AJAE APPENDIX: REGULATING NITROGEN POLLUTION WITH RISK-AVERSE FARMERS UNDER HIDDEN INFORMATION AND MORAL HAZARD

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Complement Material to "Regulating nitrogen pollution with risk-averse farmers under hidden information and moral hazard"

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This paper presents some complement materials to the article entitled "Regulating nitrogen pollution with risk-averse farmers under hidden information and moral hazard".

1 Comparison between $\nu^{LR}(\alpha)$ and $\nu^{BM}(\alpha)$

Recall that we have

$$\begin{aligned}\nu^{LR}(\alpha) - \nu^{BM}(\alpha) &= \int_{\alpha}^{\bar{\alpha}} \left(\frac{U'(\Pi(u))}{U'(CE(\Pi))} - 1 \right) g(u)du \\ &= \int_{\alpha}^{\bar{\alpha}} \left(\frac{U'(\Pi(u))}{U'(CE(\Pi))} \right) g(u)du - (1 - G(\alpha)).\end{aligned}$$

Let us denote $\phi(\alpha) = \int_{\alpha}^{\bar{\alpha}} U'(\Pi(u))g(u)du - (1 - G(\alpha))U'(CE(\Pi))$. We have $\phi(\bar{\alpha}) = 0$ and

$$\phi'(\alpha) = [U'(CE(\Pi)) - U'(\Pi(\alpha))] \, g(\alpha).$$

As $U'(\Pi(\alpha))$ is decreasing in α, $\phi'(\alpha)$ is first negative and then positive. Hence if $\phi(\underline{\alpha}) \leq 0$, then $\nu^{LR}(\alpha) \leq \nu^{BM}(\alpha) \leq 0$ for any α. Note that $\phi(\underline{\alpha}) \leq 0$ is equivalent to $\int_{\underline{\alpha}}^{\bar{\alpha}} U'(\Pi(u))g(u)du - U'(CE(\Pi)) < 0$ which in turn is equivalent to saying that the function $U'(U^{-1}(.))$ is concave, according to the Jensen inequality. Moreover $\frac{d}{d\Pi} \left[U'(U^{-1}(\Pi)) \right] = -\rho(\Pi)$ where $\rho(.) = -\frac{U''(.)}{U'(.)}$ is the Arrow-Pratt measure of local risk aversion. Hence we have that $\phi(\underline{\alpha}) \leq 0$ is finally equivalent to $U(.)$ being CARA or IARA. Similarly, $\phi(\underline{\alpha}) > 0$ is equivalent to $U(.)$ being DARA. Consequently, in the DARA case, there exists a threshold type $\tilde{\alpha}$ such that for any $\alpha \leq \tilde{\alpha}$ we have $\nu^{LR}(\alpha) \geq \nu^{BM}(\alpha)$ and for any type $\alpha \geq \tilde{\alpha}$ we have $\nu^{LR}(\alpha) \leq \nu^{BM}(\alpha)$.

2 Reduction of the agency's program to an optimal control problem

First, we reduce the complexity of incentive constraints. As it is usual in adverse selection models (see Guesnerie and Laffont), (IC) can be reduced to the following set of conditions

$$\dot{\Pi} = pf_{\alpha}(\alpha, X, Z(\alpha)), \tag{1}$$

$$f_{\alpha Z}\dot{Z} \geq 0. \tag{2}$$

Recall that f is increasing in α and that $f_{\alpha Z} = \partial^2 f / \partial\alpha\partial Z$ is strictly negative. The latter property of f corresponds to the usual single crossing condition that simplifies greatly the analysis of optimal contracts (Guesnerie and Laffont). Given that $\dot{\Pi}$ is strictly positive as indicated by (**??**), individual rationality constraints (IR) reduce to:

$$\Pi(\underline{\alpha}) \geq 0. \tag{3}$$

Hence it suffices to check for the participation constraint of the less efficient farmer. Moreover, as $f_{\alpha Z} < 0$, (**??**) reduces to $\dot{Z} \leq 0$. The optimal quota is thus non-increasing in α.

We now define $\dot{Z} = -\psi$, where ψ is a control variable and we redefine Z as a state variable. Then, constraint (**??**) reads as $\psi \geq 0$. Moreover, we redefine X as a state variable with the equation $\dot{X} = 0$ and $X(\alpha) \geq 0$, $X(\underline{\alpha}) = X$, $X(\bar{\alpha})$ free. Second, we use a first order approach for the moral hazard constraint on X.We will have to verify that local second order conditions are satisfied, i.e. $\int_{\underline{\alpha}}^{\overline{\alpha}} \left[\frac{\partial^2 U(\Pi(\alpha))}{\partial X^2}\right] dG(\alpha) \leq 0$ Actually, this condition is checked under our assumption $\frac{\partial^2 f}{\partial X^2} \leq 0$. Hence, the first order condition corresponding to the moral hazard constraint is

$$\int_{\underline{\alpha}}^{\overline{\alpha}} \left[\frac{\partial U(\Pi(\alpha))}{\partial X}\right] dG(\alpha) = \int_{\underline{\alpha}}^{\overline{\alpha}} M(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha)) dG(\alpha) = 0, \tag{4}$$

where $M = U'(\Pi(\alpha)) p f_X(\alpha, X(\alpha), Z(\alpha))$. To deal with this integral constraint, we define a new state variable

$$\dot{K} = M(X(\alpha), Z(\alpha), \Pi(\alpha), \alpha) g(\alpha) \quad \text{with } K(\underline{\alpha}) = K(\overline{\alpha}) = 0,$$

and let $\mathcal{W}(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha))$ denote the term equal to:

$$\begin{aligned} \mathcal{W}(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha)) \quad = \quad & S(\alpha, X(\alpha), Z(\alpha)) + (1+\lambda) p f(\alpha, X(\alpha), Z(\alpha)) \\ & -(1+\lambda)(\Pi(\alpha) + wZ(\alpha)). \end{aligned}$$

We then transform the agency's program (P1) into an optimal control problem:

$$\max_{\psi} CE(\Pi) + \int_{\underline{\alpha}}^{\overline{\alpha}} \mathcal{W}(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha)) dG(\alpha), \tag{5}$$

subject to

$$\begin{aligned} & \dot{X} = 0,\ \dot{\Pi} = p f_\alpha(\alpha, X(\alpha), Z(\alpha)),\ \dot{Z} = -\psi,\ \dot{K} = M(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha)) g(\alpha), \\ & Z(\alpha) - X(\alpha) \geq 0,\ \psi(\alpha) \geq 0,\ X(\alpha) \geq 0, \\ & X(\underline{\alpha}) = X,\ \Pi(\underline{\alpha}) \geq 0,\ K(\underline{\alpha}) = K(\overline{\alpha}) = 0, \end{aligned}$$

with one control variable (ψ) and four state variables (Π, X, Z, K).

3 Proof of proposition 1

The program to be solved is

$$\max_{\psi} CE(\Pi) + \int_{\underline{\alpha}}^{\overline{\alpha}} \mathcal{W}(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha)) dG(\alpha),$$

subject to

$$\begin{aligned} & \dot{X} = 0, && (\mu) \\ & \dot{\Pi} = p f_\alpha(\alpha, X(\alpha), Z(\alpha)), && (\upsilon) \\ & \dot{Z} = -\psi, && (\sigma) \\ & \dot{K} = M(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha)) g(\alpha), && (\kappa) \\ & Z(\alpha) - X(\alpha) \geq 0, && (\xi_1) \\ & \psi(\alpha) \geq 0, && (\xi_2) \\ & X(\alpha) \geq 0, && (\xi_3) \\ & X(\underline{\alpha}) = X, && \\ & \Pi(\underline{\alpha}) \geq 0, && \\ & K(\underline{\alpha}) = K(\overline{\alpha}) = 0. && \end{aligned}$$

The Lagrangian of this program reads:

$$\mathcal{L} = CE(\Pi) + \int_{\underline{\alpha}}^{\overline{\alpha}} \{ [S(\alpha, X(\alpha), Z(\alpha))) + (1+\lambda)pf(\alpha, X(\alpha), Z(\alpha))$$

$$-(1+\lambda)(\Pi(\alpha) + wZ(\alpha))]g(\alpha) - \mu(\alpha)\dot{X} + \nu(\alpha)(pf_{\alpha}(\alpha, X(\alpha), Z(\alpha)) - \dot{\Pi}(\alpha))$$
$$-\sigma(\alpha)(\psi + \dot{Z}(\alpha)) + \kappa(\alpha)(M(\alpha, X(\alpha), Z(\alpha), \Pi(\alpha))g(\alpha) - \dot{K})$$
$$+\xi_1(\alpha)(Z(\alpha) - X(\alpha)) + \xi_2(\alpha)\psi(\alpha) + \xi_3(\alpha)X(\alpha)\} \, d\alpha.$$

Integrating by parts and using initial and terminal conditions gives:

$$\begin{aligned} \int_{\underline{\alpha}}^{\overline{\alpha}} \mu(\alpha)\dot{X}(\alpha)d\alpha &= -\int_{\underline{\alpha}}^{\overline{\alpha}} \dot{\mu}(\alpha)X(\alpha)d\alpha, \\ \int_{\underline{\alpha}}^{\overline{\alpha}} \kappa(\alpha)\dot{K}(\alpha)d\alpha &= -\int_{\underline{\alpha}}^{\overline{\alpha}} \dot{\kappa}(\alpha)K(\alpha)d\alpha, \\ \int_{\underline{\alpha}}^{\overline{\alpha}} \sigma(\alpha)\dot{Z}(\alpha)d\alpha &= -\int_{\underline{\alpha}}^{\overline{\alpha}} \dot{\sigma}(\alpha)Z(\alpha)d\alpha, \end{aligned}$$

because $\sigma(\underline{\alpha}) = \sigma(\overline{\alpha}) = 0$ ($Z(\underline{\alpha})$ and $Z(\overline{\alpha})$ are free) and $\mu(\underline{\alpha}) = \mu(\overline{\alpha}) = 0$ ($X(\underline{\alpha}) = X$ and $X(\overline{\alpha})$ are free).

Thus, plugging these expressions into the Lagrangian and dropping α for the sake of clarity, $\mathcal{L}$ becomes

$$\mathcal{L} = CE(\Pi) + \int_{\underline{\alpha}}^{\overline{\alpha}} \mathcal{H}(\alpha, X, Z, \dot{\Pi}, \Pi, \psi, K)d\alpha,$$

where

$$\begin{aligned} \mathcal{H}(\alpha, X, Z, \dot{\Pi}, \Pi, \psi, K) &= \mathcal{W}(\alpha, X, Z, \Pi)g + \nu(pf_{\alpha}(\alpha, X, Z) - \dot{\Pi}) \\ &\quad +\dot{\mu}X - \sigma\psi + \dot{\sigma}Z + \kappa M(\alpha, X, Z, \Pi)g + \dot{\kappa}K \\ &\quad +\xi_1(Z - X) + \xi_2\psi + \xi_3 X. \end{aligned}$$

Pointwise maximizations give us the following necessary conditions:

- First,

  $$\frac{\partial \mathcal{H}}{\partial \psi} = -\sigma + \xi_2 \leq 0 \text{ everywhere}$$

  and $-\sigma + \xi_2 < 0 \Rightarrow \psi = 0$. Moreover, we also have the complementary slackness condition:

  $$\xi_2 \geq 0, \psi \geq 0 \text{ and } \xi_2\psi = 0.$$

  Thus, whenever $\psi(\alpha) > 0$, we have $\sigma(\alpha) = \xi_2(\alpha) = 0$.

- Moreover, we have

  $$\frac{\partial \mathcal{H}}{\partial X} = \mathcal{W}_X g + \nu p f_{\alpha X} + \dot{\mu} + \kappa M_X g - \xi_1 + \xi_3 = 0, \tag{6}$$

  $$\frac{\partial \mathcal{H}}{\partial K} = \dot{\kappa} = 0 \Rightarrow \kappa(\alpha) = \kappa \text{ everywhere}, \tag{7}$$

  $$\frac{\partial \mathcal{H}}{\partial Z} = \mathcal{W}_Z g + \nu p f_{\alpha Z} + \dot{\sigma} + \kappa M_Z g + \xi_1 = 0, \tag{8}$$

  with the following slackness conditions:

  $$\xi_1 \geq 0, Z - X \geq 0 \text{ and } \xi_1(Z - X) = 0,$$

$$\xi_3 \geq 0, X \geq 0 \text{ and } \xi_3 X = 0,$$

and the transversality conditions:

$$\begin{aligned} \sigma(\underline{\alpha}) &= \sigma(\overline{\alpha}) = 0, \\ \mu(\underline{\alpha}) &= \mu(\overline{\alpha}) = 0. \end{aligned} \tag{9}$$

With regard to Π, we have to compute the derivative of $\mathcal{L}$ with respect to Π in the direction of an arbitrary differentiable function $h(\alpha)$ satisfying $h(\underline{\alpha}) = 0$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \Pi} h &= \lim_{t \to 0} \frac{\mathcal{L}(\Pi + th) - \mathcal{L}(\Pi)}{t} \\ &= \frac{E(U'(\Pi)h)}{U'(CE(\Pi))} - \int_{\underline{\alpha}}^{\overline{\alpha}} \{(1+\lambda)hg + \nu h' - \kappa p f_X U''(\Pi)hg\} \, d\alpha \end{aligned}$$

or

$$\int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ \frac{U'(\Pi)}{U'(CE(\Pi))} - [1 + \lambda - \kappa p f_X U''(\Pi)] \right\} hg d\alpha - \int_{\underline{\alpha}}^{\overline{\alpha}} \nu h' d\alpha = 0. \tag{10}$$

Let us denote

$$B(\alpha) = \int_{\alpha}^{\bar{\alpha}} \left\{ \frac{U'(\Pi(u))}{U'(CE(\Pi))} - [1 + \lambda - \kappa p f_X(u, X(u), Z(u)) U''(\Pi(u))] \right\} g(u) du,$$

then (**??**) can be rewritten as

$$-\int_{\underline{\alpha}}^{\overline{\alpha}} B'(\alpha) h(\alpha) d\alpha - \int_{\underline{\alpha}}^{\overline{\alpha}} \nu(\alpha) h'(\alpha) d\alpha = 0.$$

Integrating by parts the first integral and rearranging terms, we then obtain:

$$-\left[B(\alpha)h(\alpha)\right]_{\underline{\alpha}}^{\overline{\alpha}} + \int_{\underline{\alpha}}^{\overline{\alpha}} \left(B(\alpha) - \nu(\alpha)\right) h'(\alpha) d\alpha = 0. \tag{11}$$

Recall that h is arbitrary. And, we can choose h so that $h(\underline{\alpha}) = 0$. Moreover $B(\bar{\alpha}) = 0$. Finally, (**??**) implies that

$$\nu(\alpha) = B(\alpha),$$

or

$$\begin{aligned} \nu(\alpha) &= \int_{\alpha}^{\bar{\alpha}} \left\{ \frac{U'(\Pi(u))}{U'(CE(\Pi))} - (1+\lambda) \right\} g(u) du \\ &\quad + \kappa \int_{\alpha}^{\bar{\alpha}} \{ p f_X(u, X(u), Z(u)) U''(\Pi(u)) \} g(u) du. \end{aligned} \tag{12}$$

Plugging (**??**) into (**??**) and assuming an interior solution for X and Z ($\xi_1 = \xi_3 = 0$) gives:

$$\begin{aligned} \dot{\mu} &= -\mathcal{W}_X g - \left[\int_{\alpha}^{\bar{\alpha}} \left\{ \frac{U'(\Pi)}{U'(CE(\Pi))} - (1+\lambda) \right\} g du \right. \\ &\quad \left. + \kappa \int_{\alpha}^{\bar{\alpha}} \{ p f_X U''(\Pi) \} g du \right] p f_{\alpha X} - \kappa M_X g. \end{aligned}$$

Integrating and using (**??**), we obtain:

$$\begin{aligned} \int_{\underline{\alpha}}^{\overline{\alpha}} \dot{\mu} d\alpha &= 0 = \int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ \mathcal{W}_X g + \left(\int_{\alpha}^{\bar{\alpha}} \left\{ \frac{U'(\Pi)}{U'(CE(\Pi))} - (1+\lambda) \right\} g du \right) p f_{\alpha X} \right\} d\alpha \\ &\quad + \kappa \int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ \left(\int_{\alpha}^{\bar{\alpha}} \{ p f_X U''(\Pi) \} g du \right) p f_{\alpha X} + M_X g \right\} d\alpha. \end{aligned}$$

Thus, the co-state variable κ is equal to:

$$\kappa = \frac{\int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ \mathcal{W}_X g + \left(\int_{\alpha}^{\bar{\alpha}} \left\{ \frac{U'(\Pi)}{U'(CE(\Pi))} - (1+\lambda) \right\} g du \right) p f_{\alpha X} \right\} d\alpha}{- \int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ \left(\int_{\alpha}^{\bar{\alpha}} \left\{ p f_X U''(\Pi) \right\} g du \right) p f_{\alpha X} + M_X g \right\} d\alpha}$$

and the co-state variable $\nu(\alpha)$ is given by (**??**). Note that $\mathcal{W}_X = S_X + (1+\lambda) p f_X$ and $\mathcal{W}_Z = S_Z + (1+\lambda) p f_Z - (1+\lambda) w$ and recall that $S_Z = 0$. Plugging these values in (**??**) and recalling that in the no bunching case $\sigma(\alpha) = 0 \Rightarrow \dot{\sigma}(\alpha) = 0$ gives (i) for an interior solution. First order condition of farmer's program w.r.t. X gives (ii). Finally, integrating the first order condition of IC gives (iii). To obtain (iv), it suffices to maximize the Lagrangian with respect to $\pi(\underline{\alpha})$ given (iii):

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi(\underline{\alpha})} = & \frac{\int_{\underline{\alpha}}^{\overline{\alpha}} U' \left(\pi(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha} p f_\alpha(u, X, Z(u)) du \right) dG(\alpha)}{U' \left(U^{-1} \left[\int_{\underline{\alpha}}^{\overline{\alpha}} U \left(\pi(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha} p f_\alpha(u, X, Z(u)) du \right) dG(\alpha) \right] \right)} \\ & - 1 - \lambda + \kappa \int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ p f_X(u, X, Z(u)) U'' \left(\pi(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha} p f_\alpha(u, X, Z(u)) du \right) \right\} dG(\alpha) = \nu(\underline{\alpha}). \end{aligned}$$

If $\pi(\underline{\alpha}) = 0$ at the optimum, then a necessary condition is $\frac{\partial \mathcal{L}}{\partial \pi(\underline{\alpha})}\Big|_{\pi(\underline{\alpha})=0} \leq 0$ or equivalently $\nu(\underline{\alpha}) \leq 0$. When U is CARA with ρ being the absolute degree of risk aversion, then from (**??**), we obtain that:

$$\nu(\underline{\alpha}) = \int_{\underline{\alpha}}^{\overline{\alpha}} \left\{ \frac{U'(\pi(u))}{U'(CE(\pi))} - (1+\lambda) \right\} f(u) du = -1 - \lambda + \int_{\underline{\alpha}}^{\overline{\alpha}} \frac{U'(\pi(u))}{U'(CE(\pi))} f(u) du,$$

because first $U'' = -\rho U'$ and second the moral hazard constraint together imply that the second term of $\nu(\underline{\alpha})$ is zero. Furthermore, we also have $U'(CE(\pi)) = E_\alpha(U'(\pi(\theta))$ and consequently, $\nu(\underline{\alpha}) = -\lambda < 0$. The conclusion follows.

Otherwise, if $\pi(\underline{\alpha}) > 0$ then it is defined by the solution of $\frac{\partial \mathcal{L}}{\partial \pi(\underline{\alpha})} = 0$, but note that the Lagrangian is not necessarily concave in $\pi(\underline{\alpha})$ due to the presence of the certainty equivalent, $CE(\pi)$ in the objective.