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AJAE Appendices for Pareto Optimal Trade in an Uncertain World: GMOs and the Precautionary Principle

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Appendix A

Proof. We first demonstrate that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$) consumption vectors in the set C (C') are Pareto optimal and then show that consumption vectors outside the set are not Paretian. Consider consumption vector $((a, a), (y_1 - a, y_2 - a))$ where $a \in [0, y_2]$. If $((a, a), (y_1 - a, y_2 - a))$ is not Paretian, there must exist $((\tilde{c}_1^E, \tilde{c}_2^E), (\tilde{c}_1^R, \tilde{c}_2^R)) \neq ((a, a), (y_1 - a, y_2 - a))$ satisfying

$$(1) \quad \tilde{c}_1^E + \tilde{c}_1^R = y_1,$$

$$(2) \quad \tilde{c}_2^E + \tilde{c}_2^R = y_2$$

and such that either

$$\begin{aligned} \tilde{c}_2^R + \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} &> y_2 - a + \min_{\pi \in P^R} \{ \pi (y_1 - y_2) \}, \\ \tilde{c}_2^E + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} &\geq a, \end{aligned}$$

or

$$\begin{aligned} \tilde{c}_2^R + \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} &\geq y_2 - a + \min_{\pi \in P^R} \{ \pi (y_1 - y_2) \}, \\ \tilde{c}_2^E + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} &> a, \end{aligned}$$

Adding the relevant inequalities in either case and using (1) and $y_1 > y_2$ gives

$$\begin{aligned} \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} + \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} &> \min_{\pi \in P^R} \{ \pi (y_1 - y_2) \} \\ &= \underline{\pi}^R (y_1 - y_2), \end{aligned}$$

which cannot be satisfied when $\underline{\pi}^E < \underline{\pi}^R < \bar{\pi}^E$.

Now suppose that $\underline{\pi}^E > 0$ and consider consumption vector $((b, y_2), (y_1 - b, 0))$ where $y_2 < b \leq y_1$. If $((b, y_2), (y_1 - b, 0))$ is not Paretian, there must exist $((\tilde{c}_1^E, \tilde{c}_2^E), (\tilde{c}_1^R, \tilde{c}_2^R))$ satisfying (1), (2) and such that either

$$\begin{aligned} \tilde{c}_2^R + \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} &> \min_{\pi \in P^R} \{ \pi (y_1 - b) \}, \\ \tilde{c}_2^E + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} &\geq y_2 + \min_{\pi \in P^E} \{ \pi (b - y_2) \}, \end{aligned}$$

or

$$\begin{aligned}\tilde{c}_2^R + \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} &\geq \min_{\pi \in P^R} \{ \pi (y_1 - b) \}, \\ \tilde{c}_2^E + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} &> y_2 + \min_{\pi \in P^E} \{ \pi (b - y_2) \},\end{aligned}$$

Adding the relevant inequalities in either case and using (1) gives

$$\begin{aligned}(3) \quad \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} &> \min_{\pi \in P^R} \{ \pi (y_1 - b) \} + \min_{\pi \in P^E} \{ \pi (b - y_2) \} \\ &= \underline{\pi}^R (y_1 - b) + \underline{\pi}^E (b - y_2).\end{aligned}$$

Because $\underline{\pi}^R (\tilde{c}_1^R - \tilde{c}_2^R) + \underline{\pi}^E (\tilde{c}_1^E - \tilde{c}_2^E) \geq \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \}$,

(3) implies that

$$(4) \quad \underline{\pi}^R (\tilde{c}_1^R - \tilde{c}_2^R) + \underline{\pi}^E (\tilde{c}_1^E - \tilde{c}_2^E) > \underline{\pi}^R (y_1 - b) + \underline{\pi}^E (b - y_2).$$

Using (1) and (2) in (4) implies

$$(5) \quad (\underline{\pi}^E - \underline{\pi}^R) (\tilde{c}_1^E + \tilde{c}_2^R - b) > 0$$

For the EU to weakly prefer $(\tilde{c}_1^E, \tilde{c}_2^E)$ to (b, y_2) , $\tilde{c}_1^E \geq b$ necessarily. This contradicts (5) because $\underline{\pi}^E - \underline{\pi}^R < 0$. Finally, note that when $\underline{\pi}^E = 0$, $((b, y_2), (y_1 - b, 0))$ is Pareto dominated by $((y_2, y_2), (y_1 - y_2, 0))$.

We now demonstrate that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$) consumption vectors $((\tilde{c}_1^E, \tilde{c}_2^E), (\tilde{c}_1^R, \tilde{c}_2^R))$ outside the set C (C') are not Pareto optimal. The Pareto problem is

$$\begin{aligned}(6) \quad &\max_{\tilde{c}_1^E, \tilde{c}_2^E, \tilde{c}_1^R, \tilde{c}_2^R \geq 0} \left\{ \tilde{c}_2^R + \min_{\pi \in P^R} \{ \pi (\tilde{c}_1^R - \tilde{c}_2^R) \} \right\} \\ &\text{subject to } \tilde{c}_2^E + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} \geq \hat{u}^E, (1) \text{ and } (2),\end{aligned}$$

where \hat{u}^E is a fixed level of the EU's utility from the interval $[0, y_2 + \underline{\pi}^E (y_1 - y_2)]$. Using the material balance conditions, we can rewrite this problem as

$$\begin{aligned}(7) \quad &\max_{\tilde{c}_1^E, \tilde{c}_2^E} \left\{ y_2 - \tilde{c}_2^E + \min_{\pi \in P^R} \{ \pi (y_1 - \tilde{c}_1^E - y_2 + \tilde{c}_2^E) \} \right\} \\ &\text{subject to } \tilde{c}_2^E + \min_{\pi \in P^E} \{ \pi (\tilde{c}_1^E - \tilde{c}_2^E) \} \geq \hat{u}^E, y_1 \geq \tilde{c}_1^E \geq 0, y_2 \geq \tilde{c}_2^E \geq 0.\end{aligned}$$

The constraint set in (7) is a polyhedral convex set while the objective function is concave. Since $\underline{\pi}^E < \underline{\pi}^R < \bar{\pi}^E$, none of the exposed faces of the constraint set is parallel to any portion of the objective's level surfaces. Hence, Pareto problem (7) has a unique solution. Note also that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$), there is a one-to-one correspondence between utility levels $\hat{u}^E \in [0, y_2 + \underline{\pi}^E (y_1 - y_2)]$ and points in the set C (C'). Combining these two facts with the observation that set $[0, y_2 + \underline{\pi}^E (y_1 - y_2)]$ coincides with the set of feasible utility levels for the EU representative agent, we obtain that when $\underline{\pi}^E > 0$ ($\underline{\pi}^E = 0$) consumption vectors outside C (C') are not Pareto optimal. ■

Appendix B

Proof. First, note that, given the relationship in (??), the three cases considered in the theorem cover all possible rankings of $z_1^E + z_1^R$, $\bar{z}_1^E + \bar{z}_1^R$, $t(z_1^E, \mathbf{x}^E) + t(z_1^R, \mathbf{x}^R)$ and $t(\bar{z}_1^E, \mathbf{x}^E) + t(\bar{z}_1^R, \mathbf{x}^R)$. The Pareto problem can be written as:

$$\begin{aligned} & \max_{(z_j^i, c_j^i)_{j=1,2}^{i=E,R} \geq 0} \left\{ \min_{\pi \in P^R} \{ \pi c_1^R + (1 - \pi) c_2^R \} + \min_{\pi \in P^E} \{ \pi c_1^E + (1 - \pi) c_2^E \} \right\} \\ & \text{subject to } c_1^R + c_1^E = z_1^R + z_1^E \text{ and } c_2^R + c_2^E = t(z_1^R, \mathbf{x}^R) + t(z_1^E, \mathbf{x}^E). \end{aligned}$$

From Theorem ??, when $P^R \subset \text{interior}(P^E)$, Pareto optimality requires that

$$\hat{c}_1^E = \hat{c}_2^E$$

for any given strictly positive aggregate production level.

Using this condition, the Pareto problem can be written as

$$\max_{(z_1^R, z_1^E) \geq 0} \left\{ \min_{\pi \in P^R} \{ \pi (z_1^R + z_1^E) + (1 - \pi) (t(z_1^R, \mathbf{x}^R) + t(z_1^E, \mathbf{x}^E)) \} \right\}.$$

Let

$$f(z_1^R, z_1^E) \equiv \min_{\pi \in P^R} \{ \pi (z_1^R + z_1^E) + (1 - \pi) (t(z_1^R, \mathbf{x}^R) + t(z_1^E, \mathbf{x}^E)) \}.$$

Since $f(\cdot, \cdot)$ is concave, it has a well-defined Gateaux directional derivative of $f(\cdot, \cdot)$ at (z_1^R, z_1^E) in the direction (v^R, v^E) given by

$$f^G((z_1^R, z_1^E); (v^R, v^E)) = \lim_{\lambda \rightarrow 0^+} \frac{f(z_1^R + \lambda v^R, z_1^E + \lambda v^E) - f(z_1^R, z_1^E)}{\lambda}.$$

Using the definition of $f(\cdot, \cdot)$, we obtain

$$(8) \quad f^G((z_1^R, z_1^E); (v^R, v^E)) = \begin{cases} \begin{bmatrix} \underline{\pi}^R (v^R + v^E) \\ + (1 - \underline{\pi}^R) [t'(z_1^R, \mathbf{x}^R) v^R + t'(z_1^E, \mathbf{x}^E) v^E] \end{bmatrix}, & \text{if } z_1^R + z_1^E > t(z_1^R, \mathbf{x}^R) + t(z_1^E, \mathbf{x}^E) \\ \begin{bmatrix} \bar{\pi}^R (v^R + v^E) \\ + (1 - \bar{\pi}^R) [t'(z_1^R, \mathbf{x}^R) v^R + t'(z_1^E, \mathbf{x}^E) v^E] \end{bmatrix}, & \text{if } z_1^R + z_1^E < t(z_1^R, \mathbf{x}^R) + t(z_1^E, \mathbf{x}^E) \\ \min_{\pi \in P^R} \left\{ \begin{bmatrix} \pi (v^R + v^E) \\ + (1 - \pi) [t'(z_1^R, \mathbf{x}^R) v^R + t'(z_1^E, \mathbf{x}^E) v^E] \end{bmatrix} \right\}, & \text{if } z_1^R + z_1^E = t(z_1^R, \mathbf{x}^R) + t(z_1^E, \mathbf{x}^E) \end{cases}.$$

Since $f(\cdot, \cdot)$ is concave in (z_1^R, z_1^E) , $(\hat{z}_1^R, \hat{z}_1^E) \in \arg \max_{(z_1^R, z_1^E) \geq 0} f(z_1^R, z_1^E)$ if and only if

$$(9) \quad f^G((\hat{z}_1^R, \hat{z}_1^E); (v^R, v^E)) \leq 0 \text{ for all } (v^R, v^E).$$

We consider each of the three cases in turn:

i) From (8),

$$f^G((\underline{z}_1^R, \underline{z}_1^E); (v^R, v^E)) = 0 \text{ for all } (v^R, v^E),$$

and, hence, $(\underline{z}_1^R, \underline{z}_1^E) = \arg \max_{(z_1^R, z_1^E) \geq 0} f(z_1^R, z_1^E)$. The second part follows directly from $t'(\underline{z}_1^E, \mathbf{x}^E) = -\frac{\underline{\pi}^R}{1 - \underline{\pi}^R}$ and the definition of z^E .

ii) From (8),

$$f^G((\bar{z}_1^R, \bar{z}_1^E); (v^R, v^E)) = 0 \text{ for all } (v^R, v^E),$$

and, hence, $(\bar{z}_1^R, \bar{z}_1^E) = \arg \max_{(z_1^R, z_1^E) \geq 0} f(z_1^R, z_1^E)$. The second part follows directly from $t'(\bar{z}_1^E, \mathbf{x}^E) = -\frac{\bar{\pi}^R}{1 - \bar{\pi}^R}$ and the definition of z^E .

iii) First, we demonstrate that $\hat{z}_1^R + \hat{z}_1^E = t(\hat{z}_1^R, \mathbf{x}^R) + t(\hat{z}_1^E, \mathbf{x}^E)$. Suppose not and consider the case

$$(10) \quad \hat{z}_1^R + \hat{z}_1^E > t(\hat{z}_1^R, \mathbf{x}^R) + t(\hat{z}_1^E, \mathbf{x}^E).$$

But then

$$f^G((\hat{z}_1^R, \hat{z}_1^E); (v^R, v^E)) = \underline{\pi}^R (v^R + v^E) + (1 - \underline{\pi}^R) [t'(\hat{z}_1^R, \mathbf{x}^R) v^R + t'(\hat{z}_1^E, \mathbf{x}^E) v^E].$$

Under (10), we have that

$$f^G((\hat{z}_1^R, \hat{z}_1^E); (v^R, v^E)) \leq 0 \text{ for all } (v^R, v^E) \text{ if and only if } \hat{z}_1^R = \underline{z}_1^R \text{ and } \hat{z}_1^E = \underline{z}_1^E,$$

which contradicts (10). Similarly, one can demonstrate that $\hat{z}_1^R + \hat{z}_1^E < t(\hat{z}_1^R, \mathbf{x}^R) + t(\hat{z}_1^E, \mathbf{x}^E)$ leads to a contradiction.

Now it is left to verify that (9) holds when $\hat{z}_1^R + \hat{z}_1^E = t(\hat{z}_1^R, \mathbf{x}^R) + t(\hat{z}_1^E, \mathbf{x}^E)$ and $t'(\hat{z}_1^R, \mathbf{x}^R) = t'(\hat{z}_1^E, \mathbf{x}^E)$. We have that in this case

$$\begin{aligned} & f^G((\hat{z}_1^R, \hat{z}_1^E); (v^R, v^E)) \\ &= \min_{\pi \in P^R} \{ \pi(v^R + v^E) + (1 - \pi)(t'(\hat{z}_1^R, \mathbf{x}^R)v^R + t'(\hat{z}_1^E, \mathbf{x}^E)v^E) \} \\ &= \begin{cases} \left[\begin{array}{c} \underline{\pi}^R(v^R + v^E) \\ + (1 - \underline{\pi}^R)(t'(\hat{z}_1^R, \mathbf{x}^R)v^R + t'(\hat{z}_1^E, \mathbf{x}^E)v^E) \end{array} \right], & \text{if } v^R + v^E \geq t'(\hat{z}_1^R, \mathbf{x}^R)v^R + t'(\hat{z}_1^E, \mathbf{x}^E)v^E \\ \left[\begin{array}{c} \overline{\pi}^R(v^R + v^E) \\ + (1 - \overline{\pi}^R)(t'(\hat{z}_1^R, \mathbf{x}^R)v^R + t'(\hat{z}_1^E, \mathbf{x}^E)v^E) \end{array} \right], & \text{if } v^R + v^E \leq t'(\hat{z}_1^R, \mathbf{x}^R)v^R + t'(\hat{z}_1^E, \mathbf{x}^E)v^E \end{cases}. \end{aligned}$$

Using this expression and (iii) it is straightforward to verify that (9) holds. If in addition $t'(z^E, \mathbf{x}^E) = t'(z^R, \mathbf{x}^R)$, then $\hat{z}_1^E = \hat{z}_2^E = z^E$ and $\hat{z}_1^R = \hat{z}_2^R = z^R$ is the unique production vector that satisfies (9). Finally, note that

$$\frac{\underline{\pi}^R}{1 - \underline{\pi}^R} \leq -t'(\hat{z}_1^R, \mathbf{x}^R) = -t'(\hat{z}_1^E, \mathbf{x}^E) \leq \frac{\overline{\pi}^R}{1 - \overline{\pi}^R}.$$

■