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AJAE Appendix for "Pricing-to-Market: Price Discrimination or Product Differentiation?"

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October 20, 2006

Note: The material contained herein is supplementary to the article named in the title and published in the *American Journal of Agricultural Economics (AJAE)*.

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Derivations of Equilibrium Prices and Quantities

Scenario 1

In country 1, the consumer indifferent between buying the low-quality product or buying nothing is defined by the value of θ solving $y + \theta q_l - p_l = y$, i.e., $\theta_{1l} = \frac{p_l}{q_l}$. Similarly, the consumer indifferent between the low- and high-quality products is defined by the value of θ solving equation $y + \theta q_h - p_h = y + \theta q_l - p_l$, i.e. $\theta_{1h} = \frac{p_h - p_l}{q_h - q_l}$.

Thus the low-quality product is purchased by consumers with $\theta \in [\theta_{1l}, \theta_{1h}]$ and the demand for the low-quality product is

$$d_{1l} = \frac{\theta_{1h} - \theta_{1l}}{\theta_1} = \frac{q_l p_h - p_l q_h}{(q_h - q_l) q_l \theta_1}. \tag{1}$$

The high-quality product is purchased by consumers with $\theta \in (\theta_{1h}, \theta_1]$ and the demand for the high-quality product is

$$d_{1h} = \frac{\theta_1 - \theta_{1h}}{\theta_1} = 1 - \frac{p_h - p_l}{(q_h - q_l)\theta_1}. \tag{2}$$

The demands for the low- and high-quality products in country 2 can be obtained in a similar manner. Note however that the demands of consumers in country 2 depend on the price of the product expressed in local currency, i.e., $p_l \cdot e$ and $p_h \cdot e$, where e is the exchange rate expressed in units of country 2's currency per unit of country 1's currency.

The demands in country 2 can be represented as

$$d_{2l} = \frac{\theta_{2h} - \theta_{2l}}{\theta_2} = e \frac{q_l p_h - p_l q_h}{(q_h - q_l) q_l \theta_2}, \text{ and} \tag{3}$$

$$d_{2h} = \frac{\theta_2 - \theta_{2h}}{\theta_2} = 1 - e \frac{p_h - p_l}{(q_h - q_l)\theta_2}. \tag{4}$$

The firm's profit is

$$\pi = (p_l - \frac{1}{2} q_l^2) \frac{q_l p_h - p_l q_h}{(q_h - q_l) q_l} \left(\frac{1}{\theta_1} + \frac{e}{\theta_2} \right) + (p_h - \frac{1}{2} q_h^2) \left[2 - \frac{p_h - p_l}{(q_h - q_l)} \left(\frac{1}{\theta_1} + \frac{e}{\theta_2} \right) \right] \tag{5}$$

with first-order conditions:

$$\frac{\partial \pi}{\partial p_l} = \frac{1}{2}\frac{(\theta_2 + e\theta_1)\left[4(p_h q_l - p_l q_h) + q_l q_h(q_l - q_h)\right]}{(q_h - q_l)q_l\theta_1\theta_2} = 0, \text{ and} \tag{6}$$

$$\frac{\partial \pi}{\partial p_h} = \frac{1}{2}\frac{(\theta_2 + e\theta_1)(4p_h - 4p_l + q_l^2 - q_h^2) - 4\theta_1\theta_2(q_h - q_l)}{(-q_h + q_l)q_l\theta_1\theta_2} = 0. \tag{7}$$

Solving these two equations simultaneously for p_l, p_h, we obtain the equilibrium prices

$$p_h^* = \frac{1}{4}\frac{[4\theta_1\theta_2 + q_h(\theta_2 + e\theta_1)]q_h}{\theta_2 + e\theta_1}, \text{ and} \tag{8}$$

$$p_l^* = \frac{1}{4}\frac{q_l[4\theta_1\theta_2 + q_l(\theta_2 + e\theta_1)]}{\theta_2 + e\theta_1}. \tag{9}$$

The equilibrium quantities are $d_{1l}^* = \frac{q_h}{4\theta_1}$, $d_{2l}^* = \frac{q_h e}{4\theta_2}$, and

$$d_{1h}^* = \frac{4e\theta_1^2 - (q_l + q_h)(\theta_2 + e\theta_1)}{4\theta_1(\theta_2 + e\theta_1)}, \text{ and} \tag{10}$$

$$d_{2h}^* = \frac{4\theta_2^2 - e(q_l + q_h)(\theta_2 + e\theta_1)}{4\theta_2(\theta_2 + e\theta_1)}. \tag{11}$$

For $d_{1h}^* > 0$ and $d_{2h}^* > 0$, $q_h + q_l < \min\left[\frac{4e\theta_1^2}{\theta_2 + e\theta_1}, \frac{4\theta_2^2}{e(\theta_2 + e\theta_1)}\right]$ must hold. We assume that this is the case throughout the article.

Scenario 2

The monopolist treats each market independently due to market segmentation and constant marginal cost. The firm's problem in country 1 is

$$\max_{p_{1l}, p_{1h}} (p_{1l} - \frac{1}{2}q_l^2)d_{1l} + (p_{1h} - \frac{1}{2}q_h^2)d_{1h}. \tag{12}$$

Similarly, the firm's problem in country 2 is

$$\max_{p_{2l}, p_{2h}} (p_{2l} - \frac{1}{2}q_l^2)d_{2l} + (p_{2h} - \frac{1}{2}q_h^2)d_{2h}. \tag{13}$$

We solve the firm's problem in the market 1 first. The marginal consumers are, $\theta_{1l} = \frac{p_{1l}}{q_l}$, $\theta_{1h} = \frac{p_{1h}-p_{1l}}{q_h-q_l}$. Thus the demands can be represented by

$$d_{1l} = \frac{\theta_{1h} - \theta_{1l}}{\theta_1} = \frac{q_l p_{1h} - p_{1l} q_h}{(q_h - q_l) q_l \theta_1}, \text{ and} \tag{14}$$

$$d_{1h} = \frac{\theta_1 - \theta_{1h}}{\theta_1} = 1 - \frac{p_{1h} - p_{1l}}{(q_h - q_l)\theta_1}. \tag{15}$$

Firm's profit is,

$$\pi_1 = (p_{1l} - \frac{1}{2}q_l^2)\frac{q_l p_{1h} - p_{1l} q_h}{(q_h - q_l) q_l \theta_1} + (p_{1h} - \frac{1}{2}q_h^2)(1 - \frac{p_{1h} - p_{1l}}{(q_h - q_l)\theta_1}). \tag{16}$$

The first order conditions are

$$\frac{\partial \pi_1}{\partial p_{1l}} = \frac{4(p_{1h} q_l - p_{1l} q_h) + q_l q_h (q_l - q_h)}{2(q_h - q_l) q_l \theta_1}, \text{ and} \tag{17}$$

$$\frac{\partial \pi_1}{\partial p_{1h}} = \frac{4(p_{1h} - p_{1l}) - (q_h - q_l)(q_l + q_h + 2\theta_1)}{2(-q_h + q_l)\theta_1}. \tag{18}$$

Solving these two equations simultaneously for p_{1l} and p_{1h}, we have, $p_{1h}^* = \frac{1}{4}q_h(2\theta_1 + q_h)$ and $p_{1l}^* = \frac{1}{4}q_l(2\theta_1 + q_l)$. Thus the equilibrium quantities are $d_{1l}^* = \frac{q_h}{4\theta_1}$ and $d_{1h}^* = \frac{2\theta_1 - q_l - q_h}{4\theta_1}$.

Similarly, by solving the maximization problem of the monopolist in country 2, we can obtain the following equilibrium prices and quantities,

$$p_{2l}^* = \frac{1}{4}q_l(2\theta_2 + eq_l)/e, \tag{19}$$

$$p_{2h}^* = \frac{1}{4}q_h(2\theta_2 + eq_h)/e, \tag{20}$$

$$d_{2l}^* = \frac{eq_h}{4\theta_2}, \text{ and} \tag{21}$$

(22)
$$d_{2h}^* = \frac{2\theta_2 - e(q_l + q_h)}{4\theta_2}.$$

For $d_{1h}^* > 0$ and $d_{2h}^* > 0$, $q_h + q_l < \min[2\theta_1, 2\theta_2/e]$ must hold. Note that this condition is less restrictive than $q_h + q_l < \min\left[\frac{4e\theta_1^2}{\theta_2+e\theta_1}, \frac{4\theta_2^2}{e(\theta_2+e\theta_1)}\right]$ established in scenario 1 for the quantities in market 2 to be positive. Thus, $d_{1h}^* > 0$ and $d_{2h}^* > 0$ in scenario 2 holds.

Derivations of Equations Associated with Corollary 3 and 4

Corollary 3

First, we determine the sign of $X - 1$. Using the equation for the domestic-export price ratio with unit values, i.e., $X = \frac{p_l^*\sigma_1 + p_h^*(1-\sigma_1)}{p_l^*\sigma_2 + p_h^*(1-\sigma_2)}$, the sign of $X - 1$ corresponds to the sign of $(p_l^* - p_h^*)(\sigma_1 - \sigma_2)$. It can be easily shown that $(p_l^* - p_h^*) < 0$ because $q_l < q_h$. Moreover,

(23)
$$\sigma_1 - \sigma_2 = \frac{4q_h(\theta_2 + e\theta_1)^2(\theta_2 - e\theta_1)}{[4e\theta_1^2 - q_l(\theta_2 + e\theta_1)][4\theta_2^2 - eq_l(\theta_2 + e\theta_1)]}$$

and the two elements of the denominator are positive given the assumption we made for all quantities to be positive in equilibrium (see scenario 1 above). Thus, the sign of $\sigma_1 - \sigma_2$ depends on the sign of $\theta_2 - e\theta_1$. When $\theta_2 < e\theta_1$, $\sigma_1 - \sigma_2 < 0$, and $X - 1 > 0$. When $\theta_2 > e\theta_1$, $\sigma_1 - \sigma_2 > 0$, and $X - 1 < 0$.

Second, we determine the sign of $\frac{\partial X}{\partial q_h}$. Because $X = \frac{P_1}{P_2}$, $\frac{\partial X}{\partial q_h} = \frac{\frac{\partial P_1}{\partial q_h}P_2 - P_1\frac{\partial P_2}{\partial q_h}}{P_2^2}$. Note that $P_1 = \sigma_1(p_l^* - p_h^*) + p_h^*$, $P_2 = \sigma_2(p_l^* - p_h^*) + p_h^*$, and q_h does not enter p_l^*. Thus,

(24)
$$\frac{\partial X}{\partial q_h} = \frac{\left[\frac{\partial \sigma_1}{\partial q_h}(p_l^* - p_h^*) + \frac{\partial p_h^*}{\partial q_h}(1 - \sigma_1)\right]P_2 - P_1\left[\frac{\partial \sigma_2}{\partial q_h}(p_l^* - p_h^*) + \frac{\partial p_h^*}{\partial q_h}(1 - \sigma_2)\right]}{P_2^2}.$$

Rearranging we obtain:

(25)
$$\frac{\partial X}{\partial q_h} = \frac{(p_l^* - p_h^*)\left(\frac{\partial \sigma_1}{\partial q_h}P_2 - \frac{\partial \sigma_2}{\partial q_h}P_1\right) - \frac{\partial p_h^*}{\partial q_h}(P_2\sigma_1 - P_1\sigma_2 - P_2 + P_1)}{P_2^2}$$

where $\sigma_1 = \frac{q_h(\theta_2+e\theta_1)}{4e\theta_1^2 - q_l(\theta_2+e\theta_1)}$, and $\sigma_2 = \frac{q_h e(\theta_2+e\theta_1)}{4\theta_2^2 - q_l e(\theta_2+e\theta_1)}$.

Given the expressions for σ_1 and σ_2, $\frac{\partial\sigma_1}{\partial q_h} = \frac{\sigma_1}{q_h}$, $\frac{\partial\sigma_2}{\partial q_h} = \frac{\sigma_2}{q_h}$. Substituting for $\frac{\partial\sigma_1}{\partial q_h}$, $\frac{\partial\sigma_2}{\partial q_h}$, P_1, and P_2, equation (25) can be re-written as

(26)
$$\frac{\partial X}{\partial q_h} = \frac{(\sigma_1 - \sigma_2)\left[(p_l^* - p_h^*)\frac{p_h^*}{q_h} - \frac{\partial p_h^*}{\partial q_h}p_l^*\right]}{P_2^2}$$

where $(p_l^* - p_h^*)\frac{p_h^*}{q_h} - \frac{\partial p_h^*}{\partial q_h}p_l^* < 0$ because $(p_l^* - p_h^*) < 0$ and $\frac{\partial p_h^*}{\partial q_h} = \frac{4\theta_1\theta_2 + 2q_h(\theta_2+e\theta 1)}{4(\theta_2+e\theta 1)} > 0$.

The above shows that the sign of $\frac{\partial X}{\partial q_h}$ also depends on the sign of $\sigma_1 - \sigma_2$, which we have already determined depends on the sign of $\theta_2 - e\theta_1$.

Thus, when $\theta_2 < e\theta_1$, $\sigma_1 - \sigma_2 < 0$, $X - 1 > 0$, and $\frac{\partial X}{\partial q_h} > 0$. When $\theta_2 > e\theta_1$, $\sigma_1 - \sigma_2 > 0$, $X - 1 < 0$, and $\frac{\partial X}{\partial q_h} < 0$.

Corollary 4

Because

(27)
$$X_l = \frac{p_{1l}^*}{p_{2l}^*} = \frac{(2\theta_1 + q_l)e}{2\theta_2 + eq_l},$$

(28)
$$X_h = \frac{p_{1h}^*}{p_{2h}^*} = \frac{(2\theta_1 + q_h)e}{2\theta_2 + eq_h}, \text{ and}$$

(29)
$$X = \frac{p_{1l}^*\sigma_1 + p_{1h}^*(1-\sigma_1)}{p_{2l}^*\sigma_2 + p_{2h}^*(1-\sigma_2)} = \frac{e(q_l^2 + 4\theta_1^2 - q_h q_l - q_h^2)(2\theta_2 - eq_l)}{(e^2q_l^2 + 4\theta_2^2 - e^2q_hq_l - e^2q_h^2)(2\theta_1 - q_l)},$$

then, when $q_l = q_h = q$, $X_l = X_h = X = \frac{(2\theta_1+q)e}{2\theta_2+eq}$.

In what follows, the equations allowing us to sign $\frac{\partial X}{\partial q_h}$ are derived. Rewrite X as $X = \frac{p_{1l}^*}{p_{2l}^*}\frac{\sigma_1}{\sigma_2}\frac{A}{B}$ where $A = 1 + \frac{p_{1h}^*}{p_{1l}^*}\left(\frac{1-\sigma_1}{\sigma_1}\right)$ and $B = 1 + \frac{p_{2h}^*}{p_{2l}^*}\left(\frac{1-\sigma_2}{\sigma_2}\right)$. Therefore,

(30)
$$\frac{\partial X}{\partial q_h} = \frac{p_{1l}^*}{p_{2l}^*}\frac{\sigma_1}{\sigma_2}\frac{1}{B^2}\left[B\left(\frac{\partial\left(\frac{p_{1h}^*}{p_{1l}^*}\right)}{\partial q_h}\left(\frac{1-\sigma_1}{\sigma_1}\right) + \frac{p_{1h}^*}{p_{1l}^*}\frac{\partial\left(\frac{1-\sigma_1}{\sigma_1}\right)}{\partial q_h}\right) - A\left(\frac{\partial\left(\frac{p_{2h}^*}{p_{2l}^*}\right)}{\partial q_h}\left(\frac{1-\sigma_2}{\sigma_2}\right) + \frac{p_{2h}^*}{p_{2l}^*}\frac{\partial\left(\frac{1-\sigma_2}{\sigma_2}\right)}{\partial q_h}\right)\right]$$

where $\frac{p_{1h}^*}{p_{1l}^*} = \frac{q_h(2\theta_1+q_h)}{q_l(2\theta_1+q_l)}$, $\frac{1-\sigma_1}{\sigma_1} = \frac{2\theta_1-q_l-q_h}{q_h}$, $\frac{p_{2h}^*}{p_{2l}^*} = \frac{q_h(2\theta_2+eq_h)}{q_l(2\theta_2+eq_l)}$, and $\frac{1-\sigma_2}{\sigma_2} = \frac{2\theta_2-e(q_l+q_h)}{eq_h}$. The derivative of these expressions with respect to q_h can be written as: $\frac{\partial\left(\frac{p_{1h}^*}{p_{1l}^*}\right)}{\partial q_h} = \frac{p_{1h}^*}{p_{1l}^* q_h} + \frac{q_h}{4p_{1l}^*}$, $\frac{\partial\left(\frac{1-\sigma_1}{\sigma_1}\right)}{\partial q_h} = \frac{-1}{\sigma_1 q_h}$, $\frac{\partial\left(\frac{p_{2h}^*}{p_{2l}^*}\right)}{\partial q_h} = \frac{p_{2h}^*}{p_{2l}^* q_h} + \frac{q_h}{4p_{2l}^*}$, and $\frac{\partial\left(\frac{1-\sigma_2}{\sigma_2}\right)}{\partial q_h} = \frac{-1}{\sigma_2 q_h}$.

Substituting these last four expressions into (30), we obtain

$$\frac{\partial X}{\partial q_h} = \frac{p_{1l}^*}{p_{2l}^*}\frac{\sigma_1}{\sigma_2}\frac{1}{B^2}\left\{B\left[-\frac{p_{1h}^*}{p_{1l}^* q_h} + \frac{q_h}{4p_{1l}^*}\left(\frac{1-\sigma_1}{\sigma_1}\right)\right] - A\left[-\frac{p_{2h}^*}{p_{2l}^* q_h} + \frac{q_h}{4p_{2l}^*}\left(\frac{1-\sigma_2}{\sigma_2}\right)\right]\right\} \tag{31}$$

which can be rewritten as

$$\frac{\partial X}{\partial q_h} = \frac{p_{1l}^*}{p_{2l}^*}\frac{\sigma_1}{\sigma_2}\frac{1}{B^2}\left\{\frac{1}{q_h}\left(\frac{p_{2h}^*}{p_{2l}^*}A - \frac{p_{1h}^*}{p_{1l}^*}B\right) + \frac{q_h}{4}\left[\frac{B}{p_{1l}^*}\left(\frac{1-\sigma_1}{\sigma_1}\right) - \frac{A}{p_{2l}^*}\left(\frac{1-\sigma_2}{\sigma_2}\right)\right]\right\}. \tag{32}$$

After substituting for A and B within the curly brackets and rearranging we obtain:

$$\frac{\partial X}{\partial q_h} = \frac{p_{1l}^*}{p_{2l}^*}\frac{\sigma_1}{\sigma_2}\frac{1}{B^2}\left(\frac{C}{q_h} + \frac{q_h}{4}D\right), \text{ where} \tag{33}$$

$$C = \left(\frac{p_{2h}^*}{p_{2l}^*} - \frac{p_{1h}^*}{p_{1l}^*}\right) + \frac{p_{1h}^* p_{2h}^*}{p_{1l}^* p_{2l}^*}\left(\frac{1-\sigma_1}{\sigma_1} - \frac{1-\sigma_2}{\sigma_2}\right) \text{ and} \tag{34}$$

$$D = \left(\frac{1-\sigma_1}{\sigma_1 p_{1l}^*} - \frac{1-\sigma_2}{\sigma_2 p_{2l}^*}\right) + \frac{p_{2h}^* - p_{1h}^*}{p_{1l}^* p_{2l}^*}\left(\frac{1-\sigma_1}{\sigma_1}\right)\left(\frac{1-\sigma_2}{\sigma_2}\right). \tag{35}$$

Then, we substitute for the equilibrium prices and market shares and simplify to obtain:

$$\frac{\partial X}{\partial q_h} = \frac{1}{p_{2l}^*}\frac{\sigma_1}{\sigma_2}\frac{1}{B^2}\frac{(e\theta_1-\theta_2)(e\theta_1+\theta_2)(2q_h+q_l)}{eq_l(2\theta_2+eq_l)}. \tag{36}$$

The sign of $\frac{\partial X}{\partial q_h}$ is the sign of $e\theta_1 - \theta_2$.