AJAE Appendices for Pareto Optimal Trade in an Uncertain World: GMOs and the Precautionary Principle

Robert G. Chambers and Tigran A. Melkonyan

Deprartment of Agricultural and Resource Economics

University of Maryland, College Park

May 22, 2007

Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

Appendix A

Proof. We first demonstrate that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$) consumption vectors in the set C (C') are Pareto optimal and then show that consumption vectors outside the set are not Paretian. Consider consumption vector $((a,a),(y_1 - a, y_2 - a))$ where $a \in [0, y_2]$. If $((a,a),(y_1 - a, y_2 - a))$ is not Paretian, there must exist $\left((\widetilde{c}_1^E, \widetilde{c}_2^E),(\widetilde{c}_1^R, \widetilde{c}_2^R)\right) \neq ((a,a),(y_1 - a, y_2 - a))$ satisfying

$$\widetilde{c}_1^E + \widetilde{c}_1^R = y_1, \tag{1}$$

$$\widetilde{c}_2^E + \widetilde{c}_2^R = y_2 \tag{2}$$

and such that either

$$\begin{aligned}
\widetilde{c}_2^R + \min_{\pi \in P^R} \left\{\pi\left(\widetilde{c}_1^R - \widetilde{c}_2^R\right)\right\} &> y_2 - a + \min_{\pi \in P^R} \left\{\pi\left(y_1 - y_2\right)\right\}, \\
\widetilde{c}_2^E + \min_{\pi \in P^E} \left\{\pi\left(\widetilde{c}_1^E - \widetilde{c}_2^E\right)\right\} &\geq a,
\end{aligned}$$

or

$$\begin{aligned}
\widetilde{c}_2^R + \min_{\pi \in P^R} \left\{\pi\left(\widetilde{c}_1^R - \widetilde{c}_2^R\right)\right\} &\geq y_2 - a + \min_{\pi \in P^R} \left\{\pi\left(y_1 - y_2\right)\right\}, \\
\widetilde{c}_2^E + \min_{\pi \in P^E} \left\{\pi\left(\widetilde{c}_1^E - \widetilde{c}_2^E\right)\right\} &> a,
\end{aligned}$$

Adding the relevant inequalities in either case and using (1) and $y_1 > y_2$ gives

$$\begin{aligned}
\min_{\pi \in P^E} \left\{\pi\left(\widetilde{c}_1^E - \widetilde{c}_2^E\right)\right\} + \min_{\pi \in P^R} \left\{\pi\left(\widetilde{c}_1^R - \widetilde{c}_2^R\right)\right\} &> \min_{\pi \in P^R} \left\{\pi\left(y_1 - y_2\right)\right\} \\
&= \underline{\pi}^R\left(y_1 - y_2\right),
\end{aligned}$$

which cannot be satisfied when $\underline{\pi}^E < \underline{\pi}^R < \overline{\pi}^E$.

Now suppose that $\underline{\pi}^E > 0$ and consider consumption vector $((b, y_2),(y_1 - b, 0))$ where $y_2 < b \leq y_1$. If $((b, y_2),(y_1 - b, 0))$ is not Paretian, there must exist $\left((\widetilde{c}_1^E, \widetilde{c}_2^E),(\widetilde{c}_1^R, \widetilde{c}_2^R)\right)$ satisfying (1), (2) and such that either

$$\begin{aligned}
\widetilde{c}_2^R + \min_{\pi \in P^R} \left\{\pi\left(\widetilde{c}_1^R - \widetilde{c}_2^R\right)\right\} &> \min_{\pi \in P^R} \left\{\pi(y_1 - b)\right\}, \\
\widetilde{c}_2^E + \min_{\pi \in P^E} \left\{\pi\left(\widetilde{c}_1^E - \widetilde{c}_2^E\right)\right\} &\geq y_2 + \min_{\pi \in P^E} \left\{\pi\left(b - y_2\right)\right\},
\end{aligned}$$

or

$$\begin{aligned}
\widetilde{c}_2^R + \min_{\pi \in P^R} \left\{ \pi \left(\widetilde{c}_1^R - \widetilde{c}_2^R \right) \right\} &\geq \min_{\pi \in P^R} \left\{ \pi (y_1 - b) \right\}, \\
\widetilde{c}_2^E + \min_{\pi \in P^E} \left\{ \pi \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) \right\} &> y_2 + \min_{\pi \in P^E} \left\{ \pi \left(b - y_2 \right) \right\},
\end{aligned}$$

Adding the relevant inequalities in either case and using (1) gives

$$\begin{aligned}
\text{(3)} \quad \min_{\pi \in P^R} \left\{ \pi \left(\widetilde{c}_1^R - \widetilde{c}_2^R \right) \right\} + \min_{\pi \in P^E} \left\{ \pi \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) \right\} &> \min_{\pi \in P^R} \left\{ \pi (y_1 - b) \right\} + \min_{\pi \in P^E} \left\{ \pi \left(b - y_2 \right) \right\} \\
&= \underline{\pi}^R (y_1 - b) + \underline{\pi}^E (b - y_2).
\end{aligned}$$

Because $\underline{\pi}^R \left(\widetilde{c}_1^R - \widetilde{c}_2^R \right) + \underline{\pi}^E \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) \geq \min_{\pi \in P^R} \left\{ \pi \left(\widetilde{c}_1^R - \widetilde{c}_2^R \right) \right\} + \min_{\pi \in P^E} \left\{ \pi \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) \right\}$, (3) implies that

$$\text{(4)} \qquad \underline{\pi}^R \left(\widetilde{c}_1^R - \widetilde{c}_2^R \right) + \underline{\pi}^E \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) > \underline{\pi}^R (y_1 - b) + \underline{\pi}^E (b - y_2).$$

Using (1) and (2) in (4) implies

$$\text{(5)} \qquad \left(\underline{\pi}^E - \underline{\pi}^R \right) \left(\widetilde{c}_1^E + \widetilde{c}_2^R - b \right) > 0$$

For the EU to weakly prefer $\left(\widetilde{c}_1^E, \widetilde{c}_2^E \right)$ to (b, y_2), $\widetilde{c}_1^E \geq b$ necessarily. This contradicts (5) because $\underline{\pi}^E - \underline{\pi}^R < 0$. Finally, note that when $\underline{\pi}^E = 0$, $((b, y_2), (y_1 - b, 0))$ is Pareto dominated by $((y_2, y_2), (y_1 - y_2, 0))$.

We now demonstrate that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$) consumption vectors $\left((\widetilde{c}_1^E, \widetilde{c}_2^E), (\widetilde{c}_1^R, \widetilde{c}_2^R) \right)$ outside the set C (C') are not Pareto optimal. The Pareto problem is

$$\text{(6)} \qquad \max_{\widetilde{c}_1^E, \widetilde{c}_2^E, \widetilde{c}_1^R, \widetilde{c}_2^R \geq 0} \left\{ \widetilde{c}_2^R + \min_{\pi \in P^R} \left\{ \pi \left(\widetilde{c}_1^R - \widetilde{c}_2^R \right) \right\} \right\}$$

$$\text{subject to } \widetilde{c}_2^E + \min_{\pi \in P^E} \left\{ \pi \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) \right\} \geq \hat{u}^E, \text{(1) and (2)},$$

where $\hat{u}^E$ is a fixed level of the EU's utility from the interval $[0, y_2 + \underline{\pi}^E (y_1 - y_2)]$. Using the material balance conditions, we can rewrite this problem as

$$\text{(7)} \qquad \max_{\widetilde{c}_1^E, \widetilde{c}_2^E} \left\{ y_2 - \widetilde{c}_2^E + \min_{\pi \in P^R} \left\{ \pi \left(y_1 - \widetilde{c}_1^E - y_2 + \widetilde{c}_2^E \right) \right\} \right\}$$

$$\text{subject to } \widetilde{c}_2^E + \min_{\pi \in P^E} \left\{ \pi \left(\widetilde{c}_1^E - \widetilde{c}_2^E \right) \right\} \geq \hat{u}^E, \; y_1 \geq \widetilde{c}_1^E \geq 0, \; y_2 \geq \widetilde{c}_2^E \geq 0.$$

The constraint set in (7) is a polyhedral convex set while the objective function is concave. Since $\underline{\pi}^E < \underline{\pi}^R < \overline{\pi}^E$, none of the exposed faces of the constraint set is parallel to any portion of the objective's level surfaces. Hence, Pareto problem (7) has a unique solution. Note also that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$), there is a one-to-one correspondence between utility levels $\hat{u}^E \in [0, y_2 + \underline{\pi}^E (y_1 - y_2)]$ and points in the set C (C'). Combining these two facts with the observation that set $[0, y_2 + \underline{\pi}^E (y_1 - y_2)]$ coincides with the set of feasible utility levels for the EU representative agent, we obtain that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$) consumption vectors outside C (C') are not Pareto optimal. ■

Appendix B

Proof. First, note that, given the relationship in (**??**), the three cases considered in the theorem cover all possible rankings of $\underline{z}_1^E + \underline{z}_1^R$, $\overline{z}_1^E + \overline{z}_1^R$, $t\left(\underline{z}_1^E, \mathbf{x}^E\right) + t\left(\underline{z}_1^R, \mathbf{x}^R\right)$ and $t\left(\overline{z}_1^E, \mathbf{x}^E\right) + t\left(\overline{z}_1^R, \mathbf{x}^R\right)$. The Pareto problem can be written as:

$$\max_{\left(z_j^i, c_j^i\right)_{j=1,2}^{i=E,R} \geq 0} \left\{ \min_{\pi \in P^R} \left\{ \pi c_1^R + (1-\pi) c_2^R \right\} + \min_{\pi \in P^E} \left\{ \pi c_1^E + (1-\pi) c_2^E \right\} \right\}$$

$$\text{subject to } c_1^R + c_1^E = z_1^R + z_1^E \text{ and } c_2^R + c_2^E = t\left(z_1^R, \mathbf{x}^R\right) + t\left(z_1^E, \mathbf{x}^E\right).$$

From Theorem **??**, when $P^R \subset interior(P^E)$, Pareto optimality requires that

$$\hat{c}_1^E = \hat{c}_2^E$$

for any given strictly positive aggregate production level.

Using this condition, the Pareto problem can be written as

$$\max_{\left(z_1^R, z_1^E\right) \geq 0} \left\{ \min_{\pi \in P^R} \left\{ \pi \left(z_1^R + z_1^E\right) + (1-\pi) \left(t\left(z_1^R, \mathbf{x}^R\right) + t\left(z_1^E, \mathbf{x}^E\right)\right) \right\} \right\}.$$

Let

$$f\left(z_1^R, z_1^E\right) \equiv \min_{\pi \in P^R} \left\{ \pi \left(z_1^R + z_1^E\right) + (1-\pi) \left(t\left(z_1^R, \mathbf{x}^R\right) + t\left(z_1^E, \mathbf{x}^E\right)\right) \right\}.$$

Since $f(\cdot,\cdot)$ is concave, it has a well-defined Gateaux directional derivative of $f(\cdot,\cdot)$ at $\left(z_1^R, z_1^E\right)$ in the direction $\left(v^R, v^E\right)$ given by

$$f^G\left(\left(z_1^R, z_1^E\right); \left(v^R, v^E\right)\right) = \lim_{\lambda \to 0^+} \frac{f\left(z_1^R + \lambda v^R, z_1^E + \lambda v^E\right) - f\left(z_1^R, z_1^E\right)}{\lambda}.$$

Using the definition of $f(\cdot,\cdot)$, we obtain

$$f^G\left(\left(z_1^R, z_1^E\right); \left(v^R, v^E\right)\right) \tag{8}$$
$$= \begin{cases} \begin{bmatrix} \underline{\pi}^R\left(v^R + v^E\right) \\ +\left(1-\underline{\pi}^R\right)\left[t'\left(z_1^R, \mathbf{x}^R\right)v^R + t'\left(z_1^E, \mathbf{x}^E\right)v^E\right] \end{bmatrix}, & \text{if } z_1^R + z_1^E > t\left(z_1^R, \mathbf{x}^R\right) + t\left(z_1^E, \mathbf{x}^E\right) \\ \begin{bmatrix} \overline{\pi}^R\left(v^R + v^E\right) \\ +\left(1-\overline{\pi}^R\right)\left[t'\left(z_1^R, \mathbf{x}^R\right)v^R + t'\left(z_1^E, \mathbf{x}^E\right)v^E\right] \end{bmatrix}, & \text{if } z_1^R + z_1^E < t\left(z_1^R, \mathbf{x}^R\right) + t\left(z_1^E, \mathbf{x}^E\right) \\ \min_{\pi\in P^R}\begin{Bmatrix} \pi\left(v^R + v^E\right) \\ +(1-\pi)\left(t'\left(z_1^R, \mathbf{x}^R\right)v^R + t'\left(z_1^E, \mathbf{x}^E\right)v^E\right) \end{Bmatrix}, & \text{if } z_1^R + z_1^E = t\left(z_1^R, \mathbf{x}^R\right) + t\left(z_1^E, \mathbf{x}^E\right) \end{cases}.$$

Since $f(\cdot,\cdot)$ is concave in $\left(z_1^R, z_1^E\right)$, $\left(\hat{z}_1^R, \hat{z}_1^E\right) \in \arg\max_{\left(z_1^R, z_1^E\right)\geq 0} f\left(z_1^R, z_1^E\right)$ if and only if

$$f^G\left(\left(\hat{z}_1^R, \hat{z}_1^E\right); \left(v^R, v^E\right)\right) \leq 0 \text{ for all } \left(v^R, v^E\right). \tag{9}$$

We consider each of the three cases in turn:

i) From (8),

$$f^G\left(\left(\underline{z}_1^R, \underline{z}_1^E\right); \left(v^R, v^E\right)\right) = 0 \text{ for all } \left(v^R, v^E\right),$$

and, hence, $\left(\underline{z}_1^R, \underline{z}_1^E\right) = \arg\max_{\left(z_1^R, z_1^E\right)\geq 0} f\left(z_1^R, z_1^E\right)$. The second part follows directly from $t'\left(\underline{z}_1^E, \mathbf{x}^E\right) = -\frac{\underline{\pi}^R}{1-\underline{\pi}^R}$ and the definition of z^E.

ii) From (8),

$$f^G\left(\left(\overline{z}_1^R, \overline{z}_1^E\right); \left(v^R, v^E\right)\right) = 0 \text{ for all } \left(v^R, v^E\right),$$

and, hence, $\left(\overline{z}_1^R, \overline{z}_1^E\right) = \arg\max_{\left(z_1^R, z_1^E\right)\geq 0} f\left(z_1^R, z_1^E\right)$. The second part follows directly from $t'\left(\overline{z}_1^E, \mathbf{x}^E\right) = -\frac{\overline{\pi}^R}{1-\overline{\pi}^R}$ and the definition of z^E.

iii) First, we demonstrate that $\hat{z}_1^R + \hat{z}_1^E = t\left(\hat{z}_1^R, \mathbf{x}^R\right) + t\left(\hat{z}_1^E, \mathbf{x}^E\right)$. Suppose not and consider the case

$$\hat{z}_1^R + \hat{z}_1^E > t\left(\hat{z}_1^R, \mathbf{x}^R\right) + t\left(\hat{z}_1^E, \mathbf{x}^E\right). \tag{10}$$

But then

$$f^G\left(\left(\hat{z}_1^R, \hat{z}_1^E\right); \left(v^R, v^E\right)\right) = \underline{\pi}^R\left(v^R + v^E\right) + \left(1-\underline{\pi}^R\right)\left[t'\left(\hat{z}_1^R, \mathbf{x}^R\right)v^R + t'\left(\hat{z}_1^E, \mathbf{x}^E\right)v^E\right].$$

Under (10), we have that

$$f^G\left(\left(\hat{z}_1^R,\hat{z}_1^E\right);\left(v^R,v^E\right)\right)\le 0 \text{ for all } \left(v^R,v^E\right) \text{ if and only if } \hat{z}_1^R=\underline{z}_1^R \text{ and } \hat{z}_1^E=\underline{z}_1^E,$$

which contradicts (10). Similarly, one can demonstrate that $\hat{z}_1^R+\hat{z}_1^E<t\left(\hat{z}_1^R,\mathbf{x}^R\right)+t\left(\hat{z}_1^E,\mathbf{x}^E\right)$ leads to a contradiction.

Now it is left to verify that (9) holds when $\hat{z}_1^R+\hat{z}_1^E=t\left(\hat{z}_1^R,\mathbf{x}^R\right)+t\left(\hat{z}_1^E,\mathbf{x}^E\right)$ and $t'\left(\hat{z}_1^R,\mathbf{x}^R\right)=t'\left(\hat{z}_1^E,\mathbf{x}^E\right)$. We have that in this case

$$\begin{aligned}
&f^G\left(\left(\hat{z}_1^R,\hat{z}_1^E\right);\left(v^R,v^E\right)\right)\\
&=\min_{\pi\in P^R}\left\{\pi\left(v^R+v^E\right)+(1-\pi)\left(t'\left(\hat{z}_1^R,\mathbf{x}^R\right)v^R+t'\left(\hat{z}_1^E,\mathbf{x}^E\right)v^E\right)\right\}
\end{aligned}$$

$$=\begin{cases}\left[\begin{array}{c}\underline{\pi}^R\left(v^R+v^E\right)\\ +\left(1-\underline{\pi}^R\right)\left(t'\left(\hat{z}_1^R,\mathbf{x}^R\right)v^R+t'\left(\hat{z}_1^E,\mathbf{x}^E\right)v^E\right)\end{array}\right], & \text{if } v^R+v^E\ge t'\left(\hat{z}_1^R,\mathbf{x}^R\right)v^R+t'\left(\hat{z}_1^E,\mathbf{x}^E\right)v^E\\ \left[\begin{array}{c}\overline{\pi}^R\left(v^R+v^E\right)\\ +\left(1-\overline{\pi}^R\right)\left(t'\left(\hat{z}_1^R,\mathbf{x}^R\right)v^R+t'\left(\hat{z}_1^E,\mathbf{x}^E\right)v^E\right)\end{array}\right], & \text{if } v^R+v^E\le t'\left(\hat{z}_1^R,\mathbf{x}^R\right)v^R+t'\left(\hat{z}_1^E,\mathbf{x}^E\right)v^E\end{cases}.$$

Using this expression and (iii) it is straightforward to verify that (9) holds. If in addition $t'\left(z^E,\mathbf{x}^E\right)=t'\left(z^R,\mathbf{x}^R\right)$, then $\hat{z}_1^E=\hat{z}_2^E=z^E$ and $\hat{z}_1^R=\hat{z}_2^R=z^R$ is the unique production vector that satisfies (9). Finally, note that

$$\frac{\underline{\pi}^R}{1-\underline{\pi}^R}\le -t'\left(\hat{z}_1^R,\mathbf{x}^R\right)=-t'\left(\hat{z}_1^E,\mathbf{x}^E\right)\le\frac{\overline{\pi}^R}{1-\overline{\pi}^R}.$$

■