\pi'_i \leq \pi'_j$. Then $\{\ell\}$ chooses to make the necessary transfer to party i and remain in office if and only if

$$U_\ell(z^\ell, -(W_i(z^\ell, \pi') - u_i(z^\ell))) \geq W_\ell(z^\ell, \pi')$$

if and only if

$$u_\ell(z^\ell) + u_i(z^\ell) \geq W_\ell(z^\ell, \pi') + W_i(z^\ell, \pi')$$

if and only if

$$-1 \geq \frac{4}{3}\pi'_\ell - 1 + \frac{1}{3}\pi'_\ell + \pi'_i - 1$$

if and only if

$$\pi'_j \geq \frac{2}{5}(1 - \pi'_i).$$

Thus, if $\pi'_i < \frac{2}{5} - \frac{2}{5}\pi'_i$ and $\pi'_i < \frac{2}{5} - \frac{2}{5}\pi'_i$, $\{\ell\}$ terminates and each party $h \in N$ receives its expected continuation payoff $W_h(z^\ell, \pi')$. Otherwise, $\{\ell\}$ remains in power so that

$$x(\{\ell\}, z^\ell) = z^\ell$$

and if $\pi'_i \geq \pi'_j$

$$r_\ell(\{\ell\}, z^\ell, \pi') = -(W_j(z^\ell, \pi') - u_j(z^\ell))$$

and

$$t_j(\{\ell\}, z^\ell, \pi') = W_j(z^\ell, \pi') - u_j(z^\ell) \text{ and } t_i(\{\ell\}, z^\ell, \pi') = 0,$$

whereas if $\pi'_i \leq \pi'_j$ transfers are equal to

$$r_\ell(\{\ell\}, z^\ell, \pi') = -(W_i(z^\ell, \pi') - u_i(z^\ell))$$

and

$$t_i(\{\ell\}, z^\ell, \pi') = W_i(z^\ell, \pi') - u_i(z^\ell) \text{ and } t_j(\{\ell\}, z^\ell, \pi') = 0,$$

which concludes the proof of the lemma.

Q.E.D.

This lemma implies that only minority governments may terminate before the expiration of the parliamentary term. It also implies that reshuffles are the norm for both minimal winning and surplus governments.

Using Lemma 4, for any given s' and for any government coalition D , we can compute the expected payoff to each party $i \in N$ in period 2 following a renegotiation by the government coalition

$$V_i(D, s') = \begin{cases} U_i(x(D, s'), r_i(D, s')) & \text{if } i \in D \text{ and } D \text{ remains in power} \\ U_i(x(D, s'), t_i(D, s')) & \text{if } i \notin D \text{ and } D \text{ remains in power} \\ W_i(s') & \text{if } D \text{ terminates} \end{cases} \quad (21)$$

The next lemma pertains to the outcome of the government formation process in period 1 for a given proto-coalition.

Lemma 5: *Suppose at the beginning of period 1 D is chosen as the proto-coalition.*

(i) *For any $s \in S$ and for any $D \subseteq N$, D forms the government and chooses policy*

$$x(D, s) = \frac{1}{|D|} \sum_{i \in D} z^i. \quad (22)$$

(ii) *For any $s \in S$, if D is a majority period 1 transfers are equal to*

$$r_i(D, s) = -\frac{1}{|D|} \sum_{\substack{j \in D \\ j \neq i}} u_j(q) + \frac{|D| - 1}{|D|} u_i(q), \quad i \in D \quad (23)$$

and

$$t_j(D, s) = 0, \quad j \in N \setminus D. \quad (24)$$

(iii) Suppose D is a minority and let $D = \{i\}$. For any $s \in S$, period 1 transfers are equal to

$$r_i(\{i\}, s) = \min\{0, -\beta E[W_j(s') - V_j(\{i\}, s')|s]\} \quad (25)$$

and

$$t_j(\{i\}, s) = \max\{0, \beta E[W_j(s') - V_j(\{i\}, s')|s]\}, \text{ and } t_\ell(\{i\}, s) = 0, \quad (26)$$

where $j \in N$, $j \neq i \neq \ell$ is such that if $q = z^i$, $\pi_j \leq \pi_\ell$, or if $q = z^h$, $j \neq h$.

Proof: The proof of the first part of the lemma follows from the same argument used in the proof of Lemma 1. Suppose first that D obtains the confidence of the parliament given transfers $t_j(D, s)$, $j \in N \setminus D$. Then, since the cake available to D in period 1 to be distributed among the coalition partners if they agree to implement a policy $x \in \mathbb{R}^2$ is

$$c(x; D, s) = \sum_{i \in D} [u_i(x) - u_i(q)] - \sum_{j \in N \setminus D} t_j(D, s)$$

and each party receives an equal share, if D forms the government it chooses policy

$$x(D, s) = \frac{1}{|D|} \sum_{i \in D} z^i.$$

To show that D wants to form the government, however, it is no longer sufficient to show that $c(x; D, s) \geq 0$. If the government formation attempt fails, each party $i \in N$ obtains a total payoff equal to

$$U_i(q, 0) + \beta E[W_i(s')|s].$$

Hence, D forms the government if and only if for all $i \in D$,

$$U_i(x(D, s), r_i(D, s)) + \beta E[V_i(D, s')|s] \geq U_i(q, 0) + \beta E[W_i(s')|s].$$

Begin by considering the case where D is a majority. If D is a majority coalition, then it does not need the support of any other party outside the proto-coalition to obtain the confidence of the parliament. Hence,

$$t_j(D, s) = 0, \quad j \in N \setminus D.$$

This implies that for all $i \in D$, $U_i(x(D, s), r_i(D, s)) > U_i(q, 0)$. Next observe that Lemma 4 implies that for all $s' \in S$, and all $i \in D$, $V_i(D, s') > W_i(s')$,

which establishes the result. Hence, D forms the government and transfers within the government coalition in period 1 are equal to

$$r_i(D, s) = -\frac{1}{|D|} \sum_{\substack{j \in D \\ i \neq j}} u_j(q) + \frac{|D| - 1}{|D|} u_i(q), i \in D$$

This completes the proof of parts (i) and (ii) of the lemma.

Next consider the case where D is a minority and let $D = \{i\}$. To obtain the confidence of a parliamentary majority, $\{i\}$ has to compute the minimum transfer required by one of the other two parties to elicit their support. For each party $h \neq i$, since the failure of the government formation attempt yields that party an expected payoff equal to

$$U_h(q, 0) + \beta E[W_h(s')|s],$$

such transfer is equal to

$$\max\{0, (u_h(q) - u_h(z^i)) + \beta E[W_h(s') - V_h(\{i\}, s')|s]\}.$$

Since q' and π' are independent, both $W_h(s')$ and $V_h(\{i\}, s')$ are linear in π' (see equations (7) and (21)), and $\pi' = \pi + \varepsilon$ with $E(\varepsilon) = 0$,

$$\begin{aligned} E[W_h(s') - V_h(\{i\}, s')|s] &= E_{q'}[E_{\pi'}[W_h(q', \pi') - V_h(\{i\}, q', \pi')|\pi]|q] \\ &= E_{q'}[W_h(q', \pi) - V_h(\{i\}, q', \pi)|q] \end{aligned}$$

and hence we can rewrite the expression for the transfer as

$$\max\{0, (u_h(q) - u_h(z^i)) + \beta E_{q'}[W_h(q', \pi) - V_h(\{i\}, q', \pi)|q]\}.$$

First, suppose that $q = z^i$. Then $u_j(q) - u_j(z^i) = u_\ell(q) - u_\ell(z^i) = 0$, $j, \ell \in N$, $i \neq j \neq \ell$. Without loss of generality, assume that $\pi_j \leq \pi_\ell$. Then, Lemma 4 implies that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) = 0$, whether $\{i\}$ remains in power or terminates in period 2. Hence,

$$t_j(\{i\}, s) = 0 \text{ and } t_\ell(\{i\}, s) = 0.$$

Together with the fact that for all $q' \in Q$, $V_i(\{i\}, q', \pi) \geq W_i(q', \pi)$ (and the inequality is strict for all period 2 states where $\{i\}$ remains in power—see Lemma 4), this implies that

$$U_i(z^i, 0) + \beta E_{q'}[V_i(\{i\}, q', \pi)|q = z^i] > U_i(z^i, 0) + \beta E_{q'}[W_i(q', \pi)|q = z^i],$$

and hence $\{i\}$ forms the government in period 1.

Suppose now that $q = z^\ell$. Then $u_j(q) - u_j(z^i) = 0$ and $u_\ell(q) - u_\ell(z^i) = 1$, $j, \ell \in N$, $i \neq j \neq \ell$. If $\pi_j \leq \pi_\ell$, Lemma 4 implies that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) = 0$, whether $\{i\}$ remains in power or terminates in period 2. Hence, again it is the case that

$$t_j(\{i\}, s) = 0 \text{ and } t_\ell(\{i\}, s) = 0,$$

and

$$U_i(z^i, 0) + \beta E_{q'}[V_i(\{i\}, q', \pi)|q = z^\ell] > U_i(z^\ell, 0) + \beta E_{q'}[W_i(q', \pi)|q = z^\ell],$$

so that $\{i\}$ forms the government in period 1.

If instead $\pi_j > \pi_\ell$, Lemma 4 implies that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) \geq 0$, (since if $\{i\}$ remains in power in period 2, it obtains the external support of party ℓ). Hence, in this case, to obtain the support of party j in period 1, $\{i\}$ needs to pay that party

$$\beta E_{q'}[W_j(q', \pi) - V_j(\{i\}, q', \pi)|q = z^\ell].$$

To obtain the support of party ℓ in period 1, $\{i\}$ needs to pay that party

$$u_\ell(z^\ell) - u_\ell(z^i) = 1,$$

since for all $q' \in Q$, $W_\ell(q', \pi) - V_\ell(\{i\}, q', \pi) = 0$. Note that this amount is always larger than what $\{i\}$ would have to pay party j . This follows from the fact that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) < 1$. Thus, if $\pi_j > \frac{2}{5}(1 - \pi_\ell)$ (i.e., $W_j(z^h, \pi) - V_j(\{i\}, z^h, \pi) > 0$ for all $h = i, j, \ell$),

$$t_j(\{i\}, z^\ell, \pi) = \beta \left(\pi_j - \frac{1}{6}\pi_\ell(1 - 3\lambda) + \frac{1}{6}(1 - \lambda) \right) \text{ and } t_\ell(\{i\}, z^\ell, \pi) = 0,$$

whereas if $\pi_j < \frac{2}{5}(1 - \pi_\ell)$ (i.e., $W_j(z^i, \pi) - V_j(\{i\}, z^i, \pi) = 0$ since $\{i\}$ would terminate if $q' = z^i$),

$$t_j(\{i\}, z^\ell, \pi) = \beta \left(\pi_j \left(\frac{2 + \lambda}{3} \right) + \pi_\ell \frac{\lambda}{3} \right) \text{ and } t_\ell(\{i\}, z^\ell, \pi) = 0.$$

The last thing we need to show is that $\{i\}$ wants to form the government. This is always the case if

$$\begin{aligned} U_i(z^i, -t_j(\{i\}, z^\ell, \pi)) + \beta E_{q'}[V_i(\{i\}, q', \pi)|q = z^\ell] &> U_i(z^\ell, 0) + \\ \beta E_{q'}[W_i(q', \pi)|q = z^\ell], \end{aligned}$$

which is always true since for all $q' \in Q$, $V_i(\{i\}, q', \pi) \geq W_i(q', \pi)$ (see Lemma 4), and $t_j(\{i\}, z^\ell, \pi) < 1$ (as shown above) implies

$$U_i(z^i, -t_j(\{i\}, z^\ell, \pi)) = -t_j(\{i\}, z^\ell, \pi) > -1 = U_i(z^\ell, 0).$$

Since the analysis of the case $q = z^j$ is totally symmetric—that is, $\{i\}$ would always seek the support of party ℓ instead of party j —this concludes the proof of part (iii) of the lemma.

Q.E.D.

When combined with Lemma 4, Lemma 5 implies that no government coalition ever changes its policy in the second period. Furthermore, the lemma identifies the conditions under which minority coalitions have to “buy” the external support of other parties to form the government.

By combining the results of the five previous lemmata, for any given s we can compute the expected utility of each party $i \in N$ in period 1 conditional on each possible government coalition forming in period 1

$$V_i(D, s) = \begin{cases} U_i(x(D, s), r_i(D, s)) + \beta E[V_i(D, s')|s] & \text{if } i \in D \\ U_i(x(D, s), t_i(D, s)) + \beta E[V_i(D, s')|s] & \text{if } i \notin D. \end{cases} \quad (27)$$

These calculations represent the basis for the main result of the paper, stated in the following proposition, which characterizes the outcome of the government formation process in period 1. This characterization hinges on the solution of the following maximization problem faced by the formateur in period 1, for any state and for any formateur:

$$\max_{D \in \Delta_{\mathbf{k}}} V_{\mathbf{k}}(D, s) \quad (28)$$

Let $D_{\mathbf{k}}(s)$ denote the solution to this optimization problem.

Proposition 1: *Consider the government formation process at the beginning of period 1 and let $\mathbf{k} \in N$ denote the identity of the formateur.*

(i) *Suppose $q = z^{\mathbf{k}}$. Then, there exist a critical value b^* and a critical decreasing function of β , $p^*(\beta)$, such that, for any π , if $\beta < b^*$, $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{1, 2, 3\}$ forms the government. If $\beta > b^*$, then for $\pi_j < \pi_\ell$ and $\pi_\ell > p^*(\beta)$, $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{\mathbf{k}, j\}$ forms the government; for $\pi_\ell < \pi_j$ and $\pi_j > p^*(\beta)$, $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{\mathbf{k}, \ell\}$ forms the government; and for $\pi_\ell < p^*(\beta)$ and $\pi_j < p^*(\beta)$, $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{1, 2, 3\}$ forms the government, $\ell, j, \mathbf{k} \in N$, $\ell \neq j \neq \mathbf{k}$.*

(ii) *Suppose $q = z^\ell$, $\ell \neq \mathbf{k}$. Then, for $\pi_j < \pi_\ell$, $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{\mathbf{k}\}$ forms the government with the external support of party j . For $\pi_\ell < \pi_j$ and*

$\pi_j > \frac{2}{5}(1 - \pi_\ell)$, there exist two critical functions of λ , $b_1^*(\lambda)$ and $b_2^*(\lambda)$, with $b_1^*(\lambda)$ decreasing in λ and $b_2^*(\lambda)$ increasing in λ , and a critical function of β and λ , $p_\ell^*(\beta, \lambda)$, which is increasing in λ and decreasing in β , such that, if $\beta < b_1^*(\lambda)$, then $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{\mathbf{k}\}$ forms the government with the external support of party j ; if $\beta > b_2^*(\lambda)$, then $D_{\mathbf{k}}(z^\ell, \pi) = \{\mathbf{k}, j\}$ forms the government; and if $b_1^*(\lambda) < \beta < b_2^*(\lambda)$, then for $\pi_j < p_j^*(\beta, \lambda)$, $D_{\mathbf{k}}(z^\ell, \pi) = \{\mathbf{k}\}$ forms the government with the external support of party j , whereas for $\pi_j > p_j^*(\beta, \lambda)$, $D_{\mathbf{k}}(z^\ell, \pi) = \{\mathbf{k}, j\}$ forms the government. Finally, for $\pi_\ell < \pi_j$ and $\pi_j < \frac{2}{5}(1 - \pi_\ell)$, there exist two critical decreasing functions of λ , $b_3^*(\lambda)$ and $b_4^*(\lambda)$, and a critical function of β , λ and π_ℓ , $p_j^*(\beta, \lambda, \pi_\ell)$, which is decreasing in β , increasing in λ , and decreasing in π_ℓ , such that, if $\beta < b_3^*(\lambda)$, then $D_{\mathbf{k}}(z^{\mathbf{k}}, \pi) = \{\mathbf{k}\}$ forms the government with the external support of party j ; if $\beta > b_4^*(\lambda)$, then $D_{\mathbf{k}}(z^\ell, \pi) = \{\mathbf{k}, j\}$ forms the government; and if $b_3^*(\lambda) < \beta < b_4^*(\lambda)$, then for $\pi_j < p_j^*(\beta, \lambda)$, $D_{\mathbf{k}}(z^\ell, \pi) = \{\mathbf{k}\}$ forms the government with the external support of party j , whereas for $\pi_j > p_j^*(\beta, \lambda)$, $D_{\mathbf{k}}(z^\ell, \pi) = \{\mathbf{k}, j\}$ forms the government, $\ell, j, \mathbf{k} \in N$, $\ell \neq j \neq \mathbf{k}$.

Proof: (i) Suppose first that $q = z^{\mathbf{k}}$. Then by Lemma 5, (using the normalization $\pi_{\mathbf{k}} = 1 - \pi_j - \pi_\ell$), for any $\pi \in \Pi$,

$$\begin{aligned} V_{\mathbf{k}}(\{1, 2, 3\}, z^{\mathbf{k}}, \pi) &= \frac{1}{3} + \beta \left(\frac{1}{3} - \pi_j - \pi_\ell \right) \\ V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^{\mathbf{k}}, \pi) &= \frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) \\ V_{\mathbf{k}}(\{\ell, \mathbf{k}\}, z^{\mathbf{k}}, \pi) &= \frac{1}{4} + \beta \left(\frac{1}{4} - \pi_\ell - \frac{1}{2}\pi_j \right) \end{aligned}$$

and for $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{5}(1 - \pi_j)$ (i.e., $\{\mathbf{k}\}$ receives the external support of party j and always survives in the second period),

$$V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta \left(-\pi_j \left(\frac{7 - 3\lambda}{6} \right) - \pi_\ell \left(\frac{1 - 3\lambda}{6} \right) - \frac{\lambda}{3} \right),$$

for $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{5}(1 - \pi_j)$ (i.e., $\{\mathbf{k}\}$ receives the external support of party j and terminates in the second period if $q' = \mathbf{k}$),

$$V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta \left(-\pi_j \left(\frac{7 + \lambda}{6} \right) - \pi_\ell \left(\frac{1 + 7\lambda}{6} \right) + \frac{\lambda}{3} \right),$$

for $\pi_j > \pi_\ell$ and $\pi_j > \frac{2}{5}(1 - \pi_\ell)$ (i.e., $\{\mathbf{k}\}$ receives the external support of party ℓ and always survives in the second period),

$$V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta \left(-\pi_\ell \left(\frac{7 - 3\lambda}{6} \right) - \pi_j \left(\frac{1 - 3\lambda}{6} \right) - \frac{\lambda}{3} \right),$$

and for $\pi_j > \pi_\ell$ and $\pi_j < \frac{2}{5}(1 - \pi_\ell)$ (i.e., $\{\mathbf{k}\}$ receives the external support of party ℓ and terminates in the second period if $q' = \mathbf{k}$),

$$V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta \left(-\pi_\ell \left(\frac{7 + \lambda}{6} \right) - \pi_j \left(\frac{1 + 7\lambda}{6} \right) + \frac{\lambda}{3} \right).$$

First, note that if $\pi_j < \pi_\ell$, then \mathbf{k} prefers $\{j, \mathbf{k}\}$ to $\{\ell, \mathbf{k}\}$, and it prefers $\{\mathbf{k}\}$ with the external support of j to $\{\mathbf{k}\}$ with the external support of ℓ , while the opposite is true if $\pi_j > \pi_\ell$.

Suppose $\pi_j < \pi_\ell$. Then

$$V_{\mathbf{k}}(\{1, 2, 3\}, z^{\mathbf{k}}, \pi) > V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$\frac{1}{3} + \beta \left(\frac{1}{3} - \pi_j - \pi_\ell \right) > \frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right)$$

if and only if

$$\pi_\ell > \frac{1 + \beta}{6\beta}.$$

Since $\pi_\ell \in (0, \frac{1}{2})$, this implies that if $\beta < b^* = \frac{1}{2}$, \mathbf{k} prefers $\{1, 2, 3\}$ to $\{\mathbf{k}, j\}$ for all $\pi \in \Pi$ where $\pi_j < \pi_\ell$. If $\beta > b^* = \frac{1}{2}$, then \mathbf{k} prefers $\{\mathbf{k}, j\}$ to $\{1, 2, 3\}$ for $\pi_\ell > p^*(\beta) = \frac{1+\beta}{6\beta}$, and it prefers $\{1, 2, 3\}$ to $\{\mathbf{k}, j\}$ for $\pi_\ell < p^*(\beta) = \frac{1+\beta}{6\beta}$. Note that $p^*(\beta)$ is a decreasing function of β .

Next, we show that for all $\pi \in \Pi$ where $\pi_j < \pi_\ell$, \mathbf{k} prefers $\{j, \mathbf{k}\}$ to $\{\mathbf{k}\}$. Note that if it were ever the case that \mathbf{k} prefers $\{\mathbf{k}\}$ to $\{j, \mathbf{k}\}$, it would have to be true that the payoff gain in period 2 is large enough to compensate the payoff loss in period 1. Hence, set $\beta = 1$ (i.e., the best case for $\{\mathbf{k}\}$). For $\pi_\ell > \frac{2}{5}(1 - \pi_j)$,

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^{\mathbf{k}}, \pi) > V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$\frac{1}{4} + \frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell > -\pi_j \left(\frac{7 - 3\lambda}{6} \right) - \pi_\ell \left(\frac{1 - 3\lambda}{6} \right) - \frac{\lambda}{3}$$

if and only if

$$\pi_\ell < \frac{\pi_j(1-3\lambda) + (3+2\lambda)}{2+3\lambda}$$

which is always true since the right hand side is greater than $\frac{1}{2}$. For $\pi_\ell < \frac{2}{5}(1-\pi_j)$,

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^{\mathbf{k}}, \pi) > V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$\frac{1}{4} + \frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell > -\pi_j\left(\frac{7+\lambda}{6}\right) - \pi_\ell\left(\frac{1+7\lambda}{6}\right) + \frac{\lambda}{3}$$

if and only if

$$\pi_j > \frac{\pi_\ell(2-7\lambda) + 2\lambda - 3}{1+\lambda}$$

which is always true since the right hand side is negative.

Since the analysis is perfectly symmetric for the case where $\pi_j > \pi_\ell$, this concludes the proof of the first part of the lemma. Hence, if $\beta < b^*$, $\{1, 2, 3\}$ forms the government. If $\beta > b^*$, then for $\pi_j < \pi_\ell$ and $\pi_\ell > p^*(\beta)$, $\{\mathbf{k}, j\}$ forms the government; for $\pi_\ell < \pi_j$ and $\pi_j > p^*(\beta)$, $\{\mathbf{k}, \ell\}$ forms the government; and for $\pi_\ell < p^*(\beta)$ and $\pi_j < p^*(\beta)$, $\{1, 2, 3\}$ forms the government.

(ii) Now suppose that $q = z^\ell$. Then by Lemma 5, (using the normalization $\pi_{\mathbf{k}} = 1 - \pi_j - \pi_\ell$), for any $\pi \in \Pi$,

$$\begin{aligned} V_{\mathbf{k}}(\{1, 2, 3\}, z^\ell, \pi) &= -\frac{2}{3} + \beta \left(\frac{1}{3} - \pi_j - \pi_\ell \right) \\ V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) &= -\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) \\ V_{\mathbf{k}}(\{\ell, \mathbf{k}\}, z^\ell, \pi) &= -\frac{3}{4} + \beta \left(\frac{1}{4} - \pi_\ell - \frac{1}{2}\pi_j \right) \end{aligned}$$

and for $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{5}(1-\pi_j)$ (i.e., $\{\mathbf{k}\}$ does not have to pay party j in period 1 to receive its support and always survives in the second period),

$$V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta \left(\pi_j + \pi_\ell \left(\frac{1-3\lambda}{6} \right) - \left(\frac{1-\lambda}{6} \right) \right),$$

for $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{5}(1-\pi_j)$ (i.e., $\{\mathbf{k}\}$ does not have to pay party j in period 1 to receive its support and terminates in the second period if $q' = \mathbf{k}$),

$$V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta \left(-\pi_j \left(\frac{4-\lambda}{3} \right) - \pi_\ell \left(\frac{2-\lambda}{3} \right) + \left(\frac{1-\lambda}{6} \right) \right),$$

for $\pi_j > \pi_\ell$ and $\pi_j > \frac{2}{5}(1 - \pi_\ell)$ (i.e., $\{\mathbf{k}\}$ pays party j in period 1 to receive its support and always survives in the second period),

$$\begin{aligned} V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = & -\beta \left(\pi_j - \pi_\ell \left(\frac{1-3\lambda}{6} \right) + \left(\frac{1-\lambda}{6} \right) \right) + \\ & \beta \left(-\pi_\ell \left(\frac{5+3\lambda}{6} \right) - \left(\frac{1-\lambda}{6} \right) \right), \end{aligned}$$

and for $\pi_j > \pi_\ell$ and $\pi_j < \frac{2}{5}(1 - \pi_\ell)$ (i.e., $\{\mathbf{k}\}$ pays party j in period 1 to receive its support and terminates in the second period if $q' = \mathbf{k}$),

$$\begin{aligned} V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi) = & -\beta \left(\pi_j \left(\frac{2+\lambda}{3} \right) + \pi_\ell \frac{\lambda}{3} \right) + \\ & \beta \left(-\pi_\ell \left(\frac{7+\lambda}{6} \right) - \pi_j \left(\frac{5-5\lambda}{6} \right) + \left(\frac{1-\lambda}{6} \right) \right). \end{aligned}$$

We first show that for all $\pi \in \Pi$, \mathbf{k} prefers $\{j, \mathbf{k}\}$ to $\{\ell, \mathbf{k}\}$ and to $\{1, 2, 3\}$.

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) > V_{\mathbf{k}}(\{\ell, \mathbf{k}\}, z^\ell, \pi)$$

if and only if

$$-\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) > -\frac{3}{4} + \beta \left(\frac{1}{4} - \pi_\ell - \frac{1}{2}\pi_j \right)$$

if and only if

$$\pi_j < \pi_\ell + \frac{1}{\beta}$$

which is always true since the right hand side is greater than 1. Furthermore,

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) > V_{\mathbf{k}}(\{1, 2, 3\}, z^\ell, \pi)$$

if and only if

$$-\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) > -\frac{2}{3} + \beta \left(\frac{1}{3} - \pi_j - \pi_\ell \right)$$

if and only if

$$\pi_\ell > \frac{\beta - 5}{6\beta}$$

which is always true since the right hand side is negative.

Next, we compare \mathbf{k} 's payoffs if it chooses $\{j, \mathbf{k}\}$ versus $\{\mathbf{k}\}$ in the different regions of the parameter space Π . Suppose that $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{5}(1 - \pi_j)$. Note that these restrictions imply that $\pi_\ell \in (\frac{2}{7}, \frac{1}{2})$ and $\pi_j \in (0, \frac{1}{2})$. Now

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) > V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) > \beta \left(\pi_j + \pi_\ell \left(\frac{1-3\lambda}{6} \right) - \left(\frac{1-\lambda}{6} \right) \right)$$

if and only if

$$\pi_j < \frac{-\pi_\ell \beta (8 - 6\lambda) + \beta (5 - 2\lambda) - 3}{24\beta}$$

which is never true since the right hand side is negative. To see this note that

$$-\pi_\ell \beta (8 - 6\lambda) + \beta (5 - 2\lambda) - 3 < 0$$

if and only if

$$\pi_\ell > \frac{\beta (5 - 2\lambda) - 3}{\beta (8 - 6\lambda)}$$

which is always true since the right hand side is smaller than $\frac{2}{7}$. Hence, for $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{5}(1 - \pi_j)$, \mathbf{k} prefers $\{\mathbf{k}\}$ to $\{j, \mathbf{k}\}$.

Next suppose that $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{5}(1 - \pi_j)$. Note that these restrictions imply that $\pi_\ell \in (\frac{1}{4}, \frac{1}{3})$ and $\pi_j \in (\frac{1}{6}, \frac{2}{7})$. Now

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) > V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) > \beta \left(-\pi_j \left(\frac{4-\lambda}{3} \right) - \pi_\ell \left(\frac{2-\lambda}{3} \right) + \left(\frac{1-\lambda}{6} \right) \right)$$

if and only if

$$\pi_j > \frac{\pi_\ell 2\beta (2\lambda - 1) - \beta (2\lambda + 1) + 3}{4\beta (1 - \lambda)}$$

which is never true since the right hand side is greater than $\frac{2}{7}$. To see this note that

$$\frac{\pi_\ell 2\beta (2\lambda - 1) - \beta (2\lambda + 1) + 3}{4\beta (1 - \lambda)} > \frac{2}{7}$$

if and only if, for $\lambda \geq \frac{1}{2}$,

$$\pi_\ell > \frac{\beta(15 + 6\lambda) - 21}{14\beta(2\lambda - 1)}$$

which is always true since the right hand side is negative, and for $\lambda < \frac{1}{2}$,

$$\pi_\ell < \frac{21 - \beta(15 + 6\lambda)}{14\beta(1 - 2\lambda)}$$

which is always true since the right hand side is greater than $\frac{1}{3}$. Hence, for $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{5}(1 - \pi_j)$, \mathbf{k} prefers $\{\mathbf{k}\}$ to $\{j, \mathbf{k}\}$. When combined with the previous findings, this result implies that whenever $\pi_j < \pi_\ell$, $\{\mathbf{k}\}$ forms the government with the support of party j .

Next consider the case where $\pi_j > \pi_\ell$ and $\pi_j > \frac{2}{5}(1 - \pi_\ell)$. Note that these restrictions imply that $\pi_j \in (\frac{2}{7}, \frac{1}{2})$ and $\pi_\ell \in (0, \frac{1}{2})$. Now

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) > V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) > -\beta \left(\pi_j - \pi_\ell \left(\frac{1 - 3\lambda}{6} \right) + \left(\frac{1 - \lambda}{6} \right) \right) + \beta \left(-\pi_\ell \left(\frac{5 + 3\lambda}{6} \right) - \left(\frac{1 - \lambda}{6} \right) \right)$$

if and only if

$$\pi_\ell > \frac{3 - \beta(7 - 4\lambda)}{2\beta(1 + 6\lambda)}$$

which for $\beta < b_1^*(\lambda) = \frac{3}{8+2\lambda}$ is never true (since the right hand side is greater than $\frac{1}{2}$), whereas for $\beta > b_2^*(\lambda) = \frac{3}{7-4\lambda}$ is always true (since the right hand side is negative). Note that $b_1^*(\lambda)$ is a decreasing function of λ , $b_2^*(\lambda)$ is an increasing function of λ , and $p_\ell^*(\beta, \lambda) = \frac{3 - \beta(7 - 4\lambda)}{2\beta(1 + 6\lambda)}$ is decreasing in β and increasing in λ . Hence, for $\pi_\ell < \pi_j$ and $\pi_j > \frac{2}{5}(1 - \pi_\ell)$, if $\beta < b_1^*(\lambda)$, then $\{\mathbf{k}\}$ forms the government with the external support of party j ; if $\beta > b_2^*(\lambda)$, then $\{\mathbf{k}, j\}$ forms the government; and if $b_1^*(\lambda) < \beta < b_2^*(\lambda)$, then for $\pi_\ell < p_\ell^*(\beta, \lambda)$, $\{\mathbf{k}\}$ forms the government with the external support of party j , whereas for $\pi_\ell > p_\ell^*(\beta, \lambda)$, $\{\mathbf{k}, j\}$ forms the government.

Finally consider the case where $\pi_j > \pi_\ell$ and $\pi_j < \frac{2}{5}(1 - \pi_\ell)$. Note that these restrictions imply that $\pi_j \in (\frac{1}{4}, \frac{1}{3})$ and $\pi_\ell \in (\frac{1}{6}, \frac{2}{7})$. Now

$$V_{\mathbf{k}}(\{j, \mathbf{k}\}, z^\ell, \pi) > V_{\mathbf{k}}(\{\mathbf{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta \left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell \right) > -\beta \left(\pi_j \left(\frac{2+\lambda}{3} \right) + \pi_\ell \frac{\lambda}{3} \right) + \beta \left(-\pi_\ell \left(\frac{7+\lambda}{6} \right) - \pi_j \left(\frac{5-5\lambda}{6} \right) + \left(\frac{1-\lambda}{6} \right) \right)$$

if and only if

$$\pi_j > \frac{-\pi_\ell \beta (8+6\lambda) - \beta (1+2\lambda) + 3}{6\beta (1-\lambda)}$$

which for $\beta < b_3^*(\lambda) = \frac{21}{37+12\lambda}$ is never true (since the right hand side is greater than $\frac{1}{3}$), whereas for $\beta > b_4^*(\lambda) = \frac{18}{23+9\lambda}$ is always true (since the right hand side is smaller than $\frac{1}{4}$). To show that this is the case, note that

$$\frac{-\pi_\ell \beta (8+6\lambda) - \beta (1+2\lambda) + 3}{6\beta (1-\lambda)} > \frac{1}{3}$$

if and only if

$$\pi_\ell < \frac{3(1-\beta)}{2\beta(4+3\lambda)}$$

which, for $\beta < \frac{21}{37+12\lambda}$, is always true since the right hand side is greater than $\frac{2}{7}$. Furthermore,

$$\frac{-\pi_\ell \beta (8+6\lambda) - \beta (1+2\lambda) + 3}{6\beta (1-\lambda)} < \frac{1}{4}$$

if and only if

$$\pi_\ell > \frac{-\beta(5+\lambda) + 6}{4\beta(4+3\lambda)}$$

which, for $\beta > \frac{18}{23+9\lambda}$, is always true since the right hand side is smaller than $\frac{1}{6}$. Note that both $b_3^*(\lambda)$ and $b_4^*(\lambda)$ are decreasing function of λ , and $p_j^*(\beta, \lambda, \pi_\ell) = \frac{-\pi_\ell \beta (8+6\lambda) - \beta (1+2\lambda) + 3}{6\beta (1-\lambda)}$ is decreasing in β , increasing in λ , and decreasing in π_ℓ . Hence, for $\pi_\ell < \pi_j$ and $\pi_j < \frac{2}{5}(1 - \pi_\ell)$, if $\beta < b_3^*(\lambda)$, then

$\{\mathbf{k}\}$ forms the government with the external support of party j ; if $\beta > b_4^*(\lambda)$, then $\{\mathbf{k}, j\}$ forms the government; and if $b_3^*(\lambda) < \beta < b_4^*(\lambda)$, then for $\pi_j < p_j^*(\beta, \lambda, \pi_\ell)$, $\{\mathbf{k}\}$ forms the government with the external support of party j , whereas for $\pi_j > p_j^*(\beta, \lambda, \pi_\ell)$, $\{\mathbf{k}, j\}$ forms the government.

Since the analysis for the case where $q = z^j$ is identical to the last case considered with j replacing ℓ and *vice versa*, this concludes the proof of part (ii) of the lemma.

Q.E.D.

To illustrate the results presented in Proposition 1, consider Figures 1 and 2.¹⁵ Without loss of generality, suppose $q = z^1$. First consider the case where party 1 is the formateur. From Lemma 2, we know that the solution to the one-period optimization problem is for party 1 to form the surplus government coalition $\{1, 2, 3\}$. Dynamic considerations, however, play an important role, since party 1's choice in period 1 also affects its period 2 payoff. In particular, while the period 1 payoff resulting from choosing coalition $\{1, 2, 3\}$ dominates the payoff induced, for example, by the choice of $\{1, 2\}$, the opposite is true with respect to period 2 payoffs. This is the case since, loosely speaking, in the renegotiation stage, party 1 would only have to compensate one party as opposed to two parties to prevent them from leaving the current coalition with the expectation of obtaining a higher payoff in a new government formation process. Since a party's future prospects improve with its share, these considerations are particularly relevant when party 1's coalition partners are relatively "big". Hence, as the discount factor converges to 1, if either party 2 or party 3 controls more than $\frac{1}{3}$ of the parliamentary seats, party 1 chooses to team up with the smaller of the two and form a minimal winning government rather than a surplus government.

Next, consider the case where party 2 is the formateur. Again, from Lemma 2 we know that the solution to the one-period optimization problem is for party 2 to form the minority government $\{2\}$ with the external support of party 3. When dynamic considerations are taken into account, however, the next best alternative from a static point of view (i.e., forming a minimal winning government with party 3) may dominate. To understand this point, recall from Lemma 5 that if π_3 is greater than π_1 , if $\{2\}$ forms the government, it has to compensate party 3 in period 1 for the fact that in the second period $\{2\}$ is likely to seek the support of party 1. The size of the transfer is increasing in the discount factor β . Hence, if the parties are sufficiently

¹⁵Both figures are drawn for the case $\beta = 1$.

forward looking, then party 2 chooses to include party 3 in the government coalition rather than elicit its external support via transfers.

4 Discussion

The results presented in the previous section fully account for the six empirical regularities described in the introduction. Moreover, our analysis generalizes existing models of government formation and termination and exhibits more clearly some of their limitations. For example, as Lupia and Strom (1995) we find that in equilibrium governments may terminate in early elections and replacements. However, our analysis suggests that their findings critically depend on the presence of transaction costs and the assumption that governments cannot reshuffle benefits and portfolios in response to exogenous changes in their bargaining environment. Once we allow for efficient bargaining and reshuffles, the Lupia and Strom framework can no longer generate early terminations. If a government commands a majority of seats—the only case considered by Lupia and Strom—reshuffles can always be used to capture any changes in the bargaining environment within the current coalition. Once efficient bargaining is not possible, however, as in the case of bargaining between the minority government and the parties in its supporting coalition, governments may fall. Lemma 5 implies that a necessary condition for any government to fall is a change in the default policy q . That is, if the default policy is perfectly persistent, (i.e., $\lambda = 1$) no government can terminate. Contrary to Lupia and Strom, changes in expected seat share (i.e. public opinion shocks) are not sufficient for cabinet terminations, unless one introduces transaction costs into the model.

In our bargaining model of parliamentary governments all three types of governments may form in equilibrium: minimal winning, minority, and surplus. Minority and surplus coalitions are not rare exceptions, but may form for all parameter values. The traditional literature views minority and surplus governments as an aberration. Our view is exactly the opposite. In a single-period model, minimal winning coalitions would never form. The party favored by the default policy would always choose a surplus coalition, and a non-favored party would always form a minority government. Minimal winning coalitions only occur when dynamic considerations are important. That is, the reason minimal winning coalitions are chosen is because it may be too expensive for a formateur to maintain surplus or minority coalitions over time, especially if the future state of the world is likely to favor a different party.

The stability and the relative occurrence of different types of governments are thus closely connected. Minimal winning coalitions are relatively cheaper to maintain than either minority or surplus coalitions. These considerations affect which kind of governments form. In the case of minority coalitions, the necessity to pay off an outside party that is willing to support the government may bring the government down or induce a minority government not to form. The price of support is determined by the outside party's continuation value that depends on its relative seat share and the state of the world. If both potential outside partners are expensive, a formateur may choose to form a minority government that is destined to fall (if the formateur's future prospects look favorable) or form a stable minimal winning government instead. In the case of surplus governments, the need to keep all members of the government in the coalition may induce a surplus government not to form. While it is always possible to make transfers within the government coalition to maintain a surplus government, if the coalition partners are large relative to the formateur party, a formateur may choose to form a minimal winning coalition instead.

5 Conclusion

In this paper we propose a legislative bargaining model that can account for some empirical regularities in the formation and termination of parliamentary governments. In our model majority governments always survive until the next regularly scheduled election, possibly by reshuffling government positions among the members of the government coalition. On the other hand, minority governments can terminate in early elections or be replaced by a new government in a vote of no-confidence. Indeed, minority governments may still form even though all parties know that there exist states of the world where a minority government would fall for sure in the next period. In contrast, surplus governments are as stable as minimal winning governments, but are prone to reshuffles.

While our model can account for these basic regularities, it is too stylized to explain cross-country differences in government formation and stability. Why are there so many surplus coalitions in Italy, but none in Denmark or Germany? Why are minority governments unheard of in Germany, but are the norm in Denmark? What explains the differences in average government duration between countries? To answer these questions our model would need to be "augmented" to capture some of the institutional details of government formation that may account for cross-country differences. The

model presented here represents a first step toward addressing these questions in a systematic fashion.

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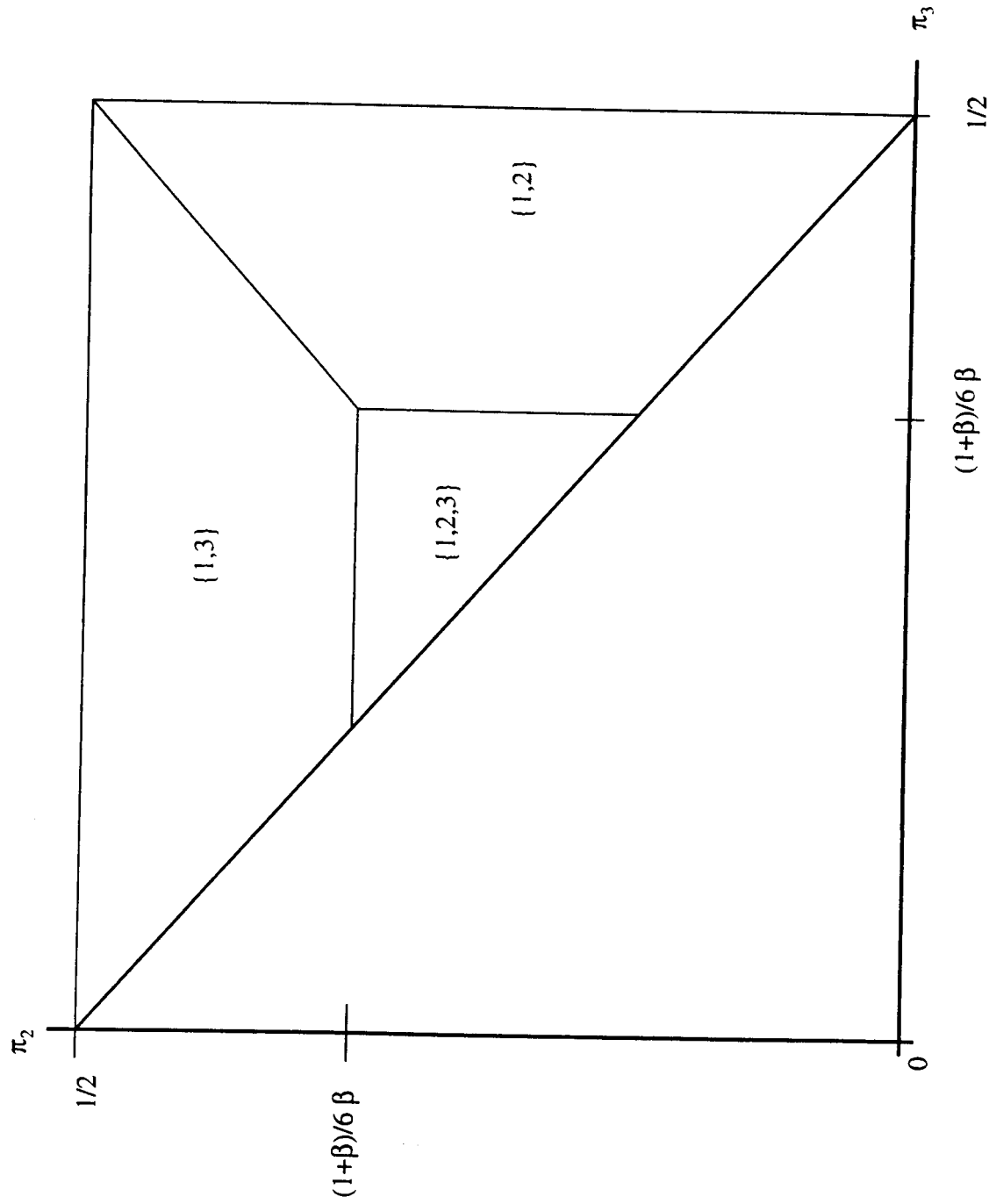
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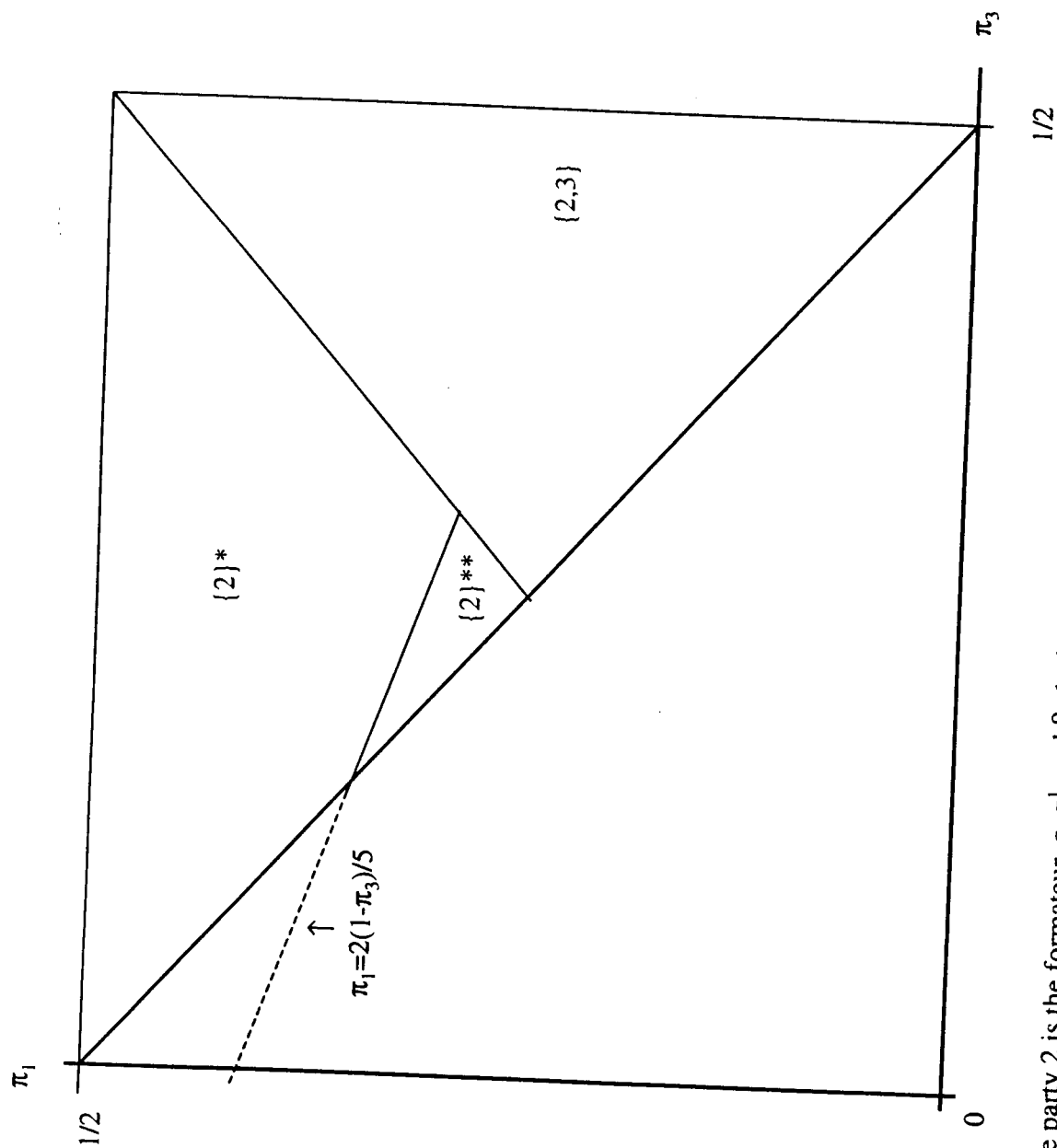
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Figure 1
Coalition formation if the formateur is the favored party



In this example party 1 is the formateur, $q=z^1$, and $\beta = 1$. The area where $\{1,2,3\}$ is chosen increases as β decreases until at $\beta=1/2$ it covers the entire upper triangle (i.e. the parameter space Π)

Figure 2
Coalition formation if the formateur is not the favored party



In this example party 2 is the formateur, $q=z^1$, and $\beta=1$; in the region indicated by * {2} always survives in period 2, while in the region indicated by ** {2} terminates with probability $(1-\lambda)/2$.

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